



CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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Question Paper With

Answer Key

PHYSICAL SCIENCE

1. Given a matrix $M = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, which of the following represents? $\cos\left(\frac{\pi M}{6}\right)$

(a) $\frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (b) $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
 (c) $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

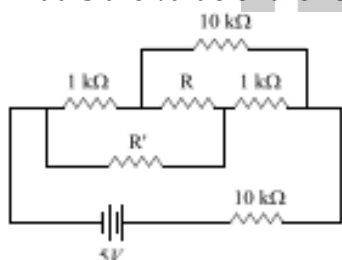
2. The wave function of a hydrogen atom is given by the following superposition of energy eigen-functions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers):

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}} \psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}} \psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}} \psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

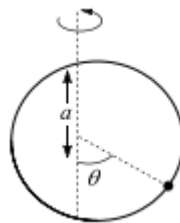
(a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$
 (c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

3. It is found that when the resistance R indicated in the figure below is changed from $1k\Omega$ to $10k\Omega$, the current flowing through the resistance R' does not change. What is the value of the resistor R' ?



(a) $5k\Omega$ (b) 100Ω
 (c) $10k\Omega$ (d) $1k\Omega$

4. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?



(a) $\tan \theta = \frac{\pi g}{\omega^2 a}$ (b) $\sin \theta = \frac{g}{\omega^2 a}$
 (c) $\cos \theta = \frac{g}{\omega^2 a}$ (d) $\tan \theta = \frac{g}{\pi \omega^2 a}$

5. A spin $\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket $-\text{state } \frac{(|1\rangle + |2\rangle)}{\sqrt{2}}$ at time $t = 0$. It evolves to the state described by the $\frac{(|1\rangle - |2\rangle)}{\sqrt{2}}$ in time T . The minimum energy difference between two levels is:

(a) $\frac{h}{6T}$ (b) $\frac{h}{4T}$
 (c) $\frac{h}{2T}$ (d) $\frac{h}{T}$

6. You receive on average 5 emails per day during a 365 days year. The number of days on average on which you do not receive any emails in that year are:

(a) more than 5 (b) more than 2
 (c) 1 (d) none of these

7. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28} \text{ kg}$ and an equilibrium inter nuclear distance $R = 0.742 \times 10^{-10} \text{ m}$. The rotational energy in terms of the rotational quantum number J is :

(a) $E_{rot}(J) = 7J(J-1)\text{meV}$
 (b) $E_{ret}(J) = \frac{5}{2}J(J+1)\text{meV}$
 (c) $E_{rot}(J) = 7J(J+1)\text{meV}$
 (d) $E_{rot}(J) = \frac{5}{2}J(J-1)\text{meV}$

8. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R . The

field is given by:

- (a) $\frac{3m_0c}{eR}$ (b) $\frac{\sqrt{2}m_0c}{eR}$
 (c) $\frac{\sqrt{3}m_0c}{eR}$ (d) $\frac{\sqrt{3}m_0c}{2eR}$

9. The central force which results in the orbit $r = a(1 + \cos \theta)$ for a particle is proportional to:

- (a) r (b) r^2
 (c) r^{-2} (d) None of the above

10. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T . The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential is $U = mgz$ where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is:

- (a) $\frac{\exp\left(-\frac{mg(z-\frac{L}{2})}{k_B T}\right)}{\sinh\left(\frac{mgL}{k_B T}\right)}$
 (b) $P(z) = \frac{N}{V} \frac{mgL}{2} \frac{\exp\left(-\frac{mg(z-\frac{L}{2})}{k_B T}\right)}{\cosh\left(\frac{mgL}{2k_B T}\right)}$
 (c) $P(z) = \frac{k_B T N}{V}$
 (d) $P(z) = \frac{N}{V} mgz$

11. A transistor in common base configuration has ratio of collector current to emitter current β and ratio of Collector to base current α . Which of the following is true?

- (a) $\beta = \frac{\alpha}{(\alpha+1)}$ (b) $\beta = \frac{(\alpha+1)}{\alpha}$
 (c) $\beta = \frac{\alpha}{(\alpha-1)}$ (d) $\beta = \frac{(\alpha-1)}{\alpha}$

12. The energy of a particle is given by $E = |p| + |q|$ where p and q are the generalized momentum and coordinate, respectively. All the states with $E \leq E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate q , with $q > 0$ is:

- (a) $\frac{(E_0+q)}{E_0^2}$ (b) $\frac{q}{E_0^2}$
 (c) $\frac{(E_0-q)}{E_0^2}$ (d) $\frac{1}{E_0^2}$

13. Consider a quantum particle of mass m in one dimension in an infinite potential well, i.e., $V(x) = 0$ for $-\frac{a}{2} < x < \frac{a}{2}$ and $V(x) = \infty$ for $|x| \geq \frac{a}{2}$. A small perturbation, $V'(x) = \frac{2\epsilon|x|}{a}$ is added. The change in the ground state energy to $O(\epsilon)$ is:

- (a) $\frac{\epsilon}{2\pi^2} (\pi^2 - 4)$ (b) $\frac{\epsilon}{2\pi^2} (\pi^2 + 4)$
 (c) $\frac{\epsilon\pi^2}{2} (\pi^2 + 4)$ (d) $\frac{\epsilon\pi^2}{2} (\pi^2 - 4)$

14. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:

- (a) $\frac{\mu_0 I}{\sqrt{3}\pi a}$ (b) $\frac{\sqrt{6}\mu_0 I}{\pi a}$
 (c) $\frac{3\mu_0 I}{\pi a}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

15. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P = P_0 - \alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to P_0/α . At what value of the volume during the process does the gas have maximum entropy?

- (a) $\frac{P_0}{\alpha(1+\gamma)}$ (b) $\frac{\gamma P_0}{\alpha(1-\gamma)}$
 (c) $\frac{\gamma P_0}{\alpha(1+\gamma)}$ (d) $\frac{P_0}{\alpha(1-\gamma)}$

16. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it take for the charge to hit the plane?

- (a) $\frac{\sqrt{2\pi^3\epsilon_0 m d^3}}{q}$ (b) $\frac{\sqrt{2\pi^3\epsilon_0 m d}}{q}$
 (c) $\frac{\sqrt{\pi^3\epsilon_0 m d^3}}{q}$ (d) $\frac{\sqrt{\pi^3\epsilon_0 m d}}{q}$

17. A two dimensional box in a uniform magnetic field B contains $N/2$ localized spin-1/2 particles with magnetic moment μ , and $N/2$ free spinless particles which do not interact with each other. The average energy of the system at a temperature T is :

- (a) $3NkT - \frac{1}{2}N\mu B \sinh\left(\frac{\mu B}{k_B T}\right)$
 (b) $NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$
 (c) $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$
 (d) $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh\left(\frac{\mu B}{k_B T}\right)$

18. If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{22} - Y_{2,-2})$ where $Y_{l,m}$ are spherical harmonics, then which of the following is true?

- (a) Y_{xy} is an eigen function of both L^2 and L_z
 (b) Y_{xy} is an eigen function of L^2 but not L_z
 (c) Y_{xy} is an eigen function both of L_z but not L^2
 (d) Y_{xy} is not an eigen function of either L^2 and L_z

19. The value of the integral $\int_0^\infty \frac{\ln x}{(x^2+1)} dx$

- (a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$
 (c) π^2 (d) 0

20. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $\left(\frac{1}{\sqrt{10}}\right)\{2, \sqrt{2}, 2i\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_z|\psi\rangle$?

- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

21. Consider N non-interacting electrons ($N \sim N_A$) in a box of sides L_x, L_y, L_z . Assume that the dispersion relation is $\epsilon(k) = Ck^4$ where C is a constant, the ratio of the ground state energy per particle to the Fermi energy is :

- (a) $\frac{3}{7}$ (b) $\frac{7}{3}$
 (c) $\frac{3}{5}$ (d) $\frac{5}{7}$

22. The Hamiltonian of a quantum particle of mass m confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m} \left(-i \frac{\partial}{\partial \theta} - \alpha \right)^2$$

Where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

- (a) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n - \alpha)^2$
 (b) $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n - \alpha)^2$
 (c) $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n - \alpha)^2$
 (d) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and

23. The sum of the infinite series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ is

- (a) 2π (b) π
 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

24. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t = 0$ at the Sun and another event occurs on Earth at $t = 1$ minute. The velocity of the inertial frame in which both these events are simultaneous is:

- (a) $c/8$ with the velocity vector pointing from Earth to Sun
 (b) $c/8$ with the velocity vector pointing from Sun to Earth
 (c) The events can never be simultaneous - no such frame exists
 (d) $c\sqrt{1 - \left(\frac{1}{8}\right)^2}$ with velocity vector pointing from Sun to Earth

25. A spherical shell of radius R carries a constant surface charge density σ and is rotating about one of its diameters with an

angular velocity ω . The magnitude of the magnetic moment of the shell is:

- (a) $4\pi\sigma\omega R^4$ (b) $\frac{4\pi\sigma\omega R^4}{3}$
 (c) $\frac{4\pi\sigma\omega R^4}{15}$ (d) $\frac{4\pi\sigma\omega R^4}{9}$

26. The adjoint of a differential operator $\frac{d}{dx}$ acting on a wave function $\psi(x)$ for a quantum mechanical system is:

- (a) $\frac{d}{dx}$ (b) $-i\hbar \frac{d}{dx}$
 (c) $-\frac{d}{dx}$ (d) $i\hbar \frac{d}{dx}$

27. In Millikan's oil-drop experiment an oil drop of radius r , mass m and charge $q = \frac{6\pi\eta r v_1}{m}$ is moving upwards with a terminal velocity V_2 due to an applied electric field of magnitude E , where η is the coefficient of viscosity. The acceleration due to gravity is given by:

- (a) $q = \frac{6\pi\eta r v_1}{m}$ (b) $q = \frac{3\pi\eta r v_1}{m}$
 (c) $q = \frac{6\pi\eta r v_2}{m}$ (d) $q = \frac{3\pi\eta r v_2}{m}$

28. The electric field $\vec{E} = E_0 \sin(\omega t - kz)x + 2E_0 \sin\left(\omega t - kz + \frac{\pi}{2}\right)y$ represents:

- (a) a linearly polarized wave
 (b) a right hand circularly polarized wave
 (c) a left hand circularly polarized wave
 (d) an elliptically polarized wave

29. An ideal gas has a specific heat ratio $\frac{C_p}{C_v} = 2$.

Starting at a temperature T_1 the gas undergoes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

- (a) $\frac{T_1}{2}$ (b) $\sqrt{2}T_1$
 (c) $2T_1$ (d) $\frac{T_1}{\sqrt{2}}$

30. Suppose yz -plane forms the boundary between two linear dielectric media I and II with dielectric constant $\epsilon_1 = 3$ and $\epsilon_2 = 4$, respectively. If the electric field in region I at the interface is given by $\vec{E}_1 = 4x + y + 5z$, then the electric field $\vec{E}_{II}$ at the interface in region II is:

- (a) $4x + 3y + 5z$
 (b) $4x + 0.75y - 1.25z$
 (c) $-3x + 3y + 5z$
 (d) $3x + 3y + 5z$

31. Given the condition $\nabla^2\phi = 0$, the solution of the equation $\nabla^2\psi = k\vec{\nabla}\phi \cdot \vec{\nabla}\phi$ is given by

- (a) $\psi = \frac{k\phi^2}{2}$ (b) $\psi = k\phi^2$
 (c) $\psi = \frac{k\phi \ln \phi}{2}$ (d) $\psi = \frac{-k\phi \ln \phi}{2}$

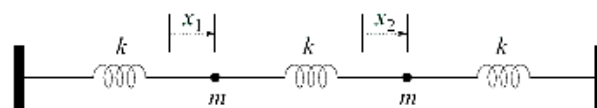
32. Circular fringes are obtained with a Michelson interferometer using 600 nm laser light. What minimum displacement of one mirror will make the central fringe from bright to dark?

- (a) 600 nm (b) 300 nm
 (c) 150 nm (d) 120 Å

33. If $\vec{k}$ is the wave vector of incident light ($|\vec{k}| = \frac{2\pi}{\lambda}$, λ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then Bragg's law can be written as:

- (a) $\vec{k} + \vec{G} = 0$ (b) $2\vec{k} \cdot \vec{G} + G^2 = 0$
 (c) $2\vec{k} \cdot \vec{G} + k^2 = 0$ (d) $\vec{k} \cdot \vec{G} = 0$

34. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$ corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$ respectively.



At $t = 0$, the displacements are $x_1 = A$, $x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time t is given by:

- (a) $x_2(t) = \frac{A}{2} (\cos(\omega_0 t) + \cos(\sqrt{3}\omega_0 t))$
 (b) $x_2(t) = \frac{A}{2} (\cos(\omega_0 t) - \cos(\sqrt{3}\omega_0 t))$
 (c) $x_2(t) = \frac{A}{2} (\sin(\omega_0 t) - \sin(\sqrt{3}\omega_0 t))$
 (d) $x_2(t) = \frac{A}{2} (\sin(\omega_0 t) - \frac{1}{\sqrt{3}} \sin(\sqrt{3}\omega_0 t))$

35. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

- (a) 6.0×10^{-9} N (b) 0.6×10^{-9} N
 (c) 0.6×10^{-8} N (d) 4.8×10^{-9} N

36. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE , then the thickness of the layer is:

- (a) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$
 (c) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

37. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is:

- (a) $\frac{7}{8}$ (b) $\frac{1}{3}$
 (c) $\frac{5}{6}$ (d) $\frac{1}{2^3}$

38. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

- (a) $\mu_0\pi R^2 n^2$ (b) $2\mu_0\pi R^2 n$
 (c) $2\mu_0\pi R^2 n^2$ (d) $\mu_0\pi R^2 n$

39. A gas contains particle of type A with fraction 0.8, and particles of type B with fraction 0.2. the probability that among 3 randomly chosen particles at least one is of type A is:

- (a) 0.8 (b) 0.25
 (c) 0.33 (d) 0.992

40. The number of different Bravais lattices possible in two dimension is:

- (a) 2 (b) 3
 (c) 5 (d) 6

41. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = \frac{\omega_0}{Q}I$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is:

- (a) Delta function (b) Gaussian
 (c) Lorentzian (d) Exponential

42. For a quantum mechanical harmonic oscillator with energies, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0, 1, 2, \dots$, the partition function is:

- (a) $\frac{\frac{\hbar\omega}{e} e^{\frac{\hbar\omega}{2k_B T}}}{e^{\frac{\hbar\omega}{2k_B T}} - 1}$ (b) $e^{\frac{\hbar\omega}{2k_B T}} - 1$
 (c) $e^{\frac{\hbar\omega}{2k_B T}} + 1$ (d) $\frac{e^{\frac{\hbar\omega}{2k_B T}}}{e^{\frac{\hbar\omega}{2k_B T}} - 1}$

43. If the direction with respect to a right-handed Cartesian coordinate system of the ket vector $|z, +\rangle$ is (0, U, 1), then the direction of the ket vector obtained by application of rotations:

$\exp\left(-i\sigma_z \frac{\pi}{2}\right) \exp\left(-i\sigma_y \frac{\pi}{4}\right)$, on the ket $|z, +\rangle$ is (σ_y, σ_z are the Pauli matrices) :

- (a) (0,1,0) (b) (1,0,0)
 (c) $\frac{(1,1,0)}{\sqrt{2}}$ (d) $\frac{(1,1,1)}{\sqrt{3}}$

44. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

- (a) $\frac{1}{2}$ (b) 1
 (c) 2 (d) $\frac{3}{2}$

45. For operator P and Q , the commutator $[P, Q^{-1}]$ is

- (a) $Q^{-1}[P, Q]Q^{-1}$ (b) $-Q^{-1}[P, Q]Q^{-1}$
 (c) $Q^{-1}[P, Q]Q$ (d) $-Q[P, Q]Q^{-1}$

46. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}}$.

$\exp\left[\frac{(x^2 + \mu x)}{(2\sigma^2)}\right]$ is:

- (a) 0 (b) $\frac{\mu}{2}$
 (c) $\frac{-\mu}{2}$ (d) σ

47. A spin $1/2$ particles is in a state $\frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstate of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

- (a) $\hbar$ (b) $-\hbar$
 (c) 0 (d) $\frac{\hbar}{2}$

48. A semicircular piece of paper is folder to make a cone with the centre of the semicircle as the apex. The half - angle of the resulting cone would be:

- (a) 90° (b) 60°
 (c) 45° (d) 30°

49. If the Rydberg constant of an atom of finite nuclear mass is αR_∞ . where R_∞ the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:

- (a) $\frac{(1-\alpha)}{\alpha}$ (b) $\frac{(\alpha-1)}{\alpha}$
 (c) $(1 - \alpha)$ (d) $\frac{1}{\alpha}$

50. A cylindrical shell of mass m has an outer radius b and an inner radius a . The moment of inertia of the shell about the axis of the cylinder is:

- (a) $\frac{1}{2}m(b^2 - a^2)$ (b) $\frac{1}{2}m(b^2 + a^2)$
 (c) $m(b^2 + a^2)$ (d) $m(b^2 - a^2)$

Answer Key

1. b	2. a	3. b	4. c	5. c
6. b	7. c	8. c	9. d	10. a
11. a	12. c	13. a	14. d	15. c
16. a	17. c	18. d	19. d	20. d
21. a	22. a	23. d	24. b	25. b
26. c	27. a	28. d	29. b	30. d
31. a	32. c	33. b	34. b	35. a
36. a	37. d	38. a	39. d	40. c
41. c	42. d	43. b	44. b	45. b
46. c	47. d	48. d	49. a	50. b