D PHYSICS

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JEST 2012

Question Paper With

Answer Key

PHYSICAL SCIENCE

1. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum p. If c dentes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v is.

(a) $\frac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\frac{(E-vp)}{\sqrt{1-(v/c)^2}}$

(c) $\frac{(E+vp)}{[1-(v/c)^2]^2}$ (d) $\frac{(E-vp)}{[1-(v/c)^2]^2}$

2. Consider a system of two particles A and B. Each particle can occupy one of three possible quantum states $|1\rangle, |2\rangle$ and $|3\rangle$. The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwll-Boltzman) particles. They are respectively

(a) 1,0 (b) 1/2,1

(c) 1,1/2 (d) 0,1/2

3. At 'equilibrium' there cannot be any free charge inside a metal. However, if you forcibly put charge in the interior then it takes some finite time to 'disappear', i.e. move to the surface. If the conductivity, σ, of a metal is $10^6(\Omega\text{m})^{-1}$ and the dielectric constant $\varepsilon_0 = 8.85 \times 10^{-12}$Farad/m, this time will be approximately:

(a) 10^{-5}sec (b) 10^{-11}sec

(c) 10^{-9}sec (d) 10^{-17}sec

4. The free fall time of a test mass on an object of mass M from a height $2R$ to R is

(a) $(\pi/2+1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$

(c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

5. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?

(a) 0.99 (b) 0.925

(c) 0.075 (d) 0.01

6. Under a Galilean transformation, the coordinates and momenta of any particle/system transform as: $t' = t, \vec{r}' = \vec{r} + \hat{v}$ and $\vec{p}' = \vec{p} + m\vec{v}$ and $\vec{v}$ is the velocity of the boosted frame with respect to the original frame. A unitary operator carrying out these transformations for a system having total mass M, total momentum $\vec{P}$ and centre of mass coordinate $\vec{X}$ is

(a) $e^{jM\hat{v}\cdot\bar{X}/h}e^{j\vec{v}\cdot\vec{P}/h}$

(b) $e^{iM\hat{V}\cdot\hat{X}/h}e^{-i\vec{v}\cdot\vec{P}/h}e^{-iN^2\lfloor(2h)}$

(d) $e^{it\vec{P}\cdot\dot{P}/h}e^{MMv^2t(2h)}$

7. A particle of mass m is contained in a one-dimensional infinite well extending from $x = -L/2$ to $x = L/2$ the particle is in its ground state given by $\Psi_0(x) = \sqrt{2/L}\cos(\pi x/L)$. The walls of the box are moved suddenly to form a box extending from $x = -L$ to $x = L$. What is the probability that the particle will be in the ground state after this sudden expansion

(a) $(8/3\pi)^2$ (b) 0

(c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

8. The electric fields outside $(r > R)$ and inside $(r < R)$ a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r\propto R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}, q$ bing the total charge. The correct ratio of the electrostatic energies for the second case to the first case is

(a) 1: 3 (b) 9: 16

(c) 3: 8 (d) 5: 6

9. A quantum mechanical particle in a harmonic oscillator potential has the initial wave function $\Psi_0(x) + \Psi_1(x)$, where Ψ_0&Ψ_1 are the real wave function in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m = h = \omega = 1$ for oscillator. What is the probability density of finding the particle at x at time $t = \pi$?

(a) $(\Psi_1(x) - \Psi_0(x))^2$

(b) $(\Psi_1(x))^2 - (\Psi_0(x))^2$

(c) $(\Psi_1(x) + \Psi_0(x))^2$

(d) $(\Psi_1(x))^2 + (\Psi_0(x))^2$

10. A spherical planet of radius R has a uniform density ρ and does not rotate. If the planet is made up of some liquid, the pressure at any point r from the center is

(a) $\frac{4\pi\rho^2 G}{3}(R^2 - r^2)$ (b) $\frac{4\pi\rho G}{3}(R^2 - r^2)$

(c) $\frac{2\pi\rho G}{3}(R^2 - r^2)$ (d) $\frac{\rho G}{3}(R^2 - r^2)$

11. Compute $\lim_{z\to 0} = \frac{\mathrm{Re}(z^2)+\ln(z^2)}{z^2}$

(a) The limit does not exist

(b) 1

(c) −i

(d) −I

12. A particle of mass m is thrown upward with velocity v and there is retarding air resistance proportional to the square of the velocity with proportionality constant k. If the particle attains a maximum height after time t, and g is the gravitational acceleration, acceleration, what is the velocity v ?

(a) $\sqrt{\frac{k}{g}\tan\left(\sqrt{\frac{g}{k}}t\right)}$ (b) $\sqrt{gk\tan\left(\sqrt{\frac{g}{k}}t\right)}$

(c) $\sqrt{\frac{k}{g}\tan(\sqrt{gkt})}$ (d) $\sqrt{\mathrm{gk}\tan(\sqrt{gkt})}$

13. Consider a uniform distribution of particles with volume density n in a box. The particles have an isotropic velocity distribution with constant magnitude v. The rate at which the particles will be emitted from a hole of area A on one side of this box is

(a) muA

(b) muA/2

(c) muA/4

(d) none of the above

14. For a diatomic ideal gas near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at constant pressure?

(a) 1/7 (b) 5/7

(c) ¾ (d) 2/7

15. A flat surface is covered with non-overlapping disks of same size. What is the largest, fraction of the area that can be covered?

(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$

(c) $\frac{6}{7}$ (d) $\frac{\pi}{2\sqrt{3}}$

16. If Jx, Jy, are angular momentum operators, the eigenvalues of the operator $(J_x + J_y)/\hbar$ are

(a) real and discrete with rational spacing

(b) real and discrete with irrational spacing

(c) real and continuous

(d) not all real

17. A metal suffers a structural phase transition form face-centred cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbor distances d_{fc} and d_{SC} for the FCC and the SC structures respectively are in the ratio (d_{fc}/d_{SC}) [Given $2^{1/3} = 1.26$]

(a) 1.029 (b) 1.122

(c) 1.374 (d) 1.130

18. If, in a Kepler potential, the pericentre distance of a particle in a parabolic orbit is $\mathbf{r}_p$ while the radius of the circular orbit with the same angular momentum is r_c, then

(a) $r_C = 2r_P$ (b) $r_C = r_P$

(c) $2r_C = r_P$ (d) $r_C = \sqrt{2r_P}$

19. A k meson (with a rest mass of 494MeV) at rest decays into a muon (with a rest mass of 106MeV) and a neutrino. The energy of the neutrino, which can be taken to be massless, is approximately

(a) 120MeV (b) 236MeV

(c) 300MeV (d) 388MeV

20. The vector field $xz\hat{\imath} + y j$ in cylindrical polar coordinates is

(a) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 - z)\hat{e}_\phi$

(b) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(c) $\rho(z\sin^2\phi + \cos^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(d) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 - z)\hat{e}_\phi$

21. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to

(a) 0.07 (b) 0.60

(c) 0.36 (d) 0.19

22. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x axis. The probability that they meet after n steps is

(a) $\frac{1}{4^n}\frac{2n!}{n!}$ (b) $\frac{1}{2^n}\frac{2n!}{n!}$

(c) $\frac{1}{2^n}2n\,!$ (d) $\frac{1}{4^n}n!$

23. The equation describing the shape of a curved mirror with the property that the light from a point source at the origin will be reflected in a beam of rays parallel to the x-axis is (with a as some constant)

(a) $y^2 = ax + a^2$ (b) $2y = x^2 + a^2$

(c) $y^2 = 2ax + a^2$ (d) $y^2 = ax^3 + 2a^2$

24. A simple model of a helium-like atom with electron-electron interaction is replaced by Hooke's law force is described by Hamiltonian $-\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + \frac{1}{2}m\omega^2(r_1^2 + r_2^2) - \frac{\lambda}{4}m\omega^2|\vec{r}_1 - \vec{r}_2|^2$. What is the exact ground state energy?

(a) $E = \frac{3}{2}\hbar\omega(1 + \sqrt{1 + \lambda})$

(b) $E = \frac{3}{2}\hbar\omega(1 + \sqrt{\lambda})$

(c) $E = \frac{3}{2}\hbar\omega\sqrt{1 - \lambda}$

(d) $E = \frac{3}{2}\hbar\omega(1 + \sqrt{1 - \lambda})$

25. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with $m = +1, 0, -1$. If L_z^2 is measured in

this state yielding a result 1 , what is the state after the measurement

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

26. A thin uniform ring carrying charge Q and mass M rotates about its axis. What is the gyro magnetic ratio (defined as ratio of magnetic dipole moment to the angular momentum) of this ring?

(a) $Q/(2\pi M)$ (b) Q/M

(c) $Q/(2M)$ (d) $Q/(\pi M)$

27. The electric and magnetic fields caused by an accelerated charged particle are found to scale as $E \propto r^{-n}$ and $B \propto r^{-m}$ at large distances. What are the values of n and m ?

(a) n = 1, m = 2 (b) n = 2, m = 1

(c) n = 1, m = 1 (d) n = 2, m = 2

28. Consider the differential equation $\frac{dG(x)}{dx} + kG(x) = \delta(x)$, where k is a constant. Which of the following statements is true?

(a) Both G(x) and G′(x) are continuous at $x = 0$.

(b) G(x) is continuous at x = 0 but G′(x) is not.

(c) G(x) is discontinuous at x = 0.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depend on the value of k.

29. A metal bullet comes to rest after hitting its target with a velocity of 80 m/s. If 50% of the heat generated remains in the bullet, what is the increase in its temperature? (The specific heat of the bullet = 160 Joule per Kg per degree. C)

(a) 14°C (b) 12.5°C

(c) 10°C (d) 8.2°C

30. What are the eigen values of the operator $H = \vec{\sigma} \cdot \vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector

(a) $a_x + a_y$ and a_z (b) $a_x + a_x \pm ia_y$

(c) $\pm(a_x + a_y + a_z)$ (d) $\pm|\vec{a}|$

31. The Hermitian conjugate of the operator $\left(-\frac{\partial}{\partial x}\right)$ is

(a) $\frac{\partial}{\partial x}$ (b) $-\frac{\partial}{\partial x}$

(c) $\frac{i\,\partial}{\partial x}$ (d) $-\frac{i\,\partial}{\partial x}$

32. If the expectation value of the momentum is $\langle p \rangle$ for the wave function $\psi(x)$, then the expectation value of momentum for the wave function $e^{ikx/\hbar}\Psi(x)$ is

(a) k (b) $\langle p \rangle - k$

(c) $\langle p \rangle + k$ (d) $\langle p \rangle$

33. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$ what would be the ground state wave function $\psi(x_1, x_2)$f both electrons are arranged to have the same spin state?

(a) $\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) + \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1, x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1, x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

34. What is the value of the following series?

$$\left(1-\frac{1}{2!}+\frac{1}{4!}-\cdots..\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}-\cdots..\right)^2$$

(a) 0 (b) e

(c) e^2 (d) 1

35. A light beam is propagating through a block of glass with index of refraction n. If glass is moving at constant velocity v in the same direction as the beam, the velocity of the light in the glass block as measured by an observer in the laboratory is approximately

(a) $u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$

(b) $u=\frac{c}{n}-v\left(1-\frac{1}{n^2}\right)$

(c) $u=\frac{c}{n}+v\left(1+\frac{1}{n^2}\right)$

(d) $u=\frac{c}{n}$

36. If the distribution function of x is $f(x)=xe^{-x/\lambda}$ over the interval $0<x<\infty$, the mean of x is

(a) λ (b) 2λ

(c) $\lambda/2$ (d) 0

37. Consider a particle with three possible spin states $s=0$ and ± 1. There is a magnetic field h present and the energy for a spin state s is $-hs$. The system is at a temperature T. Which of the following statements is true about the entropy S(T) ?

(a) $S(T) = \text{In } 3$ at $T=0$, and 3 at high T

(b) $S(T) = \text{In } 3$ at T0, and zero at high T

(c) $S(T)=0$ at $T=0$, and 3 at high T

(d) $S(T)=0$ for $T=0$, and In 3 at high T

38. The operator $\left(\frac{d}{dx}-x\right)\left(\frac{d}{dx}+x\right)$ is equivalent to

(a) $\frac{d^2}{dx^2}-x^2$ (b) $\frac{d^2}{dx^2}-x^2+1$

(c) $\frac{d^2}{dx^2}-x\frac{d}{dx}+1$ (d) $\frac{d^2}{dx^2}-2x\frac{d}{dx}-x^2$

39. Consider three situations of 4 particles in a one dimensional box of width L with hard walls. In case (i), the particles are fermions, in case (ii) they are bosons, and in case (iii) they are classical. If the total ground state energy of the four particles in these three cases are E_F, E_B and E_{cl} respectively, which of the following is true?

(a) $E_F=E_B=E_d$ (b) $E_F>E_B=E_d$

(c) $E_F<E_B<E_d$ (d) $E_F>E_B>E_d$

40. If a proton were ten times lighter, the ground state energy of the electron in a hydrogen atom would be

(a) Less

(b) more

(c) the same

(d) less, more or equal depending on the electron mass

41. If $\vec{E}_1=xy\hat{\imath}+2yzj+3xzk$ and $\vec{E}_2=y^2\hat{\imath}+(2xy+z^2)j+2yzk$ then

(a) Both are impossible electrostatic fields.

(b) Both are possible electrostatic fields.

(c) Only E_1 is a possible electrostatic field.

(d) E_2 is a possible electrostatic field.

42. ^{238}U decays with a half life of 4.51×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock sample analysis shows that the ratio of the numbers of atoms of ^{206}Pb to ^{238}U is 0.0058 . Assuming that all the ^{206}Pb has been produced by the decay of ^{238}U and that all other half-lives in the chain are negligible, the age of the rock sample is

(a) 38×10^6 years (b) 48×10^6 years

(c) 38×10^7 years (d) 48×10^7 years

43. The period of a simple pendulum inside a stationary lift is T. If the lift accelerates downwards with and acceleration g/4, the period of the pendulum will be

(a) T (b) T/4

(c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

44. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum)

(a) $\sqrt{3c}/2$ (b) $3c/4$

(c) $\sqrt{3/5c}$ (d) $c/\sqrt{2}$

45. If the Poisson bracket $\{x, p\} = -1$, then the Poisson bracket $\{x^2 + p, p\}$ is

(a) $-2x$ (b) $2x$

(c) 1 (d) -1

46. The coordinate transformation $x' = 0.8x + 0.6y, y' = 0.6x - 0.8y$ represents

(a) A translation.

(b) a proper rotation.

(c) A reflection.

(d) None of the above.

47. The binding energy of the k-shell electron in a Uranium atom (Z = 92, A = 238) will be modified due to
(i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows:

(a) Increases due to (i), remains unchanged due to (ii).

(b) Decreases due to (i), decreases due to (ii).

(c) Increases due to (i), increases due to (ii).

(d) Decreases due to (i), remains unchanged due to (ii).

48. A small mass **M** hangs from a thin string and can swing like a pendulum. It is attached above the window of a car. When the car is at rest, the string hangs vertically. The angle made by the string with the vertical when the car has a constant acceleration a = 1.2 m/s2 is approximately

(a) 1° (b) 7°

(c) 15° (d) 90°

49. A charge **q** is placed at the centre of an otherwise neurtral dielectric sphere of radius a and relative permittivity ε_r we denote the expression $q/4\pi\varepsilon_0 r^2$ by $E(r)$. Which of the following statements is false?

(a) The electric field inside the sphere, r < a, is given by E(r)ε_r

(b) The field outside the sphere, $r > a$, is given by E(r).

(c) The total charge inside a sphere of radius $r > a$ is given by q.

(d) The total charge inside a sphere of radius $r > a$ is given by q.

50. An electromagnetic wave of frequency ω travels in the x-direction through vacuum. It is polarized in the y direction and the amplitude of the electric field is E_0. With k = ω/c where c is the speed of light in vacuum, the electric and the magnetic fields are then conventionally given by

(a) $\vec{E} = E_0 \cos(ky - \omega x)x$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega x)\hat{z}$

(b) $\vec{E} = E_0\cos(kx - \omega x)x$ and $\vec{B} = \frac{E_0}{c}\cos(kx - \omega x)\hat{z}$

(c) $\vec{E} = E_0\cos(kx - \omega x)\hat{z}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega x)y$

(d) $\vec{E} = E_0\cos(kx - \omega x)x$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega x)y$

51. A dynamical system with two generalized coordinates q_1 and q_2 has Lagrangian $L = \dot{q}_1^2 + \dot{q}_2^2$. If p_1 and p_2 are the corresponding generalised momenta, the Hamiltonian is given by

(a) $(p_1^2 + p_2^2)/4$ (b) $(\dot{q}_1^2 + \dot{q}_2^2)/4$

(c) $(p_1^2 + p_2^2)/2$ (d) $(p_1\dot{q}_1 + p_2\dot{q}_2)/4$

52. In a certain inertial frame two light pulses are emitted, a distance 5 km apart and separated by $5\mu s$. An observer who is traveling, parallel to the line joining the points where the pulses are emitted, at a velocity V with respect to this frame notes that the pulses are simultaneous. Therefore V is

(a) 0.7c (b) 0.8c

(c) 0.3c (d) 0.9c

53. Suppose a spin 1/2 particle is in the state, $|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix}1+i\\2\end{bmatrix}$. If S_x(x component of the spin angular momentum operator) is measured what is the probability of getting $+\frac{\hbar}{2}$?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

54. For an optical fiber with core and cladding index of $n_1 = 1.45$ and $n_2 = 1.44$, respectively, what is the approximate cut-off angle of incidence? Cut-off angle of incidence is defined as the incidence angle below which light will be guided.

(a) 7° (b) 22°

(c) 5° (d) 0°

55. A double pendulum consists of two equal masses 'm' suspended by two strings of length l. What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos\theta = 1 - \theta^2/2$).

(a) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \omega_1^2\theta_1^2 - \frac{1}{2}\omega_1^2\theta_1^2\right)$

(b) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(c) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(b) $L \approx ml^2\left(\frac{1}{2}\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

56. Circular discs of radius 1 m each are placed on a plane so as to form a closely packed triangular lattice. The number of discs per unit area is approximately equal to

(a) 0.86 m^{-2} (b) 0.43 m^{-2}

(c) 0.29 m^{-2} (d) 0.14 m^{-2}

57. What are the solutions to $f''(x) - 2f'(x) + f(x) = 0$?

(a) c_1e^x/x (b) $c_1x + c_2/x$

(c) $c_1xe^x + c_2$ (d) $c_1e^x + c_2xe^x$

58. An ideal gas of non-relativistic fermions in 3-dimensions is at 0 K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

(a) $2^{1/2}$ (b) 1

(c) $2^{1/3}$ (d) $2^{-1/3}$

59. The value $\int_{0.2}^{2.2} xe^{x}dx$ by using the one-segment trapezoidal rule is close to

(a) 11.672 (b) 11.807

(c) 20.099 (d) 24.119

60. The Hamiltonian operator for a two-state system is given by

$$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|)$$

where a is a positive number with the dimension of energy. The energy eigenstates corresponding to the larger and smaller eigenvalues respectively are

(a) $|1\rangle - (\sqrt{2}+1)|2\rangle, |1\rangle + (\sqrt{2}-1)|2\rangle$

(b) $|1\rangle + (\sqrt{2}-1)|2\rangle, |1\rangle - (\sqrt{2}+1)|2\rangle$

(c) $|1\rangle + (\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle - |2\rangle$

(d) $|1\rangle - (\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle + |2\rangle$

61. Given the fundamental constant $\hbar$ (Planck constant), G (universal gravitation constant) and c (speed of light), which of the following has dimension of length?

(a) $\sqrt{\frac{\hbar G}{c^3}}$ (b) $\sqrt{\frac{\hbar G}{c^5}}$

(c) $\frac{\hbar G}{c^3}$ (d) $\sqrt{\frac{\hbar c}{8\pi G}}$

62. Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|\ell, m\rangle$. Let $A = \hat{n}\cdot\vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parmetrized in terms of two angles as $(n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The width ΔA in $|\ell, m\rangle$ is

(a) $\sqrt{\frac{\ell(\ell+1)-m^2}{2}}\,\hbar\cos\theta$

(b) $\sqrt{\frac{\ell(\ell+1)-m^2}{2}}\,\hbar\sin\theta$

(c) $\sqrt{\ell(\ell+1)-m^2}\,\hbar\sin\theta$

(d) $\sqrt{\ell(\ell+1)-m^2}\,\hbar\cos\theta$

63. The Laplace transformation of $e^{2t}\sin 4t$ is

(a) $\frac{4}{s^2+4s+25}$ (b) $\frac{4}{s^2+4s+20}$

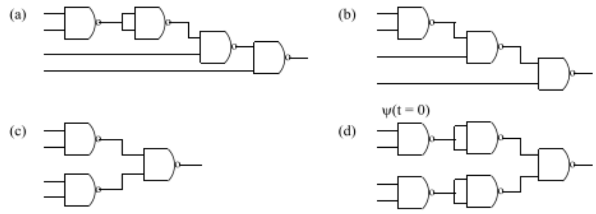
(c) $\frac{4s}{s^2+4s+20}$ (d) $\frac{4s}{2s^2+4s+20}$

64. In the mixture of isotopes normally found on the earth at the present time, ${}^{238}_{92}U$ has an abundance of 99.3% and ${}^{238}_{92}U$ has an abundance of 0.7%. The measured lifetimes of these isotopes are 6.52×10^9 years and 1.02×10^9 years, respectively. Assuming that they were equally abundant when the earth was formed, the estimated age of the earth, in years, is

(a) 6.0×10^9 (b) 1.0×10^9

(c) 6.0×10^8 (d) 1.0×10^8

65. Which of the following circuits will act like a 4-input NAND gate?



66. Consider a three-state with energies E, E and $E-3g$ (where g is a constant) and respective eigenstates

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix}, |\psi_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

If the system is initially (at $t=0$) in state

$$|\psi_1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

What is the probability that at a later time ' t ' system will be in state $|\psi_f\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(a) 0

(b) $\frac{4}{9}\sin^2\left(\frac{3gt}{2\hbar}\right)$

(c) $\frac{4}{9}\cos^2\left(\frac{3gt}{2\hbar}\right)$

(d) $\frac{4}{9}\sin^2\left(\frac{E-3gt}{2\hbar}\right)$

67. A hydrogen atom in its ground state is collided with an electron of kinetic energy 13.377eV. The maximum factor by which the radius of the atom would increase is

(a) 7

(b) 8

(c) 49

(d) 64

68. The formula for normal strain in a longitudinal bar is given by $\varepsilon = \frac{F}{AE'}$ where F is normal force applied, A is cross-sectional area of the bar, and E is Young's modulus. If $F = 50 \pm 0.5N, A = 0.2 \pm 0.002\ \mathrm{m}^2$ an $E = 210 \times 10^9 \pm 1 \times 10^9$ Pa, the maximum error in the measurement of strain is

(a) 1.0×10^{-12}

(b) 2.95×10^{-11}

(c) 1.22×10^{-9}

(d) 1.19×10^{-9}

69. A monoatomic gas consists of atoms with two internal energy levels, ground state $E_0 = 0$ an excited state $E_1 = E$. The specific heat of the gas is given by

(a) $\frac{3}{2}k$

(b) $\frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

(c) $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

(d) $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2\left(1+e^{E/kT}\right)^2}$

70. Two large no conducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are $\sigma_+ = 6.8\mu\mathrm{C/m}^2$ for the positively charged sheet and $\sigma_- = 4.3\mu\mathrm{C/m}^2$ for the negatively charged sheet. What is the electric field in the region between the sheets?

(a) 6.30×10^5 N/C

(b) 3.84×10^5 N/C

(c) 1.40×10^5 N/C

(d) 1.16×10^5 N/C

71. A monochromatic wave propagates in a direction making an angle 60° with the x-axis in the reference frame of source. The source moves at speed $v = \frac{4c}{5}$ towards the observer. The direction of the (cosine of angle) wave as seen by the observer is

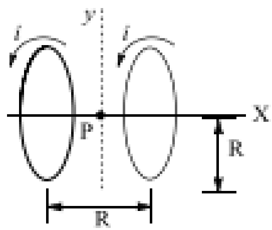
(a) $\cos\theta' = \frac{13}{14}$

(b) $\cos\theta' = \frac{3}{14}$

(c) $\cos\theta' = \frac{13}{6}$

(d) $\cos\theta' = \frac{1}{2}$

72. A system of two circular co-axial coils carrying equal currents I along same direction having equal radius R and separated by a distance R (as shown in figure below). The magnetic field at the midpoint P is given by



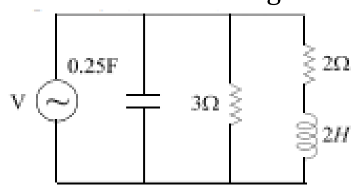
(a) $\frac{\mu_0 I}{2\sqrt{2}R}$

(b) $\frac{4\mu_0 I}{5\sqrt{5}R}$

(c) $\frac{8\mu_0 I}{5\sqrt{5}R}$

(d) 0

73. Find the resonance frequency (rad/sec) of the circle shown in the figure below



(a) 1.41

(b) 1.0

(c) 2.0

(d) 1.73

74. The temperature of a thin bulb filament (assuming that the resistance of the filament is nearly constant) of radius ' r ' and length L is proportional to

(a) $r^{1/4}L^{-1/2}$ (b) L^2r

(c) $r^{1/4}L^{-1}$ (d) r^2L^{-1}

75. Ice of density ρ_1 melts at pressure P and absolute temperature T to form water of density ρ_2. The latent heat of melting of 1 gram of ice is L. What is the change in the internal energy ΔU resulting from the melting point of 1 gram of ice?

(a) $L + P\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$ (b) $L - P\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$

(c) $L - P\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$ (d) $L + P\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$

76. A spherical air bubble is embedded in a glass slab. It will behave like a

(a) Cylindrical lens (b) Achromatic lens

(c) Converging lens (d) Diverging lens

77. The acceleration experienced by the bob of a simple pendulum is

(a) maximum at the extreme positions

(b) maximum at the lowest (central) positions

(c) maximum at a point between the above two positions

(d) same as all positions

78. A 100 ohms resistor carrying current of 1Amp is maintained at a constant temperature of 30°C by a heat bath. What is the rate of entropy increase of the resistor?

(a) 3.3Joules/K/sec

(b) 6.6Joules/K/sec

(c) 0.33Joules/K/sec

(d) None of the above

79. Consider a Hamiltonian system with a potential energy function given by $V(x) = x^2 - x^4$. which of the following is correct?

(a) The system has one stable point

(b) The system has two stable points

(c) The system has three stable points

(d) The system has four stable points

80. The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is 2eV. The energy of the quantum mechanical ground state for a system of three non-interacting spin-1/2 particles is

(a) 6eV (b) 10eV

(c) 12eV (d) 16eV

81. The value of elastic constant for copper is about 100Nm − 1 and the atomic spacing is 0.256 nm. What is the amplitude of the vibration of the Cu atoms at 300 K as a percentage of the equilibrium separation?

(a) 4.55% (b) 3.55%

(c) 2.55% (d) 1.55%

82. What is the contribution of the conduction electrons in the molar entropy of a metal with electronic coefficient of specific heat?

(a) γT (b) γT^2

(c) γT^3 (d) T^4

83. Consider a system of 2 N non-interacting spin 1/2 particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of

spin up particle for which the entropy of the system will be maximum is

(a) 0 (b) N

(c) 2 N (d) N/2

84. When two different solids are brought in contact with each other, which one of the following is true?

(a) Their Fermi energies become equal

(b) Their band gaps become equal

(c) Their chemical potentials become equal

(d) Their work functions become equal.

85. For which gas the ratio of specific heats $\left(C_p/C_v\right)$ will be largest?

(a) mono-atomic (b) di-atomic

(c) tri-atomic (d) hexa-atomic

86. A ball bounces back off earth. You are asked to solve this quantum mechanically assuming the earth is an infinitely hard sphere. Consider surface of earth as the origin implying $V(0) = \infty$ and a linear potential elsewhere (i.e. $V(x) = -mgx$ for $x > 0$). Which of the following wave function is physically admissible for this problem (with $k > 0$)

(a) $\psi(x) = \frac{e^{-kx}}{x}$ (b) $\psi(x) = xe^{-kx^2}$

(c) $\psi(x) = -Axe^{kx}$ (d) $\psi(x) = Ae^{-kx^2}$

87. Which functional form of potential best describes the interaction between a neutral atom and an ion at large distances. (i.e. much larger than their diameters)

(a) $V \propto -1/r^2$ (b) $V \propto -1/r$

(c) $V \propto -e^{-r/a}/r$ (d) $V \propto -1/r^3$

88. An electron is executing simple harmonic motion along the y-axis in right handed coordinate system. Which of the following statements is true for emitted radiation?

(a) The radiation will be most intense in xz plane

(b) The radiation will be most intense in xy plane

(c) The radiation will violate causality

(d) The electron's rest mass energy will reduce due to radiation loss

89. Two point objects A and B have masses 1000Kg and 3000Kg respectively. They are initially at rest with a separation equal to 1 m. Their mutual gravitational attraction then draws then together. How far from A's original position will they collide?

(a) 1/3 m (b) 1/2 m

(c) 2/3 m (d) 3/4 m

90. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been

(a) Less

(b) More

(c) The same

(d) Depends on the electron mass

91. Let us write down the Lagrangian of a system as $L(x, \dot{x}, \ddot{x}) = mx\ddot{x} + kx^2 + cx\dot{x}$ what is the dimension of c ?

(a) MLT^{-3} (b) MT^{-2}

(c) MT (d) ML^2T^{-1}

92. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$ for a well

behaved function $f(x)$. If x has the dimension of momentum then

(a) $\delta(x)$ has the dimension of momentum

(b) $\delta(x)$ has the dimension of (momentum) ^{2}

(c) $\delta(x)$ is dimensionless

(d) $\delta(x)$ has the dimension of (momentum) $)^{-1}$

93. The operator A and B share all the eigenstates. Then the least possible value of the product of uncertainties $\triangle A \Delta B$ is

(a) $\hbar$

(b) 0

(c) $\frac{\hbar}{2}$

(d) determinant (AB)

94. The resolving power of a grating spectrograph can be improved by

(a) recording the spectrum in the lowest order

(b) using a grating with longer grating period

(c) masking a part of the grating surface.

(d) illuminating the grating to the maximum possible extent.

95. The value of limit $\lim_{z \to i} \frac{z^{10}+1}{z^6+1}$ is equal to

(a) 1 (b) 0

(c) $-\frac{10}{3}$ (d) $\frac{5}{3}$

96. A conducting sphere of radius ' r ' has charge Q on its surface. If the charge on there sphere is doubled and its radius is halved, the energy associated with the electric field will

(a) Increase four times

(b) Increase eight times

(c) Remain the same

(d) Decrease four times

97. Three sinusoidal waves have the same frequency with amplitude $A, A/2$ and $A/3$ while their phase angles are $0, \pi/2$ and π respectively. The amplitude of the resultant wave is

(a) 11 A/6 (b) 2 A/3

(c) 5 A/6 (d) 7 A/6

98. The value of integral$I = \oint_C \frac{\sin z}{2z-\pi} dz$ with c a is circle $|z| = 2$, is

(a) 0 (b) $2\pi i$

(c) πi (d) $-\pi i$

99. Consider a square well of depth $-V_0$ and width a with $V_0 a$ fixed. Let $V_0 \to \infty$ and $a \to 0$. This potential well has

(a) No bound state

(b) 1 bound state

(c) 2 bound states

(d) Infinitely many bound states

100. If hydrogen atom is bombarded by energetic electrons, it will emit

(a) $K_\alpha X$ - Rays

(b) β-rays

(c) Neutrons

(d) none of the above.

Answer Key				
1.	2. c	3. d	4. a	5. c
6. b	7. a	8. d	9. a	10. c
11. c	12. c	13. c	14. d	15. d
16. a	17. b	18. a	19. b	20. a
21. d	22. a	23. c	24. d	25. b
26. c	27. c	28. a	29. c	30. d
31. a	32. c	33. b	34. d	35. b
36. b	37. d	38. b	39. b	40. b
41. d	42. a	43. c	44. a	45. a
46. b	47. b	48. b	49. c	50. b
51. a	52. c	53. c	54. a	55. b
56. c	57. d	58. d	59. c	60. b
61. a	62. b	63. b	64. a	65. d
66. b	67. c	68. b	69. b	70. a
71. a	72. c	73. b	74. d	75. b,d
76. d	77. a	78. c	79. a	80. c
81. b	82. a	83. b	84. c	85. a
86. b	87. a	88. a	89. d	90. a
91. c	92. d	93. b	94. d	95. d
96. b	97. c	98. c	99. b	100. a