D PHYSICS

CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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PHYSICAL SCIENCE

MATHEMATICAL PHYSICS

Previous Year Questions [Topic-Wise)

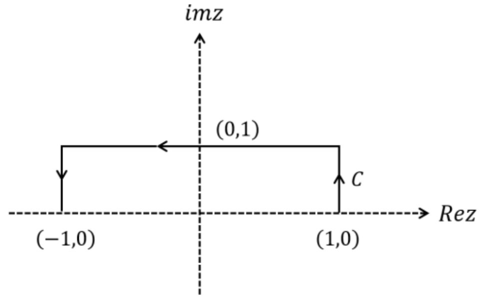
With Answer Key

CSIR-NET/JRF,GATE,JEST,TIFR

❖ CSIR-NET PYQ

1. The value of the integral $\int_C dz z^2 e^z$, where C is an open contour in the complex z-plane as shown in the figure below, is:

[CSIR JUNE 2011]



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$

(c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

2. Which of the following is an analytic function of the complex variable $z=x+iy$ in the domain $|z|<2$? **[CSIR JUNE 2011]**

(a) $(3+x-iy)^7$

(b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$

(d) $(x+iy-1)^{1/2}$

3. The principal value of the real integral

$$I=\int_{-3}^{+3}\frac{dx}{x^2+3x+2}\text{ is:}$$

[CSIR DEC 2011]

(a) $\frac{3\pi}{2}$ (b) $\ln\left(\frac{2}{5}\right)$

(c) ∞ (d) 0

4. The first few terms in the Taylor series expansion of the function $f(x)=\sin x$ around $x=\frac{\pi}{4}$ are **[CSIR DEC 2011]**

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3\ldots\right]$

(d) $\frac{1}{\sqrt{2}}\left[1-x+\frac{x^2}{2!}-\frac{X^3}{3!}\ldots\right]$

5. The first few terms in the Laurent series for

$$\frac{1}{(z-1)(z-2)}$$

in the region $1\le|z|\le 2$, and around $z=1$ is **[CSIR JUNE 2012]**

(a) $\frac{1}{2}[1+z+z^2+z^3+\cdots\ldots]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\cdots.\right]$

(b) $\frac{1}{1-z}+z-(1-z)^2+(1-z)^3+\cdots\ldots$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+\cdots\ldots\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+\cdots..\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+\cdots\ldots$

6. Let

$$u(x,y)=x+\frac{1}{2}(x^2-y^2)$$

be the real part of an analytic function f(z) of the complex variable $z=x+iy$. The imaginary part of f(z) is **[CSIR JUNE 2012]**

(a) $y+xy$ (b) xy

(c) y (d) y^2-x^2

7. The value of the integral **[CSIR JUNE 2012]**

$$\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$$

is

(a) $-2\pi/R$ (b) $-\pi/R$

(c) π/R (d) $2\pi/R$

8. The Taylor expansion of the function $ln(\cosh x)$, where ' x ' is real, about the point $x=0$ starts with the following terms:

[CSIR DEC 2012]

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+\cdots\ldots\ldots$

(b) $\frac{1}{2}x^2-\frac{1}{12}x^4+\cdots\ldots\ldots$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+\cdots\ldots$

(d) $\frac{1}{2}x^2+\frac{1}{6}x^4+\cdots\ldots\ldots$

9. The value of the integral

$$\int_c \frac{z^3 dz}{z^2-5z+6}$$

, where C is a closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is: **[CSIR DEC 2012]**

(a) $-16\pi i$ (b) $16\pi i$

(c) $8\pi i$ (d) $2\pi i$

10. With $z=x+iy$, which of the following functions $f(x,y)$ is NOT a (complex) analytic function of z ?

[CSIR JUNE 2013]

(a) $f(x,y)=(x+iy-8)^3$

(b) $f(x,y)=(x+iy)^7(1-x-iy)^3$

(c) $(4+x^2-y^2+2ixy)^7$

(d) $f(x,y)=(1-x+iy)^4(2+x+iy)^6$

11. Which of the following functions cannot be the real part of a complex analytic function of $z=x+iy$? **[CSIR DEC 2013]**

(a) x^2y (b) x^2-y^2

(c) x^3-3xy^2 (d) $3x^2y-y-y^3$

12. Given that the integral

$$\int_0^\infty \frac{dx}{y^2+x^2}=\frac{\pi}{2y}$$

,the value of

$$\int_0^\infty \frac{dx}{(y^2+x^2)^2}$$

is **[CSIR DEC 2013]**

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$

(c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

13. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral **[CSIR JUNE 2014]**

$$\oint_C \frac{dz}{\sin^2 z} \text{ is}$$

(a) ∞ (b) $2\pi i$

(c) 0 (d) πi

14. The function $\Phi(x,y,z,t)=\cos(z-vt)+\text{Re}(\sin(x+iy))$ satisfies the equation

[CSIR JUNE 2014]

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi$

(b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$

(d) $\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$

15. The principal value of the integral

$$\int_{-\infty}^{\infty}\frac{\sin(2x)}{x^3}dx$$

is **[CSIR DEC 2014]**

(a) -2π (b) $-\pi$

(c) π (d) 2π

16. The Laurent series expansion of the function $f(z)=e^z+e^{1/z}$ about $z=0$ is given by

[CSIR DEC 2014]

(a) $\sum_{n=-\infty}^{\infty}\frac{z^n}{n!}$ for all $|z|<\infty$

(b)$\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ only if $0<|x|<1$

(c)$\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ for all $0<|z|<\infty$

(d)$\sum_{n=-\infty}^{\infty}\frac{z^n}{n!}$, only if $|z|<1$

17. Consider the function

$$f(z)=\frac{1}{z}\ln(1-z)$$

of a complex variable $z=re^{i\theta}(r\geq 0,-\infty<\theta<\infty)$. The singularities of $f(z)$ are as follows:

[CSIR DEC 2014]

(a) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for $0\leq\theta<2\pi$

(b) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ other than $0\leq\theta<2\pi$

(c) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ for all θ

(d) branch points at $z=0, z=1$ and $z=\infty$

18. The value of the integral

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^4}$$

is

[CSIR JUNE 2015]

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$

(c) $\sqrt{2}\pi$ (d) 2π

19. The function

$$\frac{z}{\sin\,\pi z^2}$$

of a complex variable z has

[CSIR DEC 2015]

(a) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ for $n=1,2,3\ldots$

(b) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ and $\pm i\sqrt{n}$ for $n=1,2,3\ldots$

(c) poles of order 2 at $\pm\sqrt{n}, n=0,1,2,3\ldots$

(d) poles of order 2 at $\pm n, n=0,1,2,3\ldots$

20. The radius of convergence of the Taylor series expansion of the function

$$\frac{1}{\cosh(x)}$$

around $x=0$, is

[CSIR JUNE 2016]

(a) ∞ (b) π

(c) $\frac{\pi}{2}$ (d) 1

21. The value of the contour integral

$$\frac{1}{2\pi i}\oint_C\frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$$

around the unit circle C traversed in the anti-clockwise direction, is

[CSIR JUNE 2016]

(a) 0 (b) 2

(c) $-\frac{8}{\sqrt{3}}$ (d) $-\tanh\left(\frac{1}{2}\right)$

22. Let $u(x,y)=e^{ax}\cos(by)$ be the real part of a function $f(z)=u(x,y)+iv(x,y)$ of the complex variable $z=x+iy$, where a,b are real constants and $a\neq 0$. The function $f(z)$ is complex analytic everywhere in the complex plane if and only if

[CSIR JUNE 2017]

(a) $b=0$ (b) $b=\pm a$

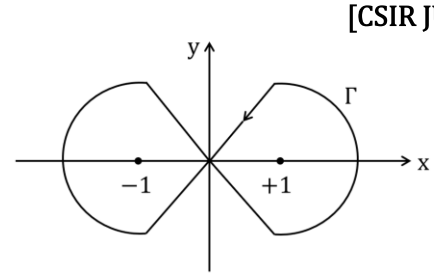
(c) $b=\pm 2\pi a$ (d) $b=a\pm 2\pi$

23. The integral

$$\oint_\Gamma\frac{ze^{i\pi z/2}}{z^2-1}dz$$

along the closed contour Γ shown in the figure is

[CSIR JUNE 2017]



(a) 0 (b) 2π

(c) -2π (d) $4\pi i$

24. Consider the real function $f(x)=\frac{1}{(x^2+4)}$. The Taylor expansion of $f(x)$ about $x=0$ converges **[CSIR DEC 2017]**

(a) for all values of x

(b) for all values of x except $x=\pm 2$

(c) in the region $-2<x<2$

(d) for $x>2$ and $x<-2$

25. What is the value of α for which

$$f(x,y)=2x+3(x^2-y^2)+2i(3xy+\alpha y)$$

is an analytic function of complex variable $z=x+iy$? **[CSIR JUNE 2018]**

(a) 1 (b) 0

(c) 3 (d) 2

26. The value of the integral

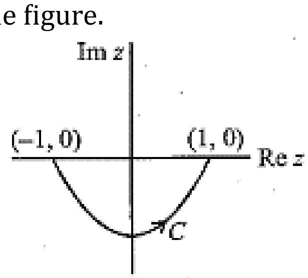
$$\oint_C \frac{dz}{z}\frac{\tanh\ 2z}{\sin\ \pi z}$$

, where C is a circle of radius $\frac{\pi}{2}$, traversed counter-clockwise, with centre at $z=0$, is **[CSIR DEC 2018]**

(a) 4 (b) $4i$

(c) $2i$ (d) 0

27. The integral $I=\int_C e^z dz$ is evaluated from the point $(-1,0)$ to $(1,0)$ along the contour C, which is an arc of the parabola $y=x^2-1$, as shown in the figure. **[CSIR DEC 2018]**



The value of I is

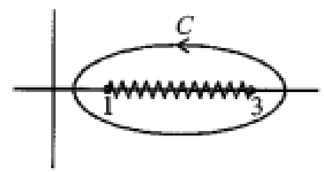
(a) 0 (b) $2\sinh\ 1$

(c) $e^{2i}\sinh\ 1$ (d) $e+e^{-1}$

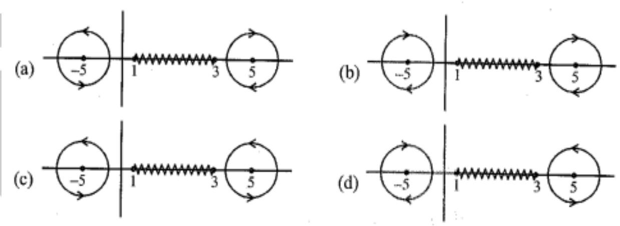
28. The contour C of the following integral

$$\oint_C dz\frac{\sqrt{(z-1)(z-3)}}{(z^2-25)^3}$$

, in the complex z-plane is shown in the figure below. **[CSIR DEC 2018]**



This integral is equivalent to an integral along the contours



29. The value of the definite integral

$$\int_0^\pi \frac{d\theta}{5+4\cos\ \theta}$$

is **[CSIR JUNE 2019]**

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) π (d) $\frac{\pi}{3}$

30. Let C be the circle of radius $\frac{\pi}{4}$, centered at $z=\frac{1}{4}$ in the complex z plane that is traversed counterclockwise. The value of the contour integral

$$\oint_C \frac{z^2}{\sin^2\ 4z}dz$$

is **[CSIR DEC 2019]**

(a) 0 (b) $i\pi^2/4$

(c) $i\pi^2/16$ (d) $i\pi/4$

31. A function of a complex variable z is defined by the integral

$$f(z)=\int_{-}\frac{w^2-2}{w-z}dw$$

, where Γ is a circular contour of radius 3 , centred at origin, running counter-clockwise in the w-plane. The value of the function at $z=(2-$

i) is [CSIR NOV 2020]

(a) 0 (b) $1-4i$

(c) $8\pi + 2\pi i$ (d) $-\frac{2}{\pi} - \frac{i}{2\pi}$

32. At $z = 0$, the function $\frac{1}{z-\sin z}$ of a complex variable z has [CSIR JUNE 2022]

(a) No singularity (b) A simple pole

(c) A pole of order 2 (d) A pole of order 3

33. If $z = i^{i^{i\cdots}}$ (note that the exponent continues indefinitely), then a possible value of $\frac{1}{z}\ln z$ is

[CSIR JUNE 2022]

(a) $2i\ln i$ (b) $\ln i$

(c) $i\ln i$ (d) $2\ln i$

34. The value of the integral

$$\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2+1} dx$$

, for $\alpha > 0$, is [CSIR JUNE 2022]

(a) πe^{α} (b) $\pi e^{-\alpha}$

(c) $\pi e^{\alpha/2}$ (d) $\pi e^{-\alpha/2}$

35. The locus of the curve

$$\operatorname{Im}\left(\frac{\pi(z-1)-1}{z-1}\right) = 1$$

in the complex z-plane is a circle centered at $(x_0 y_0)$ and R-respectively are

[CSIR JUNE 2023]

(a) $\left(1, \frac{1}{2}\right)$ and $\frac{1}{2}$ (b) $\left(1, -\frac{1}{2}\right)$ and $\frac{1}{2}$

(c) $(1,1)$ and 1 (d) $(1,-1)$ and 1

36. If z is a complex number, which among the following sets is neither open nor closed?

[CSIR DEC 2023]

(a) $\{z | 0 \le |z-1| \le 2\}$

(b) $\{z | |z| \le 1\}$

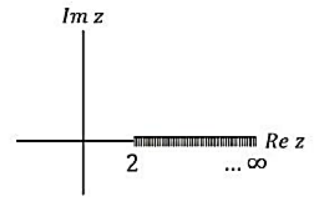
(c) $\{z \mid z \in (\mathbb{C} - \{3\}) \text{ and } |z| \le 100\}$

(d) $\left\{z \mid z = re^{i\theta}, 0 \le \theta \le \frac{\pi}{4}\right\}$

37. The branch line for the function

$$f(z) = \sqrt{\frac{z^2-5z+6}{z^2+2z+1}}$$

is [CSIR DEC 2023]

1.


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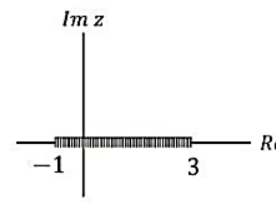

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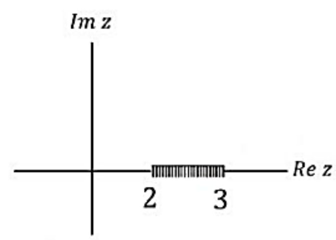

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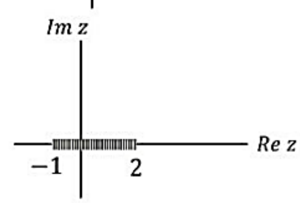


38. The function

$$f(z) = \frac{1}{(z+1)(z+3)}$$

is defined on the complex plane. The coefficient of the $(z-z_0)^2$ term of the Laurent series of $f(z)$ about $z_0 = 1$ is [CSIR DEC 2023]

(a) $\frac{7}{64}$ (b) $\frac{7}{128}$

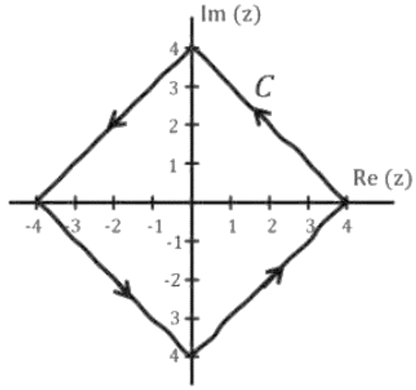
(c) $\frac{9}{64}$ (d) $\frac{9}{128}$

39. The value of the integral (where k is a constant),

$$\frac{1}{2\pi i}\oint_C \frac{5}{(z-2)^2} \sin(kz) dz$$

over the closed contour C as shown below, is

[CSIR DEC 2024]



(a) $5k\cos(2k)$ (b) $5k\sin(2k)$

(c) $5\cos(2k)$ (d) $-5k^2\sin(2k)$

40. The complex integral $\int_c z^4 \exp\left(\frac{1}{2z}\right) dz$, where C is the unit circle centered around the origin traversed counter-clock-wise, equals

[CSIR DEC 2024]

(a) $\frac{\pi i}{120}$ (b) $\frac{\pi i}{960}$

(c) 0 (d) $\frac{\pi i}{1920}$

41. Gamma function with argument z is defined as

$$\Gamma[z] = \int_0^\infty dt t^{z-1} e^{-t}$$

where z is a complex variable and $\text{Re} z \geq 0$. $\Gamma[z]$ has

[CSIR DEC 2024]

(a) a branch point at $z = 0$

(b) a simple pole at $z = 0$

(c) a removable singularity at $z = 0$

(d) an essential singularity at $z = 0$

42. For the function $f(z) = \exp\left[z - 1 + \frac{1}{z-1}\right]$

[CSIR JUNE 2025]

(a) $z = 1$ is a pole of order one.

(b) $z = 1$ is an essential singularity.

(c) $z = 1$ is a pole of order two.

(d) $z = 1$ is a removable singular point.

43. The value of the integral $\int_0^\infty \frac{\cos \alpha x}{1+x^2} dx$, where α is a positive real number, is

[CSIR JUNE 2025]

(a) $\frac{\pi}{2} e^{-\alpha}$ (b) $\pi e^{-\alpha}$

(c) $\frac{\pi}{2} e^{-(\alpha/2)}$ (d) $\pi e^{-(\alpha/2)}$

44. If C be the unit circle traversed clockwise, then the integral $\oint_C dz |1 + 2z|^2$ equals

[CSIR DEC 2025]

(a) $-4\pi i$ (b) $-\pi i$

(c) 0 (d) $-2\pi i$

45. The residue of $f(z) = \frac{\cos \pi z}{(1-z^2)^3}$ at $z = 1$ is

[CSIR DEC 2025]

(a) $\frac{\pi^2}{16}$ (b) $\frac{3}{16}$

(c) $\frac{3+\pi^2}{16}$ (d) $\frac{3-\pi^2}{16}$

❖ GATE PYQ

1. The value of the residue of $\frac{\sin z}{z^6}$ is **[GATE 2001]**

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$

(c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

2. The value of the integral $\int_C z^{10} dz$, where C is the unit circle with the origin as the centre is

[GATE 2001]

(a) 0 (b) $z^{11}/11$

(c) $2\pi i z^{11}/11$ (d) $1/11$

3. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true? **[GATE 2002]**

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$

(d) $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 v}{\partial x\,\partial y}$

4. The value of the integral $\int_C \; dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is **[GATE 2003]**

(a) 0 (b) $2\pi i$

(c) $4\pi i$ (d) $-4\pi i$

5. Using the residue theorem, compute the integral

$$I = \int_0^\infty \frac{dx}{(1+x)^4}$$

[GATE 2002]

6. The inverse of the complex number

$$\frac{3+4i}{3-4i}$$

is

[GATE 2004]

(a) $\frac{2}{25} + i\frac{24}{25}$ (b) $-\frac{7}{25} + i\frac{24}{25}$

(c) $\frac{7}{25} - i\frac{24}{25}$ (d) $-\frac{7}{25} - i\frac{25}{25}$

7. The value of

$$\int_C \frac{dz}{(z^2 + a^2)}$$

, where C is a unit circle (anti-clockwise) centered at the origin in the complex z plane is

[GATE 2004]

(a) π for $a = 2$ (b) zero for $a = \frac{1}{2}$

(c) 4π for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$

8. If $xp(x)$ and $x^2q(x)$ have the Taylor series expansions

$$xp(x) = 4 + x + x^2 + \cdots \dots.$$
$$x^2q(x) = 2 + 3x + 5x^2 + \cdots \dots.$$

then the roots of the indicial equation are

[GATE 2004]

(a) $-1,0$ (b) $-1,-2$

(c) $-1,1$ (d) $-1,2$

9. The value of the integral

$$\int_C \frac{dz}{z+3}$$

, where C is a circle (anticlockwise) with $|z| = 4$, is **[GATE 2005]**

(a) 0 (b) πi

(c) $2\pi i$ (d) $4\pi i$

10. All solutions of the equation $e^z = -3$ are

[GATE 2005]

(a) $z = \text{in } \pi \ln\ 3, n = \pm 1, \pm 2, \ldots$

(b) $z = \ln\ 3 + i(2n+1)\pi, n = 0, \pm 1, \pm 2, \ldots$

(c) $z = \ln\ 3 + i2n\pi, n = 0, \pm 1 \pm 2, \ldots$

(d) $z = i3n\pi, n = \pm 1 \pm 2, \ldots.$

11. Consider the following function: $f(z) = \frac{\sin z}{z}$. Which of the following statemens is are TRUE?

[GATE 2005]

(a) $\text{z} = 0$ is pole of order 1

(b) $\text{z} = 0$ is a removable singular point

(c) $\text{z} = 0$ is a pole order 3

(d) $z = 0$ is an essential singular point

12. The value of

$$\oint_C \frac{e^{2z}}{(z+1)^4} dz$$

, where C is a circle defined by $|z| = 3$, is **[GATE 2006]**

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$

(c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

13. The contour integral

$$\oint \frac{dz}{z^4 + a^4}$$

is to be evaluated on a circle of radius $2a$ centered at the origin. It will have contributions only from the points **[GATE 2007]**

(a) $\frac{1+i}{\sqrt{2}}a$and $-\frac{1+i}{\sqrt{2}}a$

(b) ia and -ia

(c) $ia, -ia, \frac{1-i}{\sqrt{2}}a$and $-\frac{1-i}{\sqrt{2}}a$

(d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$and $-\frac{1-i}{\sqrt{2}}a$

14. If $I = \oint_C dz \ln(z)$, where C is the unit circle taken anticlockwise and $\ln(z)$ is the principal branch of the Logarithm function, which one of the following is correct? **[GATE 2008]**

(a) $\mathrm{I} = 0$ by residue theorem

(b) I is not defined since $\ln(z)$ has a branch cut

(c) $I \neq 0$

(d) $\oint_C dz \mathrm{Ln}(z^2) = 2I$

15. The value of

$$\int_{-i}^{i} \pi(z+1)dz$$

is **[GATE 2008]**

(a) 0 (b) $2\pi i$

(c) $-2\pi i$ (d) $(-1+2i)\pi$

16. The value of the integral

$$\int_C \frac{e^x}{z^2 - 3z + 2}$$

, where the contour C is the circle $|z| = 3/2$ is **[GATE 2009]**

(a) $2\pi ie$ (b) πie

(c) $-2\pi ie$ (d) $-\pi ie$

17. The value of the integral

$$\oint_C \frac{e^z \sin(z)}{z^2}dz$$

, where the contour C is the unit circle $|z-2| = 1$ is **[GATE 2010]**

(a) $2\pi i$ (b) $4\pi i$

(c) πi (d) 0

18. For the complex function,

$$f(z) = \frac{e^{\sqrt{z}} - e^{-\sqrt{z}}}{\sin(\sqrt{z})}$$

, which of the following statements is correct? **[GATE 2010]**

(a) $z = 0$ is a branch point

(b) $z = 0$ is a pole of order one

(c) $z = 0$ is a removable singularity

(d) $z = 0$ is an essential singularity

Common Data for Questions 19 and 20:
Consider a function

$$f(z) = \frac{z\sin z}{(z-\pi)^2}$$

of a complex variable z.

19. Which of the following statements is true for the function $f(z)$? **[GATE 2011]**

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z = \pi$

(c) $f(z)$ has a pole of order 2 at $z = \pi$

(d) $f(z)$ has a simple pole at $z = \pi$

20. Consider a counter clockwise circular contour $|z| = 1$ about the origin. The integral $\oint f(z)dz$

over this contour is [GATE 2011]

(a) $-i\pi$ (b) 0

(c) $i\pi$ (d) $2i\pi$

21. The value of the integral $\oint e^{1/z}dz$, using the contour C of circle with unit radius $|z| = 1$ is **[GATE 2012]**

(a) 0 (b) $1 - 2\pi i$

(c) $1 + 2\pi i$ (d) $2\pi i$

22. For the function

$$f(z) = \frac{16z}{(z+3)(z-1)^2}$$

the residue at the pole $z = 1$ is ________(your answer should be an integer) **[GATE 2013]**

23. The value of the integral

$$\oint_C \frac{z^2}{e^z + 1} dz$$

Where C is the circle $|z| = 4$ is **[GATE 2014]**

(a) $2\pi i$ (b) $2\pi^2 i$

(c) $4\pi^3 i$ (d) $4\pi^2 i$

24. Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D. Which one of the following options is not correct? **[GATE 2015]**

(a) $u(x, y)$ satisfies Laplace equation in D

(b) $v(x, y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

25. Which of the following is an analytic function of z everywhere in the complex plane? **[GATE 2016]**

(a) z^2 (b) $(z^*)^2$

(c) $|z|^2$ (d) $\sqrt{z}$

26. Consider a complex function

$$f(z) = \frac{1}{z\left(z + \frac{1}{2}\right)\cos(z\pi)}$$

. Which one of the following statements is correct? **[GATE 2015]**

(a) $f(z)$ has simple poles at $z = 0$ and $z = -1/2$

(b) $f(z)$ has a second order pole at $z = -1/2$

(c) $f(z)$ has infinite number of sccond order poles

(d) $f(z)$ has all simple poles

27. The contour integral

$$\oint \frac{dz}{1 + z^2}$$

evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane by a half circle is equal to________ . (up to two decimal places). **[GATE 2017]**

28. The imaginary part of an analytic complex function is $v(x, y) = 2xy + 3y$. The real part of the function is zero at the origin. The value of the real part of the function at $1 + i$ is (up to two decimal places) **[GATE 2017]**

29. The absolute value of the integral

$$\int \frac{5z^3 + 3z^2}{z^2 - 4} dz$$

over the circle $|z - 1.5| = 1$ in complex plane, is (up to two decimal places). **[GATE 2018]**

30. The pole of the function $f(z) = \cot z$ at $z = 0$ is **[GATE 2019]**

(a) a removable singularity

(b) an essential singularity

(c) a simple pole

(d) a second order pole

31. The value of the integral

$$\int_{-\infty}^{\infty} \frac{\cos(kx)}{x^2 + a^2} dx$$

, where $k > 0$ and $a > 0$, is **[GATE 2019]**

(a) $\frac{\pi}{a} e^{-ka}$ (b) $\frac{2\pi}{a} e^{-ka}$

(c) $\frac{\pi}{2a} e^{-ka}$ (d) $\frac{3\pi}{2a} e^{-ka}$

32. For a complex variable z and the contour $c: |z| = 1$ taken in the counter clockwise direction,

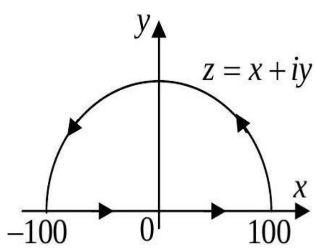
$$\frac{1}{2\pi i} \oint_c \left(z - \frac{2}{z} + \frac{3}{z^2}\right) dz =$$

[GATE 2020]

33. A contour integral is defined as

$$I_n = \oint_C \frac{dz}{(z-n)^2 + \pi^2}$$

where n is a positive integer and C is the closed contour, as shown in the figure, consisting of the line from -100 to 100 and the semicircle traversed in the counterclockwise sense.



The value of $\sum_{n=1}^{5} I_n$ (in integer) is **[GATE 2021]**

34. Complex function $f(z) = z + |z - a|^2$ (a is a real number) is **[GATE 2022]**

(a) continuous at (a, a)

(b) complex-differentiable at (a, a)

(c) complex-differentiable at $(a, 0)$

(d) analytic at $(a, 0)$

35. Consider two real functions

$U(x, y) = xy(x^2 - y^2)$, $V(x, y)$
$= ax^4 + by^4 + cx^2y^2 + k$

where k is a real constant and a, b, c are real coefficients. If $U(x, y) + iV(x, y)$ is analytic, then what is the value of $a \times b \times c$? **[GATE 2023]**

(a) $\frac{1}{8}$ (b) $\frac{3}{28}$

(c) $\frac{5}{36}$ (d) $\frac{3}{32}$

36. Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z - \pi)^4}$$

At $z = \pi$, which of the following options is (are) CORRECT? **[GATE 2023]**

(a) The order of the pole is 4

(b) The order of the pole is 3

(c) The residue at the pole is $\frac{\pi}{6}$

(d) The residue at the pole is $\frac{2\pi}{3}$

37. The complex function $e^{-\left(\frac{2}{z-1}\right)}$ has **[GATE 2024]**

(a) a simple pole at $z = 1$

(b) an essential singularity at $z = 1$

(c) a residue equal to -2 at $z = 1$

(d) a branch point at $z = 1$

38. The equation $z^2 + \bar{z}^2 = 4$ in the complex plane (where $\bar{z}$ is the complex conjugate of z) represents

(a) Ellipse (b) Hyperbola

(c) Circle of radius 2 (d) Circle of radius 4

39. The function $e^{\cos x}$ is Taylor expanded about $x = 0$. The coefficient of x^2 is

(a) $-\frac{1}{2}$ (b) $-\frac{e}{2}$

(c) $\frac{e}{2}$ (d) Zero

40. One of the roots of the equation, $z^6 - 3z^4 - 16 = 0$ is given by $z_1 = 2$. The value of the product of the other five roots is

41. The value of

$$\left|\int_0^{3+i} (\bar{z})^2 dz\right|^2$$

, along the line $3y = x$, where $z = x + iy$ is
(Round off to 1 decimal places)

42. The coefficient of x^3 in the Taylor expansion of $\sin(\sin x)$ around $x = 0$ is .
(Specify your answer upto two digits after the decimal point)

43. Consider the function $f(z) = \frac{1}{z^2(z-2)^3}$ of a complex variable z. The residues of the function at $z = 0$ and $z = 2$, respectively, are**[GATE 2025]**

(a)$-\frac{3}{8}$ and $\frac{3}{8}$ (b)$\frac{3}{8}$ and $-\frac{3}{16}$

(c)$-\frac{3}{16}$ and $\frac{3}{16}$ (d)$-\frac{3}{8}$ and $\frac{3}{16}$

44. Consider the integral $I = \frac{1}{2\pi i}\oint \frac{z^4-1}{\left(z-\frac{a}{b}\right)\left(z-\frac{b}{a}\right)} dz$

where z is a complex variable and a, b are positive real numbers. The integral is taken over a unit circle with center at the origin. Which of the following option(s) is/are correct?
[GATE 2025]

(a)$I = \frac{5}{8}$ when $a = 1, b = 2$

(b)$I = \frac{10}{3}$ when $a = 1, b = 3$

(c)$I = \frac{5}{8}$ when $a = 2, b = 1$

(d)$I = \frac{5}{8}$ when $a = 3, b = 2$

❖ JEST PYQ

1. The value of the integral

$$\int_0^\infty \frac{\ln x}{(x^2+1)^2} dx$$

is **[JEST 2012]**

(a) 0 (b) $-\pi/4$

(c) $-\pi/2$ (d) $\pi/2$

2. Compute

$$\lim_{z\to 0} = \frac{\text{Re}(z^2) + \ln(z^2)}{z^2}$$

[JEST 2013]

(a) The limit does not exist(b) 1

(c) −I (d) −I

3. The value of

$$\text{limi t}\lim_{z\to i} \frac{z^{10}+1}{z^6+1}$$

is equal to
[JEST 2014]

(a) 1 (b) 0

(c) $-\frac{10}{3}$ (d) $\frac{5}{3}$

4. The value of integral

$$I = \oint_C \frac{\sin z}{2z - \pi} dz$$

with c a is circle $|z| = 2$, is **[JEST 2014]**

(a) 0 (b) $2\pi i$

(c) πi (d) $-\pi i$

5. Given an analytic function $f(z) = \phi(x,y) + i\psi(x,y)$, where $\phi(x,y) = x^2 + 4x - y^2 + 2y$. if C is a constant, which of the following relations is true? **[JEST 2015]**

(a) $\psi(x,y) = x^2y + 4y + C$

(b) $\psi(x,y) = 2xy - 2x + C$

(c) $\psi(x,y) = 2xy + 4y - 2x + C$

(d) $\psi(x,y) = x^2y - 2x + C$

6. The value of the integral

$$\int_0^\infty \frac{\ln x}{(x^2+1)} dx$$

[JEST 2016]

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$

(c) π^2 (d) 0

7. Which one is the image of the complex domain $\{z \mid xy \geq 1, x+y>0\}$, under the mapping $f(z)=z^2$, if $z=x+iy$? **[JEST 2017]**

(a) $\{z \mid xy \geq 1, x+y>0\}$

(b) $\{z \mid x \geq 2, x+y>0\}$

(c) $\{z \mid y \geq 2\forall x\}$

(d) $\{z \mid y \geq 1\forall x\}$

8. The integral

$$I=\int_1^\infty \frac{\sqrt{x-1}}{(1+x)^2}dx$$

is **[JEST 2017]**

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$

(c) $\frac{\sqrt{\pi}}{2}$ (d) $\sqrt{\frac{\pi}{2}}$

9. The integral

$$\int_{-\infty}^{\infty} \frac{\cos\ x}{x^2+1}dx$$

is **[JEST 2018]**

(a) π/e (b) $\pi\mathrm{e}^{-2}$

(c) π (d) zero

10. Consider the function $f(x,y)=|x|-i|y|$. In which domain of the complex plane is this function analytic? **[JEST 2019]**

(a) First and second quadrants

(b) Second and third quadrants

(c) Second and fourth quadrants

(d) Nowhere

11. What is the value of the following contour integral I taken counterclockwise around the circle $|Z|=2$?

$$I=\oint_C \frac{dz}{z^3(z+4)}$$

[JEST 2020]

(a) $\frac{\pi i}{2}$ (b) $\frac{\pi i}{32}$

(c) $\frac{\pi i}{16}$ (d) $\frac{\pi i}{4}$

12. What value the following infinite series will converge to? **[JEST 2021]**

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

(a) $\frac{\pi^2}{6}$ (b) $\frac{1}{2}$

(c) 3 (d) 6

13. Consider a complex function

$$f(z)=\frac{1}{6z^3+3z^2+2z+1}.$$

What is the sum of the residues at its poles? **[JEST 2022]**

(a) $\frac{i\sqrt{3}}{7}$ (b) $\frac{4}{7}$

(c) $\frac{2}{7}$ (d) 0

14. Consider a complex number $z=x+iy$. Where do all the zeros of $\cos\ (z)$ lie? **[JEST 2022]**

(a) On the $x=0$ line.

(b) On the $y=0$ line.

(c) On the $x=y$ line.

(d) On the $x=-y$ line.

15. Calculate the contour integral

$$I=\oint_C \frac{\cos^2(z)-z^2}{(z-a)^3}dz$$

where the clockwise contour C is encircling the point $z=a$ in the complex plane.

[JEST 2023]

(a) $-(\sin\ 2a+1)2\pi i$

(c) $-(\cos\ 2a+1)2\pi i$

(b) $(\cos\ 2a+1)2\pi i$

(d) $(\sin\ 2a + 1)2\pi i$

16. Compute the contour integral:

$$I = \oint \frac{zdz}{\sinh(2\pi z)}$$

where the contour is a circle of radius $\frac{3}{4}$ centred around the origin and the direction is counterclockwise. **[JEST 2023]**

(a) 0 (b) -1

(c) π (d) 1

17. What is the value of **[JEST 2024]**

$\int_0^\infty \frac{dx}{1+x^3}$?

(a) $\frac{2\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3\sqrt{3}}$

(c) $\frac{2\pi}{3\sqrt{3}}$ (d) $\frac{\pi}{3}$

18. What is the value of

$$\int_0^\infty \frac{dx}{1+x^3}$$

? **[JEST 2024]**

(a) $\frac{2\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3\sqrt{3}}$

(c) $\frac{2\pi}{3\sqrt{3}}$ (d) $\frac{\pi}{3}$

19. What is the value of the integral

$$I = \frac{3}{2\pi}\oint_C \frac{dz}{1+z^2}$$

where the contour C is a circle of radius 2 centered at the origin? **[JEST 2025]**

❖ **TIFR PYQ**

1. Consider the integral

$$\int_{-p^2}^{+p^2} \frac{dx}{\sqrt{x^2-p^2}}$$

where p is a constant. This integral has a real, nonsingular value if **[TIFR 2012]**

(a) $p < -1$ (b) $p > 1$

(c) $p = 1$ (d) $p \to 0$

(e) $p \to \infty$

2. If $z = x + iy$ then the function

$$f(x,y) = (1+x+y)(1+x-y) + a(x^2-y^2) - 1 + 2iy(1-x-ax)$$

where a is a real parameter, is analytic in the complex z plane if $a =$ **[TIFR 2013]**

(a) -1 (b) +1

(c) 0 (d) i

3. The integral

$$\int_0^\infty \frac{dx}{4+x^4}$$

evaluates to **[TIFR 2014]**

(a)π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

4. The integral

$$\int_0^{2\pi} \frac{d\theta}{1-2a\cos\ \theta + a^2}$$

where $0 < a < 1$, evaluates to **[TIFR 2015**

(a)2π (b) $\frac{2\pi}{1+a^2}$

(c) $\frac{2\pi}{1-a^2}$ (d) $\frac{4\pi}{1-a^2}$

5. The value of the integral

$$\oint_C \frac{\sin\ z}{z^6} dz$$

where C is the circle of centre $z = 0$ and radius = 1 **[TIFR 2016]**

(a) $i\pi$ (b) $i\pi/120$

(c) $i\pi/60$ (d) $-i\pi/6$

6. The value of the integral

$$\int_0^\infty \frac{dx}{x^4+4} \text{Is}$$

[TIFR 2017]

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d)$\frac{\pi}{8}$

7. The value of the integral

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos\, x}{x^2+a^2}$$

Is

[TIFR 2018]

(a) $1/2a$ (b) $1/2\pi a$

(c) $\pi a \exp(-a)$ (d)$\exp(-a)/a$

8. Consider the complex function $f(x,y) = u(x,y) + iv(x,y)$where

$$u(x,y) = x^2(2+x) - y^2(2+3x)$$
$$v(x,y) = y(\lambda x + 3x^2 - y^2)$$

and λ is real. If it is known that $f(x,y)$ is analytic in the complex plane of $z = x+iy$, then it can be written **[TIFR 2019]**

(a) $f = z^2(2+z)$ (b) $f = 2z\bar{z} + z^2 - \bar{z}^2$

(c) $f = \bar{z}(2+\bar{z}^2)$ (d) $f = z^2 + z^3$

9. The limit

$$\lim_{x\to\infty} x\log\frac{x+1}{x-1}$$

evaluates to **[TIFR 2020]**

(a) 2 (b) 0

(c) ∞ (d) 1

10. The value of the integral is

$$\int_0^\infty \frac{dx}{x^4+4}$$

[TIFR 2020]

(a)$\frac{\pi}{8}$ (b)$\frac{3\pi}{8}$

(c)2π (d)$\frac{\pi}{4}$

11. A differentiable function $f(x)$ obeys

$$x\int_0^x \frac{f(y)}{y^2}dy = f(x)$$

If $f(1) = 1$. it follows that $f(2) =$

[TIFR 2021]

(a) 3/4 (b) 4

(c) 1 (d) 6

12. How many distinct values can the following function take at a given value of z ?

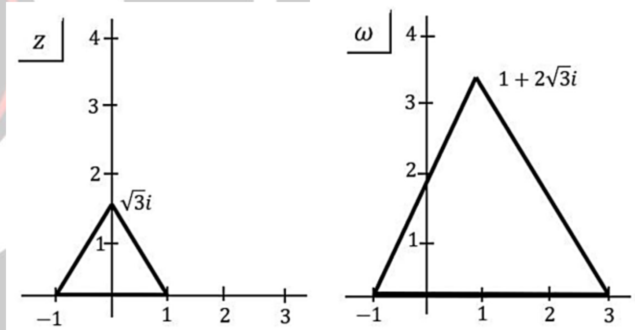
$$f(z) = \sqrt{\frac{z^2-1}{\sqrt{z}}}(z-i)^{1/3}$$

[TIFR 2021]

(a) 3 (b) 12

(c) 4 (d) 24

13. A complex analytic function $\omega = f(z)$ transforms an equilateral triangle in the complex z-plane to another equilateral triangle in the complex ω-plane as shown in the figure.



Which of the options below CANNOT be $f(z)$?

[TIFR 2023]

(a) $f(z) = 2e^{2\pi i/3}z + 2 + i\sqrt{3}$

(b) $f(z) = e^{5\pi i/6}z + 2i\sqrt{3}$

(b) $f(z) = 2ie^{5\pi i/6}z + i\sqrt{3}$

(d) $f(z) = 2z + 1$

14. Calculate the integral

$$\int_0^\infty \frac{dx}{\sqrt{x}(x^2+1)}$$

[TIFR 2023]

(a) $\frac{\pi}{\sqrt{2}}$ (a) $\pi\sqrt{2}$

(c) 2π (d) $\frac{\pi}{2}$

15. Let

$$F(\lambda) = \int_{-\infty}^{+\infty} dx e^{\lambda x - x^2}$$

If the Taylor series expansion of $F(\lambda)$ around $\lambda = 0$ is $F(\lambda) = F_0 + F_1\lambda + F_2\lambda^2 + \cdots$
then the value of F_2 is
(You might find the following integral useful:

$$\int_{-\infty}^{+\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$

for $\alpha > 0$) **[TIFR 2024]**

(a) $\sqrt{\pi}$ (b) $\sqrt{\pi}/8$

(c) $\sqrt{\pi}/2$ (d) $\sqrt{\pi}/4$

16. The integral

$$\int_{-\infty}^{+\infty} dk \frac{e^{-ikx}}{k^2+1}$$

is given by: **[TIFR 2025]**

(a) πe^{-x} (b) πe^{x}

(c) $-\pi e^{-x}$ (d) $-\pi e^{x}$

Answer Key					
CSIR-NET PYQ					
1. c	2. b	3. d	4. b	5. b	6. a
7. b	8. b	9. a	10. d	11. a	12. b
13. c	14. a	15. a	16. c	17. b	18. a
19. b	20. c	21. c	22. b	23. c	24. c
25. a	26. b	27. b	28. c	29. d	30. c
31. c	32. d	33. b	34. b	35. a	36. c
37. 3	38. b	39. a	40. d	41. b	42. b
43. a	44. a	45. d			
GATE PYQ					
1. c	2. a	3. b	4. a	5.	6. d
7. b	8. b	9. c	10. b	11. b	12. a
13. b	14. a	15. b	16. c	17. d	18. c
19. d	20. b	21. d	22. 3	23. c	24. c
25. a	26. b	27. 3.1	28. 3	29.	30. c
31. a	32. 2	33. 5	34. ac	35. d	36. b
37. bc	38.	39.	40.	41.	42.
JEST PYQ					
1. b	2. a	3. d	4. c	5. c	6. d
7. c	8. c	9. b	10. c	11. b	12. d
13. d	14. b	15. c	16. a	17. b	18. c
19. 0					
TIFR PYQ					
1. b	2. a	3. d	4. c	5. c	6. d
7. d	8. a	9. a	10. a	11. b	12. b
13. b	14. a	15. d	16. a		

GATE-Q.29:81.64

❖ **CSIR-NET PYQ's**

1. Consider a sinusoidal waveform of amplitude 1 V and frequency f_0. Starting from an arbitrary intial time, the waveform is sampled at intervals of $1/(2f_0)$. If the corresponding Fourier spectrum peaks at a frequency $\bar{f}$ and an amplitude $\bar{A}$, then

[CSIR JUNE 2012]

(a) $\bar{f} = 2f_0$ and $\bar{A} = 1V$

(b) $\bar{f} = f_0$ and $0 \le \bar{A} \le 1V$

(c) $\bar{f} = 0$ and $\bar{A} = 1V$

(d) $\bar{f} = \frac{f_0}{2}$ and $A = \frac{1}{\sqrt{2}}V$

2. Fourier transform of the derivative of the Dirac δ-function, namely $\delta'(x)$, is proportional to

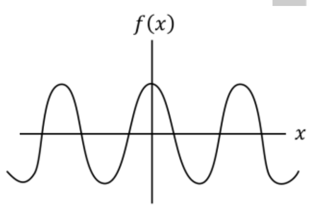
[CSIR DEC 2013]

(a) 0 (b) 1

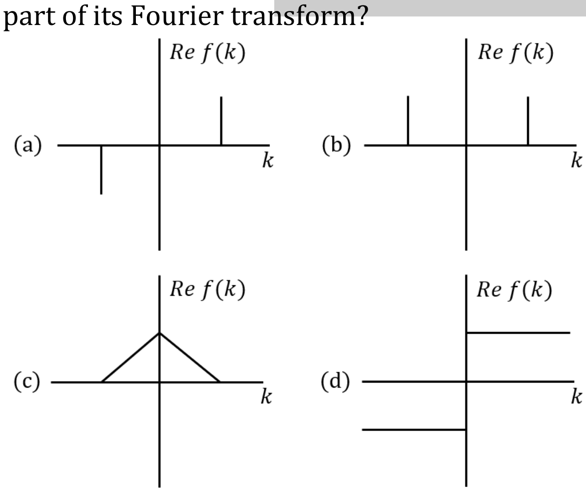
(c) sink (d) ik

3. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

[CSIR JUNE 2014]



Which of the following graphs represents the real part of its Fourier transform?



4. The Fourier transform of $f(x)$ is

$$\tilde{f}(k) = \int_{-\infty}^{+\infty} dx e^{ikx} f(x)$$

. If $f(x) = \alpha\delta(x) + \beta\delta'(x) + \gamma\delta''(x)$, where $\delta(x)$ is the Dirac delta-function (and prime denotes derivative), what is $\tilde{f}(k)$?

[CSIR DEC 2015]

(a) $\alpha + i\beta k + i\gamma k^2$ (b) $\alpha + \beta k - \gamma k^2$

(c) $\alpha - i\beta k - \gamma k^2$ (d) $i\alpha + \beta k - i\gamma k^2$

5. A function $f(x)$ satisfies the differential equation

$$\frac{d^2 f}{dx^2} - \omega^2 f = -\delta(x-a)$$

, where ω is positive. The Fourier transform

$$\tilde{f}(k) = \int_{-\infty}^{+\infty} dx e^{ikx} f(x)$$

of f, and the solution of the equation are, respectively,

[CSIR DEC 2015]

(a) $\frac{e^{ikx}}{k^2+\omega^2}$ and $\frac{1}{2\omega}\left(e^{-\text{eq}|x-a|} + e^{\omega|x-a|}\right)$

(b) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega} e^{-\omega|x-a|}$

(c) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left(e^{-j\omega|x-a|} + e^{i\omega|x-a|}\right)$

(d) $\frac{e^{ika}}{k^2-\omega^2}$, and $\frac{1}{2i\omega}\left(e^{-i\omega|\pi-a|} - e^{i\omega|x-a|}\right)$

6. What is the Fourier transform $\int dx e^{ikx} f(x)$ of

$$f(x) = \delta(x) + \sum_{n=1}^{\infty} \frac{d^n}{dx^n}\delta(x)$$

where $\delta(x)$ is the Dirac delta-function?

[CSIR JUNE 2016]

(a) $\frac{1}{1-ik}$ (b) $\frac{1}{1+ik}$

(c) $\frac{1}{k+i}$ (d) $\frac{1}{k-i}$

7. The Fourier transform $\int_{-\infty}^{\infty} dx f(x) e^{ikx}$ of the function

$$f(x) = \frac{1}{x^2+2}$$

is

[CSIR DEC 2016]

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$ (b) $\sqrt{2}\pi e^{-\sqrt{2}k}$

(c) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2k}}$ (d) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2}|k|}$

8. The Fourier transform $\int_{-\infty}^{\infty} dx f(x)e^{ikx}$ of the function $f(x) = e^{-|x|}$ is

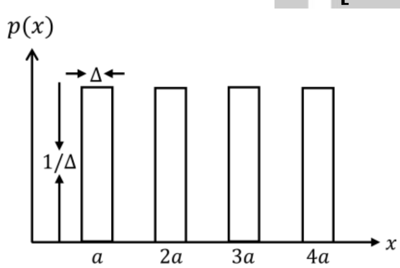
[CSIR JUNE 2018]

(a) $-\frac{2}{1+k^2}$ (b) $-\frac{1}{2(1+k^2)}$

(c) $\frac{2}{1+k^2}$ (d) $\frac{2}{(2+k^2)}$.

9. Consider an array of atoms in one dimension with an ensemble averaged periodic density distribution as shown in the figure. If k is the wave number and $S(k,\Delta)$ denotes the Fourier transform of the density density correlation function, the ratio $S(k,\Delta)/S(k,0)$ is

[CSIR JUNE 2019]



(a) $\cos\left(\frac{k\Delta}{2}\right)$ (b) $\cos^2\left(\frac{k\Delta}{2}\right)$

(c) $\frac{2}{k\Delta}\sin\left(\frac{k\Delta}{2}\right)$ (d) $\frac{4}{k^2\Delta^2}\sin^2\left(\frac{k\Delta}{2}\right)$

10. An integral transform $\tilde{f}(x)$ of a function $f(x)$ can be regarded as a result of applying an operator F to the function such that

$$(Ff)(x) \equiv \tilde{f}(x) = \int_{-\infty}^{\infty} dy e^{-ixy} f(y)$$

If I is the identity operator, then the operator F^4 is given by

[CSIR JUNE 2024]

(a) $(2\pi)^4 I$ (b) $(2\pi)I$

(c) I (d) $(2\pi)^2 I$

❖ GATE PYQ's

1. If

$$F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-kkx}dx$$

, then $\hat{F}^2[f(x)]$ is equal to

[GATE 2001]

(a) $f(x)$ (b) $-f(x)$

(c) $f(-x)$ (d) $[f(x)+f(-x)]/2$

2. Fourier transform of which of the following functions does not exist?

[GATE 2002]

(a) $e^{-|x|}$ (b) xe^{-x^2}

(c) e^{x^2} (d) e^{-x^2}

3. The Fourier transform of the function $f(x)$ is

$$F(k) = \int e^{ikx} f(x)dx$$

. The Fourier transform of $df(x)/dx$ is

[GATE 2003]

(a) $dF(k)/dk$ (b) $\int F(k)/dk$

(c) $-ikF(k)$ (d) $ikF(k)$

4. The Fourier transform $F(k)$ of a function $f(x)$ is defined as

$$F(k) = \int_{-\infty}^{\infty} dx f(x) \exp(ikx)$$

. The $F(k)$ for

$$f(x) = \exp(-x^2)$$

is [Given

: $\int_{-\infty}^{\infty}\exp(-x^2)dx = \sqrt{x}$]

[GATE 2004]

(a) $\pi\exp(-k)$ (b) $\sqrt{\pi}\exp\left(\frac{-k^2}{4}\right)$

(c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi}\exp(-k^2)$

5. If $xp(x)$ and $x^2q(x)$ have the Taylor series expansions

$$xp(x) = 4 + x + x^2 + \cdots \ldots.$$
$$x^2q(x) = 2 + 3x + 5x^2 + \cdots \ldots.$$

then the roots of the indicial equation are

[GATE 2004]

(a) $-1,0$ (b) $-1,-2$

(c) $-1,1$ (d) $-1,2$

6. The k th Fourier component of $f(x) = \delta(x)$ is **[GATE 2006]**

(a) 1 (b) 0

(c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

7. If the Fourier transform
$$F[\delta(x-a)] = \exp(-i2\pi\nu a)$$
, then $F^{-1}(\cos\ 2\pi a\nu)$ will correspond to

[GATE 2008]

(a) $\delta(x-a) - \delta(x+a)$

(b) a constant

(c) $\frac{1}{2}[\delta(x-a) + i\delta(x+a)]$

(d) $\frac{1}{2}[\delta(x-a) + \delta(x+a)]$

8. The Heaviside function is defined as
$$H(t) = \begin{cases} +1 & \text{for} \quad t > 0 \\ -1 & \text{for} \quad t < 0 \end{cases}$$
and its Fourier transform is given by $-2i/\omega$. The Fourier transform of
$$\frac{1}{2}[H(t+1/2) - H(t-1/2)]$$
is **[GATE 2015]**

(a) $\dfrac{\sin\left(\frac{\omega}{2}\right)}{\omega/2}$ (b) $\dfrac{\cos\left(\frac{\omega}{2}\right)}{\omega/2}$

(c) $\sin\left(\frac{\omega}{2}\right)$ (d) 0

9. Let
$$f_n(x) = \begin{cases} 0, & x < -\frac{1}{2n} \\ n, & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} < x \end{cases}$$
The value of
$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x) \sin\ x dx$$
is

[GATE 2020]

10. A function $f(t)$ is defined only for $t \geq 0$. The Laplace transform of $f(t)$ is
$$L(f;s) = \int_0^\infty e^{-st} f(t) dt$$
whereas the Fourier transform of $f(t)$ is
$$\tilde{f}(\omega) = \int_0^\infty f(t) e^{-i\omega t} dt$$
The correct statement(s) is(are) **[GATE 2021]**

(a) The variable s is always real.

(b)The variable s can be complex.

(c) $L(f;s)$ and $\tilde{f}(\omega)$ can never be made connected.

(d) $L(f;s)$ and $\tilde{f}(\omega)$ can be made connected

11. If $g(k)$ is the Fourier transform of $f(x)$ then which of the following are true? **[GATE 2022]**

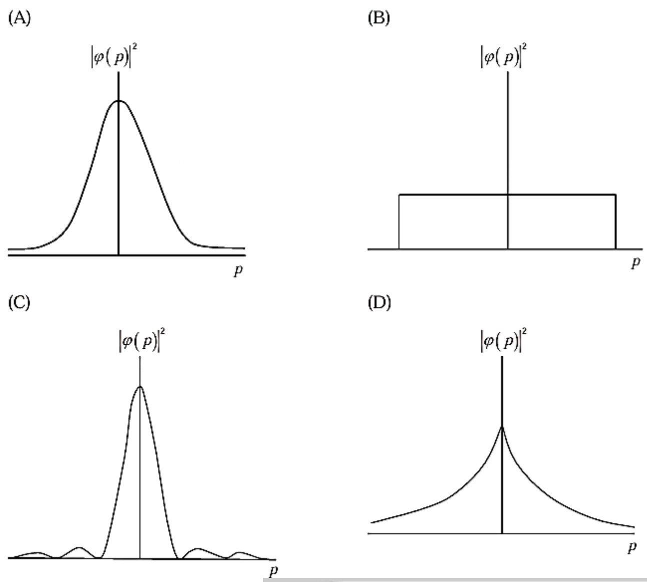
(a) $g(-k) = +g*(k)$ implies $f(x)$ is real

(b) $g(-k) = -g*(k)$ implies $f(x)$ is purely imaginary

(c) $g(-k) = +g*(k)$ implies $f(x)$ is purely imaginary

(d) $g(-k) = -g*(k)$ implies $f(x)$ is real

12. The wavefunction of a particle in one dimension is given by
$$\psi(x) = \begin{cases} M, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$
Here M and a are positive constants. If $\varphi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\varphi(p)|^2$? **[GATE 2023]**



13. The Fourier transform and its inverse transform are respectively defined as

$$\tilde{f}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{i\omega x}dx$$

and

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{f}(\omega)e^{-i\omega x}d\omega.$$

Consider two functions f and g. Another function f^*g is defined as

$$(f^*g)(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(y)g(x-y)dy$$

Which of the following relation is/are true?
Note: Tilde (~) denote the Fourier transform.

[GATE 2024]

(a) $f*g=g*f$ (b) $\widetilde{f*g}=\widetilde{g*f}$

(c) $\widetilde{f\cdot g}=\tilde{f}\tilde{g}$ (d) $\widetilde{f*g}=\tilde{f}\tilde{g}$

❖ JEST PYQ's

1. the output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I=\frac{\omega_0}{Q}I$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is:

[JEST 2016]

(a) Delta function (b) Gaussian

(c) Lorentzian (d) Exponential

2. The Fourier transform of the function

$$\frac{1}{x^4+3x^2+2}$$

up to proportionality constant is

[JEST 2017]

(a) $\sqrt{2}\exp(-k^2)-\exp(-2k^2)$

(b) $\sqrt{2}\exp(-|k|)-\exp(-2|k|)$

(c) $\sqrt{2}\exp(-\sqrt{|k|})-\exp(-\sqrt{2|k|})$

(d) $\sqrt{2}\exp\left(-\sqrt{2}k^2\right)-\exp(-2k^2)$

3. If $f(t)$ is a real and even function of t, which one of the following statements is true about its Fourier transform $F(\omega)$ (here $*$ indicates complex conjugation)?

[JEST 2020]

(a) $F^*(\omega)=-F(\omega)$ (b) $F^*(\omega)=F(\omega)$

(c) $F(-\omega)=F(\omega)$ (d) $F(-\omega)=F^*(\omega)$

4. Consider the Fourier transform of a function $f(x)$ defined as

$$g(p)=\int_{-\infty}^{\infty} f(x)\exp(ipx)dx,\ \text{where } f(x)=\frac{1}{\sqrt{|x|}}.$$

Which of the following is the correct form of $g(p)$ for some constant β ?

[JEST 2024]

(a) $g(p)=\frac{\beta}{p^2}$ (b) $g(p)=\frac{\beta}{p}$

(c) $g(p)=\frac{\beta}{\sqrt{|p|}}$ (d) $g(p)=\frac{\beta}{|p|}$

❖ TIFR PYQ's

1. The integral

$$\int_0^{\infty}\frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right)-\exp\left(-\frac{x}{\sqrt{2}}\right)\right]$$

evaluates to **[TIFR 2016]**

(a) zero (b) 2.03×10^{-2}

(c) 2.03×10^{-1} (d) 2.03

2. Evaluate the integral

$$\int_{-\infty}^{+\infty} dx \exp(-x^2) \cos(\sqrt{2}x)$$

[TIFR 2018]

3. The value of the integral

$$\int_{-\pi/2}^{+\pi/2} dx \cosh\, kx^2 \sin^2\, x$$

in the large- k limit, will be

[TIFR 2022]

(a) $\frac{1}{2k\pi} e^{k\pi^2/4}$

(b) $\cos h\left(\frac{\pi^2}{4}\right)$

(c) $\frac{1}{k^2\pi^2} \cos h\left(\frac{\pi^2}{4}\right)$

(d) $\frac{1}{k\pi} e^{k\pi^2/4}$

Answer Key				
CSIR-NET PYQ				
1. d	2. d	3. b	4. c	5. b
6. b	7. d	8. c	9. d	10. d
GATE PYQ				
1. a	2. c	3. c	4. b	5. b
6. c	7. d	8. a	9. 0	10. b
11. ab	12. c	13. abd		
JEST PYQ				
1. c	2. b	3. b	4. c	
TIFR PYQ				
1. a	2. 1.074	3. d		

❖ CSIR-NET PYQ's

1. Which of the following matrices is an element of the group $SU(2)$?

[CSIR JUNE 2011]

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$

(c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

2. The character table of C_3, the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1\Gamma_1$	1	1	b
$3\Gamma_2$	1	a	c
$2\Gamma_3$	1	1	d

In the above C_1, C_2, C_3 denotes the three classes of C_{3v}, containing 1,3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v}.

[CSIR JUNE 2011]

(A) The entries a, b, c and d in this table are, respectively

(a) 2,1, −1,0 (b) −1,2,0, −1

(c) −1,1,0, −1 (d) −1,1,1, −1

3. The reducible representation Γ of C_{3v} with character $\chi = (4,0,1)$ decomposes into its

irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as

[CSIR JUNE 2011]

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$

(c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

4. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ig} = A_i B_j$

[CSIR DEC 2013]

(a) reduces to a direct sum of three 3-dimensional representations

(b) is an irreducible 9-dimensional representation

(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5 -dimensional irreducible representations

(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

5. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication?

[CSIR DEC 2014]

(a) $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}$, where $\alpha\beta \neq 1$

(c) $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real

(d) $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

6. The rank-2 tensor $x_i x_j$, where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set) containing

[CSIR JUNE 2015]

(a) 4 and 2 elements

(b) 5 and 1 elements

(c) 3, 2 and 1 elements

(d) 4, 1 and 1 elements

7. A part of the group multiplication table for a six element group $G = \{e, a, b, c, d, f\}$ is shown below. (In the following e is the identity element of G).

[CSIR JUNE 2016]

s.	e	a	b	c	d	$\vec{f}$
e	e	a	b	c	d	f
a	a	b	e	d		
b	b	e	x	f	y	z
c	c					
d	d					
f	f					

The entries x, y and z should be

(a) $x = a, y = d$ and $z = c$

(b) $x = c, y = a$ and $z = d$

(c) $x = c, y = d$ and $z = a$

(d) $x = a, y = c$ and $z = d$

8. The 2×2 identity matrix I and the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ do not form a group under matrix multiplication. The minimum number of 2×2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is

[CSIR DEC 2016]

(a) 20 (b) 8

(c) 12 (d) 16 .

9. Which of the following sets of 3×3 matrices (in which a and b are real numbers) form a group under matrix multiplication?

[CSIR JUNE 2017]

(a) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ b & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(b) $\left\{\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(c) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(d) $\left\{\begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

10. Consider an element $U(\varphi)$ of the group SU(2), where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as $U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(\varphi)$. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be

[CSIR DEC 2017]

(a) positive definite (b) real symmetric

(c) Hermitian (d) anti-hermitian

11. The regular representation of two nonidentity elements of the group of order 3 are given by

[CSIR DEC 2023]

(a) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

12. The following four matrices form a representation of a group

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Which of the following represents the multiplication table for the same group?

1.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	A	I
C	C	B	I	A

2.

	I	A	B	C
I	I	A	B	C
A	A	B	C	I
B	B	C	I	A
C	C	I	A	B

3.

	I	A	B	C
I	I	A	B	C
A	A	C	I	B
B	B	I	C	A
C	C	B	A	I

4.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I

❖ JEST PYQ'S

1. If an abelian group is constructed with two distinct elements a and b such that $a^2 = b^2 = I$, where I is the group identity. What is the order of the smallest abelian group containing a, b and I ?

[JEST 2018]

2. $G = \{e, a, a^2, b, ba, ba^2\}$ is a group of order 6. e is the identity element and a is of order 3. What could be the order of the element b ?

[JEST 2022]

(a) 2

(b) 3

(c) 1

(d) Can't be determined

3. Let $(G,\circ)$ be a discrete group of order 4 where the group operation ' o ' among the various elements of $G = \{e, a, b, c\}$ is given by the following multiplication table:

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Which of the following is correct?

[JEST 2024]

(a) $(G,\circ)$ is cyclic and abelian.

(b) $(G,\circ)$ is non-cyclic and abelian.

(c) $(G,\circ)$ is cyclic and non-abelian.

(d) $(G,\circ)$ is non-cyclic and non-abelian.

Answer Key				
CSIR-NET PYQ				
1. b	2. b	3. c	4. c	5. d
6. b	7. d	8. d	9. c	10. d
11. c	12. 4			
JEST PYQ				
1. 0004	2. a	3. b		

❖ CSIR-NET PYQ's

1. The value of the integral

$$\int_{-\pi/2}^{+\pi/2} dx \int_{-1}^{+1} dy\, \delta(\sin\ 2x)\delta(x-y)$$

is

[CSIR JUNE 2018]

(a) 0 (b) ½

(c) $1/\sqrt{2}$ (d) 1

2. The operator $x\frac{d}{dx}\delta(x)$, where $\delta(x)$ is the Dirac delta function, acts on the space of real-valued square -integrable functions on the real line. This operator is equivalent to

[CSIR JUNE 2019]

(a) $-\delta(x)$ (b) $\delta(x)$

(c) x (d) 0

3. The value of the integral

$$\int_{-\infty}^{\infty} dx\, 2^{-\frac{|x|}{\pi}}\delta(\sin\ x)$$

where $\delta(x)$ is the Dirac delta function, is

[CSIR JUNE2023]

(a) 3 (b) 0
(c) 5 (d) 1

4. The value of the integral

$$\int_1^e dy \int_0^5 dx\, \delta(x^2-y^2)\ln(xy)$$

Is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{e}$ (d) $\frac{e}{5}$

❖ JEST PYQ's

1. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$ for a well behaved function $f(x)$. If x has the dimension of momentum then

[JEST 2014]

(a) $\delta(x)$ has the dimension of momentum

(b) $\delta(x)$ has the dimension of (momentum) ^{2}

(c) $\delta(x)$ is dimensionless

(d) $\delta(x)$ has the dimension of (momentum) $^{-1}$

2. Consider the differential equation $G'(x) + kG(x) + \delta(x)$; where k is a constant. Which following statements are true?

[JEST 2015]

(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$.

(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not

(c) $G(x)$ is discontinuous at $x = 0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at x = 0 depends on the value of k.

3. $\int_{-\infty}^{+\infty}(x^2+1)\delta(x^2-3x+2)dx = ?$

[JEST 2017]

(a) 1 (b) 2

(c) 5 (d) 7

4. $\pi\int_{-\infty}^{\infty}\exp(-|x|)\delta(\sin(\pi x))dx$, where $\delta(\ldots)$ is Dirac delta distribution, is

[JEST 2018]

(a) 1 (b) $\frac{e+1}{e-1}$

(c) $\frac{e-1}{e+1}$ (d) $\frac{e}{e+1}$

5. What is the value of the integral $\int_{-\infty}^{+\infty} dx\, \delta(x^2-\pi^2)\cos\ x$?

[JEST 2019]

(a) π (b) $-\frac{1}{2\pi}$

(c) $-\frac{1}{\pi}$ (d) 0

6. What is the value of the following integral?

$$I = \frac{100\sqrt{2}}{\pi}\int_0^{\pi/2} x\delta(2\sin\ x - \sqrt{2})dx$$

[JEST 2020]

❖ TIFR PYQ's

1. The function $f(x)$ represents the nearest integer less than x, e.g.

$$f(3.14) = 3.$$

The derivative of this function (for arbitrary x) will be given in terms of the integers n as $f'(x) =$

[TIFR 2012]

(a) 0 (b) $\sum_n \delta(x-n)$

(c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

2. The integral

$$\int_{-\infty}^{\infty} dx\delta(x^2 - \pi^2)\cos\ x$$

evaluates to

[TIFR 2013]

(a) -1 (b) 0

(c) $1/\pi$ (d) $-1/\pi$

3. In spherical polar coordinates $\vec{r} = (r, \theta, \varphi)$ the delta function $\delta(\vec{r}_1 - \vec{r}_2)$ can be written as

[TIFR 2014]

(a) $\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2)$

(b) $\frac{1}{r_1^2}\delta(r_1 - r_2)\delta(\cos\ \theta_1 - \cos\ \theta_2)\delta(\varphi_1 - \varphi_2)$

(c) $\frac{1}{|\vec{r}_1 - \vec{r}_2|^2}\delta(r_1 - r_2)\delta(\cos\ \theta_1 - \cos\ \theta_2)\delta(\varphi_1 - \varphi_2)$

(d) $\frac{1}{r_1^2\cos\ \theta_1}\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2)$

4. The integral

$$I = \int_0^{\infty} dxe^{-x}\delta(\sin\ x)$$

where $\delta(x)$ denotes the Dirac delta function, i

[TIFR 2019]

(a) 1 (b) $\frac{\exp\pi}{\exp\pi - 1}$

(c) $\frac{\exp\pi}{\exp\pi + 1}$ (d) $\frac{1}{\exp\ \pi - 1}$

Answer Key				
CSIR-NET PYQ				
1. b	2. a	3. a	4. a	
JEST PYQ				
1. d	2. a	3. d	4. a	5. c
6. 0025				
TIFR PYQ				
1. b	2. d	3. b	4. c	

❖ **CSIR-NET PYQ's**

1. Let $p_n(x)$ (where $n = 0,1,2 \dots\dots$) be a polynomial of degree n with real coefficients, defined in the interval $2 \le n \le 4$. If $\int_2^4 p_n(x)p_m(x)dx = \delta_{nm}$, then

[CSIR JUNE 2011]

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$

(b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

(d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

2. The generating function

$$F(x,t) \equiv \sum_{n=0}^{\infty} P_n(x)t^n$$

for the Legendre polynomials $P_n(x)$ is $F(x,t) = (1-2xt+t^2)^{-1/2}$ The value of $P_3(-1)$ is:

[CSIR DEC 2011]

(a) 5/2 (b) 3/2

(c) +1 (d) -1

3. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$$

, and let

$$w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$$

If $w(0) = 1$, then $w(1)$ is given by

[CSIR DEC 2011]

(a) 1 (b) e^2

(c) $1/e$ (d) $1/e^2$

4. Let $y(x)$ be a continuous real function in the range 0 and 2π, satisfying the inhomogeneous differential equaltion:

$$\sin x\frac{d^2y}{dx^2} + \cos x\frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$$

. The value of dy/dx at the point $x = \pi/2$

[CSIR JUNE 2012]

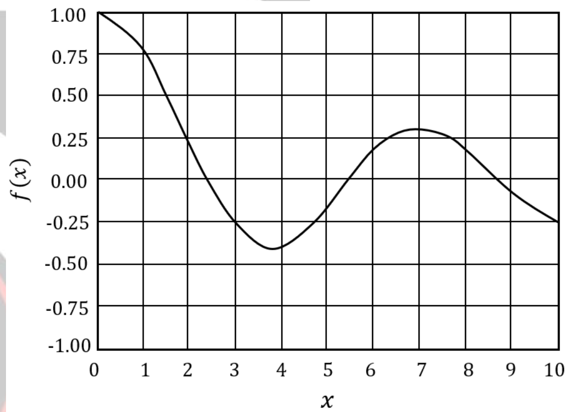
(a) is continuous

(c) has a discontinuity of 1/3

(b) has a discontinuity of 3

(d) has a discontinuity of 1

5. The graph of the function $f(x)$ as shown below is best described by **[CSIR DEC 2012]**



(a) The Bessel function $J_0(x)$

(b) $\cos x$

(b) $e^{-x}\cos x$

(d) $\frac{1}{x}\cos x$

6. A function $f(x)$ obeys the differential equation

$$\frac{d^2f}{dx^2} - (3-2i)f = 0$$

and satisfies the conditions $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is:

[CSIR DEC 2012]

(a) $e^{2\pi}$ (b) $e^{-2\pi}$

(c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$

7. Given that

$$\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$$

the value of $H_4(0)$ is

[CSIR JUNE 2013]

(a) 12 (b) 6

(c) 24 (d) ÷ 6

8. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 0$$

satisfying the boundary conditions $u(0,t) = 0 = u(L,t)$ and initial conditions $u(x,0) = \sin(\pi x/\text{L})$ and

$$\frac{\partial}{\partial t}u(x,t)_{t=0} = \sin(2\pi x/L)$$

is

[CSIR JUNE 2013]

(a) $\sin(\pi x/L)\cos(\pi t/L) + \frac{L}{2\pi}\sin(2\pi x/L)\cos(2\pi t/L)$

(b) $2\sin(\pi x/L)\cos(\pi t/L) - \sin(\pi x/L)\cos(2\pi t/L)$

(c) $\sin(\pi x/L)\cos(2\pi t/L) + \frac{L}{\pi}\sin(2\pi x/L)\sin(\pi t/L)$

(d) $\sin(\pi x/L)\cos(\pi t/L) + \frac{L}{2\pi}\sin(2\pi x/L)\sin(2\pi t/L)$

9. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tends to

[CSIR JUNE 2013]

(a) 1 (b) 2

(c) 1/2 (d) ∞

10. Given,

$$\sum_{n=0}^{\infty} P_n(x)t^n = (1 - 2xt + t^2)^{-1/2}$$

, for $|t| < 1$, the value of $P_5(-1)$ is

[CSIR JUNE 2014]

(a) 0.26 (b) 1

(c) 0.5 (d) -1

11. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with the initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when ' t ' is

[CSIR JUNE 2014]

(a) 1/2 (b) 1

(c) 2 (d) ∞

12. The function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$$

satisfies the differential equation

[CSIR DEC 2014]

(a) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$

(b) $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$

(c) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$

(d) $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

13. Consider the differential equation

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$$

. If $x = 0$ at $t = 0$ and $x = 1$ at $t = 1$, the value of x at $t = 2$ is

[CSIR JUNE 2015]

(a) $e^2 + 1$ (b) $e^2 + e$

(c) $e + 2$ (d) $2e$

14. Let $f(x,t)$ be a solution of the wave equation

$$\frac{\partial^2 f}{\partial t^2} = v^2\frac{\partial^2 f}{\partial x^2}$$

in 1-dimension. If at $t = 0, f(x,0) = e^{-x^2}$ and

$$\frac{\partial f}{\partial t}(x,0) = 0$$

for all x, then $f(x,t)$ for all future times $t > 0$ is described by

[CSIR JUNE 2015]

(a) $e^{-(x^2 - v^2t^2)}$

(b) $e^{-(x-v)^2}$

(c) $\frac{1}{4}e^{-(x-v)^2}+\frac{3}{4}e^{-(x+w)^2}$

(d) $\frac{1}{2}\left[e^{-(x-w)^2}+e^{-(x+v)^2}\right]$

15. The solution of the differential equation

$$\frac{dx}{dt}=2\sqrt{1-x^2}$$

, with initial condition $x=0$ at $t=0$ is

[CSIR DEC 2015]

(a) $x=\begin{cases}\sin\ 2t, & 0\le t<\frac{\pi}{4}\\ \sinh\ 2t, & t\ge\frac{\pi}{4}\end{cases}$

(b) $x=\begin{cases}\sin\ 2t, & 0\le t<\frac{\pi}{2}\\ 1, & t\ge\frac{\pi}{2}\end{cases}$

(c) $x=\begin{cases}\sin\ 2t, & 0\le t<\frac{\pi}{4}\\ 1, & t\ge\frac{\pi}{4}\end{cases}$

(d) $x=1-\cos\ 2t, t\ge 0$

16. The Hermite polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2}-2x\frac{dH_n}{dx}+2nH_n(x)=0$$

. The corresponding generating function

$$G(t,x)=\sum_{n=0}^{\infty}\frac{1}{n!}H_n(x)t^n$$

satisfies the equation

[CSIR DEC 2015]

(a) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2t\frac{\partial G}{\partial t}=0$

(b) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}-2t^2\frac{\partial G}{\partial t}=0$

(c) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2\frac{\partial G}{\partial t}=0$

(d) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2\frac{\partial^2G}{\partial x\,\partial t}=0$

17. The integral equation $\phi(x,t)=\lambda\int dx'dt'$

$$\int\frac{d\omega dk}{(2\pi)^2}\frac{e^{-ik(x-x')+i\omega(t-t')}}{\omega^2-k^2-m^2+i\varepsilon}\phi^3(x',t')$$

is equivalent to the differential equation

[CSIR JUNE 2016]

(a) $\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}-m^2+i\varepsilon\right)\phi(x,t)=-\frac{1}{6}\lambda\phi^3(x,t)$

(b) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=\lambda\phi^2(x,t)$

(c) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\varepsilon\right)\phi(x,t)=-3\lambda\phi^2(x,t)$

(d) $\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+m^2-i\delta\right)\phi(x,t)=-\lambda\phi^3(x,t)$

18. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values $m=10$ kg, $\gamma=10$ kg/s, and $g\approx 10$ m/s^2, the distance travelled (in meters) in time t in seconds, is

[CSIR DEC 2016]

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$

(c) $5t^2-(1-e^t)$ (d) $5t^2$

19. The function $y(x)$ satisfies the differential equation

$$x\frac{dy}{dx}+2y=\frac{\cos\ \pi x}{x}$$

. If $y(1)=1$, the value of $y(2)$ is

[CSIR JUNE 2017]

(a) π (b) 1

(c) ½ (d) 1/4

20. The Green's function satisfying

$$\frac{d^2}{dx^2}g(x,x_0)=\delta(x-x_0)$$

, with the boundary conditions $g(-L,x_0)=0=g(L,x_0)$, is

[CSIR JUNE 2017]

(a) $\begin{cases} \frac{1}{2L}(x_0 - L)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 + L)(x - L), & x_0 \le x \le L \end{cases}$

(b) $\begin{cases} \frac{1}{2L}(x_0 + L)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 - L)(x - L), & x_0 \le x \le L \end{cases}$

(c) $\begin{cases} \frac{1}{2L}(L - x_0)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 + L)(L - x), & x_0 \le x \le L \end{cases}$

(d) $\frac{1}{2L}(x - L)(x + L), -L \le x \le L$

21. The number of linearly independent power series solutions, around $x = 0$, of the second order linear differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

is

[CSIR DEC 2017]

(a) 0 (this equation does not have a power series solution)
(b) 1
(c) 2
(d) 3

22. The generating function $G(t,x)$ for the Legendre polynomials $P_n(t)$ is

$$G(t,x) = \frac{1}{\sqrt{1 - 2xt + x^2}} = \sum_{n=0}^{\infty} x^n P_n(t), \text{ for } |x| < 1.$$

If the function $f(x)$ is defined by the integral equation

$$\int_0^x f(x')dx' = xG(1,x),$$

it can be expressed as

[CSIR DEC 2017]

(a) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m\left(\frac{1}{2}\right)$

(b) $\sum_{n,m=0}^{\infty} x^{n+n} P_n(1) P_{m+1}(1)$

(c) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(1) P_m(1)$

(d) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(0) P_m(1)$

23. Consider the following ordinary differential equation:

$$\frac{d^2x}{dt^2} + \frac{1}{x}\left(\frac{dx}{dt}\right)^2 - \frac{dx}{dt} = 0$$

with the boundary conditions $x(t = 0) = 0$ and $x(t = 1) = 1$. The value of $x(t)$ at t & is

[CSIR JUNE 2018]

(a) $\sqrt{e - 1}$
(b) $\sqrt{e^2 + 1}$
(c) $\sqrt{e + 1}$
(d) $\sqrt{e^2 - 1}$

24. In the function $P_n(x)e^{-x^2}$ of a real variable $x, P_n(x)$ is a polynomial of degree n. The maximum number of extrema that this function can have is

[CSIR JUNE 2018]

(a) $n + 2$
(b) $n - 1$
(c) $n + 1$
(d) n

25. The Green's function $G(x, x^2)$ for the equation

$$\frac{d^2y(x)}{dx^2} + y(x) = f(x)$$

, with the boundary values $y(0) = y\left(\frac{\pi}{2}\right) = 0$, is

[CSIR JUNE 2018]

(a) $G(x,x') = \begin{cases} x\left(x' - \frac{\pi}{2}\right), & 0 < x < x' < \frac{\pi}{2} \\ \left(x - \frac{\pi}{2}\right), & 0 < x' < x < \frac{\pi}{2} \end{cases}$

(b) $G(x,x') = \begin{cases} -\cos x' \sin x, & 0 < x < x' < \frac{\pi}{2} \\ -\sin x' \cos x, & 0 < x' < x < \frac{\pi}{2} \end{cases}$

(c) $G(x,x') = \begin{cases} \cos x' \sin x, & 0 < x < x' < \frac{\pi}{2} \\ \sin x' \cos x, & 0 < x' < x < \frac{\pi}{2} \end{cases}$

(d) $G(x,x') = \begin{cases} x\left(\frac{\pi}{2} - x'\right), & 0 < x < x' < \frac{\pi}{2} \\ x'\left(\frac{\pi}{2} - x\right), & 0 < x' < x < \frac{\pi}{2} \end{cases}$

26. The polynomial $f(x) = 1 + 5x + 3x^2$ is written as a linear combination of the Legendre polynomials

$$\left(P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1)\right)$$

as $\sum_n c_n P_n(x)$. The value of c_0 is

[CSIR DEC 2018]

(a) 1/4 (b) ½

(c) 2 . (d) 4

27. In terms of arbitrary constant A and B, the general solution to the differential equation

$$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 3y = 0, \text{ is}$$

[CSIR DEC 2018]

(a) $y = \frac{A}{x} + Bx^3$ (b) $y = Ax + \frac{B}{x^3}$

(c) $y = Ax + Bx^3$ (d) $y = \frac{A}{x} + \frac{B}{x^3}$

28. The Green's function $G(x, x')$ for the equation

$$\frac{d^2y(x)}{dx^2} = f(x)$$

, with the boundary values $y(0) = 0$ and $y(1) = 0$, is

[CSIR DEC 2018]

(a) $G(x,x') = \begin{cases} \frac{1}{2}x(1-x'), & 0 < x < x' < 1 \\ \frac{1}{2}x'(1-x), & 0 < x' < x < 1 \end{cases}$

(b) $G(x,x') = \begin{cases} x(x'-1), & 0 < x < x' < 1 \\ x'(1-x), & 0 < x' < x < 1 \end{cases}$

(c) $G(x,x') = \begin{cases} -\frac{1}{2}x(1-x), & 0 < x < x' < 1 \\ \frac{1}{2}x'(1-x), & 0 < x' < x < 1 \end{cases}$

(d) $G(x,x') = \begin{cases} x(x'-1), & 0 < x < x' < 1 \\ x'(x-1), & 0 < x' < x < 1 \end{cases}$

29. A particle of mass m, moving along the x-direction, experiences a damping force $-\gamma v^2$, where γ is a constant and v is its instantaneous speed. If the speed at $t = 0$ is v_0, the speed at time t is

[CSIR DEC 2018}

(a) $v_0 e^{-\frac{\gamma v_0 t}{m}}$ (b) $\frac{v_0}{1 + \ln\left(1 + \frac{\gamma v_0 t}{m}\right)}$

(c) $\frac{m v_0}{m + \gamma v_0 t}$ (d) $\frac{2v_0}{1 + e^{\frac{\gamma v_0 l}{m}}}$

30. The solution of the differential equation

$$x\frac{dy}{dx} + (1 + x)y = e^{-x}$$

with the boundary condition $y(x = 1) = 0$, is

[CSIR JUNE 2019]

(a) $\frac{(x-1)}{x}e^{-x}$ (b) $\frac{(x-1)}{x^2}e^{-x}$

(c) $\frac{(1-x)}{x^2}e^{-x}$ (d) $(x-1)^2 e^{-x}$

31. The Green's function for the differential equation

$$\frac{d^2x}{dt^2} + x = f(t)$$

, satisfying the initial conditions $x(0) = \frac{dx}{dt}(0) = 0$ is

[CSIR JUNE 2020]

$$G(t,\tau) = \begin{cases} 0 & \text{for } \quad 0 < t < \tau \\ \sin(t - \tau) & \text{for } \quad t > \tau \end{cases}$$

The solution of the differential equation when the source $f(t) = \theta(t)$ (the Heaviside step function) is

(a) $\sin\ t$ (b) $1 - \sin\ t$

(c) $1 - \cos\ t$ (d) $\cos^2\ t - 1$

32. The solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} = e^y$$

, with the boundary conditions $y(0) = 0$ and $y'(0) = -1$, is

[CSIR NOV 2020]

(a) $-\ln\left(\frac{x^2}{2} + x + 1\right)$ (b) $-x\ln(e + x)$

(c) $-xe^{-x^2}$ (d) $-x(x+1)e^{-x}$

33. The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a dissipative force is described by $\frac{d^2x}{dt^2}+10\frac{dx}{dt}+16x=6te^{-8t}+4t^2e^{-2t}$. The general form of the particular solution, in terms of constants A, B etc, is

[CSIR JUNE 2021]

(a) $t(At^2+Bt+C)e^{-2t}+(Dt+E)e^{-8t}$

(b) $(At^2+Bt+C)e^{-2t}+(Dt+E)e^{-8t}$

(c) $t(At^2+Bt+C)e^{-2t}+t(Dt+E)e^{-8t}$

(d) $(At^2+Bt+C)e^{-2t}+t(Dt+E)e^{-8t}$

34. The Legendre polynomials $P_n(x), n=0,1,2,\ldots,$ satisfying the orthogonality condition

$$\int_{-1}^{1}P_n(x)P_m(x)dx=\frac{2}{2n+1}\delta_{nm}$$

on the interval $[-1,+1]$ may be defined by the Rodrigues formula

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

. The value of the definite integral $\int_{-1}^{1}(4+2x-3x^2+4x^3)P_3(x)dx$ is

[CSIR JUNE 2021]

(a) $\frac{3}{5}$ (b) $\frac{11}{15}$

(c) $\frac{23}{32}$ (d) $\frac{16}{35}$

35. The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a dissipative force is described by

$$\frac{d^2x}{dt^2}+10\frac{dx}{dt}+16x=6te^{-8t}+4t^2e^{-2t}$$

. The general form of the particular solution, in terms of constants A, B etc, is

[CSIR FEB 2022]

(a) $t(At^2+Bt+C)e^{-2t}+(Dt+E)e^{-8t}$

(b) $(At^2+Bt+C)e^{-2t}+(Dt+E)e^{-8t}$

(c) $t(At^2+Bt+C)e^{-2t}+t(Dt+E)e^{-8t}$

(d) $(At^2+Bt+C)e^{-2t}+t(Dt+E)e^{-8t}$

36. If we use the Fourier transform $\phi(x,y)=\int e^{ikx}\phi_k(y)dk$ to solve the partial differential equation

$$-\frac{\partial^2\phi(x,y)}{\partial y^2}-\frac{1}{y^2}\frac{\partial^2\phi(x,y)}{\partial x^2}+\frac{m^2}{y^2}\phi(x,y)=0$$

in the half-plane $\{(x,y): -\infty<x<\infty, 0<y<\infty\}$ the Fourier modes $\phi_k(y)$ depend on y as y^α and y^β. The values of α and β are

[CSIR FEB 2022]

(a) $\frac{1}{2}+\sqrt{1+4(k^2+m^2)}$ and $\frac{1}{2}-\sqrt{1+4(k^2+m^2)}$

(b) $1+\sqrt{1+4(k^2+m^2)}$ and $1-\sqrt{1+4(k^2+m^2)}$

(c) $\frac{1}{2}+\frac{1}{2}\sqrt{1+4(k^2+m^2)}$ and $\frac{1}{2}-\frac{1}{2}\sqrt{1+4(k^2+m^2)}$

(d) $1+\frac{1}{2}\sqrt{1+4(k^2+m^2)}$ and $1-\frac{1}{2}\sqrt{1+4(k^2+m^2)}$

37. The Legendre polynomials $P_n(x), n=0,1,2,\ldots,$ satisfying the orthogonality condition

$$\int_{-1}^{1}P_n(x)P_m(x)dx=\frac{2}{2n+1}\delta_{nm}$$

on the interval $[-1,+1]$ may be defined by the Rodrigues formula

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

. The value of the definite integral

$$\int_{-1}^{1}(4+2x-3x^2+4x^3)P_3(x)dx$$

is

[CSIR FEB 2022]

(a) $\frac{3}{5}$ (b) $\frac{11}{15}$

(c) $\frac{23}{32}$ (d) $\frac{16}{35}$

38. If the Bessel function of integer order n is defined as

$$J_n(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,(n+k)!}\left(\frac{x}{2}\right)^{2k+n}$$

then

$$\frac{d}{dx}[x^{-n}J_n(x)]$$

is

[CSIR JUNE 2023]

(a) $-x^{n+1}J_{n+1}(x)$ (b) $-x^{n+1}J_{n-1}(x)$

(c) $-x^nJ_{n-1}(x)$ (d) $-x^nJ_{n+1}(x)$

39. The solution $y(x)$ of the differential equation

$$y'' + \frac{y}{4} = \frac{x}{2}$$

, where $0 \le x \le \pi$, together with the boundary conditions $y(0) = y(\pi) = 0$ is

[CSIR DEC 2023]

(a) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi \sin nx}{n\frac{1}{4} - n^2}$

(b) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}n^2}$

(c) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi \sin nx}{n\frac{1}{4} - n^2}$

(d) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4} - n^2}$

40. The general solution for the second order differential equation

$$\frac{d^2y}{dx^2} - y = x\sin x$$

will be **[CSIR JUNE2024]**

(a) $C_1e^x + C_2e^{-x} - \frac{1}{2}(x\sin x + \cos x)$

(b) $C_1e^x + C_2e^{-x} - \frac{1}{2}(\sin x - x\cos x)$

(c) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x - \cos x)$

(d) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x + \cos x)$

(where C_1 and C_2 are arbitrary constants)

41. The solutions of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

[CSIR JUNE 2025]

(a)ellipses with different eccentricities

(b)circles with different centers

(c)circles with different radii

(d)ellipses with different foci

42. Let $P_n(x)$ be a polynomial of degree n with real coefficients, where $n = 0,1,2,3,\cdots$. If $\int_2^4 P_n(x)P_m(x)dx = \delta_{mn}$, then

[CSIR JUNE 2025]

(a)$P_1(x) = \pm\sqrt{\frac{3}{2}}(3 - x)$

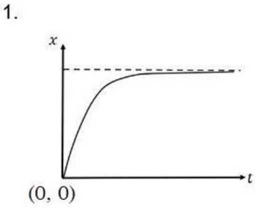
(b)$P_1(x) = \pm\sqrt{\frac{3}{2}}(2 - x)$

(c)$P_1(x) = \pm\sqrt{\frac{3}{2}}(1 - x)$

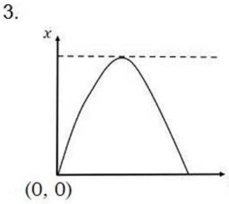
(d) $P_1(x) = \pm\sqrt{3}(3 + x)$

43. Which one of the following curves best represents the solution of the differential equation $\frac{dx}{dt} + x = 1$, with the initial condition $x(0) = 0$? **[CSIR JUNE 2025]**

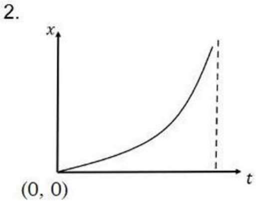
1.



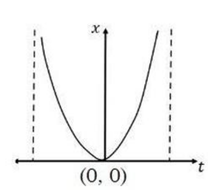
3.



2.



4.



44. A sequence of polynomial $Q_n(x)[n = 0,1,2 ....]$satisfies there cursion relation

$$Q_{n+1}(x) - 2xQ_n(x) + 2nQ_{n-1}(x) = 0, \text{ for all } n \geq 0[\text{ here } Q_{-1}(x) = 0].$$

The generating function for the polynomials, $g(x,t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} Q_n(x)$, satisfies

[CSIR DEC 2025]

(a) $\frac{\partial g}{\partial t} = 2(t+x)g$ (b) $\frac{\partial g}{\partial t} = 2(x-t)g$

(c) $\frac{\partial g}{\partial t} = \frac{2(x-t)}{t}g$ (d) $\frac{\partial g}{\partial t} = 2 + (x+t)g$

45. Find the curve that extremizes the functional

$$I(y) = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$$

for the given boundary conditions $y(0) = 0$ and $y(1) = 1$

[CSIR DEC 2025]

(a)$y = x^3$ (b)$y = x^2$

(c)$y = 2x^2 - x$ (d)$y = 3x^3 - 2x^2$

❖ GATE PYQ's

1. The solution of the system of differential equations

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

[GATE 2001]

(a) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} + 2Be^{-x}$

(b) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} + 2Be^{-x}$

(c) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} - 2Be^{-x}$

(d) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} - 2Be^{-x}$

2. If $u(x,y,z,t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave function

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

then β is **[GATE 2001]**

(a) $\left(1 - \frac{v}{c}\right)^{1/2}$ (b) $\left(1 - \frac{v}{c}\right)$

(c) $\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1 - \frac{v^2}{c^2}\right)$

3. Find the general solution of

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

, using the Frobenius power series method.

[GATE 2001]

4. The solution of the differential equation

$$(1+x)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx} - y(x) = 0 \text{ is}$$

[GATE 2002]

(a) $Ax^2 + B$ (b) $Ax + Be^{-x}$

(c) $Ax + Be^x$ (d) $Ax + Bx^2$

where A and B are constants

5. Given the differential equation

$$\frac{d^2y(x)}{dx^2} + 2\frac{dy(x)}{dx} + 5y(x) = 0$$

find its solution that satisfies the initial conditions $y = 0$ and $x = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$

[GATE 2002]

6. Consider the differential equation $d^2x/dt^2 + 2dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by **[GATE 2003]**

(a) $1/e$ (b) $2/e$

(c) 1 (d) $3/e$

7. If $p(x) = 0$ with the Wronskian at $x = 0$ as $W(x = 0) = 1$ and one of the solutions is x, then the other linearly independent solution which vanishes at $x = 1/2$ is

[GATE 2004]

(a) 1 (b) $1 - 4x^2$

(c) x (d) $-1 + 2x$

Statement for Linked Answer Q. 8 and Q.9:
For the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

[GATE 2005]

8. One of the solutions is:

(a) e^x (b) $\ln\ x$

(c) e^{-x^2} (d) e^{x^2}

9. The second linearly independent solution is:

(a) e^{-x} (b) xe^x

(c) x^2e^x (d) x^2e^{-x}

10. The points, where the series solution of the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0$$

will diverge, are located at

[GATE 2007]

(a) 0 and 1 (b) 0 and -1

(c) -1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

11. Solution of the differential equation

$$x\frac{dy}{dx} + y = x^4$$

, with the boundary condition that $y = 1$, at $x = 1$, is **[GATE 2007]**

(a) $y = 5x^4 - 4$ (b) $y = \frac{x^4}{5} + \frac{4x}{5}$

(c) $y = \frac{4x^4}{5} + \frac{1}{5x}$ (d) $y = \frac{x^4}{5} + \frac{4}{5x}$

12. Consider the Bessel equation

$$\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + y = 0, \qquad (v = 0),$$

. Which one of the following statements is correct?

[GATE 2008]

(a) equation has regular singular points at $z = 0$ and $z = \infty$

(b) equation has 2 linearly independent solutions that are entire

(c) equation has an entire solution and a second linearly independent solution singular at $z = 0$

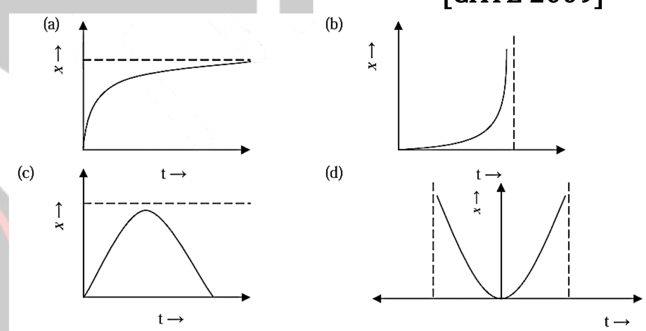
(d) limit $z \to \infty$, taken along x axis, exists for both the linearly independent solutions

13. Which one of the following curves gives the solution of the differential equation

$$k_1\frac{dx}{dt} + k_2x = k_3$$

, where k_1, k_2 and k_1, k_2 and k_3 are positive constants with initial conditions $x = 0$ at $t = 0$

[GATE 2009]



14. The solution of the differential equation for $y(t)$: $\frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\left.\frac{dy}{dr}\right|_{l=0} = 0$ is

[GATE 2010]

(a) $\frac{1}{2}\cosh(t) + t\sinh(t)$

(b) $-\sinh(t) + t\cosh(t)$

(c) $t\cosh(t)$

(d) $t\sinh(t)$

15. Given the recurrence relation for the Legendre polynomials $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ which of the following integrals has a non-zero value

[GATE 2010]

(a) $\int_{-1}^{-1} x^2 p_n(x)P_{n+1}(x)dx$

(b) $\int_{-1}^{+1} xp_n(x)P_{n+2}(x)dx$

(c) $\int_{-1}^{+1} x[p_n(x)]^2dx$

(d) $\int_{-1}^{+1} x^2P_n(x)P_{n+2}(x)dx$

16. The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of **[GATE 2011]**

(a) circles with different radii

(b) circles with different Centre

(c) straight lines with different slopes

(d) straight lines with different intercepts on the y-axis

17. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

Subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$

[GATE 2014]

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$

(c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

18. A function $y(z)$ satisfies the ordinary differential equation

$$y'' + \frac{1}{2}y' - \frac{m^2}{z^2}y = 0$$

, where $m = 0,1,2,3 \ldots \ldots$ consider the four statements P, Q, R, S as given below,

P: $\mathrm{Z^m}$ and $\mathrm{Z^{-m}}$ are linearly independent solutions for all values of m

Q: $\mathrm{7^m}$ and $\mathrm{7^{-m}}$ are linearly independent solutions for all values of $m > 0$

R: $\ln z$ and 1 are linearly independent solutions for $m = 0$

S: $\mathrm{z^m}$ and $\ln z$ are linearly independent solutions for all values of m

The correct option for the combination of valid statement is

[GATE 2015]

(a) P, R and S only (b) P and R only

(c) Q and R only (d) R and S only

19. Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x = 2$ is given by

[GATE 2016]

(a) e^{-2} (b) $2e^{-2}$

(c) e^2 (d) $2e^2$

20. Consider the differential equation $dy/dx + y\tan(x) = \cos(x)$. If $y(0) = 0, y(\pi/3)$ is______ . (up to two decimal places). **[GATE 2017]**

21. Given

$$\frac{d^2f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0,$$

and boundary conditions $f(0) = 1$ and $f(1) = 0$, the value of $f(0.5)$ is (up to two decimal places)

[GATE 2018]

22. For the differential equation

$$\frac{d^2y}{dx^2} - n(n+1)\frac{y}{x^2} = 0$$

, where n is a constant, the product of its two independent solutions is

[GATE 2019]

(a) $\frac{1}{x}$ (b) x

(c) x^n (d) $\frac{1}{x^{n+1}}$

23. Which one of the following is a solution of

$$\frac{d^2u(x)}{dx^2} = k^2u(x)$$

, for k real?

[GATE 2020]

(a) e^{-kx} (b) $\sin kx$

(c) $\cos kx$ (d) $\sinh x$

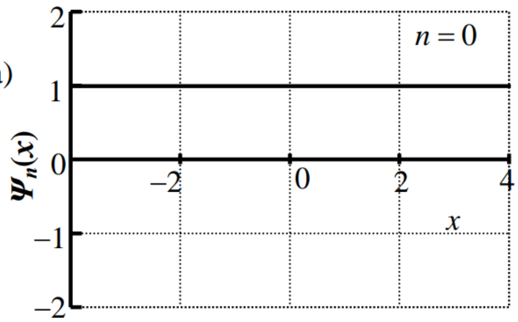
24. If $y_n(x)$ is a solution of the differential equation

$$y'' - 2xy' + 2ny = 0$$

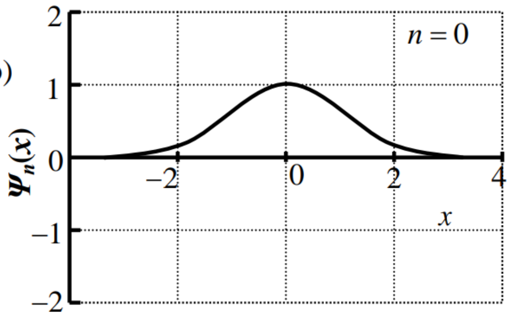
where n is an integer and the prime (') denotes differentiation with respect to x, then acceptable plot(s) of $\psi_n(x) = e^{-x^2/2}y_n(x)$, is(are)

[GATE 2021]

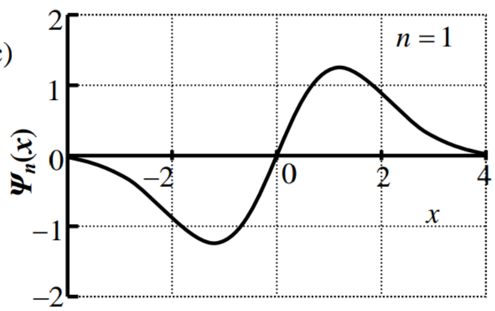
(a)



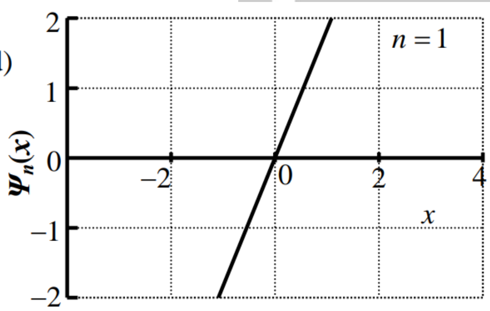
(b)



(c)



(d)



25. Consider the ordinary differential equation

$$y'' - 2xy' + 4y = 0$$

and its solution $y(x) = a + bx + cx^2$. Then

[GATE 2022]

(a) $a = 0, c = -2b \neq 0$

(b) $c = -2a \neq 0, b = 0$

(c) $b = -2a \neq 0, c = 0$

(d) $c = 2a \neq 0, b = 0$

26. The ordinary differential equation

$$(1 - x^2)y'' - xy' + 9y = 0$$

has a regular singularity at

[GATE 2022]

(a) -1 (b) 0

(c) +1 (d) no finite value of x

27. The equation of motion for the forced simple harmonic oscillator is

$$\ddot{x}(t) + \omega^2 x(t) = F\cos(\omega t)$$

where $x(t = 0) = 0$ and $\dot{x}(t = 0) = 0$. Which one of the following options is correct?

[GATE 2024]

(a) $x(t) \propto t\sin(\omega t)$ (b) $x(t) \propto t\cos(\omega t)$

(c) $x(t) = \infty$ (d) $x(t) \propto e^{\omega t}$

❖ JEST PYQ's

1. Consider the differential equation

$$\frac{dG(x)}{dx} + kG(x) = \delta(x)$$

where k is a constant. Which of the following statement is true? **[JEST 2013, 2015]**

(a) Both G(x) and G$'(x)$ are continuous at $x = 0$

(b) G(x) is continuous at $x = 0$ but G$'(x)$ is not.

(c) G(x) is discontinuous at $x = 0$

(d) The continuity properties of G(x) and G$'(x)$ at $x = 0$ depend on the value of k.

2. What are the solutions to $f''(x) - 2f'(x) + f(x) = 0$?

[JEST 2014]

(a) c_1e^x/x (b) $c_1x + c_2/x$

(c) $c_1xe^x + c_2$ (d) $c_1e^x + c_2xe^x$

3. Consider the differential equation $G'(x) + kG(x) + \delta(x)$; where k is a constant. Which following statements are true? **[JEST 2015]**

(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$.

(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not

(c) $G(x)$ is discontinuous at $x = 0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depends on the value of k.

4. What is the maximum number of extrema of the function

$$f(x) = P_k(x)e^{\left(\frac{x^2}{4}+\frac{x^2}{2}\right)}$$

where $x \in (-\infty, \infty)$ and $P_k(x)$ is an arbitrary polynomial of degree k ?

[JEST 2015]

(a) $k+2$ (b) $k+6$

(c) $k+3$ (d) k

5. The Bernoulli polynomials $B_n(s)$ are defined by,

$$\frac{xe^{xs}}{e^x - 1} = \sum B_n(s)\frac{x^n}{n!}$$

. which one of the following relations is true?

[JEST 2015]

(a) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)\frac{x^n}{(n+1)!}$

(b) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)(-1)^n\frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(-s)(-1)^n\frac{x^n}{n!}$

(d) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)(-1)^n\frac{x^n}{n!}$

6. Given the condition $\nabla^2\phi = 0$, the solution of the equation $\nabla^2\psi = k\vec{\nabla}\phi \cdot \vec{\nabla}\phi$ is given by

[JEST 2016]

(a) $\psi = \frac{k\phi^2}{2}$ (b) $\psi = k\phi^2$

(c) $\psi = \frac{k\phi \ln\, \phi}{2}$ (d) $\psi = \frac{-k\phi \ln\, \phi}{2}$

7. For which of the following condition does the integral $\int_0^1 P_m(x)P_n(x)dx$ vanish for $m \neq n$, where $P_m(x)$ and $P_n(x)$ are the Legendre polynomials of order m and n respectively?

[JEST 2018]

(a) All $m, m \neq n$

(b) $m - n$ is an odd integer

(c) $m - n$ is a non zero even integer

(d) $n = m \pm 1$

8. If $y(x)$ satisfies

$$\frac{dy}{dx} = y[1 + (\log\, y)^2]$$

And $y(0) = 1$ for $x \geq 0$, then $y(\pi/2)$ is

[JEST 2018]

(a) 0 (b) 1

(c) $\pi/2$ (d) infinity

9. Consider a function $f(x) = P_k(x)e^{-(x^4+2x^2)}$ in the domain $x \in (-\infty, \infty)$, where P_k is any polynomial of degree k. What is the maximum possible number of extrema of the function?

[JEST 2019]

(a) $k+3$ (b) $k-3$

(c) $k+2$ (d) $k+1$

10. The solution of the differential equation $y'' - 2y' - 3y = e^u$ is given as $C_1e^{-t} + C_2e^{2t} + C_3e^{3t}$. The values of the coefficients C_1, C_2 and C_3 are:

[JEST 2020]

(a) C_1, C_2 and C_3 are arbitrary

(b) C_1, C_3 are arbitrary and $C_2 = \frac{-1}{3}$

(c) C_2, C_3 are arbitrary and $C_1 = \frac{-1}{3}$

(d) C_1, C_2 are arbitrary and $C_3 = \frac{-1}{3}$

11. A particle moving in two dimensions satisfies the equations of motion

$$\dot{x}(t) = x(t) + y(t)$$
$$\dot{y}(t) = x(t) - y(t)$$

with $\dot{x}(0) = 0$. What is the ratio of $\frac{x(\infty)}{y(\infty)}$?

[JEST 2020]

(a) $1 - \frac{1}{\sqrt{2}}$ (b) $1 + \frac{1}{\sqrt{2}}$

(c) $\sqrt{2} - 1$ (d) $\sqrt{2} + 1$

12. Some bacteria are added to a bucket at time 10am. The number of bacteria doubles every minute and reaches a number 16×10^{15} at 10: 18am. How many seconds after 10 am were there 25×10^{13} bacteria?

[JEST 2020]

13. Consider the differential operators given below:

$$J^+ = x^2 \frac{d}{dx} + \mu x, J^0 = x\frac{d}{dx} + \rho$$

that act on the set of monomials $\{x^m\}$. Here, μ and ρ are constants. Which one the following is equal to $(J^0 J^+ - J^+ J^0)x^m$?

[JEST 2022]

(a) $-(m+1)J^+ x^{(m-1)}$ (b) $mJ^+ x^{(m-1)}$

(c) $J^+ x^m$ (d) $-J^+ x^m$

14. If three real variables x, y and z evolve with time t following

$$\frac{dx}{dt} = x(y-z), \frac{dy}{dt} = y(z-x), \frac{dz}{dt} = z(x-y),$$

then which of the following quantities remains invariant in time?

[JEST 2022]

(a) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ (b) $x^2 + y^2 + z^2$

(c) $xy + yz + zx$ (d) $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

15. Solve the differential equation,

$$\frac{dy}{dx} = xy + xy^2$$

If

$$y(x = \sqrt{2}) = \frac{e}{2-e}$$

where e is the base of natural logarithms, compute $y(x = 0)$.

[JEST 2023]

(a) -1 (b) 1

(c) e (d) 0

16. If a power series

$$y = \sum_{j=0}^{\infty} a_j x^j$$

analysis is carried out of the following differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{x^2}\frac{dy}{dx} - \frac{4}{x^2}y = 0,$$

which of the following recurrence relations results?

[JEST 2023]

(a) $a_{j+1} = a_j \frac{4 - j(j+1)}{j+1}, j = 0,1,2,\cdots$

(c) $a_{j+2} = a_j \frac{4 - j(j+1)}{j+1}, j = 0,1,2,\cdots$

(b) $a_{j+2} = a_j \frac{4 - j(j-1)}{j+1}, j = 0,1,2,\cdots$

(d) $a_{j+1} = a_j \frac{4 - j(j-1)}{j+1}, j = 0,1,2,\cdots$

17. A polynomial $C_n(x)$ of degree n defined on the domain $x \in [-1,1]$ satisfies the differential equation

$$(1-x^2)\frac{d^2C_n}{dx^2} - x\frac{dC_n}{dx} + n^2 C_n = 0.$$

The polynomials satisfy the orthogonality relation

$$\int_{-1}^{1} \sigma(x) C_n(x) C_m(x) dx = 0$$

for $n \neq m$. What is $\sigma(x)$?

[JEST 2024]

(a) $(1-x^2)^{-1/2}$ (b) $(1-x^2)$

(c) 1 (d) $\exp(-x^2)$

❖ TIFR PYQ's

1. The differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

has the complete solution, in terms of arbitrary constants A and B, **[TIFR 2013]**

(a) $A\exp\ x + Bx\exp\ x$

(c) $A\exp\ x + Bx\exp\ (-x)$

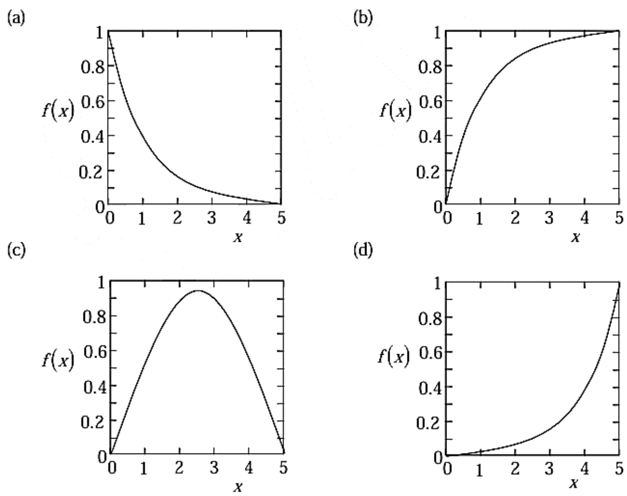
(b) $A\exp\ x + B\exp\ (-x)$

(d) $x\{A\exp\ x + B\exp\ (-x)$

2. The solution of the integral equation

$$f(x) = x - \int_0^x dt f(t)$$

has the graphical form **[TIFR 2014]**



3. Consider the differential equation

$$\frac{d^2y}{dx^2} = -4\left(y + \frac{dy}{dx}\right)$$

with the boundary condition that $y(x) = 0$ at $x = 1/5$. When plotted as a function of x, for $x \geq 0$, we can say with certainty that the value of y

[TIFR 2015]

(a) oscillates from positive to negative with amplitude decreasing to zero

(b) has an extremum in the range $0 < x < 1$

(c) first increases, then decreases to zero

(d) first decreases, then increases to zero

4. The Bernoulli polynomials $B_n(s)$ are defined by,

$$\frac{xe^{xs}}{e^x - 1} = \sum B_n(s)\frac{x^n}{n!}$$

. which one of the following relations is true? **[TIFR 2015]**

(a) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)\frac{x^n}{(n+1)!}$

(b) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n\frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(-s)(-1)^n\frac{x^n}{n!}$

(d) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n\frac{x^n}{n!}$

5. The generating function for a set of polynomials in x is given by

$$f(x,t) = (1 - 2xt + t^2)^{-1}$$

The third polynomial (order x^2) in this set is **[TIFR 2015]**

(a) $2x^2 + 1$ (b) $2x^2 - x$

(c) $4x^2 + 1$ (d) $4x^2 - 1$

6. The function $y(x)$ satisfies the differential equation

$$x\frac{dy}{dx} = y(\ln\ y - \ln\ x + 1)$$

with the initial condition $y(1) = 3$. What will be the value of $y(3)$? **[TIFR 2015]**

7. Write down $x(t)$, where $x(t)$ is the solution of the following differential equation

$$\left(\frac{d}{dt} + 2\right)\left(\frac{d}{dt} + 1\right)x = 1,$$

with the boundary conditions **[TIFR 2017]**

$$\left.\frac{dx}{dt}\right|_{t=0} = 0,\ x(t)|_{t=0} = -\frac{1}{2}$$

Ans: exp(-2t) -2exp(-t) + 1/2

8. Consider the two equations

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$
$$x^3 - y = 1$$

How many simultaneous real solutions does this pair of equations have?

9. If $y(x)$ satisfies the differential equation

$$y'' - 4y' + 4y = 0$$

with boundary conditions $y(0) = 1$ and $y'(0) = 0$, then $y\left(-\frac{1}{2}\right) =$

[TIFR 2018]

(a) $\frac{2}{e}$

(b) $\frac{1}{2}\left(e + \frac{1}{e}\right)$

(c) $\frac{1}{e}$

(d) $-\frac{e}{2}$

10. The differential equation $x\frac{dy}{dx} - xy = \exp(x)$, where $y = e^2$ at $x = 1$, has the solution $y =$

[TIFR 2019]

(a) $\exp(x^2 + x)$

(b) $(1 - x)\exp(x) + \exp(1 + x)$

(c) $\exp(1 + x)(1 + \ln x)$

(d) $\exp(x)\ln x + \exp(1 + x)$

11. A set of polynomials of order n are given by the formula

$$P_n(x) = (-1)^n \exp\left(\frac{x^2}{2}\right)\frac{d^n}{dx^n}\exp\left(-\frac{x^2}{2}\right)$$

The polynomial $P_7(x)$ of order $n = 7$ is

[TIFR 2019]

(a) $x^7 - 21x^5 + 105x^4 + 35x^3 - 105x$

(c) $x^7 - 21x^5 + 105x^3 - 105x + 21$

(b) $x^6 - 21x^5 + 105x^4 - 105x^3 + 21x^2 + x$

(d) $x^7 - 21x^5 + 105x^3 - 105x$

12. Consider the improper differential

$$đs = (1 + y^2)dx + xydy$$

An integrating factor for this is

[TIFR 2020]

(a) $-x$

(b) $1 + x^2$

(c) xy

(d) $-1 + y^2$

13. The solution of the differential equation

$$\frac{dy}{dx} = 1 + \frac{y}{x} - \frac{y^2}{x^2}$$

for $x > 0$ with the boundary condition $y = 0$ at $x = 1$. is given by $y(x) =$

[TIFR 2020]

(a) $\frac{x(x^2 - 1)}{x^2 + 1}$

(b) $\frac{x(x - 1)}{x + 1}$

(c) $\frac{x - 1}{x + 1}$

(d) $\frac{x^2 - 1}{x^2 + 1}$

14. If $y(x)$ satisfies the following differential equation

$$x\frac{dy}{dx} = \cot y - \operatorname{cosec} y \cos x$$

and we have $\lim_{x\to 0} y(x) = 0$ then $y(\pi/2) =$

[TIFR 2021]

(a) $-\cos^{-1}(2/\pi - 2)$

(b) $\sin^{-1}(2/\pi)$

(c) $\pi/2$

(d) 0

15. Consider the following differential equations:

$$\frac{dx}{dt} = ay(t), \frac{dy}{dt} = a$$

where a is a positive constant. The solutions to these equations define a family of curves in the x, y plane. What are these curves?

[TIFR 2024]

(a) Ellipses

(b) Circles

(c) Hyperbolas

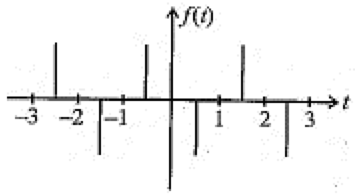
(d) Parabolas

Answers key				
CSIR-NET				
1. d	2. d	3. d	4. d	5. a
6. c	7. a	8. d	9. a	10. d
11. b	12. c	13. b	14. d	15. c
16. a	17. d	18. b	19. d	20. a
21. c	22. b	23. c	24. c	25. b
26. c	27. d	28. d	29. c	30. a
31. c	32. a	33. c	34. d	35. c
36. c	37. d	38. a	39. d	40. a
41. c	42. a	43. a	44. b	45. a
GATE				
1. a	2. c	3.	4. b	5.
6. b	7. d	8. a,b	9. b	10. d
11. d	12. c	13. a	14. d	15. d
16. a	17. d	18. c	19. d	20. 05233
21. 0.81	22. b	23. a	24. bc	25. b
26. a,c	27. a			
JEST				
1. a	2. d	3. a	4. c	5. d
6. a	7. c	8. d	9. a	10. b
11. d	12. 0720	13. c	14. d	15. b
16. d	17. a			
TIFR				
1. a	2. b	3. b	4. d	5. d
6. 081	7.	8. 002	9. a	10. a
11. d	12. a	13. a	14. a	15. d

7. TIFR:- exp(-2t) -2exp(-t) +1/2 (TIFR)

❖ CSIR-NET PYQ's

1. Consider the periodic function $f(t)$ with time period T as shown in the figure below:



The spikes, located at

$$t = \frac{1}{2}(2n-1)$$

where $n = 0, \pm1, \pm2, \ldots$, are Dirac-delta functions of strength ±1. The amplitudes a_n in the Fourier expansion

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{2\pi i n t/T}$$

are given by

[CSIR JUNE 2015]

(a) $(-1)^n$ (b) $\frac{1}{n\pi}\sin\frac{n\pi}{2}$

(c) $i\sin\frac{n\pi}{2}$ (d) $n\pi$

2. The function $f(t)$ is a periodic function of period 2π. In the range $(-\pi, \pi)$, it equals e^{-t}, If

$$f(t) = \sum_{-\infty}^{\infty} c_n e^{int}$$

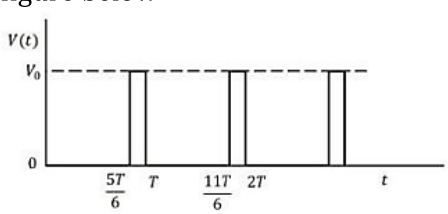
denotes its Fourier series expansion, the sum $\sum_{-\infty}^{\infty}|c_n|^2$ is

[CSIR DEC 2019]

(a) 1 (b) $\frac{1}{2\pi}$

(c) $\frac{1}{2\pi}\cosh(2\pi)$ (d) $\frac{1}{2\pi}\sinh(2\pi)$

3. An infinite waveform $V(t)$ varies as shown in the figure below



The lowest harmonic that vanishes in the Fourier series of $V(t)$ is **[CSIR DEC 2023]**

(a)2 (b)3

(c)6 (d)None

❖ GATE PYQ's

1. A periodic function $f(x) = x$ for $-\pi < x < +\pi$ has the Fourier series representation

$$f(x) = \sum_{n=1}^{\infty}\left(-\frac{2}{n}\right)(-1)^n \sin nx$$

Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be

[GATE 2004]

(a) $2\ln 2$ (b) $\frac{\pi^2}{3}$

(c) $\frac{\pi^2}{6}$ (d) $\pi\ln 2$

2. The k th Fourier component of $f(x) = \delta(x)$ is

(a) 1 (b) 0

(c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

3. $f(x)$ is a symmetric periodic function of x i.e. $f(x) = f(-x)$. Then in general, the Fourier series of the function $f(x)$ will be of the form

[GATE 2013]

(a) $f(x) = \sum_{n=1}^{\infty}(a_n(nkx) + b_n \sin(nkx))$

(b) $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nkx))$

(c) $f(x) = \sum_{n=1}^{\infty}(b_n \sin(nkx))$

(d) $f(x) = a_0 + \sum_{n=1}^{\infty}(b_n \sin(nkx))$

4. A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi < x < \pi)$ as:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

The appropriate Fourier series expansion for $f(x)$ is

[GATE 2016]

(a) $f(x) = (4/\pi)[\sin x + (\sin 3x)/3 + (\sin 5x)/5 + \cdots\ldots]$

(b) $f(x) = (4/\pi)[\sin\ x - (\sin\ 3x)/3 + (\sin\ 5x)/5 - \cdots \ldots]$

(c) $f(x) = (4/\pi)[\cos\ x + (\sin\ 3x)/3 + (\cos\ 5x)/5 + \cdots \ldots]$

(d) $f(x) = (4/\pi)[\cos\ x - (\cos\ 3x)/3 + (\cos\ 5x)/5 - \cdots..$

5. Let θ be a variable in the range $-\pi \le \theta < \pi$. Now consider a function

$$\psi(\theta) = 1 \quad \text{for } -\pi/2 \le \theta < \pi/2$$
$$= 0 \quad \text{otherwise.}$$

If its Fourier-series is written as $\psi(\theta) = \sum_{m=-\infty}^{\infty} C_m e^{-im\theta}$, then the value of $|C_3|^2$ (rounded off to three decimal places) is **[GATE 2019]**

6. If $x = \sum_{k=1}^{\infty} a_k \sin\ kx$, for $-\pi \le x \le \pi$, the value of a_2 is **[GATE 2020]**

❖ JEST PYQ's

1. The function $f(x) = \cosh\ x$ which exists in the range $-\pi \le x \le \pi$ is periodically repeated between $x = (2m-1)\pi$ and $(2m+1)\pi$, where $m = -\infty$ to $+\infty$. Using Fourier series, indicate the correct relation at $x = 0$. **[JEST 2017]**

(a) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = \frac{1}{2}\left(\frac{\pi}{\cosh\ \pi} - 1\right)$

(b) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1-n^2} = 2\frac{\pi}{\cosh\ \pi}$

(c) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1+n^2} = 2\frac{\pi}{\sinh\ \pi}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2} = \frac{1}{2}\left(\frac{\pi}{\sinh\ \pi} - 1\right)$

❖ TIFR PYQ's

1. A function $f(x)$ is defined in the range $-1 \le x \le 1$ by

$$f(x) = \begin{matrix} 1 - x \text{ for } x \ge 0 \\ 1 + x \text{ for } x < 0 \end{matrix}$$

The first few terms in the Fourier series approximating this function are

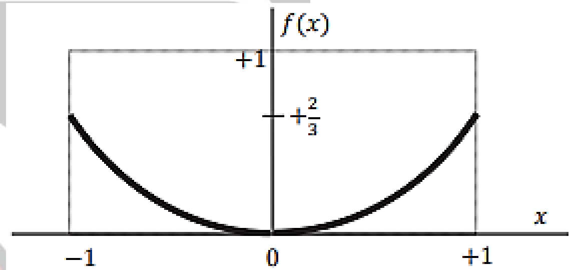
(a) $\frac{1}{2} + \frac{4}{\pi^2}\cos\ \pi x + \frac{4}{9\pi^2}\cos\ 3\pi x + \cdots$

(b) $\frac{1}{2} + \frac{4}{\pi^2}\sin\ \pi x + \frac{4}{9\pi^2}\sin\ 3\pi x + \cdots$

(c) $\frac{4}{\pi^2}\cos\ \pi x + \frac{4}{9\pi^2}\cos\ 3\pi x + \cdots$

(d) $\frac{1}{2} - \frac{4}{\pi^2}\cos\ \pi x + \frac{4}{9\pi^2}\cos\ 3\pi x - \cdots$

2. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1 \le x \le +1$, as indicated by the thick line in the figure below. **[TIFR 2013]**



The student represents the function by a sum of three terms

$$f(x) \approx a_0 + a_1\cos\frac{\pi x}{2} + a_2\sin\frac{\pi x}{2}$$

Which of the following would be the best choices for the coefficients a_0, a_1 and a_2 ?

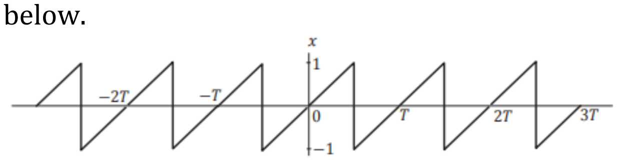
(a) $a_0 = 1, a_1 = -\frac{1}{3}, a_2 = 0$

(b) $a_0 = \frac{2}{3}, a_1 = -\frac{2}{3}, a_2 = 0$

(c) $a_0 = \frac{2}{3}, a_1 = 0, a_2 = -\frac{2}{3}$

(d) $a_0 = -\frac{1}{3}, a_1 = 0, a_2 = -1$

3. Consider the waveform $x(t)$ shown in the diagram below.



The Fourier series for $x(t)$ which gives the closest approximation to this waveform is **[TIFR 2017]**

(a) $x(t) = \frac{2}{\pi}\left[\cos\frac{\pi t}{T} - \frac{1}{2}\cos\frac{4\pi t}{T} + \frac{1}{3}\cos\frac{3\pi t}{T} + \cdots\right]$

(b) $x(t) = \frac{2}{\pi}\left[-\sin\frac{\pi t}{T} + \frac{1}{2}\sin\frac{2\pi t}{T} - \frac{1}{3}\sin\frac{3\pi t}{T} + \cdots\right]$

(c) $x(t) = \frac{2}{\pi}\left[\sin\frac{\pi t}{T} - \frac{1}{2}\sin\frac{2\pi t}{T} + \frac{1}{3}\sin\frac{3\pi t}{T} + \cdots\right]$

(d) $x(t) = \frac{2}{\pi}\left[-\cos\frac{2\pi t}{T} + \frac{1}{2}\cos\frac{4\pi t}{T} - \frac{1}{3}\cos\frac{6\pi t}{T} + \cdots\right]$

4. The Fourier series which reproduces, in the interval $0 \le x < 1$, the function

$$f(x) = \sum_{n=-\infty}^{+\infty} \delta(x-n)$$

where n is an integer, is **[TIFR 2018]**

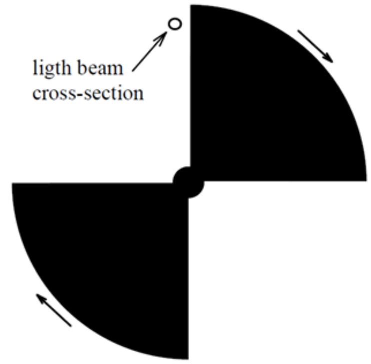
(a) $\cos\pi x + \cos 2\pi x + \cos 3\pi x + \cdots$ (to ∞)

(b) $1 + 2\cos 2\pi x + 2\cos 4\pi x + 2\cos 6\pi x + \cdots +$ (to ∞)

(c) $1 + \cos\pi x + \cos 2\pi x + \cos 3\pi x + \cdots$ (to ∞)

(d) $(\cos\pi x + \sin\pi x) + \frac{1}{2}(\cos 2\pi x + \sin 2\pi x) + \frac{1}{3}(\cos 3\pi x + \sin 3\pi x) + \cdots$ (to ∞)

5. Consider a fan with blades rotating with frequency f, as shown in the Figure. It is used to periodically block a light beam of intensity I_0. The beam has a very small cross-sectional area and hits the blade near its outer edge, as shown. The transmitted beam is detected by a photo-detection unit which gives out a voltage signal V proportional to the transmitted intensity I. If this voltage signal pattern is displayed on an oscilloscope, what would best describe the signal pattern? **[TIFR 2025]**



(a) $V_0\left[\frac{1}{2} + \sum_n \frac{4}{\pi n}\sin(2n\pi f t)\right], n = 2,6,10,14\ldots$

(b) $V_0 \sum_n [\cos^2(2n\pi f t) - \sin^2(2n\pi f t)], n = 2,6,10,14\ldots$

(c) $V_0\left[\frac{1}{2} + \frac{1}{2}\sin(4\pi f t)\right]$

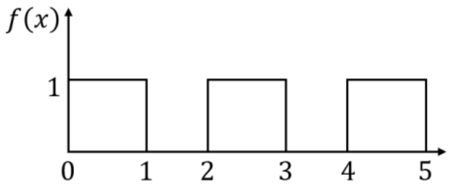
(d) $V_0[\cos^2(4\pi f t)]$

❖ Answer Key				
CSIR-NET				
1. c	2. d	3. c		
GATE				
1. c	2. c	3. b	4. a	5. 0.011
6. -1				
JEST				
1. d				
TIFR				
1. a	2. b	3. b	4. b	5. a

❖ CSIR-NET PYQ's

1. The graph of the function

$$f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$$



(Where $n=0,1,2,\ldots\ldots$) is shown below. Its Laplace transform $\tilde{f}(s)$ is

[CSIR DEC 2011]

(a) $\frac{1+e^{-s}}{s}$

(b) $\frac{1-e^{-s}}{s}$

(c) $\frac{1}{s(1+e^{-s})}$

(d) $\frac{1}{s(1-e^{-s})}$

2. The inverse Laplace transform of $\frac{1}{s^2(s+1)}$ is

[CSIR JUNE 2013]

(a) $\frac{1}{2}t^2e^{-t}$

(b) $\frac{1}{2}t^2+1-e^{-t}$

(c) $t-1+e^{-t}$

(d) $\frac{1}{2}t^2(1-e^{-t})$

3. The Laplace transform of $6t^3+3\sin 4t$ is

[CSIR JUNE 2015]

(a) $\frac{36}{s^4}+\frac{12}{s^2+16}$

(b) $\frac{36}{s^4}+\frac{12}{s^2-16}$

(c) $\frac{18}{s^4}+\frac{12}{s^2-16}$

(d) $\frac{36}{s^3}+\frac{12}{s^2+16}$

4. The Laplace transform of $f(t)=\begin{cases}\frac{t}{T}, & 0<t<T\\ 1, & t>T\end{cases}$, is

[CSIR JUNE 2016]

(a) $-\frac{(1-e^{-sT})}{s^2T}$

(b) $\frac{(1-e^{-sT})}{s^2T}$

(c) $\frac{(1+e^{-sT})}{s^2T}$

(d) $\frac{(1-e^{sT})}{s^2T}$

5. Consider the differential equation $\frac{dy}{dx}+ay=e^{-bt}$ with the initial condition $y(0)=0$. Then the Laplace transform $Y(s)$ of the solution $y(t)$ is

[CSIR DEC 2017]

(a) $\frac{1}{(s+a)(s+b)}$

(b) $\frac{1}{b(s+a)}$

(c) $\frac{1}{a(s+b)}$

(d) $\frac{e^{-a}-e^{-b}}{b-a}$

6. The Laplace transform L[f](y) of the function $f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1\\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$, $n=0,1,2,\cdots$ is

[CSIR JUNE 2022]

(a) $\frac{e^{-y}(e^{-y}+1)}{y(e^{-2y}+1)}$

(b) $\frac{e^{y}-e^{-y}}{y}$

(c) $\frac{e^{y}+e^{-y}}{y}$

(d) $\frac{e^{y}(e^{y}-1)}{y(e^{2y}-1)}$

❖ GATE PYQ's

1. The Laplace transform $f(t)=\sin \pi t$ is

$$F(s)=\frac{\pi}{(s^2+\pi^2)}, s>0$$

. Therefore, the Laplace transform of $t\sin \pi t$ is

[GATE 2004]

(a) $\frac{\pi}{s^2(s^2+\pi^2)}$

(b) $\frac{2\pi}{s^2(s^2+\pi^2)^2}$

(c) $\frac{2\pi s}{(s^2+\pi^2)^2}$

(d) $\frac{2\pi}{(s^2+\pi^2)^2}$

2. If $\bar{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constant, is

[GATE 2005]

(a) $\frac{1}{a}\bar{f}(s)$

(b) $\frac{1}{a}\bar{f}(s/a)$

(c) $\bar{f}(s)$

(d) $\bar{f}(s/a)$

3. Inverse Laplace transform of

$$\frac{s+1}{s^2-4}$$

is

[GATE 2007]

(a) $\cos\ 2x + \frac{1}{2}\sin\ 2x$

(b) $\cos\ x + \frac{1}{2}\sin\ x$

(c) $\cosh\ x + \frac{1}{2}\sinh\ x$

(d) $\cosh\ 2x + \frac{1}{2}\sinh\ 2x$

4. If $f(x) = \begin{cases} 0 & \text{for } x < 3 \\ x-3 & \text{for } x \geq 3 \end{cases}$, then the Laplace transform of $f(x)$ is

[GATE 2010]

(a) $s^{-2}e^{3s}$ (b) s^2e^{-3s}

(c) s^{-2} (d) $s^{-2}e^{-3s}$

5. Which of the followings pairs of the given function f(t) and its Laplace transforms f(s) is not correct?

[GATE 2013]

(a) $f(t) = \delta(t), f(s) = 1$ (Singularity at +0)

(b) $f(t) = 1, f(s) = \frac{1}{s}, (s > 0)$

(c) $f(t) = \sin\ kt, f(s) = \frac{s}{s^2 + k^2}, (s > 0)$

(d) $f(t) = te^{kt}, f(s) = \frac{1}{(s-k)^2}, (s > k, s > 0)$

6. A function $f(t)$ is defined only for $t \geq 0$. The Laplace transform of $f(t)$ is

$$L(f; s) = \int_0^\infty e^{-st} f(t) dt$$

whereas the Fourier transform of $f(t)$ is

$$\tilde{f}(\omega) = \int_0^\infty f(t) e^{-i\omega t} dt$$

The correct statement(s) is(are)

[GATE 2021]

(a) The variable s is always real.

(b) The variable s can be complex.

(c) $L(f; s)$ and $\tilde{f}(\omega)$ can never be made connected.

(d) $L(f; s)$ and $\tilde{f}(\omega)$ can be made connected

❖ JEST PYQ's

1. The Laplace transformation of $e^{2t}\sin\ 4t$ is

[JEST 2014]

(a) $\frac{4}{s^2 + 4s + 25}$ (b) $\frac{4}{s^2 - 4s + 20}$

(c) $\frac{4s}{s^2 + 4s + 20}$ (d) $\frac{4s}{2s^2 + 4s + 20}$

2. The Laplace transform of $(\sin(at) - at\cos(at))/(2a^3)$ is

[JEST 2018]

(a) $\frac{2as}{(s^2 + a^2)^2}$ (b) $\frac{s^2 - a^2}{(s^2 + a^2)^2}$

(c) $\frac{1}{(s + a)^2}$ (d) $\frac{1}{(s^2 + a^2)^2}$

❖ Answer Key				
CSIR-NET				
1. c	2. c	3. a	4. b	5. a
6. d				
GATE				
1. c	2. b	3. d	4. d	5. c
6. b,d				
JEST				
1. b	2. d			

❖ **CSIR-NET PYQ's**

Common data Q.1 Q.2

Consider the matrix $M=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$

1. The eigenvalues of M are

[CSIR JUNE 2011]

(a) 0,1,2 (b) 0,0,3

(c) 1,1,1 (d) −1,1,3

2. The exponential of M simplifies to (I is the 3×3 identity matrix)

[CSIR JUNE 2011]

(a) $e^{M}=I+\left(\frac{e^{3}-1}{3}\right)M$ (b) $e^{M}=I+M+\frac{M^{2}}{2!}$

(c) $e^{M}=I+3^{3}M$ (d) $e^{M}=(e-1)M$

3. A 3×3 matrix M has $\operatorname{Tr}[M]=6$, $\operatorname{Tr}[M^{2}]=26$ and $\operatorname{Tr}[M^{3}]=90$. Which of the following can be a possible set of eigenvalues of M ?

[CSIR DEC 2011]

(a) {1,1,4} (c) {−1,3,4}

(b) {−1,0,7} (d) {2,2,2}

4. The eigenvalues of the matrix, $A=\begin{pmatrix}1&2&3\\2&4&6\\3&6&9\end{pmatrix}$ are

[CSIR JUNE 2012]

(a) (1,4,9) (b) (0,7,7)

(c) (0,1,13) (d) (0,0,14)

5. The eigenvalues of the antisymmetric matrix; $A=\begin{pmatrix}0&-n_3&n_2\\n_3&0&-n_1\\-n_2&n_1&0\end{pmatrix}$ where n_1,n_2 and n_3 are the components of a unit vector, are

[CSIR JUNE 2012]

(a) 0, i, −I (b) 0,1, −1

(c) 0,1 + i, −1 − I (d) 0,0,0

6. A 2×2 matrix A has eigenvalues $e^{in/5}$ and $e^{i\pi/6}$. The smallest value of ' n ' such that $A^{n}=1$ is:

[CSIR DEC 2012]

(a) 20 (b) 30

(c) 60 (d) 120

7. Given a 2×2 unitary matrix U satisfying $U'U=U'=I$ with $\det U=e^{iq}$, one can construct a unitary matrix $V(V'V=VV'=1)$ with $\det V=1$ from it by

[CSIR DEC 2012]

(a) Multiplying U by $e^{-i//2}$

(b) Multiplying any single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column of U by $e^{-i\varphi/2}$

(d) Multiplying U by $e^{-i\varphi}$.

8. Consider an $n\times n(n>1)$ matrix A , in which A_{ij} is the product of the indices i and j (namely $A_{ij}=ij$). The matrix A

[CSIR DEC 2013]

(a) has one degenerate eigenvalue with degeneracy ($n-1$)

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalue with degeneracy n

(d) does not have any degenerate eigenvalue

9. Consider the matrix

$$M=\begin{pmatrix}0&2i&3i\\-2i&0&6i\\-3i&-6i&0\end{pmatrix}$$

The eigenvalues of M are

[CSIR JUNE 2014]

(a) −5, −2,7 (b) −7,0,7

(c) −4i, 2i, 2i (d) 2,3,6

10. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relations

[CSIR JUNE 2014]

(a) $[A, B] = B + C, [B, C] = 0, [C, A] = B + C$

(b) $[A, B] = C, [B, C] = A, [C, A] = B$

(c) $[A, B] = B, [B, C] = 0, [C, A] = A$

(d) $[A, B] = C, [B, C] = 0, [C, A] = B$

11. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

[CSIR DEC 2014]

(a) $b = 0$ or $a = 0$

(b) $b = a$ or $b = -2a$

(c) $b = 2a$ or $b = -a$

(d) $b = a/2$ or $b = -a/2$

12. The Gauss hypergeometric function $F(a, b, c; z)$, defined by the Taylor series expansion around $z = 0$ as

$$F(a, b, c; z) = \sum_{n=0}^{\infty} \frac{a(a+1)\ldots(a+n-1)b(b+1)\ldots(b+n-1)}{c(c+1)\ldots(c+n-1)n!} z^n$$

satisfies the equation relation

[CSIR JUNE 2016]

(a) $\frac{d}{dz} F(a, b, c; z) = \frac{c}{ab} F(a-1, b-1, c-1; z)$

(b) $\frac{d}{dz} F(a, b, c; z) = \frac{c}{ab} F(a+1, b+1, c+1; z)$

(c) $\frac{d}{dz} F(a, b, c; z) = \frac{ab}{c} F(a-1, b-1, c=1; z)$

(d) $\frac{d}{dz} F(a, b, c; z) = \frac{ab}{c} F(a+1, b+1, c+1; z)$

13. The matrix $M = \begin{pmatrix} 1 & 3 & 2 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ satisfies the equation

[CSIR DEC 2016]

(a) $M^3 - M^2 - 10M + 12I = 0$

(b) $M^3 + M^2 - 12M + 10I = 0$

(c) $M^3 - M^2 - 10M + 10I = 0$

(d) $M^3 + M^2 - 10M + 10I = 0$

14. The 2×2 identity matrix I and the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ do not form a group under matrix multiplication. The minimum number of 2×2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is

[CSIR DEC 2016]

(a) 20 (b) 8

(c) 12 (d) 16

15. Which of the following cannot be eigen values of a real 3×3 matrix

[CSIR JUNE 2017]

(a) $2i, 0, -2i$ (b) 1,1,1

(c) $e^{i\theta}, e^{-i\theta}, 1$ (d) $i, 1, 0$

16. Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and

$$x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times [x\sigma_x + y\sigma_y + z\sigma_z] \exp\left(-\frac{i\theta\sigma_z}{2}\right)$$

Then the coordinates are related as follows

[CSIR JUNE 2017]

(a) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(b) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(c) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(d) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

17. Consider the matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution, and the corresponding normalized solution (up to a sign) is

[CSIR DEC 2017]

(a) $b = 2c$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$

(b) $c = 2b$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, 1, -2)$

(c) $c = b + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(2, -1, -1)$

(d) $b = c + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$

18. Consider an element $U(\varphi)$ of the group SU(2), where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as

$$U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(\varphi)$$

. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be

[CSIR DEC 2017]

(a) positive definite (b) real symmetric

(c) Hermitian (d) anti-Hermitian

19. Which of the following statements is true for a 3×3 real orthogonal matrix with determinant $+1$?

[CSIR JUNE 2018]

(a) the modulus of each of its eigenvalues need not be 1, but their product must be 1.

(b) at least one of its eigenvalues is $+1$.

(c) all of its eigenvalues must be real.

(d) none of its eigenvalues need be real.

20. One of the eigenvalues of the matrix e^A is e^a, where $A = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix}$. The product of the other two eigenvalues of e^A is

[CSIR DEC 2018]

(a) e^{2a} (b) e^{-a}

(c) e^{-2a} (d) 1

21. A 4×4 complex matrix A satisfies the relation $A^\dagger A = 4I$, where I is the 4×4 identify matrix. The number of independent real parameters of A is

[CSIR DEC 2018]

(a) 32 (b) 10

(c) 12 (d) 16

22. The elements of a 3×3 matrix A are the products of its row and column indices $A_{ij} = ij$ (where $i, j = 1,2,3$). The eigenvalues of A are

[CSIR JUNE 2019]

(a) (7,7,0) (b) (7,4,3)

(c)(14,0,0) (d) $\left(\frac{14}{3}, \frac{14}{3}, \frac{14}{3}\right)$

23. The operator A has a matrix representation $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ in the basis spanned by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In another basis spanned by $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, the matrix representation of A is

[CSIR JUNE 2019]

(a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$

24. If the rank of an $n \times n$ matrix A is m, where m and n are positive integers with $1 \le m \le n$, then the rank of the matrix A^2 is

[CSIR DEC 2019]

(a) m (b) $m - 1$

(c) $2m$ (d) $m-2$

25. The eigenvalues of the 3×3 matrix $M = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ are

[CSIR NOV 2020]

(a) $a^2+b^2+c^2, 0, 0$ (b) $b^2+c^2, a^2, 0$

(c) $a^2+b^2, c^2, 0$ (d) $a^2+c^2, b^2, 0$

26. A generic 3×3 real matrix A has eigenvalues 0,1 and 6 and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the

[CSIR JUNE 2021]

(a) eigenvalues $(I+A)^{-1}$

(b) eigenvalues of $(I+A^TA)$

(c) determinant of A^TA

(d) rank of A

27. Two $n\times n$ invertible real matrices A and B satisfy the relation

$$(AB)^T = -(A^{-1}B)^{-1}$$

If B is orthogonal then A must be

[CSIR JUNE 2022]

(a) Lower triangle (b) Orthogonal

(c) Symmetric (d) Anti-Symmetric

28. The matrix corresponding to the differential operator

$$\left(1+\frac{d}{dx}\right)$$

in the space of polynomials of degree at most two, in the basis spanned by $f_1 = 1, f_2 = x$ and $f_3 = x^2$, is **[CSIR JUNE 2023]**

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

29. The matrix $M = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ satisfies the equation

$M^3 + \alpha M^2 + \beta M + 3 = 0$ if (α, β) are

[CSIR JUNE 2023]

(a) $(-2,2)$ (b) $(-3,3)$

(c) $(-6,6)$ (d) $(-4,4)$

30. The matrix $R_{\hat{n}}(\theta)$ represents a rotation by an angle θ about the axis $\hat{n}$. The value of θ and $\hat{n}$ corresponding to the matrix

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}$$

respectively, are

[CSIR JUNE 2023]

(a) $\pi/2$ and $\left(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$

(b) $\pi/2$ and $\left(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$

(c) π and $\left(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$

(d) π and $\left(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$

31. Let M be a 3×3 real matrix such that

$$e^{M\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where θ is a real parameter. Then M is given by

[CSIR DEC 2023]

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

32. The matrix A is given by

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

The eigenvalues of $3A^3 + 5A^2 - 6A + 2I$, where I is the identity matrix, are

[CSIR JUNE 2024]

(a)4,9,27 (b)1,9,44

(c)1,110,8 (d)4,110,10

33. If I is an $n \times n$ identity matrix and $\text{adj}(2I) = 2^k I$, then k is equal to **[CSIR DEC 2024]**

(a)1 (b)n

(c)$n-1$ (d)2

34. For the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, which of the following is true?

[CSIR JUNE 2025]

(a)$A^3 = 5A^2 - 4A - 2$

(b)$A^3 = 4A^2 - 6A + 3$

(c)$A^3 = 5A^2 - 5A - 1$

(d)$A^3 = 8A^2 + 3A - 4$

❖ GATE PYQ's

1. Find the matrix of the linear transformation T on $V_3(R)$ (i.e., three dimensional real vector space) defined as $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ b+c \\ c+a \end{pmatrix}$, with respect to the basis $B = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$, where $\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Also calculate the matrix representation of T^{-1}. **[GATE 2001]**

2. For any operator A, $i(A^+ - A)$ is

[GATE 2001]

(a) Hermitian (b) anti-Hermitian

(c) unitary (d) orthogonal

3. If two matrices A and B can be diagonalized simultaneously, which of the following is true?

[GATE 2002]

(a) $A^2 B = B^2 A$ (b) $A^2 B^2 = B^2 A$

(c) $AB = BA$ (d) $AB^2AB = BABA^2$

4. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?

[GATE 2002]

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

5. Find the matrix that diagonalizes the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **[GATE 2002]**

6. A 3×3 matrix has eigenvalues $0, 2+i$ and $2-i$. Which one of the following statements is correct?

[GATE 2003]

(a) The matrix is Hermitian

(b) The matrix is unitary

(c) The inverse of the matrix exists

(d) The determinant of the matrix is zero

7. A real traceless 4×4 unitary matrix has two eigenvalues -1 and +1. The other eigenvalues are

[GATE 2004]

(a) zero and +2 (b) zero and +1

(c) zero and +2 (d) -1 and +1

8. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are

[GATE 2004]

(a) +1 and +1 (b) zero and +1

(c) zero and +2 (d) -1 and +1

9. The determinant of a 3×3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvalue is

[GATE 2005]

(a) 4 (b) 6

(c) 8 (d) 9

10. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are **[GATE 2005]**

(a) $-2,-1,1,2$ (b) $-1,1,0,2$

(c) 1,0,2,3 (d) $-1,1,0,3$

11. The trace of a 3×3 matrix is 2. Two of its eigenvalues are 1 and 2. The third eigenvalue is **[GATE 2006]**

(a) -1 (b) 0

(c) 1 (d) 2

Common Data for Q. 12 and Q. 13:

One of the eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5.

12. The other two eigenvalues are **[GATE 2006]**

(a) 0 and 0 (b) 1 and 1

(c) 1 and -1 (d) -1 and -1

13. The normalized eigenvector corresponding to the eigenvalue 5 is **[GATE 2006]**

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

14. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are **[GATE 2007]**

(a) 6,1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix},\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2,5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix},\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6,1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix},\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2,5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix},\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

15. The eigenvalues of a matrix are $i,-2i$ and $3i$. The matrix is **[GATE 2007]**

(a) unitary (b) anti-unitary

(c) Hermitian (d) anti-Hermitian

16. For arbitrary matrices E, F, G and H, if EF − FE = 0 then Trace (EFGH) is equal to **[GATE 2008]**

(a) Trace (HGFE)

(b) Trace (E) Trace (F) Trace (G) Trace (H)

(c) Trace (GFEH)

(d) Trace (EGHF)

17. An unitary matrix $\begin{pmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{pmatrix}$ is given, where a,b,c,d,α and β are real. The inverse of the matrix is **[GATE 2008]**

(a) $\begin{pmatrix} ae^{i\alpha} & -ce^{jp} \\ b & d \end{pmatrix}$ (b) $\begin{pmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{pmatrix}$

(c) $\begin{pmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{pmatrix}$ (d) $\begin{pmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{pmatrix}$

18. The eigen values of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are **[GATE 2008]**

(a) $\frac{1}{2}(\sqrt{3}\pm i)$when $\theta=45°$

(b) $\frac{1}{2}(\sqrt{3}\pm i)$when $\theta=30°$

(c) $\pm$ 1 since the matrix is unitary

(d) $\frac{1}{2}(1\pm i)$ when $\theta=30°$

19. The eigen values of the matrix $A=\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ are **[GATE 2009]**

(a) real and distinct

(b) complex and distinct

(c) complex and coinciding

(d) real and coinciding

20. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

[GATE 2010]

(a) $5,2,-2$ (b) $-5,-1,-1$

(c) $5,1,-1$ (d) $-5,1,1$

21. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is not true?

[GATE 2011]

(a) Det A = Det B

(b) Trace of A = Trace of B

(c) A and B have the same eigen vectors

(d) A and B have the same eigen values

22. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigen-values of the matrix are all known to be positive integers. The largest eigen-value of the matrix is

[GATE 2011]

(a) 18 (b) 12

(c) 9 (d) 6

23. The eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

[GATE 2012]

(a) 0,1,1 (b) $0,-\sqrt{2},\sqrt{2}$

(c) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0$ (d)

24. The degenerate eigen value of the matrix $M = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$ is (your answer should be an integer)____________.

[GATE 2013]

25. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

[GATE 2014]

(a) orthogonal (b) symmetric

(c) anti-symmetric (d) unitary

26. The eigenvalues of a Hermitian matrix are all

[GATE 2018]

(a) real (b) imaginary

(c) of modulus one (d) real and positive

27. During a rotation, vectors along the axis of rotation remain unchanged. For the rotation matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, the unit vector along the axis of rotation is **[GATE 2019]**

(a) $\frac{1}{3}(2\hat{\imath} - \hat{\jmath} + 2\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} - \hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} - \hat{k})$ (d) $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

28. A real, invertible 3×3 matrix M has eigenvalues $\lambda_i, (i = 1,2,3)$ and the corresponding eigenvectors are $|e_i\rangle, (i = 1,2,3)$ respectively. Which one of the following is correct?

[GATE 2020]

(a) $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1,2,3$

(b) $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1,2,3$

(c) $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for $i = 1,2,3$

(d) The eigenvalues of M and M^{-1} are not related

29. The product of eigenvalues of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is

[GATE 2020]

(a) -1 (b) 1

(c) 0 (d) 2

30. Which one of the following matrices does NOT represent a proper rotation in a plane?

[GATE 2020]

(a) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

(d) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

31. P and Q are two Hermitian matrices and there exists a matrix R, which diagonalizes both of them, such that $RPR^{-1} = S_1$ and $RQR^{-1} = S_2$, where S_1 and S_2 are diagonal matrices. The correct statement(s) is (are)

[GATE 2021]

(a) All the elements of both matrices S_1 and S_2 are real

(b) The matrix PQ can have complex eigenvalues.

(c) The matrix QP can have complex eigenvalues.

(d) The matrices P and Q commute

32. What is the maximum number of free independent real parameters specifying an n dimensional orthogonal matrix?

[GATE 2022]

(a) $n(n-2)$ (b) $(n-1)^2$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

33. A 4×4 matrix M has the property $M^\dagger = -M$ and $M^4 = 1$, where 1 is the 4×4 identity matrix. Which one of the following is the CORRECT set of eigenvalues of the matrix M?

[GATE 2023]

(a) $(1,1,-1,-1)$ (b) $(i,i,-i,-i)$

(c) $(i,i,i,-i)$ (d) $(1,1,-i,-i)$

34. Consider two matrices: $P = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Which of the following statement is/are true?

[GATE 2024]

(a) P and Q have same set of eigenvalues

(b) P and Q commute with each other

(c) P and Q have different sets of linearly independent eigenvectors

(d) P is diagonalizable

❖ JEST PYQ's

1. For an N × N matrix consisting of all ones,

[JEST 2012]

(a) All eigenvalues = 1

(b) all eigenvalues = 0

(c) The eigenvalues are 1,2, ……, N

(d) one eigenvalue = N, the others = 0

2. The coordinate transformation $x' = 0.8x + 0.6y, y' = 0.6x - 0.8y$ represents [JEST 2013]

(a) A translation.

(b) a proper rotation.

(c) A reflection.

(d) None of the above.

3. Given a matrix $M = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, which of the following represents? $\cos\left(\frac{\pi M}{6}\right)$

[JEST 2016]

(a) $\frac{1}{2}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

(c) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

4. Let $\wedge = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$ similarity transformation of M to ∧ can be performed by

[JEST 2017]

(a) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$

(c) $\frac{1}{\sqrt{10}}\begin{pmatrix}1 & 3i\\ -3i & 11\end{pmatrix}$ (d) $\frac{1}{\sqrt{9}}\begin{pmatrix}1 & 3i\\ -3i & 1\end{pmatrix}$

5. Two of the eigenvalues of the matrix

$$A=\begin{pmatrix}a & 3 & 0\\ 3 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Are 1 and -1. What is the third eigenvalue?

[JEST 2018]

(a) 2 (b) 5

(c) -2 (d) -5

6. Consider two $n\times n$ matrices, A and B such that $A+B$ is invertible. Define two matrices, $C=A(A+B)^{-1}B$ and $D=B(A+B)^{-1}A$. Which of the following relations always hold true?

[JEST 2019]

(a) $C=D$ (b) $C^{-1}=D$

(c) $BCA=ADB$ (d) $C\neq D$

7. Let A be a hermitian matrix, and C and D be the unitary matrices. Which one of the following matrices is unitary?

[JEST 2019]

(a) $C^{-1}AC$ (b) $C^{-1}DC$

(c) $C^{-1}AD$ (d) $A^{-1}CD$

8. Consider a 2×2 matrix $A=\begin{pmatrix}1 & 13\\ 0 & 1\end{pmatrix}$ what is A^{27} ?

[JEST 2019]

(a) $\begin{pmatrix}1 & 13\\ 0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}1 & 13^{27}\\ 0 & 1\end{pmatrix}$

(c) $\begin{pmatrix}1 & 27\\ 0 & 1\end{pmatrix}$ (d) $\begin{pmatrix}1 & 351\\ 0 & 1\end{pmatrix}$

9. An $n\times n$ Hermitian matrix A is not a multiple of the identity matrix. Which one of the following statements is always true?

[JEST 2020]

(a) $n\mathrm{Tr}\,(A^2)=(\mathrm{Tr}\ A)^2$

(b) $n\mathrm{Tr}\,(A^2)<(\mathrm{Tr}\ A)^2$

(c) $n\mathrm{Tr}\,(A^2)>(\mathrm{Tr}\ A)^2$

(d) $\mathrm{Tr}\,(A^2)=n(\mathrm{Tr}\ A)^2$

10. Consider the matrix $A=\begin{pmatrix}1 & 0 & 0 & 1\\ 0 & -2 & 0 & 0\\ 0 & 0 & -3 & 0\\ 1 & 0 & 0 & 4\end{pmatrix}$

What is the determinant of the matrix exp (A) ?

[JEST 2021]

(a) 1 (b) exp (24)

(c) 24 (d) 0

11. How many independent real parameters are required to describe an arbitrary $N\times N$ Hermitian matrix?

[JEST 2023]

(a) N^2-N (b) N^2

(c) $2N$ (d) N^2-1

12. Choose the largest eigenvalue of the matrix $M=\begin{pmatrix}1 & 1 & 1 & 2 & 3\\ 12 & 2 & 3 & 2 & 1\\ 0 & 0 & 0 & 2 & 2\\ 0 & 0 & 3 & 0 & 3\\ 0 & 0 & 0 & 0 & 1\end{pmatrix}$. **[JEST 2023]**

(a) 3 (b) 5

(c) 8 (d) 10

13. A and B are 2×2 Hermitian matrices. $|a_1\rangle$ and $|a_2\rangle$ are two linearly independent eigenvectors of A. Consider the following statements:

1.If $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B, then $[A,B]=0$.

2.If $[A,B]=0$, then $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B.

Mark the correct option. **[JEST 2024]**

(a) Both statements 1 and 2 are true.

(b) Statement 2 is true but statement 1 is false.

(c) Statement 1 is true but statement 2 is false.

(d) Both statements 1 and 2 are false.

14. The singular matrix $A=\begin{pmatrix}2 & 3 & 3\\ 3 & 6 & 3\\ 3 & 3 & 6\end{pmatrix}$ commutes with the matrix $B=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$.

The eigenvalues of A are **[JEST 2024]**

(a) (0,0,12) (b) (0,3,13)

(c) (0,3,11) (d) (0,2,5)

15. Consider the rotation matrix $R=\begin{pmatrix}2/3 & -1/3 & 2/3\\ 2/3 & 2/3 & -1/3\\ -1/3 & 2/3 & 2/3\end{pmatrix}$.

Let ϕ be the angle of rotation. What is the value of $\sec^2\phi$?

[JEST 2024]

16. The number of independent real numbers that parameterize any (3×3) Hermitian matrix is **[JEST 2025]**

(a) 6 (b) 9

(c) 3 (d) 8

17. Consider a 2×2 matrix $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ which has eigenvalues $\lambda_1=\frac{1+\sqrt{5}}{2}$ and $\lambda_2=\frac{1-\sqrt{5}}{2}$. For any natural number n which of the following is correct ? **[JEST 2025]**

(a) $A^n=\frac{1}{\sqrt{5}}\begin{bmatrix}\lambda_1^{n-1}-\lambda_2^{n-1} & \lambda_1^n+\lambda_2^n\\ \lambda_1^n+\lambda_2^n & \lambda_1^{n+1}-\lambda_2^{n+1}\end{bmatrix}$

(b) $A^n=\frac{1}{\sqrt{5}}\begin{bmatrix}\lambda_1^{n-1}-\lambda_2^{n-1} & \lambda_1^n-\lambda_2^n\\ \lambda_1^n-\lambda_2^n & \lambda_1^{n+1}-\lambda_2^{n+1}\end{bmatrix}$

(c) $A^n=\frac{1}{\sqrt{5}}\begin{bmatrix}\lambda_1^{n-1}+\lambda_2^{n-1} & \lambda_1^n+\lambda_2^n\\ \lambda_1^n+\lambda_2^n & \lambda_1^{n+1}+\lambda_2^{n+1}\end{bmatrix}$

(d) $A^n=\frac{1}{\sqrt{5}}\begin{bmatrix}\lambda_1^{n-1}+\lambda_2^{n-1} & \lambda_1^n-\lambda_2^n\\ \lambda_1^n-\lambda_2^n & \lambda_1^{n+1}+\lambda_2^{n+1}\end{bmatrix}$

18. A 3×3 matrix M satisfies $M^2-3M+2I=0$. Find the determinant of the matrix M if its trace is 6 .

[JEST 2025]

❖ TIFR PYQ's

1. The matrix $\begin{pmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{pmatrix}$ can be related by a similarity transformation to the matrix

[TIFR 2010]

(a) $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & 0\end{pmatrix}$ (b) $\begin{pmatrix}2 & 1 & 0\\ 1 & -1 & -1\\ 0 & -1 & 2\end{pmatrix}$

(c) $\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 1\\ 0 & 1 & -1\end{pmatrix}$ (d) $\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 1\\ 0 & 1 & 1\end{pmatrix}$

2. Consider the matrix

$$\mathbb{M}=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{pmatrix}$$

A 3-dimensional basis formed by eigenvectors of $\mathbb{M}$ is

[TIFR 2011]

(a) $\begin{pmatrix}2\\ 1\\ -1\end{pmatrix},\begin{pmatrix}1\\ -1\\ 1\end{pmatrix}$ and $\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}$

(b) $\begin{pmatrix}1\\ 1\\ -1\end{pmatrix},\begin{pmatrix}0\\ 1\\ 1\end{pmatrix}$ and $\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}$

(c) $\begin{pmatrix}0\\ 0\\ 0\end{pmatrix},\begin{pmatrix}0\\ 1\\ 1\end{pmatrix}$ and $\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}$

(d) $\begin{pmatrix}2\\ 1\\ -1\end{pmatrix},\begin{pmatrix}0\\ 1\\ 1\end{pmatrix}$ and $\begin{pmatrix}-1\\ 1\\ 1\end{pmatrix}$

3. The trace of the real 4×4 matrix $\mathbb{U}=\exp(\mathbb{A})$, where

$$\mathbb{A}=\begin{pmatrix}0 & 0 & 0 & \pi/4\\ 0 & 0 & -\pi/4 & 0\\ 0 & \pi/4 & 0 & 0\\ -\pi/4 & 0 & 0 & 0\end{pmatrix}$$

is equal to

[TIFR 2011]

(a) $2\sqrt{2}$ (b) $\pi/4$

(c) $\exp(i\varphi)$ for $\varphi=0,\pi$ (d) zero

(e) $\pi/2$ (f) 2

4. Two different 2×2 matrices A and B are found to have the same eigenvalues. It is then correct to state that $A=SBS^{-1}$ where S can be a

[TIFR 2012]

(a) traceless 2×2 matrix

(b) Hermitian 2×2 matrix

(c) unitary 2×2 matrix

(d) arbitrary 2×2 matrix

5. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that

[TIFR 2014]

(a) $\{M, N\} = 0$ (b) $[M, N] = 0$

(c) $M^\dagger = N$ (d) $M^\dagger = N^{-1}$

6. If the eigenvalues of a symmetric 3×3 matrix $\mathbf{A}$ are 0,1,3 and the corresponding eigenvectors can be written as

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

respectively, then the matrix $\mathbf{A}^4$ is

[TIFR 2016]

(a) $\begin{pmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{pmatrix}$

(b) $\begin{pmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{pmatrix}$

(c) $\begin{pmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{pmatrix}$

(d) $\begin{pmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{pmatrix}$

7. Denote the commutator of two matrices A and B by $[A, B] = AB - BA$ and the anti-commutator by $\{A, B\} = AB + BA$.
If $\{A, B\} = 0$, we can write $[A, BC] =$

[TIFR 2017]

(a) $-B[A, C]$ (b) $B\{A, C\}$

(c) $-B\{A, C\}$ (d) $[A, C]B$

8. The matrix

$$\begin{pmatrix} 100\sqrt{2} & x & 0 \\ -x & 0 & -x \\ 0 & x & 100\sqrt{2} \end{pmatrix}$$

where $x > 0$, is known to have two equal eigenvalues. Find the value of x. **[TIFR 2017]**

9. A unitary matrix U is expanded in terms of a Hermitian matrix H, such that

$$U = e^{i\pi H/2}$$

If we know that $H = \begin{pmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & -1/2 \end{pmatrix}$

then U must be

[TIFR 2017]

(a) $\begin{pmatrix} i & 1/2 & \sqrt{3}/2 \\ 1/2 & i & 1/2 \\ \sqrt{3}/2 & 1/2 & i \end{pmatrix}$

(b) $\begin{pmatrix} i/2 & 0 & i\sqrt{3}/2 \\ 0 & i & 0 \\ i\sqrt{3}/2 & 0 & -i/2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ \sqrt{3} & 0 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} 2i & 1 & \sqrt{3}/2 \\ 1 & 2i & 0 \\ \sqrt{3}/2 & 0 & 2i \end{pmatrix}$

10. If a 2×2 matrix $\mathbb{M}$ is given by

$$\mathbb{M} = \begin{pmatrix} 1 & (1-i)/\sqrt{2} \\ (1+i)/\sqrt{2} & 0 \end{pmatrix}$$

then $\det \exp \mathbb{M} =$ **[TIFR 2018]**

(a) e (b) e^2

(c) $2i\sin\sqrt{2}$ (d) $\exp(-2\sqrt{2})$

11. The eigenvalues of a 3×3 matrix $\mathbb{M}$ are

$$\lambda_1 = 2\ \lambda_2 = -1\ \lambda_3 = 1$$

and the eigenvectors are

$$e_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\ e_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\ e_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

The matrix $\mathbb{M}$ is **[TIFR 2019]**

(a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

12. The eigenvector e_1 corresponding to the smallest eigenvalue of the matrix

$$\begin{pmatrix} 2a^2 & a & 0 \\ a & 1 & a \\ 0 & a & 2a^2 \end{pmatrix}$$

where $a = \sqrt{\frac{3}{2}}$, is given (in terms of its transpose) by

[TIFR 2020]

(a) $e_1^T = \frac{1}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\sqrt{3} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(b) $e_1^T = \frac{1}{2}\begin{pmatrix} \sqrt{\frac{3}{2}} & 1 & \sqrt{\frac{3}{2}} \end{pmatrix}$

(c) $e_1^T = \frac{1}{\sqrt{2}}(1 \quad 0 \quad -1)$

(d) $e_1^T = \frac{1}{\sqrt{2}}(1 \quad 0 \quad 1)$

13. A unitary matrix U is expressed in terms of a Hermitian matrix H, such that $\mathrm{U} = e^{i\pi \mathrm{H}/2}$
If the matrix H is given by

$$\mathrm{H} = \sqrt{3}\begin{pmatrix} 1/3 & 0 & \sqrt{2}/3 \\ 0 & 1/\sqrt{3} & 0 \\ \sqrt{2}/3 & 0 & -1/3 \end{pmatrix}$$

then **U** will have the form **[TIFR 2021]**

(a) $\begin{pmatrix} 3\sqrt{3}i & \sqrt{3} & 3/2 \\ \sqrt{3} & i & 0 \\ \sqrt{2}/\sqrt{3} & 0 & 3\sqrt{3}i \end{pmatrix}$

(b) $\begin{pmatrix} \sqrt{3} & 0 & \sqrt{6} \\ 0 & 3\sqrt{3} & 0 \\ \sqrt{6} & 0 & -\sqrt{3} \end{pmatrix}$

(c) $\begin{pmatrix} i\sqrt{3} & 1/\sqrt{3} & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} & i & 1/\sqrt{3} \\ \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & i\sqrt{3} \end{pmatrix}$

(d) $\begin{pmatrix} i/\sqrt{3} & 0 & i\sqrt{2}/\sqrt{3} \\ 0 & i & 0 \\ i\sqrt{2}/\sqrt{3} & 0 & -i/\sqrt{3} \end{pmatrix}$

14. Consider a symmetric matrix

$$M = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \end{pmatrix}$$

An orthogonal matrix O which can diagonalize this matrix by an orthogonal transformation $O^T M O$ is given by $O =$ **[TIFR 2023]**

(a) $\begin{pmatrix} \sqrt{1/3} & 0 & \sqrt{2/3} \\ 0 & 1 & 0 \\ \sqrt{2/3} & 0 & -\sqrt{1/3} \end{pmatrix}$

(b) $\begin{pmatrix} \sqrt{2/3} & 0 & \sqrt{1/3} \\ 0 & 1 & 0 \\ \sqrt{1/3} & 0 & -\sqrt{2/3} \end{pmatrix}$

(c) $\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$

(d) $\begin{pmatrix} 1/\sqrt{2} & 0 & i/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \end{pmatrix}$

15. Consider the following matrix

$$\mathbb{M} = \begin{pmatrix} 1 & 5 & -7 & 1 \\ 1 & 0 & 2 & 2 \\ 9 & -1 & 3 & 1 \\ 9 & 6 & -7 & -4 \end{pmatrix}$$

What is det $e^{\mathbb{M}}$? **[TIFR 2024]**

(a) e^{1210} (b) e

(c) 1 (d) e^{-1210}

16. The $n \times n (n > 4)$ matrix M, with all entries equal to 1 has: **[TIFR 2025]**

(a) Precisely $n-1$ degenerate eigenvalues and one other non-degenerate eigenvalue

(b) Precisely $n-2$ degenerate eigenvalues and two other non-degenerate eigenvalues

(c) Precisely 2 degenerate eigenvalues and $n-2$ other non-degenerate eigenvalues

(d) No degenerate eigenvalues

❖ Answer Key				
CSIR-NET PYQ				
1. b	2. a	3. c	4. d	5. a
6. c	7. a	8. a	9. b	10. d
11. b	12. d	13. c	14. d	15. d
16. b	17. d	18. d	19. b	20. d
21. d	22. c	23. b	24. a,b	25. a
26. b	27. d	28. a	29. c	30. d
31. b	32. d	33. c	34. a	
GATE PYQ				
1.	2. a	3. c	4. c	5.
6. d	7. d	8. c	9. b	10. a
11. a	12. c	13. d	14. a	15. d
16. a	17. d	18. b	19. b	20. c
21. c	22. d	23. b	24. 5	25. d
26. a	27. b	28. b	29. a	30. d
31. a,d	32. c	33. b	34. abc	
JEST PYQ				
1. d	2. c	3. b	4. a	5. b
6. a	7. b	8. d	9. c	10. a
11. b	12. b	13. c	14. c	15. 4
16. b	17. b			
TIFR PYQ				
1.	2.	3.	4. c	5. a
6. c	7. c	8. 050	9. b	10. a
11. a	12. a	13. d	14. c	15. c
16. a				

❖ CSIR-NET PYQ's

1. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0,1] is divided into 4 equal parts, the correct result is

[CSIR JUNE 2014]

(a) 0.683 (b) 0.667

(c) 0.657 (d) 0.638

2. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h=2$, is

[CSIR DEC 2015]

(a) 0.565 (b) 0.620

(c) 0.698 (d) 0.736

3. Consider the differential equation

$$\frac{dy}{dx} = x^2 - y$$

with the initial condition $y=2$ at $x=0$. Let $y_{(1)}$ and $y_{(1}$ 2) be the solutions at $x=1$ obtained using Euler's forward algorithm with step size 1 and 1/2 respectively.

[CSIR JUNE 2015]

The value of $\left(y_{(1)} - y_{(1/2)}\right)/y_{(1/2)}$ is

(a) $-\frac{1}{2}$ (b) -1

(c) $\frac{1}{2}$ (d) 1

4. In finding the roots of the polynomial $f(x) = 3x^3 - 4x - 5$ using the iterative Newton-Raphson method, the initial guess is taken to be $x=2$. In the next iteration its value is nearest to

[CSIR JUNE 2016]

(a) 1.671 (b) 1.656

(c) 1.559 (d) 1.551

5. A stable asymptotic solution of the equation

$$x_{n+1} = 1 + \frac{3}{1+x_n}$$

is $x=2$. If we take $x_n = 2+\varepsilon_n$ and $x_{n+1} = 2+\varepsilon_{n+1}$, where ε_n and ε_{n+1} are both small, the ratio $\varepsilon_{n+1}/\varepsilon_n$ is approximately

[CSIR DEC 2016]

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

6. Given the values $\sin 45^\circ = 0.7071, \sin 50^\circ - 0.7660, \sin 55^\circ = 0.8192$ and $\sin 60^\circ = 0.8660$, the approximate value of $\sin 52^\circ$, computed by Newton's forward difference method, is

[CSIR DEC 2016]

(a) 0.804 (b) 0.776

(c) 0.788 (d) 0.798

7. The interval [0,1] is divided into $2n$ parts of equal length to calculate the integral $\int_0^1 e^{2ix\pi}dx$ using Simpson's $\frac{1}{3}$-rule. What is the minimum value of n for the result to be exact?

[CSIR JUNE 2017]

(a) ∞ (b) 2

(c) 3 (d) 4

8. The differential equation

$$\frac{dy(x)}{dx} = \alpha x^2$$

, with the initial condition $y(0)=0$, is solved using Euler's method. If $y_E(x)$ is the exact solution and $y_N(x)$ the numerical solution obtained using n steps of equal length, then the relative error

$$\left|\frac{(y_N(x) - y_E(x))}{y_E(x)}\right|$$

is proportional to

[CSIR DEC 2017]

(a) $\frac{1}{n^2}$ (b) $\frac{1}{n^3}$

(c) $\frac{1}{n^4}$ (d) $\frac{1}{n}$

9. The interval [0,1] is divided into n parts of equal length to calculate the integral $\int_0^1 e^{i2\pi x}dx$ using the trapezoidal rule. The minimum value of n for which the result is exact, is

[CSIR DEC 2017]

(a) 2 (b) 3

(c) 4 (d) ∞

10. The fractional error in estimating the integral $\int_0^1 x dx$ using Simpson's $\frac{1}{3}$-rule, using a step size 0.1, is nearest to

[CSIR JUNE 2018]

(a) 10^{-4} (b) 0

(c) 10^{-2} (d) 3×10^{-4}

11. The value of the integral $\int_0^1 x^2 dx$, evaluated using the trapezoidal rule with a step size of 0.2, is

[CSIR DEC 2018]

(a) 0.30 (b) 0.39

(c) 0.34 (d) 0.27

12. If the Newton-Raphson method is used to find the positive root of the equation $x = 2\sin x$, the iteration equation is

[CSIR JUNE 2019]

(a) $x_{n+1} = \dfrac{2x_n - 2(\sin x_n + x_n \cos x_n)}{1 - 2\cos x_n}$

(b) $x_{n+1} = \dfrac{2(\sin x_n - x_n \cos x_n)}{1 - 2\cos x_n}$

(c) $x_{n+1} = \dfrac{x_n^2 - 1 + 2(\cos x_n - x_n \sin x_n)}{x_n - 2\sin x_n}$

(d) $x_{n+1} = \dfrac{x_n^2 - 1 - 2(\cos x_n + \sin x_n)}{x_n - 2\sin x_n}$

13. The positive zero of the polynomial $f(x) = x^2 - 4$ is determined using Newton-Raphson method, using an initial guess $x = 1$. Let the estimate, after two iterations, be $x^{(2)}$. The percentage error $\left|\frac{x^{(2)}-2}{2}\right| \times 100\%$ is

[CSIR DEC 2019]

(a) 7.5% (b) 5.0%

(c) 1.0% (d) 2.5%

14. Using the following values of x and $f(x)$

x	0	0.5	1.0	1.5
$f(x)$	1	a	0	$-5/4$

the integral $I = \int_0^{1.5} f(x)dx$, evaluated by the Trapezoidal rule, is $5/16$. The value of a is

[CSIR NOV 2020]

(a) 3/4 (b) 3/2

(c) 7/4 (d) 19/24

15. The Newton-Raphson method is to be used to determine the reciprocal of the number $x = 4$. If we start with the initial guess 0.20 then after the first iteration the reciprocal is

[CSIR NOV 2021]

(a) 0.23 (b) 0.24

(c) 0.25 (d) 0.26

16. The Newton-Raphson method is to be used to determine the reciprocal of the number $x = 4$. If we start with the initial guess 0.20 then after the first iteration the reciprocal is

[CSIR FEB 2022]

(a) 0.23 (b) 0.24

(c) 0.25 (d) 0.26

17. The bisection method is used to find a zero x_0 of the polynomial $f(x) = x^3 - x^2 - 1$. Since $f(1) = -1$, while $f(2) = 3$ the values $a = 1$ and $b = 2$ are chosen as the boundaries of the interval in which the x_0 lies. If the bisection method is iterated three times, the resulting value of x_0 is

[CSIR JUNE2023]

(a) $\frac{15}{8}$ (b) $\frac{13}{8}$

(c) $\frac{11}{8}$ (d) $\frac{9}{8}$

18. Given the data points

x	1	3	5
y	4	28	92

using Lagrange's method of interpolation, the value of y at $x=4$ is closest to

[CSIR DEC 2023]

(a)54 (b)55

(c)53 (d)56

19. A set of 100 data points yields an average $\bar{x}=9$ and a standard deviation $\sigma_x=4$. The error in the estimated mean is closest to

[CSIR JUNE 2024]

(a)3.0 (b)0.4

(c)4.0 (d)0.3

20. The integral $I=\int_0^1 \frac{2x}{1+x^2} dx$ is estimated using Simpson's $1/3^{\text{rd}}$ rule with a grid value of $h=0.5$. The difference $(I_{\text{estimated}}-I_{\text{exact}})$ is closest to

[CSIR JUNE 2024]

(a)0.007 (b)0.001

(c)0.0007 (d)-0.005

21. The following table shows the relationship between an independent quantity x and an experimentally measured quantity y.

x	0	1	2	3	4	5
y	0.1	2.1	8.1	17.9	32.2	49.7

The relationship between x and y is best represented by

[CSIR DEC 2024]

(a)$y\propto x^3$ (b)$y\propto e^x$

(c)$y\propto x^2$ (d)$y\propto\sqrt{x}$

❖ JEST PYQ's

1. The value $\int_{0.2}^{2.2} xe^x dx$ by using the one-segment trapezoidal rule is close to

[JEST 2014]

(a) 11.672 (b) 11.807

(c) 20.099 (d) 24.119

❖ TIFR PYQ's

1. Given the following xy data

x	1.0	2.0	3.0	4.0	5.0
y	0.002	0.601	0.948	1.21	1.42

which of the following would be the best curve, with constant positive parameters a and b, to fit this data?

[TIFR 2018]

(a) $y=ax-b$

(b) $y=a+\exp\ bx$

(c) $y=a\log_{10}\ bx$

(d) $y=a-\exp(-bx)$

2. The integral

$$I=\int_{1/2}^{3/4} dx \exp\left\{-\exp\left(\frac{1}{x}\right)\right\}$$

evaluates to $I=$

[TIFR 2021]

(a) $\exp\sqrt{2}$ (b) 0.00215

(c) 1.762633 (d) $-\exp(-1$

3. Given the following $x-y$ data table

x	1.0	2.0	3.0	4.0	5.0	6.0
y	0.602	0.984	1.315	1.615	1.894	2.157

which would be the best-fit curve, where a and b are constant positive parameters?

[TIFR 2021]

(a) $y=ax-b$ (b) $y=bx^{1/(1+a)}$

(c) $y=a+e^{bx}$ (d) $y=a\log_{10}\ bx$

❖ Answer Key				
CSIR-NET PYQ				
1. c	2. a	3. b	4. b	5. c
6. c	7. b	8. d	9. a	10. b
11. c	12. b	13. d	14. a	15. b
16. b	17. c	18. b	19. b	20. a
21. c				
JEST PYQ				
1. c				
TIFR PYQ				
1. c	2. b	3. b		

❖ **CSIR-NET PYQ's**

1. An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is

[CSIR DEC 2011]

(a) $\frac{1}{16}$ (b) $\frac{1}{36}$

(c) $\frac{1}{108}$ (d) $\frac{1}{216}$

2. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ?

[CSIR JUNE 2012]

(a) $100/\lambda$ (b) $10^4/\lambda^2$

(c) $10^4/\lambda$ (d) $1/\lambda$

3. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white?

[CSIR JUNE 2012]

(a) 34/70 (b) 41/70

(c) 36/70 (d) 29/70

4. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to

[CSIR JUNE 2012]

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$

(c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

5. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is

[CSIR DEC 2012]

(a) 1/8 (b) 5/8

(c) 3/16 (d) 5/16

6. Consider three particles A, B and C, each with an attribute S that can take two value ± 1. Let $S_A = 1$, $S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to $-S$ with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is

[CSIR JUNE 2013]

(a) 2/3 (b) 1/3

(c) 2/9 (d) 4/9

7. Let u be a random variable uniformly distributed in the interval [0,1] and $V = -c\ln u$, where c is a real constant. If V is to be exponentially distributed in the interval $[0, \infty)$ with unit standard deviation, then the value of c should be

[CSIR JUNE 2013]

(a) ln 2 (b) ½

(c) 1 (d) -1

8. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}$ and $\frac{6}{21}$ of turning up 1,2,3,4,5 and 6 , respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?

[CSIR DEC 2013]

(a) $\frac{144}{441}$ (b) $\frac{225}{441}$

(c) $\frac{221}{441}$ (d) $\frac{220}{441}$

9. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255,0.255,0.245, and 0.245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance

[CSIR DEC 2013]

(a) $\sqrt{2} \times 10^{-2}$Na in the north-cast direction

(b) $\sqrt{2N} \times 10^{-2}a$ in the north-east direction

(c) $2\sqrt{2} \times 10^{-2}$Na in the south-east direction

(d) 0

10. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?

[CSIR JUNE 2014]

(a) 3/8 (b) 5/16

(c) 1/4 (d) 1/16

11. Let

$$y = \frac{1}{2}(x_1 + x_2) - \mu$$

, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ. Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is

[CSIR JUNE 2014]

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

12. Two independent random variables m and n, which can take the integer values $0,1,2,\ldots,\infty$, follow the Poisson distribution, with distinct mean values μ and ν respectively. Then

[CSIR DEC 2014]

(a) the probability distribution of the random variable $l = m + n$ is a binomial distribution

(b) the probability distribution of the random variable $r = m - n$ is also a Poisson distribution

(c) the variance of the random variable $l = m + n$ is equal to $\mu + v$

(d) the mean value of the random variable $r = m - n$ is equal to 0.

13. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is

[CSIR DEC 2014]

(a) $n/3$ (b) $n/8$

(c) $2n/3$ (d) 0

14. Three real variables a, b and c are each randomly chosen from a uniform probability distribution in the interval [0,1]. The probability that $a + b > 2c$ is

[CSIR JUNE 2015]

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

15. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbor site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is

[CSIR DEC 2015]

(a) 12/27 (b) 27/64

(c) 3/8 (d) 9/16

16. Let X and Y be two independent random variables, each of which follow a normal distribution with the same standard deviation σ, but with means $+\mu$ and $-\mu$, respectively. Then the sum $X + Y$ follows a

[CSIR JUNE 2016]

(a) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$

(b) normal distribution with mean 0 and standard deviation 2σ

(c) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation 2σ

(d) normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$

17. A box of volume V containing N molecules of an ideal gas, is divided by a wall with a hole into two compartments. If the volume of the smaller compartment is $V/3$, the variance of the number of particles in it, is

[CSIR DEC 2016]

(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$

(c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$

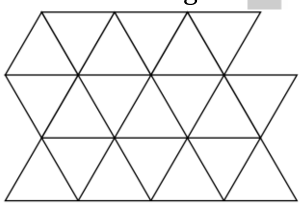
18. Consider two radioactive atoms, each of which has a decay rate of 1 per year. The probability that at least one of them decays in the first two years is

[CSIR DEC 2016]

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $1-e^{-4}$ (d) $(1-e^{-2})^2$

19. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities of moving to any of the nearest neighbor sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps?

[CSIR DEC 2016]

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$

(c) $\frac{1}{18}$ (d) $\frac{1}{12}$

20. The random variable $x(-\infty < x < \infty)$ is distributed according to the normal distribution

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

. The
probability density of the random variable $y = mx^2$ is

[CSIR JUNE 2017]

(a) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(b) $\frac{1}{2\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(c) $\frac{1}{\sqrt{2\sigma^2}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(d) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{\sigma^2}}, 0 \le y < \infty$

21. A random variable n obeys Poisson statistics. The probability of finding n 0 is 10^{-6}. The expectation value of n is nearest to

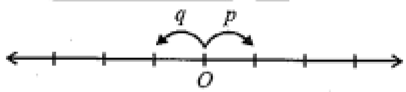
[CSIR JUNE 2017]

(a) 14 (b) 10^6

(c) e (d) 10^2

22. A particle hops on a one-dimensional lattice with lattice spacing a. The probability of the particle to hop to the neighbouring site to its right is p, while the corresponding probability to hop to the left is $q = 1-p$. The root-mean-squared deviation $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ in displacement after N steps, is

[CSIR DEC 2018]



(a) $a\sqrt{Npq}$ (b) $aN\sqrt{pq}$

(c) $2a\sqrt{Npq}$ (d) $a\sqrt{N}$

23. The standard deviation of the following set of data : {10.0,10.0,9.9,9.9,9.8,9.9,9.9,9.9,9.8,9.9}

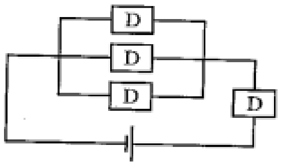
[CSIR DEC 2018]

(a) 0.10 (b) 0.07

(c) 0.01 (d) 0.04

24. In the following circuit, each device D may be an insulator with probability p, or a conductor with probability $(1-p)$.

[CSIR JUNE 2019]



The probability that a non-zero current flows through the circuit is

[CSIR JUNE 2019]

(a) $2-p-p^3$ (b) $(1-p)^4$

(c) $(1-p)^2p^2$ (d) $(1-p)(1-p^3)$

25. At each time step, a random walker in one-dimension either remains at the same point with probability $\frac{1}{4}$, or moves by a distance Δ to the right or left with probabilities 3/8 each. After N time steps, its root mean squared displacement is

[CSIR JUNE 2019]

(a)$\Delta\sqrt{N}$ (b)$\Delta\sqrt{\frac{9N}{16}}$

(c)$\Delta\sqrt{\frac{3N}{4}}$ (d) $\Delta\sqrt{\frac{3N}{8}}$

26. A box contains 5 white and 4 black balls. Two balls are picked together at random from the box. What is the probability that these two balls are of different colours?

[CSIR DEC 2019]

(a) 1/2 (b) 5/18

(c) 1/3 (d) 5/9

27. A particle hops randomly from a site to its nearest neighbor in each step on a square lattice of unit lattice constant. The probability of hopping to the positive x-direction is 0.3 , to the negative x direction is 0.2 , to the positive y-direction is 0.2 and to the negative y-direction is 0.3 . If a particle starts from the origin, its mean position after N steps is

[CSIR DEC 2019]

(a) $\frac{1}{10}N(-\hat{\imath}+\hat{\jmath})$ (b) $\frac{1}{10}N(\hat{\imath}-\hat{\jmath})$

(c) $N(0.3\hat{\imath}-0.2\hat{\jmath})$ (d) $N(0.2\hat{\imath}-0.3\hat{\imath})$

28. A basket consists of an infinite number of red and black balls in the proportion $p:(1-p)$. Three balls are drawn at random without replacement. The probability of their being two red and one black is a maximum for

[CSIR JUNE 2020]

(a) $p=\frac{3}{4}$ (b) $p=\frac{3}{5}$

(c)$p=\frac{1}{2}$ (d) $p=\frac{2}{3}$

29. A discrete random variable X takes a value from the set $\{-1,0,1,2\}$ with the corresponding probabilities $p(X)=\frac{3}{10},\frac{2}{10},\frac{2}{10}$ and $\frac{3}{10}$, respectively. The probability distribution $q(Y)=(q(0),q(1),q(4))$ of the random variable $Y=X^2$ is

[CSIR JUNE 2021]

(a) $\left(\frac{1}{5},\frac{3}{5},\frac{1}{5}\right)$ (b) $\left(\frac{1}{5},\frac{1}{2},\frac{3}{10}\right)$

(c) $\left(\frac{2}{5},\frac{2}{5},\frac{1}{5}\right)$ (d) $\left(\frac{3}{10},\frac{3}{10},\frac{2}{5}\right)$

30. The position of a particle in one dimension changes in discrete steps. With each step it moves to the right, however, the length of the step is drawn from a uniform distribution from the interval

$$\left[\lambda-\frac{1}{2}w,\lambda+\frac{1}{2}w\right]$$

, where λ and w are positive constants. If X denotes the distance from the starting point after N steps, the standard deviation $\sqrt{\langle X^2\rangle-\langle X\rangle^2}$ for large values of N is

[CSIR FEB 2021]

(a) $\frac{\lambda}{2}\times\sqrt{N}$ (b) $\frac{\lambda}{2}\times\sqrt{\frac{N}{3}}$

(c) $\frac{w}{2}\times\sqrt{N}$ (d) $\frac{w}{2}\times\sqrt{\frac{N}{3}}$

31. A walker takes steps, each of length L, randomly in the directions along east, west, north and south. After four steps its distance from the starting point is d. The probability that d $\leq$ 3L is

[CSIR JUNE 2022]

(a)63/64, (b)57/64

(c)59/64 (d)55/64

32. A bucket contains 6 red and 4 blue balls. A ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

[CSIR JUNE 2022]

(a)4/11 (b)2/11

(c)1/4 (d)3/4

33. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from J1 is

[CSIR JUNE2023]

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{4}{7}$

34. Two random walkers A and B walk on a one-dimensional lattice. The length of each step taken by A is one, while the same for B is two, however, both move towards right or left with equal probability. If they start at the same point, the probability that they meet after 4 steps, is

[CSIR JUNE2023]

(a) $\frac{9}{64}$ (b) $\frac{5}{32}$

(c) $\frac{11}{64}$ (d) $\frac{3}{16}$

35. A random variable Y obeys a normal distribution

$$P(y)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(Y-\mu)^2/2\sigma^2}$$

The mean value of e^Y is

[CSIR JUNE2023]

(a)$e^{\mu+\frac{\sigma^2}{2}}$ (b) $e^{\mu-\sigma^2}$

(c)$e^{\mu+\sigma^2}$ (d) $e^{\mu-\frac{\sigma^2}{2}}$

36. Probability density function of a variable x is given by

$$P(x)=\frac{1}{2}[\delta(x-a)+\delta(x+a)]$$

. The variance of x is **[CSIR JUNE 2024]**

(a)a^2 (b)0

(c)$2a^2$ (d)$\frac{a^2}{2}$

37. A class has 60% boys and 40% girls. In an examination 8% of the boys and 12% of the girls got an ' A ' grade. If a randomly selected student had an ' A ' grade, what is the probability that the student is male?

s[CSIR DEC 2024]

(a)0.7 (b)0.6

(c)0.4 (d)0.5

38. A random walker takes a step of unit length towards right or left at any discrete time step. Starting from $x=0$ at time $t=0$, it goes right to reach $x=1$ at $t=1$. Hereafter if it repeats the direction taken in the previous step with probability p, the probability that it is again at $x=1$ at $t=3$ is

[CSIR JUNE 2024]

(a)$1-p$ (b)$(1-p)^2$

(c)$2p(1-p)$ (d)$4p^2(1-p)$

39. A bucket contains 6 red and 4 blue balls. A ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

(a)4/11 (b)2/11

(c)1/4 (d)3/4

40. From a straight-line segment of unit length, three points are chosen at random, one after another. The probability that they are in increasing order is

[CSIR JUNE 2025]

(a) $\frac{1}{3}$ (b) $\frac{1}{8}$

(c) $\frac{1}{9}$ (d) $\frac{1}{6}$

41. Two discrete time random walkers start from the point $x = 0$ at time $t = 0$ taking discrete steps of unit length along the x axis. The first walker is unbiased and the second walker is biased to move towards the right with probability p. The probability that they are at a distance of 2 units from each other at both time steps $t = 1$ and $t = 2$ is

[CSIR JUNE 2025]

(a) $\frac{1}{4}$ (b) $\frac{1}{2} - \frac{p}{2}$

(c) $1 - \frac{3p}{4}$ (d) $\frac{p}{2}$

42. Let $p(x)$ be the probability density function for a positive real variable x, and $g(\alpha) = \int_0^\infty p(x)e^{-\alpha x}dx$. If $g'(\alpha)$ and $g''(\alpha)$ are respectively first and second derivatives of $g(\alpha)$ with respect to α, which of the following gives the variance of x ?

[CSIR DEC 2025]

(a) $g''(0) - [g'(0)]^2$ (b) $g''(0) + [g'(0)]^2$

(c) $[g''(0) - g'(0)]^2$ (d) $\frac{g''(0)}{g'(0)g(0)}$

❖ JEST PYQ's

1. An unbiased die is cast twice. The probability that the positive difference (bigger-smaller) between the two numbers is 2 is

[JEST 2012]

(a) 1/9 (b) 2/9

(c) 1/6 (d) 1/3

2. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?

[JEST 2013]

(a) 0.99 (b) 0.925

(c) 0.075 (d) 0.01

3. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to

[JEST 2013]

(a) 0.07 (b) 0.60

(c) 0.36 (d) 0.19

4. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, the mean of x is

(a) λ (b) 2λ

(c) $\lambda/2$ (d) 0

5. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x axis. The probability that they meet after n steps is

[JEST 2013]

(a) $\frac{1}{4^n}\frac{2n!}{n!}$ (b) $\frac{1}{2^n}\frac{2n!}{n!}$

(c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

6. If two ideal dice are rolled once, what is the probability of getting at least one ' 6 "?

[JEST 2015]

(a) $\frac{11}{36}$ (b) $\frac{1}{36}$

(c) $\frac{10}{36}$ (d) $\frac{5}{36}$

7. You receive on average 5 emails per day during a 365 days year. The number of days on average on which you do not receive any emails in that year are:

[JEST 2016]

(a) more than 5 (b) more than 2

(c) 1 (d) none of these

8. A gas contains particle of type A with fraction 0.8, and particles of type B with fraction 0.2. the probability that among 3 randomly chosen particles at least one is of type A is:

[JEST 2016]

(a) 0.8 (b) 0.25

(c) 0.33 (d) 0.992

9. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[\frac{(x^2+\mu x)}{(2\sigma^2)}\right]$ is:

[JEST 2016]

(a) 0 (b) $\frac{\mu}{2}$

(c) $\frac{-\mu}{2}$ (d) σ

10. Suppose that we toss two fair coins hundred times each. The probability that the same number of heads occur for both coins at the end of the experiment is

[JEST 2017]

$(a)\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}$

(b)$2\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

(c)$\frac{1}{2}\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

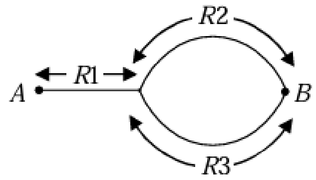
(d) $\left(\frac{1}{4}\right)^{100} \sum_{n=0}^{100} \binom{100}{n}^2$

11. An electronic circuit with 10000 components performs its intended function successfully with a probability 0.99 if there are no faulty components in the circuit. The probability that there are faulty components is 0.05. if there are faulty components, the circuit perform successfully with a probability 0. . the probability that the circuit performs successfully is x/10000. What is x ?

[JEST 2018]

12. A person plans to go from town A to town B by taking either the route $(R1 + R2)$ with probability $\frac{1}{2}$ or the route $(R1 + R3)$ with probability $\frac{1}{3}$ that $R2$ is blocked, and a probability $\frac{1}{3}$ that $R3$ is blocked. What is the probability that he/she would reach town B ?

[JEST 2019]



(a) $\frac{8}{9}$ (b) $\frac{1}{3}$

(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

13. The persons named $A, B, C, D, E, F, G, H, I, J$ have come for an interview. They are being called one by one to the interview panel at random. What is the probability that C gives interview before A and A gives before F ?

[JEST 2020]

(a) $\frac{1}{6}$ (b) $\frac{1}{10}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

14. If x and y have the joint probability distribution

$$f(x, y) = \frac{3}{4} + xy$$

for $0 < x, y < 1$, and $f(x, y) = 0$ otherwise. What is the probability that y assumes a value greater than $\frac{1}{2}$, given that x is equal to $\frac{1}{2}$

[JEST 2020]

(a) $\frac{6}{16}$ (b) $\frac{7}{16}$

(c) $\frac{8}{16}$ (d) $\frac{9}{16}$

15. A particle is moving on a one-dimensional discrete lattice with lattice spacing unity. It can move from a site to its nearest neighbour site every 1/5 seconds with p being the probability to move right and $q = (1 - p)$ being the probability to move left. Consider that the particle starts at origin, $x = 0$ at time $t = 0$. Taking $p = \frac{3}{4}$, calculate the variance $\langle(x - \langle x\rangle)^2\rangle$ at time $t = 5 \times 10^4$ seconds, where $\langle x\rangle$ is the average position.

[JEST 2020]

16. The six faces of a cube are painted violet, blue, red, green, yellow and orange. If the cube is rolled 4 times, what is the probability that the green face appears exactly 3 times?

[JEST 2021]

(a) $\frac{3}{24}$ (b) $\frac{5}{124}$

(c) $\frac{5}{324}$ (d) $\frac{15}{222}$

17. The probability that you get a sum m from a throw of two identical fair dice is P_m. If the dice have 6 (six) faces labeled by 1,2, … 6, which of the following statements is correct?

[JEST 2022]

(a) $P_9 = P_3$ (b) $P_9 = P_4$

(c) $P_9 = P_5$ (d) $P_9 = P_6$

18. If θ and ϕ are respectively the polar and azimuthal angles on the unit sphere, what is $\langle\cos^2(\theta)\rangle$ and $\langle\sin^2(\theta)\rangle$, where $\langle\mathcal{O}\rangle$ denotes the average of $\mathcal{O}$?

(a) $\langle\cos^2(\theta)\rangle = 3/4$ and $\langle\sin^2(\theta)\rangle = 1/4$

(b) $\langle\cos^2(\theta)\rangle = 1/2$ and $\langle\sin^2(\theta)\rangle = 1/2$

(c) $\langle\cos^2(\theta)\rangle = 1/3$ and $\langle\sin^2(\theta)\rangle = 2/3$

(d) $\langle\cos^2(\theta)\rangle = 2/3$ and $\langle\sin^2(\theta)\rangle = 1/3$

19. Two fair six-faced dice are thrown simultaneously. The probability that one of the dice yields an outcome that is a multiple of 2 and the other yields a multiple of 3 is:

[JEST 2023]

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{11}{36}$

20. A classical particle undergoing simple harmonic motion is confined to the region $(-a, a)$ on the X-axis. If a snapshot of the particle is taken at a random instant of time, what is the probability that it would be found in the region $\left(\frac{a}{2}, a\right)$?

[JEST 2024]

(a) $\frac{2}{5}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

21. Consider the rotation matrix $R = \begin{pmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix}$.

Let ϕ be the angle of rotation. What is the value of $\sec^2 \phi$?

[JEST 2024]

❖ TIFR PYQ's

1. A 100 page book is known to have 200 printing errors distributed randomly through the pages. The probability that one of the pages will be found to be completely free of errors is closest to

[TIFR 2011]

(a) 67% (b) 50%

(c) 25% (d) 13%

2. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle/\hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$

(c) $\frac{1}{\lambda}$ (d) 1

3. A random number generator outputs +1 or -1 with equal probability every time it is run. After it is run 6 times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other.

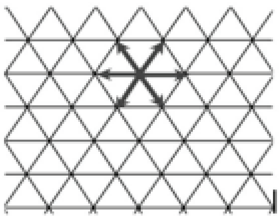
[TIFR 2015]

(a) 1/2 (b) 5/6

(c) 5/16 (d) 15/32

4. In a triangular lattice a particle moves from a lattice point to any of its 6 neighboring points with equal probability, as shown in the figure on the right.
The probability that the particle is back at its starting point after 3 moves is

[TIFR 2016]



(a) 5/18 (b) 1/6

(c) 1/18 (d) 1/36

5. A British coin has a portrait of Queen Elizabeth II on the 'heads' side and 'ONE POUND' written on the 'tails' side, while an Indian coin has a portrait of Mahatma Gandhi on the 'heads' side and '10 RUPEES' written on the 'tails' side (see below). **[TIFR 2019]**

These two coins are tossed simultaneously twice in succession.
The result of the first toss was 'heads' for both the coins. The probability that the result of the second toss had a '10 RUPEES' side is

(a) 1/2 (b) 4/7

(c) 3/5 (d) 2/3

6. In a country, the fraction of population infected with Covid-19 is 0.2. It is also known that out of the people who are infected with Covid-19, only a fraction 0.3 show symptoms of the disease, while the rest do not show any symptoms.
If you randomly select a citizen of this country, the probability that this person will NOT show symptoms of Covid-19 is **[TIFR 2021]**

(a) 0.94 (b) 0.56

(c) 0.86 (d) 0.80

7. Two students A and B, measure the time period of a simple pendulum in the laboratory using the same stopwatch but following two different methods.

- Student A measures the time taken for one oscillation and repeats it for N_A number of times and finds the average.
- Student B, on the other hand, measures the time taken for N_B number of oscillations and then computes the period.

Given that $N_A, N_B \gg 1$, to ensure that both students measure the time period with the same uncertainty, the relation between N_A and N_B must be

[TIFR 2022]

(a) $N_A = N_B^2$ (b) $N_A = \sqrt{N_B}$

(c) $N_A = N_B$ (d) $\ln\ 2N_A = N_B$

8. In a standardized entrance exam, the passing rates for the past 10 years are tabulated below.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Passing Rate	22 %	16 %	23 %	21%	22 %	14 %	17 %	20 %	24 %	21 %

If 1000 candidates appear for the exam every year, the probability that more than 250 students will pass the exam this year is about

[TIFR 2022]

(a) 6% (b) 20%

(c) 25% (d) 0.1%

9. A random positive variable x follows an exponential distribution

$$p(x) \propto e^{-\lambda x}$$

with $\lambda > 0$. The probability of observing an event $x > 3\langle x \rangle$, where $\langle x \rangle$ represents the average value of x, is

[TIFR 2023]

(a) $1/e^3$ (b) $1/e^4$

(c) $1/e$ (d) $1/e^2$

10. A student measures the radioactive decay of a material with a half-life of 13,000 years with a Geiger counter. In the laboratory notebook, the student records the following number of decays every 10 seconds:

158, 146, 145, 163, 154, 163, 160, 160, 152, 157, 154, 156, 149, 168, 152

The teacher suspects that the experiment was not done properly and the student created the numbers manually.

Why would the teacher have such a suspicion? **[TIFR 2023]**

(a) The variance is much less than the mean, unlike what is expected for a Poisson distribution.

(b) The standard deviation is much less than the variance, as expected for a Poisson distribution.

(c) The median is less than the mean, unlike what is expected for a Poisson distribution.

(d) The median is greater than the mean, as expected for a Poisson distribution.

11. Consider two random variables x and y described by the joint distribution

$$P(x,y) = \frac{1}{2\pi\sqrt{1-a^2}} e^{\frac{2axy-x^2-y^2}{2(1-a^2)}}$$

with $0 < a < 1$. If the above distribution is written in terms of orthogonal coordinates $z = x - y$ and $u = x + y$, the probability distribution in z is given by: **[TIFR 2025]**

(a) A Gaussian with mean 0 and standard deviation $\sqrt{2(1-a)}$

(b) A Gaussian with mean $\sqrt{a}$ and standard deviation $\sqrt{2(1-a)}$

(c) A Gaussian with mean 0 and standard deviation $\sqrt{2(1-a^2)}$

(d) Not a Gaussian distribution

12. Consider a particle P moving on a one-dimensional discrete lattice with lattice constant a. P can hop from one site to a neighbouring site. The probabilities of moving to the right and left are p and $q = 1 - p$, respectively. Starting from the origin $x = 0$ at time $t = 0$, what is the mean square displacement $\langle(x - \langle x\rangle)^2\rangle$ after N steps, where $\langle x\rangle$ is the average position at time t ?

(a) $4Na^2pq$ (b) $4Na^2(p-q)$

(c) $2Na^2pq$ (d) $2Na^2(p-q)$

❖ Answer Key

CSIR-NET PYQ				
1. b	2. c	3. b	4. a	5. d
6. d	7. c	8. b	9. a	10. a
11. b	12. c	13. a	14. c	15. d
16. d	17. b	18. c	19. c	20. a
21. a	22. c	23. b	24. d	25. c
26. d	27. b	28. d	29. b	30. d
31. d	32. b	33. d	34. c	35. a
36. a	37. d	38. a	39. b	40. d
41. a	42. a			
JEST PYQ				
1. a	2. c	3. d	4. b	5. a
6. a	7. b	8. d	9. c	10. d
11. 9555	12. c	13. a	14. d	15. 46875
16. c	17. c	18. a	19. d	20. d
21. 4				
TIFR PYQ				
1. d	2. a	3. c	4. c	5. b
6. a	7. d	8. a	9. a	10. a
11. a	12. a			

❖ **CSIR-NET PYQ's**

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by

[CSIR JUNE 2011]

(a) $\dfrac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ (b) $\dfrac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$

(c) $\dfrac{(\vec{a}\cdot\vec{b})\vec{b}}{b^2}$ (d) $\dfrac{(\vec{b}\cdot\vec{a})\vec{a}}{a^2}$

2. The equation of the plane that is tangent to the surface xyz = 8 at the point (1,2,4) is

[CSIR DEC 2011]

(a) $x+2y+4z=12$ (c) $x+4y+2z=12$

(b) $4x+2y+z=12$ (d) $x+y+z=7$

3. A vector perpendicular to any vector that lies on the plane defined by $x+y+z=5$, is

[CSIR JUNE 2012]

(a) $\hat{\imath}+\hat{\jmath}$ (b) $\hat{\jmath}+\hat{k}$

(c) $\hat{\imath}+\hat{\jmath}+\hat{k}$ (d) $2\hat{\imath}+3\hat{\jmath}+5\hat{k}$

4. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

, is **[CSIR DEC 2012]**

(a) $\dfrac{bc\hat{\imath}+ca\hat{\jmath}+ab\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (b) $\dfrac{a\hat{\imath}+b\hat{\jmath}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

(c) $\dfrac{b\hat{\imath}+c\hat{\jmath}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\dfrac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$

5. A unit vector n̂ on the xy-plane is an angle of 720° with respect to i. The angle between the vectors $\vec{u}=a\hat{\imath}+b\hat{n}$ and $\vec{v}=a\hat{n}+b\hat{\imath}$ will be 60° if

[CSIR JUNE 2013]

(a) $b=\sqrt{3}a/2$ (b) $b=2a/\sqrt{3}$

(c) $b=a/2$ (d) $b=a$

6. If $\mathbf{A}=\hat{\imath}yz+\hat{\jmath}xz+\hat{k}xy$, then the integral $\oint_C A dl$ (where C is along the perimeter of a rectangular area bounded by $x=0, x=a$ and $y=0, y=b$) is **[CSIR DEC 2013]**

(a) $\frac{1}{2}(a^3+b^3)$ (b) $\pi(ab^2+a^2b)$

(c) $\pi(a^3+b^3)$ (d) 0

7. If $\vec{A}=yz\hat{\imath}+zx\hat{\jmath}+xy\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}\cdot\vec{d\ell}$ is

[CSIR JUNE 2014]

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

8. Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and $r=|\vec{r}|$. Then

[CSIR DEC 2014]

(a) $\vec{\nabla}\cdot\bar{r}=0$ and $\vec{\nabla}\times\vec{r}=\vec{r}/r$

(b) $\vec{\nabla}\cdot\vec{r}=0$ and $\nabla^2 r=0$

(c) $\vec{\nabla}\cdot\bar{r}=3$ and $\nabla^2\bar{r}=\vec{r}/r^2$

(d) $\vec{\nabla}\cdot\vec{r}=3$ and $\vec{\nabla}\times\vec{r}=0$

9. The two vectors $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}b\\c\end{pmatrix}$ are orthogonal if

[CSIR JUNE 2017]

(a) $a=\pm1, b=\pm1/\sqrt{2}, c=\pm1/\sqrt{2}$

(b) $a=\pm1, b=\pm1, c=0$

(c) $a=\pm1, b=0, c=\pm1$

(d) $a=\pm1, b=\pm1/2, c=1/2$

10. Let A be a non-singular 3×3 matrix, the columns of which are denoted by the vectors $\vec{a},\vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u},\vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $(A^T)^{-1}$. Which of the following is true?

[CSIR DEC 2017]

(a) $\vec{u}\cdot\vec{a}=0, \vec{u}\cdot\vec{b}=0, \vec{u}\cdot\vec{c}=1$

(b) $\vec{u}\cdot\vec{a}=0, \vec{u}\cdot\vec{b}=1, \vec{u}\cdot\vec{c}=0$

(c) $\vec{u}\cdot\vec{a}=1, \vec{u}\cdot\vec{b}=0, \vec{u}\cdot\vec{c}=0$

(d) $\vec{u}\cdot\vec{a}=0, \vec{u}\cdot\vec{b}=0, \vec{u}\cdot\vec{c}=0$

11. Consider the three vectors $\vec{v}_1 = 2\hat{\imath}+3\hat{k}, \vec{v}_2 = \hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{v}_3 = 5\hat{\imath}+\hat{\jmath}+\alpha\hat{k}$, where $\hat{\imath},\hat{\jmath}$ and $\hat{k}$ are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of α is

[CSIR JUNE 2018]

(a) 31/4 (b) 23/4

(c) 27/4 (d) 0

12. Consider the set of polynomials $\{x(t)=a_0+a_1t+\cdots+a_{n-1}t^{n-1}\}$ in t of degrees less than n, such that $x(0)=0$ and $x(1)=1$. This set

[CSIR DEC 2019]

(a) constitutes a vector space of dimension n

(b) constitutes a vector space of dimension $n-1$

(c) constitutes a vector space of dimension $n-2$

(d) does not constitute a vector space

13. Two time dependent non-zero vectors $\vec{u}(t)$ and $\vec{v}(t)$, which are not initially parallel to each other, satisfy

$$\vec{u}\times\frac{d\vec{v}}{dt}-\vec{v}\times\frac{d\vec{u}}{dt}=0$$

at all time t. If the area of the parallelogram formed by $\vec{u}(t)$ and $\vec{v}(t)$ be $A(t)$ and the unit normal vector to it be $\hat{n}(t)$, then

[CSIR JUNE 2020]

(a) $A(t)$ increases linearly with t, but $\hat{n}(t)$ is a constant

(b) $A(t)$ increases linearly with t, and $\hat{n}(t)$ rotates about $\vec{u}(t)\times\vec{v}(t)$

(c) $A(t)$ is a constant, but $\hat{n}(t)$ rotates about $\vec{u}(t)\times\vec{v}(t)$

(d) $A(t)$ and $\hat{n}(t)$ are constants

14. The volume integral

$$I=\iiint_V A\cdot(\nabla\times A)d^3x$$

is over a region V bounded by a surface $\sum$ (an infinitesimal area element being $\hat{\mathbf{n}}ds$, where $\hat{\mathbf{n}}$ is the outward unit normal). If it changes to $I+\Delta I$, when the vector $\mathbf{A}$ is changed to $\mathbf{A}+\nabla\Lambda$, then ΔI can be expressed as

[CSIR FEB 2021]

(a) $\iiint_V \nabla\cdot(\nabla\Lambda\times A)d^3x$ (b) $-\iiint_V \nabla^2\Lambda d^3x$

(c) $-\oiint_\Sigma(\nabla\Lambda\times\mathbf{A})\cdot\hat{\mathbf{n}}ds$ (d) $\oiint_z \nabla\Lambda.\hat{\mathbf{n}}ds$

15. The volume of the region common to the interiors of two infinitely long cylinders defined by $x^2+y^2=25$ and $x^2+4z^2=25$ is best approximated by

[CSIR FEB 2022]

(a) 225 (b) 333

(c) 423 (d) 625

16. n integral is given by

$$\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\exp\left[-(x^2+y^2+2axy)\right]$$

where a is a real parameter. The full range of values of a for which the integral is finite, is

[CSIR JUNE 2024]

(a) $-\infty<a<\infty$ (b) $-2<a<2$

(c) $-1<a<1$ (d) $-1\le a\le 1$

17. Vorticity of a vector field $\vec{B}$ is defined as

$$\vec{V}=\vec{\nabla}\times\vec{B}$$

. Given $\vec{B}=kxyz\hat{r}$, where k is a constant, which one of the following is correct?

[CSIR JUNE 2024]

(a)Vorticity is a null vector for all finite x,y,z

(b)Vorticity is parallel to the vector field everywhere

(c)The angle between vorticity and vector field depends on x,y,z

(d)Vorticity is perpendicular to the vector field everywhere

❖ GATE PYQ's

1. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the position vector, then the value of the following integral $\iint_S \vec{r}\cdot\hat{n}dS$ is

[GATE 2001]

(a) V (b) 2 V

(c) 0 (d) 3 V

2. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

[GATE 2001]

(a) The three vectors are orthonormal

(b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as a linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

3. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals

[GATE 2001]

(a) 1 (b) 3

(c) 0 (d) -3

4. Given $\vec{A} = y^2\hat{e}_x + 2yx\hat{e}_y + (xye^z - \sin x)\hat{e}_z$, calculate the value of $\iint_S(\nabla\times\vec{A})\cdot\hat{n}ds$ over the part of the sphere $x^2+y^2+z^2=1$ above the xoy plane.

[GATE 2001]

5. A vector $\vec{A} = (5x+2y)\hat{\imath} + (3y-z)\hat{\jmath} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value

[GATE 2002]

(a) 4 (b) -4

(c) 8 (d) -8

6. Which of the following vectors is orthogonal to the vector $(a\hat{\imath} + b\hat{\jmath})$, where a and $b(a \neq b)$ are constants, and $\hat{\imath}$ and $\hat{\jmath}$ are unit orthogonal vectors?

[GATE 2002]

(a) $-b\hat{\imath} + a\hat{\jmath}$ (b) $-a\hat{\imath} + b\hat{\jmath}$

(c) $-a\hat{\imath} - b\hat{\jmath}$ (d) $-b\hat{\imath} - a\hat{\jmath}$

7. The unit vector normal to the surface $3x^2 + 4y = z$ at the point $(1,1,7)$ is

[GATE 2002]

(a) $(-6\hat{\imath} + 4\hat{\jmath} + \hat{k})/\sqrt{53}$

(b) $(4\hat{\imath} + 6\hat{\jmath} - \hat{k})/\sqrt{53}$

(c) $(6\hat{\imath} + 4\hat{\jmath} - \hat{k})/\sqrt{53}$

(d) $(4\hat{\imath} + 6\hat{\jmath} + \hat{k})/\sqrt{53}$

8. The two vectors $P = i, q = (i+j)/\sqrt{2}$ are

[GATE 2003]

(a) related by a rotation

(b) related by a reflection through the xy-plane

(c) related by an inversion

(d) not linearly independent

9. The curl of the vector $A = zi + xj + yk$ is given by

[GATE 2003]

(a) $i + j + k$ (b) $i - j + k$

(c) $i + j - k$ (d) $-i - j - k$

10. The surface integral of this vector over the surface of a cube of size a and centered at the origin

[GATE 2003]

(a) 0 (b) 2π

(c) $2\pi a^3$ (d) 4π

11. Which one of the following is NOT correct?

[GATE 2003]

(a) Value of the line integral of this vector around any closed curve is zero

(b) This vector can be written as the gradient of some scalar function

(c) The line integral of this vector from point P to point Q is independent of the path taken

(d) This vector can represent the magnetic field of some current distribution

Data for Q. No. 12 to 13

Consider the vector $V = r/r^3$

12. The surface integral of this vector over the surface of a cube of size a and centered at the origin

[GATE 2003]

(a) 0 (b) 2π

(c) $2\pi a^3$ (d) 4π

13. Which one of the following is NOT correct?

[GATE 2003]

(a) Value of the line integral of this vector around any closed curve is zero

(b) This vector can be written as the gradient of some scalar function

(c) The line integral of this vector from point P to point Q is independent of the path taken

(d) This vector can represent the magnetic field of some current distribution

14. For the function $\phi = x^2y + xy$, the value of $|\vec{\nabla}\phi|$ at $x = y = 1$ is

[GATE 2004]

(a) 5 (b) $\sqrt{5}$

(c) 13 (d) $\sqrt{13}$

15. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2,0) is

[GATE 2005]

(a) $\frac{(\hat{\imath} + \hat{\jmath})}{\sqrt{2}}$ (b) $-\hat{\imath}$

(c) $-\hat{\jmath}$ (d) $\hat{\jmath}$

16. If a vector field $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is

[GATE 2005]

(a) 0 (b) $\hat{\imath}$

(c) $2\hat{\jmath}$ (d) $3\hat{k}$

17. Given the four vectors

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}, u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$$

the linearly dependent pair is

[GATE 2005]

(a) u_1u_2 (b) u_1u_3

(c) u_1u_4 (d) u_3u_4

18. The value of $\oint_c \vec{A} \cdot d\vec{l}$ along a square loop of side L in a uniform field $\vec{A}$ is

[GATE 2006]

(a) 0 (b) 2LA

(c) 4LA (d) L^2 A

19. A linear transformation T, defined as

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$$

, transform a vector $\vec{x}$ for a 3-dimensional real space to a 2-dimensional real space. The transformation matrix T is

[GATE 2006]

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

20. The value of

$$\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$$

where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is

[GATE 2006]

(a) 0 (b) π

(c) 4π (d) 8π

21. If $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, then

[GATE 2007]

(a) $\vec{\nabla} \cdot \vec{r} = 0$ and $\vec{\nabla}|\vec{r}| = \vec{r}$

(b) $\vec{\nabla} \cdot \vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \hat{r}$

(c) $\vec{\nabla} \cdot \vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

(d) $\vec{\nabla} \cdot \vec{r} = 3$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

22. A vector field is defined everywhere as

$$\vec{F} = \frac{y^2}{L}\hat{\imath} + z\hat{k}$$

The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is

(a) $2L^3$ (b) $4L^3$

(c) $8L^3$ (d) $10L^3$

23. Consider a vector $\vec{p} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ in the coordinate system $(\hat{\imath}, \hat{\jmath}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{\imath}', \hat{\jmath}', \hat{k}')$ is

[GATE 2007]

(a) $(1-\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{\imath}' + (3+\sqrt{3})\hat{\jmath}' + 2\hat{k}'$

(d) $(1-\sqrt{3})\hat{\imath}' + (3-\sqrt{3})\hat{\jmath}' + 2\hat{k}'$

24. The curl of a vector field $\vec{F}$ is $2\hat{x}$. Identify the appropriate vector field $\vec{F}$ from the choices given below.

[GATE 2008]

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2\hat{x} + 5y\hat{z}$

25. Using the scalar product obtained in the above question, identify the subspace of V that is orthogonal to $(1 + x)$:

[GATE 2008]

(a) $\{f(x): b(1-x) + cx^2; b, c \in R\}$

(b) $\{f(x): b(1-2x) + cx^2; b, c \in R\}$

(c) $\{f(x): b + cx^2; b, c \in R\}$

(d) $\{f(x): bx + cx^2; b, c \in R\}$

26. The value of the contour integral, $\left|\int_c \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with centre at the origin is

[GATE 2009]

(a) 2π (b) $r^2/2$

(c) πr^2 (d) r

27. Consider the set of vectors in three dimensional real vector space $\mathbb{R}^3, S = \{(1,1),, (1,-1,1), (1,1,-1)\}$. Which one of the following statements is true? S is not a linearly independent set

[GATE 2009]

(a) S is a basis for $\mathbb{R}^3$

(b) The vectors in S are orthogonal

(c) An orthogonal set of vectors cannot be generated from S

(d) Consider the set of vectors in 3 - dimensional real vector space $\mathbb{R}^3$

28. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement **[GATE 2009]**

(a) Circulation of $\vec{E}$ is zero

(b) $\vec{E}$ can always be expressed as the gradient of a scalar field

(c) The potential difference between any two arbitrary points in the region R is zero

(d) The word done in a closed path lying entirely in R is zero

29. Consider an anti-symmetric tensor P_0 with the indices i and j running from 1 to 5. The number of independent components of the tensor is

[GATE 2010]

(a) 3 (b) 10

(c) 9 (d) 6

30. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that

[GATE 2011]

(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla} \cdot \vec{F} = 0$

(c) $\vec{\nabla} V = 0$ (d) $\nabla^2 V = 0$

31. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1,1,1)$ is

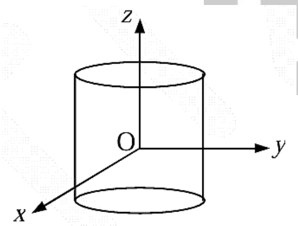
[GATE 2011]

(a) $\frac{\hat{\imath} + \hat{\jmath} - \hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{\imath} + \hat{\jmath} - \hat{k}}{\sqrt{6}}$

(c) $\frac{\hat{\imath} + 2\hat{\jmath} - \hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{\imath} + 2\hat{\jmath} - \hat{k}}{3}$

32. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_S \vec{r} \cdot \hat{n} ds$ over the closed surface of the cylinder is

[GATE 2011]



(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$

(c) $2\pi a^2 h$ (d) 0

33. Identify the correct statement for the following vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath}$

[GATE 2012]

(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

34. The number of independent components of the symmetric tensor A_{ij} with indices $i, j = 1,2,3$ is

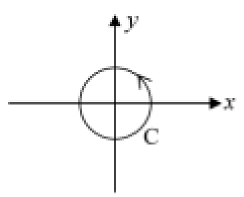
[GATE 2012]

(a) 1 (b) 3

(c) 6 (d) 9

35. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F} \cdot d\vec{r}$, where C is a circle of unit radius centered at origin is

[GATE 2012]



(a) 0 (b) $2\pi B_0$

(c) $-2\pi B_0$ (d) 1

36. If $\vec{A}$ and $\vec{B}$ are constants vectors then $\nabla(\vec{A} \cdot \vec{B} \times \vec{r})$ is

[GATE 2013]

(a) $\vec{A} \cdot \vec{B}$ (b) $\vec{A} \times \vec{B}$

(c) $\vec{r}$ (d) zero

37. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1) is

[GATE 2014]

(a) $\frac{\hat{x} + \hat{y} - \hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x} - \hat{y} - \hat{z}}{\sqrt{3}}$

(c) $\frac{\hat{x} - \hat{y} + \hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}}$

38. The length element ds of an arc is given by, $(ds)^2 = 2(dx^1)^2 + (dx^2)^2 + \sqrt{3}dx^1dx^2$. The metric tensor g_{ij} is

[GATE 2014]

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

39. Four forces are given below in Cartesian and spherical polar coordinates.

(i) $\vec{F}_1 = K\exp\,(-r^2/R^2)\hat{r}$

(ii) $\vec{F}_2 = K(x^3\hat{y} - y^3\hat{z})$

(iii) $\vec{F}_3 = K(x^3\hat{x} + y^3\hat{y})$

(iv) $\vec{F}_4 = K(\hat{\phi}/r)$

Where K is a constant. Identify the correct option

[GATE 2015]

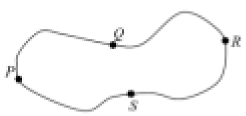
(a) (iii) and (iv) are conservative but (i) and (ii) are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

40. Given that the magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot d\vec{l} = \phi_{\text{along}}$ PQR the value of $\int_P^R \vec{A}\cdot d\vec{l}$ along PSR is



(a) $\phi - \phi_1$ (b) $\phi - \phi$

(c) $-\phi_1$ (d) ϕ

41. The direction of $\vec{\nabla} f$ for a scalar field

$$f(x_i, y_* z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$$

at the point $P(1,1,2)$ is

[GATE 2016]

(a) $\frac{(-\hat{j} - 2\hat{k})}{\sqrt{5}}$ (b) $\frac{(-\hat{j} + 2\hat{k})}{\sqrt{5}}$

(c) $\frac{(\hat{j} - 2\hat{k})}{\sqrt{5}}$ (d) $\frac{(\hat{j} + 2\hat{k})}{\sqrt{5}}$

42. In spherical polar coordinates (r, θ, ϕ), the unit vector $\hat{\theta}$ at $(10, \pi/4, \pi/2)$ is'

[GATE 2018]

(a) $\hat{k}$ (b) $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$

(c) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$

43. The scale factors corresponding to the covariant metric tensor g_{ij} in spherical polar coordinates are

[GATE 2018]

(a) $1, r^2, r^2\sin^2\,\theta$ (b) $1, r^2, \sin^2\,\theta$

(c) 1,1,1 (d) $1, r, r\sin\,\theta$

44. Given $\vec{V}_1 = \hat{\imath} - \hat{j}$ and $\vec{V}_2 = -2\hat{\imath} + 3\hat{j} + 2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1, \vec{V}_2, \vec{V}_3)$ a complete set for a three dimensional real linear vector space?

[GATE 2018]

(a) $\vec{V}_3 = \hat{\imath} + \hat{j} + 4\hat{k}$ (b) $\vec{V}_3 = 2\hat{\imath} - \hat{j} + 2\hat{k}$

(c) $\vec{V}_3 = \hat{\imath} + 2\hat{j} + 6\hat{k}$ (d) $\vec{V}_3 = 2\hat{\imath} + \hat{j} + 4\hat{k}$

45. During a rotation, vectors along the axis of rotation remain unchanged. For the rotation matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, the unit vector along the axis of rotation is

[GATE 2019]

(a) $\frac{1}{3}(2\hat{\imath} - \hat{j} + 2\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{j} - \hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{j} - \hat{k})$ (d) $\frac{1}{3}(2\hat{\imath} + 2\hat{j} - \hat{k})$

46. Let $|e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_2\rangle = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, |e_3\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let $S = \{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$. Let $\mathbb{R}^3$ denote the three-dimensional real vector space. Which one of the following is correct? **[GATE 2020]**

(a) S is an orthonormal set

(b) S is a linearly dependent set

(c) S is a basis for $\mathbb{R}^3$

(d) $\sum_{i=1}^{3} |e_i\rangle\langle e_i| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

47. Let u^μ denote the 4 -velocity of a relativistic particle whose square $u^\mu u_\mu = 1$. If $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor then the value of $\varepsilon_{\mu\nu\rho\sigma} u^\mu u^\nu u^\rho u^\sigma$ is **[GATE 2020]**

48. Which of the following options represent(s) linearly independent pair(s) of functions of a real variable x ?

[GATE 2023]

(a) e^{ix} and e^{-ix} (b) x and e^x

(c) 2^x and 2^{-3+x} (d) e^{ix} and $\sin x$

49. Consider a volume integral $I = \int_V \nabla^2 \left(\frac{1}{r}\right) dV$ over a volume V, where $r = \sqrt{x^2 + y^2 + z^2}$. Which of the following statement is/are correct?

[GATE 2024]

(a) $I = -4\pi$, if $r = 0$ is inside the volume V

(b) Integrand vanishes for $r \neq 0$

(c) $I = 0$, if $r = 0$ is not inside the volume V

(d) Integrand diverges as $r \to \infty$

50. Consider the set $\{1, x, x^2\}$. An orthonormal basis in $x \in [-1,1]$ is formed from these three terms, where the normalization of a function $f(x)$ is defined via $\int_{-1}^{1} x^2 [f(x)]^2 dx = 1$. If the orthonormal basis set is

$$\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}x, \frac{1}{2}\sqrt{\frac{21}{N}}(5x^2 - 3)\right)$$

then the value of N (in integer) is **[GATE 2025]**

❖ JEST PYQ's

1. The vector field $xz\hat{\imath} + yj$ in cylindrical polar coordinates is

[JEST 2013]

(a) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 - z)\hat{e}_\phi$

(b) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(c) $\rho(z\sin^2\phi + \cos^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(d) $\rho(z\cos^2\phi + \sin^2\phi)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 - z)\hat{e}_\phi$

2. What is the equation of the plane which is tangent to the surface $xyz = 4$ at the point (1,2,2) ?

[JEST 2017]

(a) $x + 2y + 4z = 12$

(b) $4x + 2y + z = 12$

(c) $x + 4y + z = 0$

(d) $2x + y + z = 6$

3. The temperature in a rectangular plate bounded by the lines $x = 0, y = 0, x = 3$ and $y = 5$ is $T = xy^2 - x^2y + 100$. What is the maximum temperature difference between two points on the plate? **[JEST 2017]**

4. Let $\vec{r}$ be the position vector of a point on a closed contour C. What is the value of the line integral $\oint \vec{r} \cdot d\vec{r}$?

[JEST 2019]

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) π

5. Which one of the following vectors lie along the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$?

[JEST 2019]

(a) $10\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (b) $10\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$

(c) $10\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ (d) $10\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

6. What is the angle (in degrees) between the surfaces $y^2 + z^2 = 2$ and $y^2 - x^2 = 0$ at the point $(1, -1, 1)$

[JEST 2019]

7. Suppose $\psi\vec{A}$ is a conservative vector, $\vec{A}$ is a non-conservative vector and ψ is non-zero scalar everywhere. Which one of the following is true?

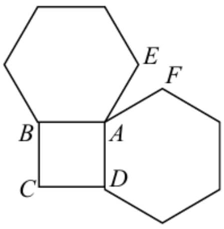
[JEST 2019]

(a) $(\nabla \times \vec{A}) \cdot \vec{A} = 0$ (b) $\vec{A} \times \nabla\psi = \vec{0}$

(c) $\vec{A} \cdot \nabla\psi = 0$ (d) $(\nabla \times \vec{A}) \times \vec{A} = \vec{0}$

8. A paper has been cut into the shape given in figure ($ABCD$ is a square and the two hexagonal flaps are regular) and placed on the table. The square base lies flat on the table. The hexagonal flaps are then folded upwards along the edges AB and AD such that edges AE and AF of the two hexagons coincide. What is the minimum angle (in degrees) made by the edge AE (or AF) with the surface of the table?

[JEST 2021]



(a) 120 (b) 85

(c) 60 (d) 45

9. Consider a real tensor T_{ijk} with $i, j, k = 1, \ldots, 5$. It has the following properties:

$$T_{ijk} = T_{jik} = T_{ikj}, \sum_i T_{iik} = 0$$

What is the number of independent real components of this tensor? **[JEST 2021]**

10. The position and velocity vectors of a particle changes from $\vec{R}_1$ to $\vec{R}_2$ and $\vec{v}_1$ to $\vec{v}_2$ respectively as time flows from t_1 to t_2. If $\vec{r}(t), \vec{v}(t)$, and $\vec{a}(t)$ are the instantaneous position, velocity and acceleration vectors of the particle, compute the integral:

$$\vec{I} = \int_{t_1}^{t_2} \vec{r} \times \vec{a} dt.$$

[JEST 2023]

(a) $\vec{I} = \vec{R}_2 \times \vec{v}_2 - \vec{R}_1 \times \vec{v}_1$

(b) $\vec{I} = \vec{R}_1 \times \vec{v}_1 - \vec{R}_2 \times \vec{v}_2$

(c) $\vec{I} = 0$

(d) $|\vec{I}| = |\vec{R}_1 \times \vec{v}_1| + |\vec{R}_2 \times \vec{v}_2|$

11. Given the vector $\vec{v} = y\hat{\imath} + 3x\hat{\jmath}$, what is the value of the line integral

$$\oint \vec{v} \cdot d\vec{r}$$

along the unit circle (centered at the origin) in an anti-clockwise direction?

[JEST 2023]

(a) $\frac{2\pi}{3}$ (b) π

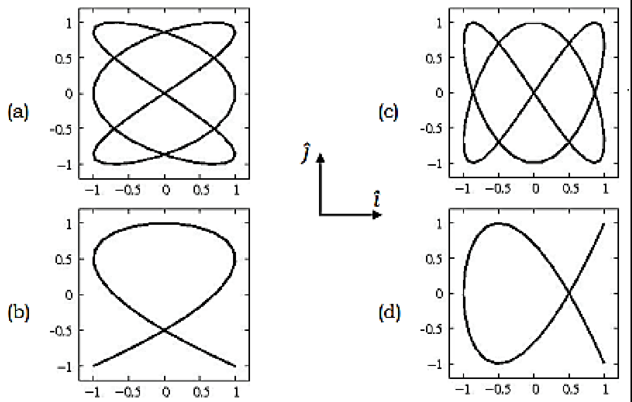
(c) 0 (d) 2π

❖ TIFR PYQ's

1. A two-dimensional vector $\vec{A}(t)$ is given by

$\vec{A}(t) = \hat{\imath}\sin 2t + \hat{\jmath}\cos 3t$.

Which of the following graphs best describes the locus of the tip of the vector, as t is varied from 0 to 2π ? **[TIFR 2013]**



2. Consider the surface corresponding to the equation

$$4x^2 + y^2 + z = 0$$

A possible unit tangent to this surface at the point $(1, 2, -8)$ is

[TIFR 2013]

(a) $\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$

(b) $\frac{4}{9}\hat{\imath} - \frac{8}{9}\hat{\jmath} + \frac{1}{9}\hat{k}$

(c) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$

(d) $-\frac{1}{\sqrt{5}}\hat{\imath} + \frac{3}{\sqrt{5}}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

3. Which of the following vectors is parallel to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$? **[TIFR 2015]**

(a) $+6\hat{\imath} - 2\hat{j} - 5\hat{k}$ (b) $+6\hat{\imath} + 2\hat{j} + 5\hat{k}$

(c) $-6\hat{\imath} - 2\hat{j} + 5\hat{k}$ (d) $+6\hat{\imath} - 2\hat{j} + 5\hat{k}$

4. A fourth rank Cartesian tensor $T_{ijk\ell}$ satisfies the following identities **[TIFR 2018]**

(i) $T_{ijk\ell} = T_{jik\ell}$

(ii) $T_{ijk\ell} = T_{ij\ell k}$

(iii) $T_{ijk\ell} = T_{k\ell ij}$

Assuming a space of three dimensions (i.e. $i, j, k = 1,2,3$), what is the number of independent components of $T_{ijk\ell}$?

5. Consider the surface defined by $ax^2 + by^2 + cz + d = 0$, where a, b, c and d are constants. If $\hat{n}_1$ and $\hat{n}_2$ are unit normal vectors to the surface at the points $(x, y, z) = (1,1,0)$ and $(0,0,1)$ respectively, and $\hat{m}$ is a unit vector normal to both $\hat{n}_1$ and $\hat{n}_2$, then $\hat{m} =$ **[TIFR 2019]**

(a) $\frac{-a\hat{\imath} + b\hat{\boldsymbol{j}}}{\sqrt{a^2+b^2}}$ (b) $\frac{2a\hat{\imath} + 2b\hat{\boldsymbol{j}} - c\hat{\boldsymbol{k}}}{\sqrt{4a^2+4b^2+c^2}}$

(c) $\frac{b\hat{\imath} - a\hat{j}}{\sqrt{a^2+b^2}}$ (d) $\frac{a\hat{\imath} - b\hat{\boldsymbol{j}} + c\hat{\boldsymbol{k}}}{\sqrt{a^2+b^2+c^2}}$

6. Consider a set of three 3-dimensional vectors

$$A = \begin{pmatrix}1\\0\\0\end{pmatrix}\ B = \begin{pmatrix}0\\1\\0\end{pmatrix}\ C = \begin{pmatrix}1\\1\\2\end{pmatrix}$$

These vectors undergo a linear transformation

$A \to A' = \mathbb{M}A\ B \to B' = \mathbb{M}B\ C \to C' = \mathbb{M}C$

where $\mathbb{M}$ is given by $\mathbb{M} = \begin{pmatrix}1 & 1 & 4\\1 & 0 & 1\\2 & 1 & 1\end{pmatrix}$

The volume of a parallelopiped whose sides are given by the transformed vectors A', B' and C' is **[TIFR 2022]**

(a) 16 (b) 4

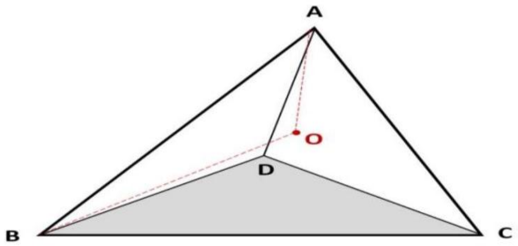
(c) 2 (d) 8

7. A surface is given by $4x^2y - 2xy^2 + 3z^3 = 0$ Which of the following is a vector normal to it at the point (2,3,1) ? **[TIFR 2023]**

(a) $30\hat{\imath} + 8\hat{\jmath} - 9\hat{k}$ (b) $15\hat{\imath} - 4\hat{\jmath} + 18\hat{k}$

(c) $30\hat{\imath} - 8\hat{\jmath} + 9\hat{k}$ (d) $30\hat{\imath} - 8\hat{\jmath} - 9\hat{k}$

8. Consider a regular tetrahedron ABCD, as shown in the figure below. Let the point O be its centre.



The value of the angle AOB must be **[TIFR 2023]**

(a) $\cos^{-1}(-4/5)$ (b) $\cos^{-1}(-1/3)$

(c) $\cos^{-1}(-\sqrt{4/5})$ (d) $2\pi/3$

9. A surface is given by $2x^3z + 4y^2z + 3z^2 = 81$ Which of the following is a vector tangential to it at the point on the surface with coordinates $(x, y, z) = (1,2,3)$? **[TIFR 2024]**

(a) $-3\hat{\imath} + 2\hat{j} + 6\hat{k}$ (b) $18\hat{\imath} + 48\hat{j} + 36\hat{k}$

(c) $2\hat{\imath} - 3\hat{j} + 3\hat{k}$ (d) $-3\hat{\imath} - 2\hat{j} + 6\hat{k}$

10. Consider the triangle subtended on the surface of a sphere of radius 1 by joining the points

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

, and (0,0,1) with arcs of great circles. The area subtended by this triangle on the surface of the sphere is given by:

(Hint: Drawing a figure might help.)

(a) $\pi/3$ (b) $\sqrt{3}\pi/2$

(c) $\sqrt{3}\pi$ (d) $2\pi/3$

❖ Answer Key

CSIR-NET PYQ				
1. a	2. b	3. c	4. a	5. c
6. d	7. d	8. d	9. c	10. c
11. a	12. d	13. d	14. c	15. b
16. c	17. d			
GATE PYQ				
1. d	2. b	3. c	4. 0	5. c
6. a	7. c	8. a	9. a	10. d
11. d	12. a	13. d	14. d	15. d
16. a	17. d	18. a	19. a	20. c
21. c	22. a	23. a	24. b	25. a
26. a	27. b	28. c	29. b	30. a
31. b	32. b	33. a	34. c	35. c
36. b	37. d	38. b	39. d	40. b
41. b	42. d	43. d	44. d	45. b
46. c	47. 0	48. abd	49. abc	50. 6to6
JEST PYQ				
1. a	2. d	3. 0038	4. a	5. b
6. 60	7. a	8. b	9.	10. a
11. d				
TIFR PYQ				
1. c	2. a	3. d	4.	5. b
6. d	7. c	8. b	9. c	10. a

❖ CSIR-NET PYQ's

1. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ?

[CSIR JUNE 2012]

(a) $100/\lambda$ (b) $10^4/\lambda^2$

(c) $10^4/\lambda$ (d) $1/\lambda$

2. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than: (take 180°) π = 57.29°). **[CSIR JUNE 2013]**

(a) 1.28° (b) 1.81°

(c) 3.28° (d) 4.01°

3. The viscosity η of a liquid is given by Poiseuille's formula $\eta = \frac{\pi P a^4}{8lV}$. Assume that l and V can be measured very accurately, but the pressure P has an rms error of 1% and the radius a has an independent rms error of 3%. The rms error of the viscosity is closest to

[CSIR JUNE 2015]

(a) 2% (b) 4%

(c) 12% (d) 13%

4. Two data sets A and B consist of 60 and 10 readings of a voltage measured using voltmeters of resolution of 1mV and 0.5mV respectively. The uncertainty in the mean voltage obtained from the data sets A and B are U_A and U_B, respectively. If the uncertainty of the mean of the combined data sets is U_{AB}, then which of the following statements is correct?

[CSIR DEC 2015]

(a) $U_{AB} < U_A$ and $U_{AB} > U_B$

(b) $U_{AB} < U_A$ and $U_{AB} < U_B$

(c) $U_{AB} > U_A$ and $U_{AB} < U_B$

(d) $U_{AB} > U_A$ and $U_{AB} > U_B$

5. The decay constants f_p of the heavy pseudoscalar mesons, in the heavy quark limit,

are related to their masses m_p by the relation

$$f_P = \frac{a}{\sqrt{m_p}}$$

, where a is an empirical parameter to be determined. The values $m_p = 6400 \pm 160\text{MeV}$ and $f_p = 180 \pm 15\text{MeV}$ correspond to uncorrelated measurement of a meson. The error on the estimate of a is

[CSIR JUNE 2016]

(a) $175(\text{MeV})^{3/2}$ (b) $900(\text{MeV})^{3/2}$

(c) $1200(\text{MeV})^{3/2}$ (d) $2400(\text{MeV})^{3/2}$

6. The resistance of a sample is measured as a function of temperature, and the data are shown below.

$T(^\circ\text{C})$	2	4	6	8
$R(\Omega)$	90	105	110	115

The slope of R vs T graphs, using a linear least-squares fit to the data, will be

[CSIR DEC 2016]

(a) $6\Omega/^\circ\text{C}$ (b) $4\Omega\,^\circ\text{C}$

(c) $2\Omega/^\circ\text{C}$ (d) $8\Omega/^\circ\text{C}$

7. The experimentally measured values of the variables x and y are 2.00 ± 0.05 and 3.00 ± 0.02, respectively. What is the error in the calculated value of $z = 3y - 2x$ from the measurements?

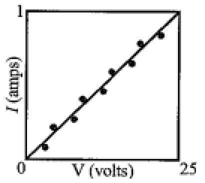
[CSIR JUNE 2017]

(a) 0.12 (b) 0.05

(c) 0.03 (d) 0.07

8. Both the data points and a linear fit to the current vs voltage of a resistor are shown in the graph below.
If the error in the slope is $1.255 \times 10^{-3}\Omega^{-1}$, then the value of resistance estimated from the graph is

[CSIR JUNE 2017]



(a) $(0.04 \pm 0.8)\Omega$ (b) $(25.0 \pm 0.8)\Omega$

(c) $(25 \pm 1.25)\Omega$ (d) $(25 \pm 0.0125)\Omega$

9. Two physical quantities T and M are related by the equation

$$T = \frac{2\pi}{a}\sqrt{\frac{M+b}{2}},$$

where a and b are constant parameters. The variation of T as a function of M was recorded in an experiment to determine the value of a graphically. Let m be the slope of the straight line when T^2 is plotted vs. M, and δm be the uncertainty in determining it. The uncertainty in determining a is

[CSIR DEC 2017]

(a) $\frac{a}{2}\left(\frac{\delta m}{m}\right)$ (b) $a\left(\frac{\delta m}{m}\right)$

(c) $\frac{b}{2a}\left(\frac{\delta m}{m}\right)$ (d) $\frac{2\pi}{a}\left(\frac{\delta m}{m}\right)$

10. In an experiment to measure the acceleration due to gravity g using a simple pendulum, the length and time period of the pendulum are measured to three significant figures. The mean value of g and the uncertainty δg of the measurements are then estimated using a calculator from a large number of measurements and found to be 9.82147 m/s and 0.02357 m/s^2, respectively. Which of the following is the most accurate way of presenting the experimentally determined value of g ?

[CSIR JUNE 2019]

(a) $9.82 \pm 0.02\ m/s^2$

(b) $9.8215 \pm 0.02\ m/s^2$

(c) $9.82147 \pm 0.02357\ m/s^2$

(d) $9.82 \pm 0.02357\ m/s^2$

11. A student measures the displacement x from the equilibrium of a stretched spring and reports it be 100μm with a 1% error. The spring constant k is known to be 10 N/m with 0.5% error. The percentage error in the estimate of the potential energy $V = \frac{1}{2}kx^2$ is

[CSIR DEC 2019]

(a) 0.8% (b) 2.5%

(c) 1.5% (d) 3.0%

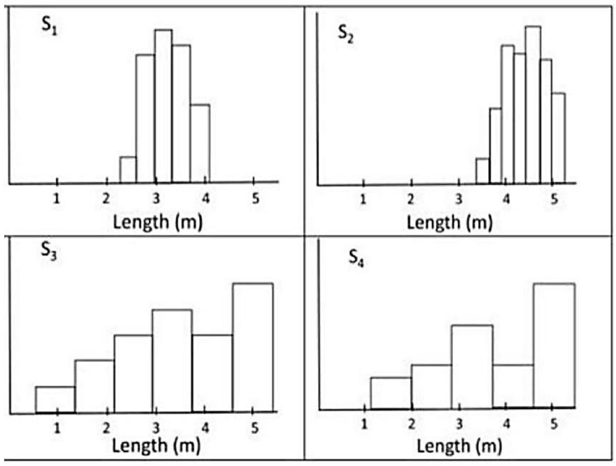
12. In an experiment, the velocity of a non-relativistic neutron is determined by measuring the time (~ 50 ns) it takes to travel from the source to the detector kept at a distance L. Assume that the error in the measurement of L is negligibly small. If we want to estimate the kinetic energy T of the neutron to within 5% accuracy i.e., $\left|\frac{\delta T}{T}\right| \leq 0.05$, the maximum permissible error $|\delta T|$ in measuring the time of flight is nearest to

[CSIR JUNE 2021]

(a) 1.75 ns (b) 0.75 ns

(c) 2.25 ns (d) 1.25 ns

13. Four students (S_1, S_2, S_3 and S_4) make multiple measurements on the length of a table. The binned data are plotted as histograms in the following figures



(a)S_2 (b)S_4

(c)S_1 (d)S

14. A DC motor is used to lift a mass M to a height H from the ground. The electric energy delivered to the motor is VIt, where V is the applied voltage, I is the current and t the time for which the motor runs. The efficiency e of the motor is the ratio between the work done by the motor and the energy delivered to it. If $M = 2.0 \pm 0.02$ kg, $h = 1.00 \pm 0.01$ m, $V = 10.0 \pm 0.1$ V, $I = 2.00 \pm 0.02$ A and $t = 300 \pm 15$ s, then the fractional error $|\delta e/e|$ in the efficiency of the motor is closest to

[CSIR JUNE2023]

(a) 0.05 (b) 0.09

(c) 0.12 (d) 0.15

15. In the measurement of a radioactive sample, the measured counts with and without the sample for equal time intervals are $C = 500$ and $B = 100$, respectively. The errors in the measurements of C and B are $|\Delta C| = 20$ and $|\Delta B| = 10$, respectively. The net error $|\Delta Y|$ in the measured counts from the sample $Y = C - B$, is closest to

[CSIR DEC 2023]

(a)22 (b)10

(c)30 (d)43

16. A set of 100 data points yields an average $\bar{x} = 9$ and a standard deviation $\sigma_x = 4$. The error in the estimated mean is closest to

[CSIR DEC 2024]

(a)3.0 (b)0.4

(c)4.0 (d)0.3

❖ JEST PYQ's

1. The length and radius of a perfect cylinder are each measured with an RMS error of 1%. The RMS error on the inferred volume of the cylinder is roughly

[JEST 2012]

(a) 1.7% (b) 3.3%

(c) 0.5% (d) 1%

2. An astrophysical observation measured the mass of a star as $(12.41 \pm 1.12)M_{\square}$, where $M_{\square}$ is the mass of the Sun. Another independent observation measured the mass of the same star as $(8.40 \pm \Delta)M_{\square}$. Assuming the errors to have Gaussian distributions, one concluded that the two measurements differed by 3 standard deviations. The value of Δ was approximately

[JEST 2021]

(a) 0.22 (b) 0.73

(c) 1.04 (d) 1.55

❖ TIFR PYQ's

1. A detector is used to count the number of γ rays emitted by a radioactive source. If the number of counts recorded in exactly 20 seconds is 10000, the error in the counting rate per second is **[TIFR 2010]**

(a) ± 5.0 (b) ± 22.4

(c) ± 44.7 (d) ± 220.0

2. A liquid is flowing through a capillary tube of inner radius r under the influence of an external pressure P. The uncertainties in the measurements of P and r are found to be 2% and 1%, respectively. The uncertainty in the flow of liquid per second is

[TIFR 2017]

(a) 4.47% (b) 2.23%

(c) 2.83% (d) 3.61%

3. In an experiment that measures the resistivity ρ of a substance it was observed that ρ varies with temperature T and a parameter Δ, as

$$\rho = \rho_0 e^{\Delta/T}$$

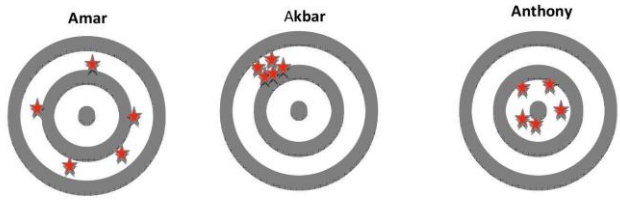
where ρ_0 is a constant. In one measurement, made at $T = 100$ K and $\Delta = 50$, the percentage error in Δ is found to be 2% while the percentage error in T was 3%. What was the approximate percentage error for the resistivity ρ ?

[TIFR 2021]

(a) 18% (b) 3.6%

(c) 9% (d) 1.8%

4. In an archery contest, the aim is to shoot arrows at the center of a board. Three archers, Amar, Akbar and Anthony each shot 5 arrows at the board. The locations of their arrow hits are shown in the figures with red stars. Which of the following statements are true? **[TIFR 2021]**



(a) Amar has more accuracy than Anthony

(b) Amar has more precision than Akbar

(c) Akbar has more accuracy than Anthony

(d) Akbar has more precision than Anthony

5. A very sensitive spring balance with spring constant $k = 2 \times 10^8 \text{Nm}^{-1}$ is operating at a temperature of 300 K. The thermal fluctuations can lead to an error in the measurement of mass. If you are trying to measure a mass of 1mg, the relative error in the measurement is closest to **[TIFR 2021]**

(a) 0.01% (b) 10.0%

(c) 20.0% (d) 0.9%

6. Two students A and B, measure the time period of a simple pendulum in the laboratory using the same stopwatch but following two different methods.

- Student A measures the time taken for one oscillation and repeats it for N_A number of times and finds the average.
- Student B, on the other hand, measures the time taken for N_B number of oscillations and then computes the period.

Given that $N_\text{A}, N_\text{B} \gg 1$, to ensure that both students measure the time period with the same uncertainty, the relation between N_A and N_B must be **[TIFR 2022]**

(a) $N_\text{A} = \sqrt{N_\text{B}}$ (b) $N_\text{A} = N_\text{B}^2$

(c) $N_\text{A} = N_\text{B}$ (d) $\ln\, 2N_\text{A} = N_\text{B}$

7. In a standardized entrance exam, the passing rates for the past 10 years are tabulated below.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Passing Rate	22 %	16 %	23 %	21%	22 %	14 %	17 %	20 %	24 %	21 %

If 1000 candidates appear for the exam every year, the probability that more than 250 students will pass the exam this year is about **[TIFR 2022]**

(a) 20% (b) 6%

(c) 25% (d) 0.1%

8. A spectrographic method to search for exoplanets is by measuring its velocity along the line of sight, using the Doppler shift in the spectrum. If a star of mass M and a planet of mass m are moving around their common centre of mass, this component of velocity will vary periodically with an amplitude.

$$A = \left(\frac{2\pi G_N}{T}\right)^{1/3} \frac{m}{M^{2/3}}$$

For a particular planet-star system, if the time period is $T = (12 \pm 0.3)$ years, and A and M are measured with an accuracy of 3% each, then the error in the measurement of the mass m is **[TIFR 2022]**

(a) 3.7% (b) 8.5%

(c) 5.8% (d) 6.3%

9. A faint star is known to emit light of a given frequency at an average rate of 10 photons per minute. An astronomer plans to measure this rate using a photon-counting detector. If she wants to measure the rate to an accuracy of 5%, approximately how long should be the exposure time? **[TIFR 2023]**

(a) 40 minutes (b) 1 hour

(c) 20 minutes (d) 10 minutes

10. A particular counting system has an average background rate of 50 counts/min. A decaying radioisotope source was introduced and the total 168 counts were measured in one minute. After a delay of 24 hrs, the system measured total 91 counts in one minute. If these measurements were used for determining the half-life (τ) of the source and if the average background rate, and the time have no errors, the % error ($100 \times \sigma_\tau/\tau$) in the calculated half-life value due to counting statistics would be: **[TIFR 2024]**

(a) 24.3% (b) 21.2%

(c) 25.7% (d) 18.2%

11. Consider a universe that always expands with a scale factor a that increases with time as $a(t) = Ct^{2/3}$ where C is a constant. Its expansion rate at time t is defined by the Hubble parameter

$$H(t) = \frac{1}{a(t)}\frac{da(t)}{dt}$$

The current value of $H(t)$ in the universe is given by $H_0 = 975 \text{ km s}^{-1}\text{Mpc}^{-1}$ where $1\text{Mpc} = 3.1 \times 10^{22}$ m. What is the approximate age of this universe?

[TIFR 2024]

(a) 10^9 years (b) 10^7 years

(c) 10^{11} years (d) 10^{13} years

12. For a given measurement of particles in a counter, a 10-minute data collection resulted in a statistical uncertainty of 2.5%. How much additional time must be allocated to reduce the statistical uncertainty to 0.5% ? **[TIFR 2025]**

(a) 240 minutes (b) 40 minutes

(c) 250 minutes (d) 50 minutes

13. Two students perform a counting experiment independently. Student A measures the counts for 1-minute intervals each and repeats the measurement five times. The obtained counts are given below.

Measurement turn	Counts
1	25
2	35
3	30
4	23
5	27

This student then takes the mean of these counts and reports the count rate (counts/min). The second student (B) makes one measurement for five minutes. She measures 145 counts and reports the count rate (counts /min). If the clock used for all these measurements is accurate up to 0.1 minutes, and there are no other sources of uncertainties, we can conclude that:

(a) The count rate reported by student A will have a larger uncertainty than that reported by student B.

(b) The count rate reported by student B will have a larger uncertainty than that reported by student A.

(c) The reported uncertainty in both results would be identical

(d) Nothing may be concluded about the relative uncertainties between A and B

❖ Answer Key

CSIR-NET PYQ

1. c	2. b	3. c	4. a	5. c
6. b	7. a	8. b	9. a	10. a
11. b	12. d	13. c	14. a	15. a
16. b				

JEST PYQ

1. a	2. b			

TIFR PYQ

1.	2. a	3. d	4. d	5. d
6. b	7. b	8. a	9. a	10. a
11. a	12. a	13. a		

❖ CSIR-NET PYQ's

1. Which of the following limits exists?

[CSIR JUNE 2012]

(a) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}+\ln\ N\right)$

(b) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}-\ln\ N\right)$

(c) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{\sqrt{m}}-\ln\ N\right)$

(d) $\lim_{N\to\infty}\sum_{m=1}^{N}\frac{1}{m}$

2. The expression

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$

is proportional to

[CSIR DEC 2013]

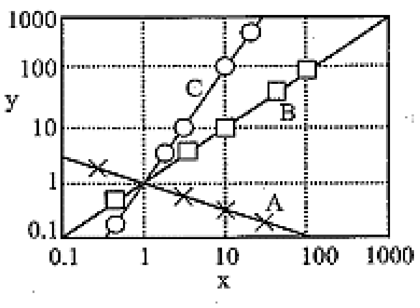
(a) $\delta(x_1+x_2+x_3+x_4)$

(b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$

(d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

3. Three sets of data A, B and C from an experiment, represented by ×,□ and O, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.



The functional dependence $y(x)$ for the sets A, B and C are, respectively

[CSIR DEC 2013]

(a) $\sqrt{x}, x$ and x^2

(b) $-\frac{x}{2}, x$and $2x$

(c) $\frac{1}{x^2}, x$and x^2

(d) $\frac{1}{\sqrt{x}}, x$and x^2

4. The following data is obtained in an experiment that measures the viscosity η as a function of molecular weight M for a set of polymers. (Diagram missing)

The relation that best describes the dependence of η on M is

[CSIR JUNE 2014]

(a) $\eta \sim M^{4/9}$

(b) $\eta \sim M^{3/2}$

(c) $\eta \sim M^2$

(d) $\eta \sim M^3$

5. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by

[CSIR DEC 2015]

(a) $\sqrt{\frac{hc}{E}}$

(b) $\frac{hc}{E^{3/2}}$

(c) $\left(\frac{hc}{E}\right)^2$

(d) $\frac{hc}{E}$

6. If

$$y=\frac{1}{\tan\ \mathrm{h}(x)}$$

, then x is

[CSIR DEC 2015]

(a) $\ln\left(\frac{y+1}{y-1}\right)$

(b) $\ln\left(\frac{y-1}{y+1}\right)$

(c) $\ln\sqrt{\frac{y-1}{y+1}}$

(d) $\ln\sqrt{\frac{y+1}{y-1}}$

7. For a dynamical system governed by the equation

$$\frac{dx}{dt}=2\sqrt{1-x^2}$$

with $|x|\le 1$,

[CSIR DEC 2015]

(a) $x=-1$ and $x=1$ are both unstable fixed points

(b) $x=-1$ and $x=1$ are both stable fixed points

(c) $x = -1$ is an unstable fixed point and $x = 1$ is a stable fixed point

(d) $x = -1$ is stable fixed point and $x = 1$ is an unstable fixed point

8. Using dimensional analysis, Planck defined a characteristic temperature T_P from powers of the gravitional constant G, Planck's constant h, Boltzmann constant k_k and the speed of light c in vacuum. The expression for T_P is proportional to

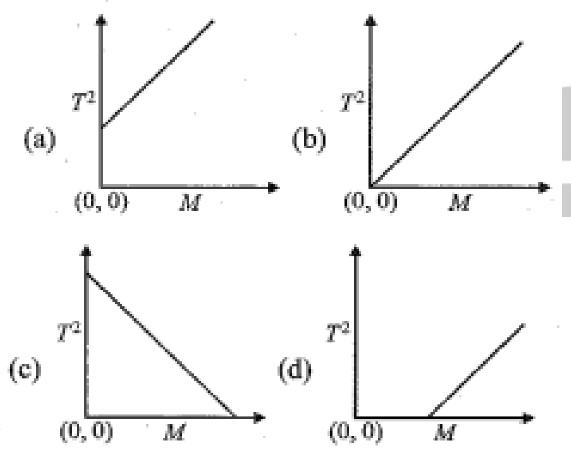
[CSIR JUNE 2016]

(a) $\sqrt{\frac{hc^5}{k_B^2 G}}$ (b) $\sqrt{\frac{hc^3}{k_B^2 G}}$

(c) $\sqrt{\frac{G}{hc^4 k_b^2}}$ (d) $\sqrt{\frac{hk_\beta^2}{Gc^3}}$

9. The spring constant k, of a spring of a mass m_s, is determined experimentally by loading the spring with mass M and recording the time period T, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is

[CSIR DEC 2017]



A

10. The value of the integral $\int_0^\infty dxe^{-x^{2m}}$, where m is a positive integer, is

[CSIR JUNE 2022]

(a) $\Gamma\left(\frac{m+1}{2m}\right)$ (b) $\Gamma\left(\frac{m-1}{2m}\right)$

(c) $\Gamma\left(\frac{2m+1}{2m}\right)$ (d) $\Gamma\left(\frac{2m-1}{2m}\right)$

11. The infinite series $\sum_{n=0}^{\infty}(n^2+3n+2)x^n$ evaluated at $x = \frac{1}{2}$, is

[CSIR JUNE 2022]

(a) 16 (b) 8

(c) 32 (d) 24

12. The Beta function is defined as

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$$

Then $B(x, y+1) + B(x+1, y)$ can be expressed as

[CSIR DEC 2023]

(a) $B(x, y-1)$ (b) $B(x+y, 1)$

(c) $B(x+y, x-y)$ (d) $B(x, y)$

❖ GATE PYQ's

1. Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statements is correct?

P the plot of $f(x)$ versus x has two maxima and two minima

Q the plot of $f(x)$ versus x cuts the x axis at four points

R the plot of $f(x)$ versus x has three extrema

S no part of the plot $f(x)$ versus x lies in the fourth quadrant

Pick the right combination of correct choices from those given below

[GATE 2003]

(a) P and R (b) R only

(c) R and S (d) P and Q

2. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is

[GATE 2004]

(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$

(c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

3. If $xp(x)$ and $x^2q(x)$ have the Taylor series expansions

$$xp(x) = 4 + x + x^2 + \cdots \ldots.$$
$$x^2q(x) = 2 + 3x + 5x^2 + \cdots \ldots.$$

then the roots of the incidicial equation are

[GATE 2004]

(a) $-1,0$ (b) $-1,-2$

(c) $-1,1$ (d) $-1,2$

4. The average value of the function $f(x) = 4x^3$ inthe interval 1 to 3 is

[GATE 2005]

(a) 15 (b) 20

(c) 40 (d) 80

5. For $f(x) = a_0 + a_1x + a_2x^2 \in V$ and $g(x) = b_0 + b_1x + b_2x^2 \in V$, which one of the following constitutes an acceptable scalar product?

[GATE 2008]

(a) $(f,g) = a_0^2b_0 + a_1^2b_1 + a_2^2b_2$

(b) $(f,g) = a_0^2b_0^2 + a_1^2b_1^2 + a_2^2b_2^2$

(c) $(f,g) = a_0b_0 - a_1b_1 - a_2b_2$

(d) $(f,g) = a_0b_0 + \frac{a_1b_1}{2} + \frac{a_2b_2}{3}$

6. $\Gamma\left(n+\frac{1}{2}\right)$ is equal to [Given $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$]

[GATE 2013]

(a) $\frac{n!}{2^n}\sqrt{\pi}$ (b) $\frac{2n!}{n!\,2^n}\sqrt{\pi}$

(c) $\frac{2n!}{n!\,2^{2n}}\sqrt{\pi}$ (d) $\frac{n!}{2^{2n}}\sqrt{\pi}$

7. The integral $\int_0^\infty x^2e^{-x^2}dx$ is equal to_______. (up to two decimal places) **[GATE 2017]**

❖ JEST PYQ's

1. If $[x]$ denotes the greatest integer not exceeding x, then $\int_0^\infty [x]e^{-x}dx$

[JEST 2012]

(a) $\frac{1}{e-1}$ (b) 1

(c) $\frac{e-1}{e}$ (d) $\frac{e}{e^2-1}$

2. As $x \to 1$, the infinite series

$$x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots ..$$

[JEST 2012]

(a) diverges

(b) converges to unity

(c) converges to $\frac{\pi}{4}$

(d) none of the above.

3. What is the value of the following series?

$$\left(-1 - \frac{1}{2!} + \frac{1}{4!} + \cdots ..\right)^2 - \left(1 + \frac{1}{3!} + \frac{1}{5!} + \cdots ..\right)^2$$

[JEST 2012]

(a) 0 (b) e

(c) e^2 (d) 1

4. What is the value of the following series?

$$\left(1 - \frac{1}{2!} + \frac{1}{4!} - \cdots ..\right)^2 + \left(1 - \frac{1}{3!} + \frac{1}{5!} - \cdots ..\right)^2$$

[JEST 2013]

(a) 0 (b) e

(c) e^2 (d) 1

5. The Bernoulli polynomials $B_n(s)$ are defined by,

$$\frac{xe^{xs}}{e^x - 1} = \sum B_n(s)\frac{x^n}{n!}$$

. which one of the following relations is true?

[JEST 2015]

(a) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)\frac{x^n}{(n+1)!}$

(b) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)(-1)^n\frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(-s)(-1)^n\frac{x^n}{n!}$

(d) $\frac{xe^{x(1-s)}}{e^x-1} = \sum B_n(s)(-1)^n\frac{x^n}{n!}$

6. The sum $\sum_{m=1}^{99} \frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to

[JEST 2015]

(a) 9 (b) $\sqrt{99}-1$

(c) $\frac{1}{(\sqrt{99}-1)}$ (d) 11

7. The sum of the infinite series

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\ldots$$

is **[JEST 2016]**

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

8. The Euler polynomials are defined by

$$\frac{2e^{xs}}{e^x+1}=\sum_{n=0}^{\infty} E_n(s)\frac{x^n}{n!}$$

What is the value of $E_5(2)+E_5(3)$?

[JEST 2019]

9. What value the following infinite series will converge to? **[JEST 2021]**

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

(a) $\frac{\pi^2}{6}$ (b) $\frac{1}{2}$

(c) 3 (d) 6

10. Consider the infinite series

$$\exp\left[\left(x+\frac{x^3}{3}+\cdots\ldots\right)^2-\left(\frac{x^2}{2}+\frac{x^4}{4}+\cdots\ldots\right)^2\right]$$

Which one of the following represents this series?

[JEST 2021]

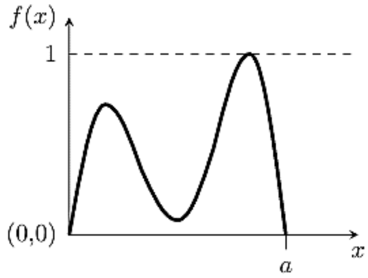
(a) $(1+x)^{\ln(1-x)}$

(b) $\exp\left[\sin^2 x-(\cos x-1)^2\right]$

(c) $\exp\left(xe^x\right)$

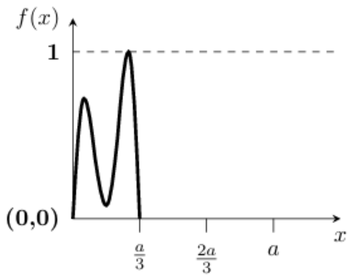
(d) $(1-x)^{-\ln(1+x)}$

11. The function $f(x)$ shown below has non-zero values only in the range $0<x<a$.

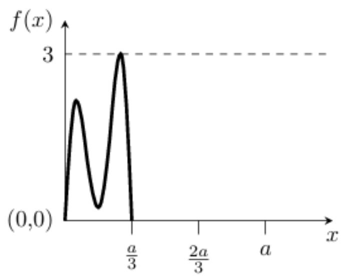


Which of the following figure represents $f(3x)$?

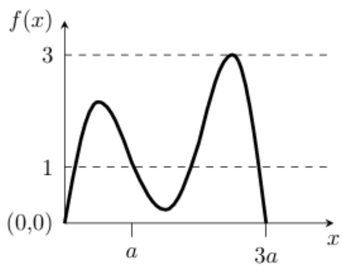
[JEST 2022]



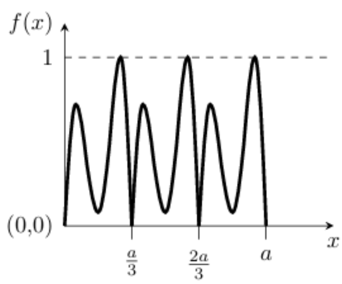
A.



B.



C.



D.

❖ TIFR PYQ's

1. Consider a standard chess board with 8×8 squares. A piece starts from the lower left corner, which we shall call Square (1,1). A single move of this piece corresponds to either one step right, i.e. to Square (1,2) or one step forwards, i.e. to Square (2,1). If it continues to move according to these rules, the number of different paths by which the piece can reach the Square (5,5) starting from the Square (1,1) is

[TIFR 2010]

(a) 120 (b) 72

(c) 70 (d) 45

2. The infinite series

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\cdots$$

where $-1<x<+1$, can be summed to the value

[TIFR 2011]

(a) $\tanh x$

(b) $\ln\left(1-\frac{4}{\pi}\tan^{-1} x\right)$

(c) $\frac{1}{2}\ln[(1+x)/(1-x)]$

(d) $\frac{1}{2}\ln[(1-x)/(1+x)]$

3. The value of the integral $\int_0^\infty dx x^9 \exp(-x^2)$ is **[TIFR 2013]**

(a) 20160 (b) 12

(c) 18 (d) 24

4. The integral evaluates to

$$\int_0^\infty \frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right)-\exp\left(-\frac{x}{\sqrt{2}}\right)\right]$$

[TIFR 2016]

(a) zero (b) 2.03×10^{-2}

(c) 2.03×10^{-1} (d) 2.03

5. Given the infinite series

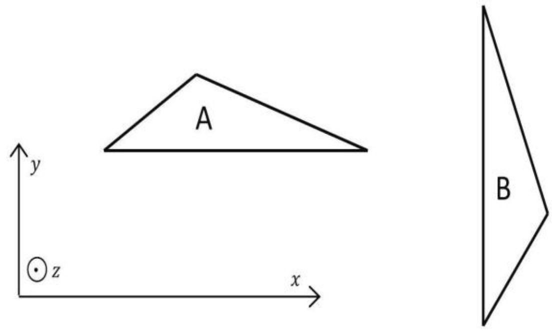
$$y(x)=1+3x+6x^2+10x^3+\cdots+\frac{(n+1)(n+2)}{2}x^n+\cdots$$

find the value of $y(x)$ for $x=6/7$. **[TIFR 2016]**

6. Evaluate the expression

$$n!\int_0^A dx_{n-1}\int_0^{x_{n-1}} dx_{n-2}\int_0^{x_{n-2}} dx_{n-3}\cdots\int_0^{x_3} dx_2\int_0^{x_2} dx_1\int_0^{x_1} dx_0$$

7.



Refer to the figure above. If the z-axis points out of the plane of the paper towards you, the triangle marked ' A ' can be transformed (and suitably re-positioned) to the triangle marked 'B' by

[TIFR 2018]

(a) rotation about x-direction by $\pi/2$, then rotation by $-\pi/2$ in the yz-plane

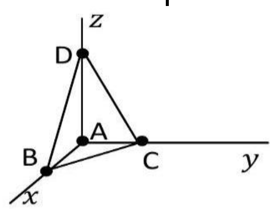
(b) rotation about z-direction by $\pi/2$, then reflection in the yz-plane

(c) reflection in the yz-plane, then rotation by $\pi/2$ about z-direction

(d) reflection in the xz-plane, then rotation by $-\pi/2$ about z-direction

8. Which of the following operations will transform a tetrahedron $ABCD$ with vertices as listed below

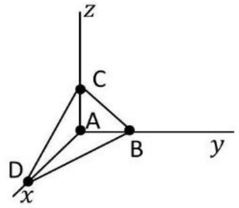
	x	y	z
A	0	0	0
B	1	0	0
C	0	1	0
D	0	0	2



into a tetrahedron ABCD with vertices as listed below

	x	y	z

A	0	0	0
B	0	1	0
C	0	0	1
D	2	0	0



up to suitable translation?

[TIFR 2019]

(a) A rotation about x axis by $\pi/2$, then a rotation about z axis by $\pi/2$

(b) A reflection in the xy plane, then a rotation about x axis by $\pi/2$

(c) A reflection in the yz plane, then a reflection in the xy plane

(d) A rotation about y axis by $\pi/2$, then a reflection in the xz plane

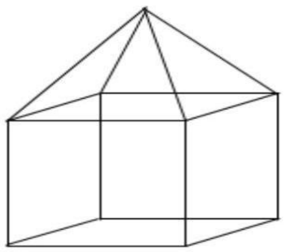
9. The sum of the infinite series is given by **[TIFR 2020]**

$$S = 1 + \frac{3}{5} + \frac{6}{25} + \frac{10}{125} + \frac{15}{625} + \cdots$$

(a) $S = \frac{125}{64}$ (b) $S = \frac{25}{16}$

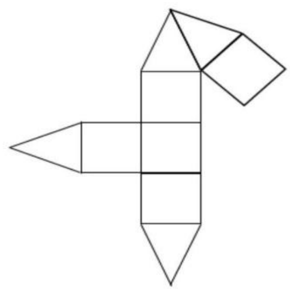
(c) $S = \frac{25}{24}$ (d) $S = \frac{16}{25}$

10. A 3-dimensional view of a polygon, whose faces are either squares or isosceles triangles, is sketched below.

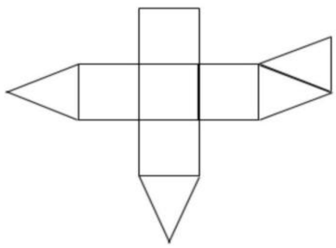


Which of the following 2-dimensional figures represents it after flattening? **[TIFR 2021]**

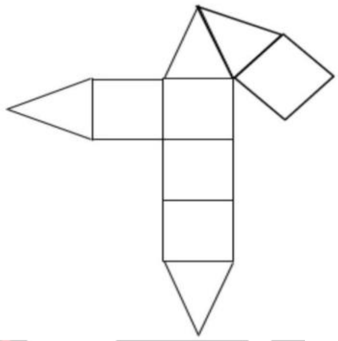
(a)



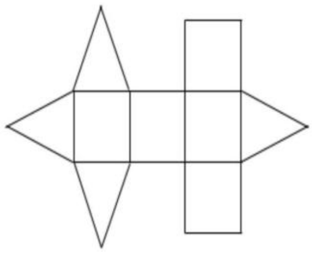
(b



(c)



(d)



11. Consider a square which can undergo rotations and reflections about its centre, where making no transformation at all is counted as a rotation by 0^0. The total number of such distinct rotations and reflections which will keep the square unchanged is

[TIFR 2022]

(a) 8 (b) 4

(c) 16 (d) 32

12. Consider the two-dimensional polar integral

$$P = \int drd\theta r^{19} e^{-r^2} \sin^8 \theta \cos^{11} \theta$$

If the integration is over only the first quadrant $(0 \leq \theta \leq \pi/2)$, the value of P is **[TIFR 2022]**

(a) 20160 (b) 88π

(c) 180 (d) 16π

13. Consider the inner product in the space of normal sable functions defined on the interval $[-1,1]$ $\langle f \mid g \rangle = \int_{-1}^{1} dx(1+x^2)f(x)g(x)$

The projection of the vector 1 along the vector x^2 is

[TIFR 2022]

(a) $\frac{16}{15}x^2$ (b) $\frac{16}{15}\sqrt{\frac{35}{24}}x^2$

(c) $\frac{14}{9}x^2$ (d) $\sqrt{\frac{35}{24}}x^2$

14. The value of the first derivative of the function at $x = 0$ is $f'(0) =$

$$f(x) = \frac{2}{\sqrt{3}}e^{-\sqrt{3}x^2|x|}$$

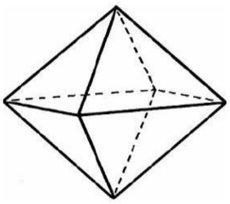
[TIFR 2023]

(a) 0 (b) 2

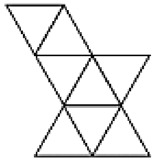
(c) $2/\sqrt{3}$ (d) undefined

15. Which of the following sheets of paper can be turned into a regular octahedron (a three-dimensional regular polyhedron with eight triangular faces, as shown on the right) by folding along the marked lines?

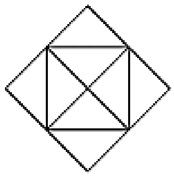
[TIFR 2024]



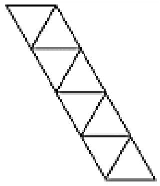
(a)



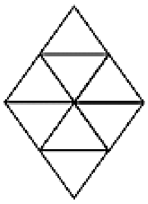
(c)



(b)



(d)



16. The asymptotic expansion of the following function for $x \to \infty$

$$x\tanh^{-1}\frac{1}{x}$$

is given by:

(a) $1 + \frac{1}{3x^2} + \frac{1}{5x^4} + \frac{1}{7x^6} + \cdots$

(b) $1 - \frac{1}{3x^2} + \frac{1}{5x^4} - \frac{1}{7x^6} + \cdots$

(c) $x + \frac{1}{2x} + \frac{1}{4x^3} + \frac{1}{6x^5} + \cdots$

(d) $1 + \frac{1}{2x^2} + \frac{1}{4x^4} + \frac{1}{6x^6} + \cdots$

❖ Answer Key

CSIR-NET				
1. b	2. b	3. d	4. d	5. c
6. d	7. c	8. a	9. a	10. c
11. a	12. d			
GATE				
1. b	2. b	3. b	4. c	5. c
6. c	7. 0.44			
JEST				
1. a	2. b	3. d	4. d	5. d
6. a	7. d	8. 64	9. d	10. d
11. a				
TIFR				
1.	2.	3. b	4. c	5. 343
6. A^n	7. b	8. a	9. a	10. a
11. a	12. c	13. c	14. a	15. a
16. a				