D PHYSICS

CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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# GATE 2014

## Question Paper With

## Answer Key

## PHYSICAL SCIENCE

1. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1) is
   (a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$
   (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

2. Which one of the following quantities is invariant under Lorentz transformation?
   (a) Charge density (b) Charge
   (c) Current (d) Electric field

3. The number of normal Zeeman splitting components of $^1\text{P} \rightarrow {}^1\text{D}$ transition is
   (a) 3 (b) 4
   (c) 8 (d) 9

4. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is $5 \times 10^{-8}$ s, then the proper half-life is…… $\times 10^{-8} s \cdot (c = 3 \times 10^8 \text{ m/s})$

5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is
   (a) 0° (b) 45°
   (c) 90° (d) 120°

6. Two masses m and 3 m are attached to the two ends of a massless spring with force constant K. If $m = 100$ g and $K = 0.3$Nlm, then the natural angular frequency of oscillation is $Hz$

7. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by
   $\vec{E} = \hat{x}10\cos(16\pi \times 10^7 t - 0.4\pi z)\text{V/m}$
   The phase velocity of the wave is……
   $\times 10^8$ m/s.

8. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is
   (a) orthogonal (b) symmetric
   (c) anti-symmetric (d) unitary

9. The recoil momentum of an atom is $p_A$ when it emits an infrared photon of wavelength 1500 nm, and it is $p_B$ when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is
   (a) 1: 1 (b) 1: $\sqrt{3}$
   (c) 1: 3 (d) 3: 2

10. For a gas under isothermal conditions, its pressure $P$ varies with volume $V$ as $P \propto V^{-5/3}$. The bulk modulus B is proportional to
    (a) $V^{-1/2}$ (b) $V^{-2/3}$
    (c) $V^{-3/5}$ (d) $V^{-5/3}$

11. Which one of the following high energy processes is allowed by conservation laws?
    (a) $p + \bar{p} \rightarrow \Lambda^\circ + \Lambda^\circ$
    (b) $\pi^- + p \rightarrow \pi^\circ + n$
    (c) $n \rightarrow p + e^- + v_e$
    (d) $\mu^+ \rightarrow e^+ + \gamma$

12. The length element $ds$ of an arc is given by, $(ds)^2 = 2(dx^1)^2 + (dx^2)^2 + \sqrt{3}dx^1dx^2$. The metric tensor $g_{ij}$ is
    (a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$
    (c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

13. The ground state and the first excited state wave functions of a one dimensional infinite potential well are $\psi_1$ and $\Psi_2$, respectively. When two spin-up electrons are placed in this potential, which one of the following with $x_1$ and $x_2$ denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?
    (a) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) - \psi_2(x_2)\psi_2(x_2)]$

(b) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

(c) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$

(d) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

14. If the vector potential

$$\vec{A} = \alpha x\hat{x} + 2y\hat{y} - 3z\hat{z}$$

satisfies the Coulomb gauge, the value of the constant $\alpha$ is……

15. At a given temperature $T$ the average energy per particle of a non-interacting gas of two dimensional classical harmonic oscillator is……. $k_BT$ ( $k_B$ is the Boltzmann constant)

16. Which one of the following is a fermions?

(a) $\alpha$ particle (b) $_4Be^7$ nucleus

(c) Hydrogen atom (d) Deuteron

17. Which one of the following three-quark states (qqq) denoted by X, cannot be a possible baryon? The corresponding electric charge is indicated in the superscript.

(a) $X^{++}$ (b) $X^{+}$

(c) $X^{-}$ (d) $X^{--}$.

18. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

(a) $\dot{q} = \{q, H\}; \dot{p} = \{p, H\}$

(b) $\dot{q} = \{H, q\}; \dot{p} = \{H, p\}$

(c) $\dot{q} = \{H, p\}; \dot{p} = \{H, q\}$

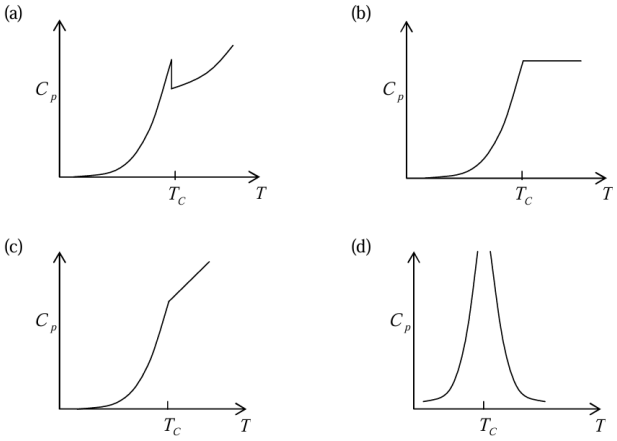
(d) $\dot{q} = \{p, H\}; \dot{p} = \{q, H\}$

19. The Miller indices of a plane passing through the three points having coordinates (0,0,1), (1,0,0), $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$ are

(a) (212) (b) (111)

(c) (121) (d) (211)

20. The plot of specific heat versus temperature across the superconducting transition temperature ($T_C$) is most appropriately represented by

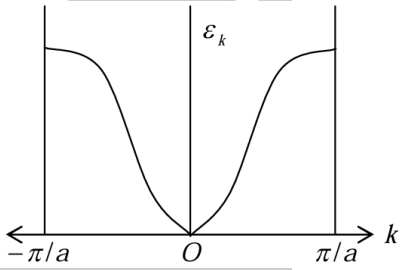



21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum then $\vec{L} \cdot \vec{S}$ does not commute with

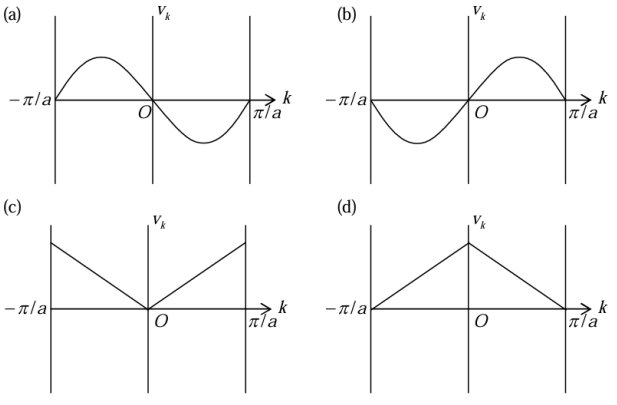
(a) $S_z$ (b) $L^2$

(c) $S^2$ (d) $(\vec{L} + \vec{S})^2$

22. The energy, $\varepsilon_k$ for band electrons as a function of the wave vector, k in the first Brillouin zone $\left(-\frac{\pi}{a} \leq k \leq \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as ( $a$ is lattice constant)



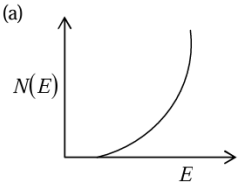

The variation of the group velocity $v_k$ is most appropriately represented by



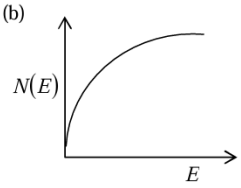

23. For a free electron gas in two dimensions, the variation of the density of states, $N(E)$

as a function of energy $E$, is best represented by

(a)




(b)

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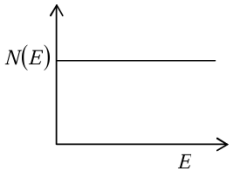


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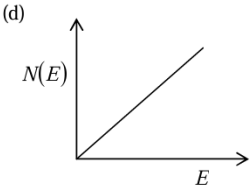
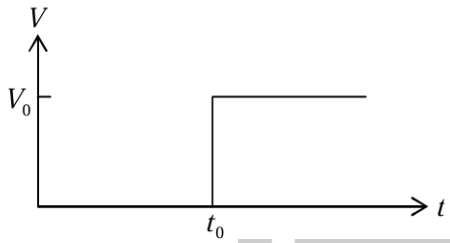
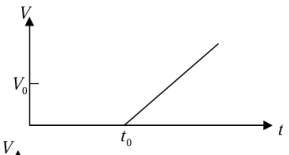



24. The input given to an ideal OP-AMP integrator circuit is

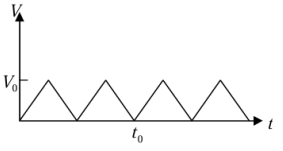



The correct output of the integrator circuit is

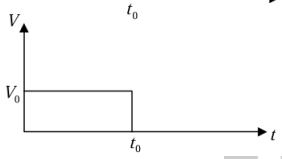
(a)



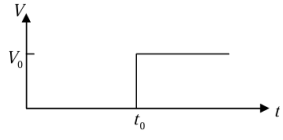

(b)




(c)

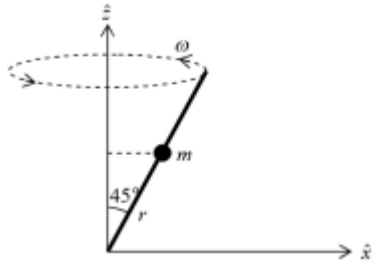



(d)




25. The minimum number of flip-flops required to construct a mod- 75 counter is……..

26. A bead of mass $m$ can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed $\omega$. At any instant, $r$ is the distance of the bead from the origin. The momentum conjugate to $r$ is




(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$

(c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

27. An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

Where $a_0$ is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is

(Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n+1)!}{\alpha^{n+1}}$)

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$

(c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

28. For Nickel, the number density is $8 \times 10^{23}$ atom /cm$^3$ and electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic state is_____ $\times$ $10^9$ A/m.

(Given the value of Bohr magneton $\mu_B = 9.21 \times 10^{-21}\text{Am}^2$ )

29. A particle of mass $m$ is in a potential given by

$$V(r) - \frac{a}{r} + \frac{ar_0^2}{3r^3},$$

Where $a$ and $r_0$ are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$

(c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

30. The donor concentration in a sample of n-type silicon is increased by a factor 100 .

The shift in the position of the Fermi level at $300K$, assuming the sample to be non degenerate is_____ MeV. ($k_BT = 25meV$ at 300 K)

31. A particle of mass $m$ is subjected to a potential

$$V(x,y) = \frac{1}{2}m\omega^2(x^2+y^2), -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is $g$-fold degenerate. The value of $g$ is…….

32. A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\Psi_{200} - \sqrt{\frac{3}{7}}\Psi_{310} + \sqrt{\frac{4}{21}}\Psi_{321},$$

Where $n, l, m$ in $\Psi_{nlm}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of $L^2$ is_____ $\hbar^2$.

33. A planet of mass m moves in a circular orbit of radius $r_0$ in the gravitational potential $V(r) = -\frac{k}{r}$, where $k$ is a positive constant. The orbital angular momentum of the planet is

(a) $2r_0$ km (b) $\sqrt{2r_0km}$

(c) $r_0$ km (d) $\sqrt{r_0km}$

34. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecules B. If the rotational energies of the two molecules are equal then the corresponding values of the rotational quantum numbers $J_A$ and $J_B$ are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$

(c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

35. The value of the integral

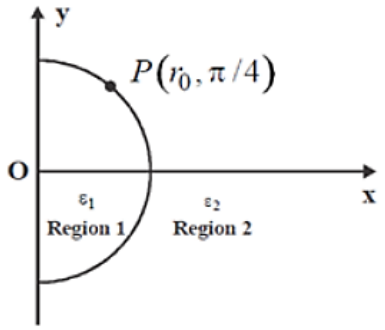
$$\oint_C \frac{z^2}{e^z+1}dz$$

Where C is the circle $|z| = 4$ is

(a) $2\pi i$ (b) $2\pi^2 i$

(c) $4\pi^3 i$ (d) $4\pi^2 i$

36. A ray of light inside Region 1 in the $xy$-plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to $x$-axis. If $\varepsilon_1$ and $\varepsilon_2$ are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is_____




37. The solution of the differential equation

$\frac{d^2y}{dt^2} - y = 0$

Subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$

(c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

38. Given that the linear transformation of generalized coordinate $q$ and the corresponding momentum p,

$$Q = q + 4ap$$
$$P = q + 2p$$

Is canonical, the value of the constant $a$ is_____.

39. The value of the magnetic field required to maintain non-relativistic protons of energy 1MeV in a circular orbit of radius 100 nm is-_____ Tesla

(Given: $m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$C )

40. For a system of two bosons, each of which can occupy any of the two energy level 0

and $\varepsilon$, the mean energy of the system at a temperature $T$ with $\beta = \frac{1}{k_B T}$ is given by

(a) $\frac{\varepsilon e^{-\beta e} + 2\varepsilon e^{-2\beta e}}{1 + 2e^{-\beta e} + e^{-2\beta e}}$ (b) $\frac{1 + \varepsilon e^{-\beta z}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta c} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta c}}$

41. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and $I_0$ respectively. The intensities of the individual waves are

(a) $3I_0$ and $I_0$ (b) $4I_0$ and $I_0$

(c) $5I_0$ and $4I_0$ (d) $9I_0$ and $I_0$

42. $\Psi_1$ and $\Psi_2$ are two orthogonal states of a spin 1/2 system. It is given that

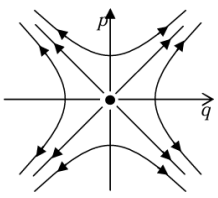
$$\Psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix},$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin down states, respectively. When the system is in the state $\Psi_2$ its probability to be in the spin-up state is……..

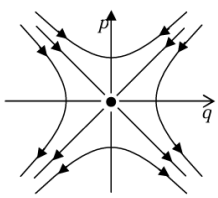
43. Neutrons moving with speed $10^3$ m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30°, the inter-planar spacing of the crystal is_____ $\dot{A}$.

44. The Hamiltonian of a particle of mass m is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which one of the following figures describes the motion of the particle in phase space?

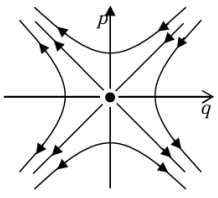
(a)



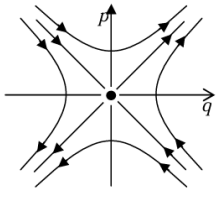

(b)




(c)




(d)




45. The intensity of a laser in free space is 150 mW/m$^2$. The corresponding amplitude of the electric field of the laser is _____$V/m$
($\varepsilon_0 = 8.854 \times 10^{-12} C^2/N.m^2$)

46. The emission wavelength for the transition $^1D_2 \rightarrow {}^1F_3$ is 3122$\dot{A}$. The ratio of populations of the final to the initial states at a temperature 5000 K is ($h = 6.626 \times 10^{-34}$ J.$s$, $c = 3 \times 10^8$ m/s, $k_B = 1.380 \times 10^{-23}$ J/k)

(a) $2.03 \times 10^{-5}$ (b) $4.02 \times 10^{-5}$

(c) $7.02 \times 10^{-5}$ (d) $9.83 \times 10^{-5}$

47. Consider a system of 3 fermions which can occupy any of the 4 available energy with equal probability. The energy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$

(c) $2k_B \ln 4$ (d) $3k_B \ln 4$

48. A particle is confined to a one dimensional potential box with the potential

$V(x) = 0, 0 < x < a$

$= \infty$, otherwise

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where $\beta$ is small constant, the first order correction to the ground state energy is

(a) 0 (b) $\frac{\alpha\beta}{4}$

(c) $\frac{\alpha\beta}{2}$ (d) $\alpha\beta$

49. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons ($\mu$) in the centre of mass frame required to produce the pion ($\pi$) pairs at rest is_____ MeV
(Given: $m_\mu = 105\text{MeV/c}^2, m_x = 140\text{MeV/c}^2$ )

50. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\Psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

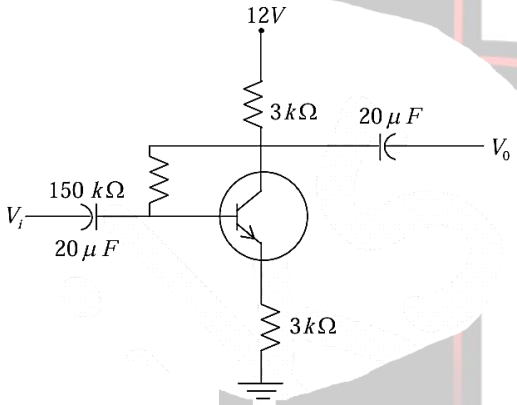
The average energy of the oscillator in the given state is_____ $\hbar\omega$

51. A nucleus X undergoes a first forbidden $\beta$-decay to a nucleus Y. If the angular momentum (I) and parity (P) denoted by $I^P$ as $\frac{7^-}{2}$ for X, which of the following is a possible $I^P$ value for Y ?

(a) $\frac{1}{2}$ (b) $\frac{1^-}{2}$

(c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current $I_C$ is




53. In order to measure a maximum of W with a resolution of 1mV using a n bit A/D converter working under the principle of ladder network, the minimum value of n is……..

54. If $L_+$and $L_-$are angular momentum ladder operators, then the expectation value of $(L_+L_- + L_-L_+)$in the state $|l = 1, m = 1\rangle$ of an atom is……. $\hbar^2$.

55. A low pass filter is formed by a resistance $R$ and a capacitance C. At the cut off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively are

(a) 0.71 and 45° (b) 0.71 and −45°

(c) 0.5 and −90° (d) 0.5 and 90°s

| Answer Key | | | | |
|---|---|---|---|---|
| 1. d | 2. b | 3. a | 4. 2.18 | 5. c |
| 6. 0.32 | 7. 1.5 | 8. d | 9. c | 10. d |
| 11. b | 12. b | 13. d | 14. 1 | 15. 2 |
| 16. b | 17. d | 18. a | 19. a | 20. a |
| 21. d | 22. b | 23. c | 24. a | 25. 7 |
| 26. a | 27. d | 28. 4.4 | 29. a | 30. 115 |
| 31. 4 | 32. 2 | 33. d | 34. b | 35. c |
| 36. 2.33 | 37. d | 38. 0.25 | 39. 1.4450 | 40. |
| 41. b | 42. 0.66 | 43. 3.96 | 44. d | 45. 10.62 |
| 46. c | 47. b | 48. c | 49. 35 | 50. 3.25 |
| 51. c | 52. 1.5 | 53. 10 | 54. 2 | 55. b |