GATE 2023

Quantum Mechanics

1. H is the Hamiltonian, $\vec{L}$ the orbital angular momentum and L_z is the z-component of $\vec{L}$. The $1s$ state of the hydrogen atom in the non-relativistic formalism is an eigen function of which one of the following sets of operators?

(a) H, L^2 and L_z (b) $H, \vec{L}, L^2$ and L_z
(c) L^2 and L_z only (d) H and L_z only

2. A particle has wavefunction $\psi(x,y,z) = Nze^{-\alpha(x^2+y^2+z^2)}$
where N is a normalization constant and α is a positive constant. In this state, which one of the following options represents the eigenvalues of L^2 and L_z respectively?

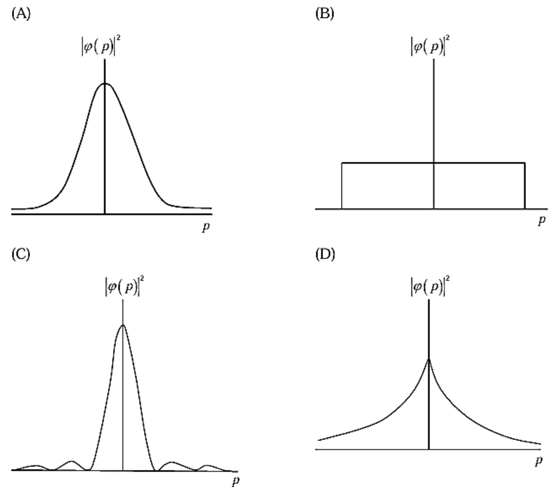
Some values of Y_ℓ^m are: $Y_0^0 = \sqrt{\frac{1}{4\pi}}, Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta, Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$

(a) 0 and 0 (b) $\hbar^2$ and $-\hbar$
(c) $2\hbar^2$ and 0 (d) $\hbar^2$ and $\hbar$

3. The wavefunction of a particle in one dimension is given by

$$\psi(x) = \begin{cases} M, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

Here M and a are positive constants. If $\varphi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\varphi(p)|^2$?



4. Consider a particle in a two-dimensional infinite square well potential of side L, with $0 \le x \le L$ and $0 \le y \le L$. The wavefunction of the particle is zero only along the line $y = \frac{L}{2}$, apart from the boundaries of the well. If the energy of the particle in this state is E, what is the energy of the ground state?

(a) $\frac{1}{4}E$ (b) $\frac{2}{5}E$
(c) $\frac{3}{8}E$ (d) $\frac{1}{2}E$

5. Consider two non-identical spin $\frac{1}{2}$ particles labelled 1 and 2 in the spin product state $\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$. The Hamiltonian of the system is

$$H = \frac{4\lambda}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spin operators of particles 1 and 2 , respectively, and λ is a constant with appropriate dimensions. What is the expectation value of H in the above state?

(a) $-\lambda$ (b) -2λ
(c) λ (d) 2λ

6. A spin $\frac{1}{2}$ particle is in a spin up state along the x-axis (with unit vector $\hat{x}$) and is denoted as $\left|\frac{1}{2},\frac{1}{2}\right\rangle_x$. What is the probability of finding the particle to be in a spin up state along the direction $\hat{x}'$, which lies in the xy-plane and makes an angle θ with respect to the positive x-axis, if such a measurement is made?

(a) $\frac{1}{2}\cos^2\frac{\theta}{4}$ (b) $\cos^2\frac{\theta}{4}$
(c) $\frac{1}{2}\cos^2\frac{\theta}{2}$ (d) $\cos^2\frac{\theta}{2}$

Mathematical Physics

7. A 4×4 matrix M has the property $M^\dagger = -M$ and $M^4 = 1$, where 1 is the 4×4 identity matrix. Which one of the following is the CORRECT set of eigenvalues of the

matrix M?

(a) $(1,1,-1,-1)$ (b) $(i,i,-i,-i)$
(c) $(i,i,i,-i)$ (d) $(1,1,-i,-i)$

8. Which of the following options represent(s) linearly independent pair(s) of functions of a real variable x ?
(a) e^{ix} and e^{-ix} (b) x and e^{x}
(c) 2^{x} and 2^{-3+x} (d) e^{ix} and $\sin\ x$

9. Consider two real functions
$U(x,y)=xy(x^2-y^2)$, $V(x,y)$
$= ax^4+by^4+cx^2y^2+k$
where k is a real constant and a,b,c are real coefficients. If $U(x,y)+iV(x,y)$ is analytic, then what is the value of $a\times b\times c$?
(a) $\frac{1}{8}$ (b) $\frac{3}{28}$
(c) $\frac{5}{36}$ (d) $\frac{3}{32}$

10. Consider the complex function $f(z)=\frac{z^2\sin\ z}{(z-\pi)^4}$
At $z=\pi$, which of the following options is (are) CORRECT?
(a) The order of the pole is 4
(b) The order of the pole is 3
(c) The residue at the pole is $\frac{\pi}{6}$
(d) The residue at the pole is $\frac{2\pi}{3}$

11. Consider the vector field $\vec{V}$ consisting of the velocities of points on a thin horizontal disc of radius $R=2$ m, moving anticlockwise with uniform angular speed $\omega=2$rad/sec about an axis passing through its center. If $V=|\vec{V}|$, then which of the following options is(are) CORRECT ? (In the options, $\hat{r}$ and $\hat{\theta}$ are unit vectors corresponding to the plane polar coordinates r and θ).
You may use the fact that in cylindrical coordinates (s,ϕ,z) (s is the distance from the zaxis), the gradient, divergence, curl and Laplacian operators are:

$$\vec{\nabla}f=\frac{\partial f}{\partial s}\hat{S}+\frac{1}{s}\frac{\partial f}{\partial\phi}\hat{\phi}+\frac{\partial f}{\partial z}\hat{Z}$$

$$\vec{\nabla}\cdot\vec{A}=\frac{1}{s}\frac{\partial}{\partial s}(sA_s)+\frac{1}{s}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A}=\left(\frac{1}{s}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z}\right)\hat{S}+\left(\frac{\partial A_s}{\partial z}-\frac{\partial A_z}{\partial s}\right)\hat{\phi}+\frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\phi)-\frac{\partial A_s}{\partial\phi}\right)\hat{Z}$$

$$\vec{\nabla}^2f=\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right)+\frac{1}{s^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}$$

(a) $\vec{\nabla}V=2\hat{r}$
(b) $\vec{\nabla}\cdot\vec{V}=2$
(c) $\vec{\nabla}\times\vec{V}=4\hat{Z}$, where $\hat{Z}$ is a unit vector perpendicular to the (r,θ) plane
(d) $\vec{\nabla}^2V=\frac{4}{3}$ at $r=1.5$ m

12. Consider a two dimensional Cartesian coordinate system in which a rank 2 contravariant tensor is represented by the matrix $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. The coordinate system is rotated anticlockwise by an acute angle θ with the origin fixed. Which one of the following matrices represents the tensor in the new coordinate system?
(a) $\begin{pmatrix}0 & \cos\ 2\theta\\ -\sin\ 2\theta & 0\end{pmatrix}$
(b) $\begin{pmatrix}\sin\ 2\theta & \cos\ 2\theta\\ \cos\ 2\theta & -\sin\ 2\theta\end{pmatrix}$
(c) $\begin{pmatrix}\sin\ 2\theta & -\cos\ 2\theta\\ \cos\ 2\theta & \sin\ 2\theta\end{pmatrix}$
(d) $\begin{pmatrix}\sin\ 2\theta & 0\\ 0 & -\cos\ 2\theta\end{pmatrix}$

Electromagnetic Theory

13. Which one of the following is a dimensionless constant?
(a) Permittivity of free space
(b) Permeability of free space
(c) Bohr magneton
(d) Fine structure constant

14. Consider an isolated magnetized sphere of radius R with a uniform magnetization $\vec{M}$ along the positive z-direction, with the north and south poles of the sphere lying on the z axis. It is given that the magnetic field inside the sphere is $\vec{B}=\frac{2\mu_0}{3}\vec{M}$, where μ_0 is

the permeability of vacuum. Which of the following statements is (are) CORRECT?
(a) The bound volume current density is zero
(b) The bound surface current density has maximum magnitude at the equator, where this magnitude equals $|\vec{M}|$
(c) The auxiliary field $\vec{H} = -\frac{2}{3}\vec{M}$
(d) Far from the sphere, the magnetic field is due to a dipole of moment $\vec{m}$, where $\frac{\vec{m}}{4\pi R^3} = \frac{B}{2\mu_0}\hat{z}$

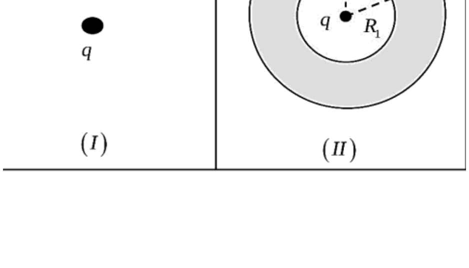
15. An electric field as a function of radial coordinate r has the form $\vec{E} = \alpha\frac{e^{-r^2}}{r}\hat{r}$, where α is a constant. Assume that dimensions are appropriately taken care of. The electric flux through a sphere of radius $\sqrt{2}$, centered at the origin, is Φ. What is the value of $\frac{\Phi}{2\pi\alpha}$ (rounded off to two decimal places)?

16. Young's double slit experiment is performed using a beam of C_{60} (fullerene) molecules, each molecule being made up of 60 carbon atoms. When the slit separation is 50 nm, fringes are formed on a screen kept at a distance of 1 m from the slits. Now, the experiment is repeated with C_{70} molecules with a slit separation of 92.5 nm. The kinetic energies of both the beams are the same. The position of the 4^{th} bright fringe for C_{60} will correspond to the n^{th} bright fringe for C_{70}. What is the value of n (rounded off to the nearest integer)?
(a) 5 (b) 6
(c) 7 (d)

17. Different spectral lines of the Balmer series (transitions $n \to 2$, with n being the principal quantum number) fall one at a time on a Young's double slit apparatus. The separation between the slits is d and the screen is placed at a constant distance from the slits. What factor should d be multiplied by to maintain a constant fringe width for various lines, as n takes different allowed values?
(a) $\frac{n^2-4}{4n^2}$ (b) $\frac{n^2+4}{4n^2}$
(c) $\frac{4n^2}{n^2-4}$ (d) $\frac{4n^2}{n^2+4}$

18. Under parity and time reversal transformations, which of the following statements is (are) TRUE about the electric dipole moment $\vec{p}$ and the magnetic dipole moment $\vec{\mu}$?
(a) $\vec{p}$ is odd under parity and $\vec{\mu}$ is odd under time reversal
(b) $\vec{p}$ is odd under parity and $\vec{\mu}$ is even under time reversal
(c) $\vec{p}$ is even under parity and $\vec{\mu}$ is odd under time reversal
(d) $\vec{p}$ is even under parity and $\vec{\mu}$ is even under time reversal

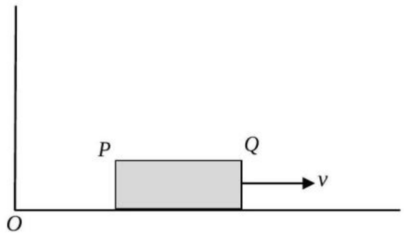
19. Two independent electrostatic configurations are shown in the figure. Configuration (I) consists of an isolated point charge $q = 1\text{C}$, C, and configuration (II) consists of another identical charge surrounded by a thick conducting shell of inner radius $R_1 = 1$ m and outer radius $R_2 = 2$ m, with the charge being at the centre of the shell. $W_I = \frac{\epsilon_0}{2}\int E_I^2 dV$ and $W_{II} = \frac{\epsilon_0}{2}\int E_{II}^2 dV$, where E_I and E_{II} are the magnitudes of the electric fields for configurations (I) and (II) respectively, ϵ_0 is the permittivity of vacuum, and the volume integrations are carried out over all space. If $\frac{8\pi}{\epsilon_0}|W_I - W_{II}| = \frac{1}{2}$, what is the value of the integer n ?



20. Consider an electromagnetic wave propagating in the z-direction in vacuum, with the magnetic field given by $\vec{B} = \vec{B}_0 e^{i(kz-\omega t)}$. If $B_0 = 10^{-8}$ T, the average power passing through a circle of radius 1.0 m placed in the xy plane is P (in Watts). Using $\epsilon_0 = 10^{-11} \frac{c^2}{\text{Nm}^2}$, what is the value of $\frac{10^3 P}{\pi}$ (rounded off to one decimal place)?

Classical Mechanics

21. A rod PQ of proper length L lies along the x-axis and moves towards the positive x direction with speed $v = \frac{3c}{5}$ with respect to the ground (see figure), where c is the speed of light in vacuum. An observer on the ground measures the positions of P and Q at different times t_P and t_Q respectively in the ground frame, and finds the difference between them to be $\frac{9L}{10}$. What is the value of $t_Q - t_P$?



(a) $\frac{L}{3c}$ (b) $\frac{L}{5c}$
(c) $\frac{L}{6c}$ (d) $\frac{2L}{3c}$

22. A symmetric top has principal moments of inertia $I_1 = I_2 = \frac{2\alpha}{3}, I_3 = 2\alpha$ about a set of principal axes 1,2,3 respectively, passing through its center of mass, where α is a positive constant. There is no force acting on the body and the angular speed of the body about the 3-axis is $\omega_3 = \frac{1}{8}$ rad/s. With what angular frequency in rad/s does the angular velocity vector $\vec{\omega}_1$ precess about the 3 -axis?

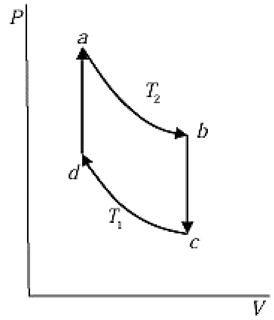
(a) 2 (b) 3
(c) 5 (d) 7

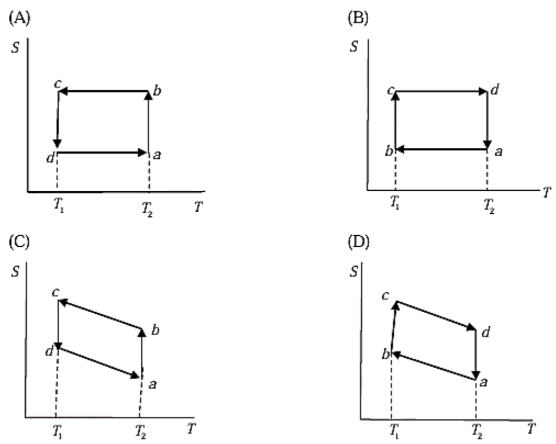
23. A particle of mass m is free to move on a frictionless horizontal two dimensional (r, θ) plane, and is acted upon by a force $\vec{F} = -\frac{k}{2r^3}\hat{r}$ with k being a positive constant. If p_r and p_θ are the generalized momenta corresponding to r and θ respectively, then what is the value of $\frac{dp_r}{dt}$?

(a) $\frac{p_\theta^2 - 2mk}{2mr^3}$ (b) $\frac{2p_\theta^2 - mk}{mr^3}$
(c) $\frac{p_\theta^2 - 2mk}{mr^3}$ (d) $\frac{2p_\theta^2 - mk}{2mr^3}$

Statistical Mechanics

24. Which one of the following entropy (S) - temperature (T) diagrams CORRECTLY represents the Carnot cycle $abcda$ shown in the $P - V$ diagram?’





25. Which of the following is (are) the CORRECT option(s) for the Joule-Thomson effect?
(a) It is an isentropic process

(b) It is an isenthalpic process
(c) It can result in cooling as well as heating
(d) For an ideal gas it always results in cooling

26. A simple harmonic oscillator with an angular frequency ω is in thermal equilibrium with a reservoir at absolute temperature T, with $\omega = \frac{2k_BT}{\hbar}$. Which one of the following is the partition function of the system?
(a) $\frac{e}{e^2-1}$ (b) $\frac{e}{e^2+1}$
(c) $\frac{e}{e-1}$ (d) $\frac{e}{e+1}$

27. Two identical, non-interacting $^4He_2$ atoms are distributed among 4 different nondegenerate energy levels. The probability that they occupy different energy levels is p. Similarly, two $^3He_2$ atoms are distributed among 4 different non-degenerate energy levels, and the probability that they occupy different levels is q. What is the value of $\frac{p}{q}$ (rounded off to one decimal place)?

28. Two identical bodies kept at temperatures 800 K and 200 K act as the hot and the cold reservoirs of an ideal heat engine, respectively. Assume that their heat capacity (c) in Joules/K is independent of temperature and that they do not undergo any phase change. Then, the maximum work that can be obtained from the heat engine is $n \times C$ Joules. What is the value of n (in integer)?

Atomic Molecular Physics

29. The atomic number of an atom is 6 . What is the spectroscopic notation of its ground state, according to Hund's rules?
(a) $^3P_0$ (b) $^3P_1$
(c) $^3D_3$ (d) $^3S_1$

30. An atom with non-zero magnetic moment has an angular momentum of magnitude $\sqrt{12}\hbar$. When a beam of such atoms is passed through a Stern-Gerlach apparatus, how many beams does it split into?
(a) 3 (b) 7
(c) 9 (d) 25

31. The deuteron is a bound state of a neutron and a proton. Which of the following statements is(are) CORRECT?
(a) The deuteron has a finite value of electric quadrupole moment due to nonspherical electronic charge distribution
(b) The magnetic moment of the deuteron is equal to the sum of the magnetic moments of the neutron and the proton
(c) The deuteron state is an admixture of $^3S_1$ and $^3D_1$ states
(d) The deuteron state is an admixture of $^3S_1$ and $^3P_1$ states

32. The Geiger-Muller counter is a device to detect α, β and γ radiations. It is a cylindrical tube filled with monatomic gases like argon, and polyatomic gases such as ethyl alcohol. The inner electrode is along the axis of the cylindrical tube and the outer electrode is the tube. Which of the following statements is (are) CORRECT?
(a) Argon is used so that ambient light coming from the surroundings do not produce any signal in the detector
(b) Ethyl alcohol is used as a quenching gas
(c) The electric field strength decreases from the axis to the edge of the tube and the direction of the field is radially outward
(d) The electric field increases from the axis to the edge of the tube and the field

33. In the vector model of angular momentum applied to atoms, what is the minimum angle in degrees (in integer) made by the orbital angular momentum vector and the positive z axis for a $2p$ electron?

34. It is given that the electronic ground state of a diatomic molecule X_2 has even parity and the nuclear spin of X is 0 . Which one of the following is the CORRECT statement with regard to the rotational Raman spectrum (J is the rotational quantum number) of this molecule?
(a) Lines of all J values are present
(b) Lines have alternating intensity in the ratio of 3:1
(c) Lines of only even J values are present
(d) Lines of only odd J values are present

35. Which one of the following options is the most appropriate match between the items given in Column 1 and Column 2?

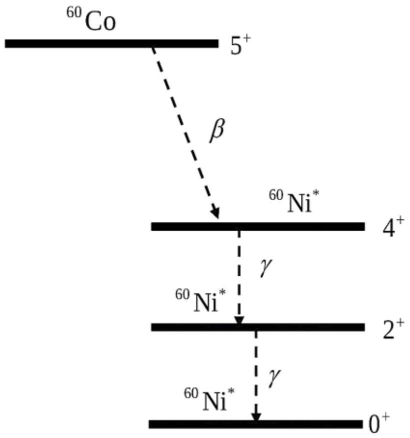
Column 1	Column 2
(i) Visible light	P. Transition between core energy levels of atoms
(ii) X-rays	Q. Transition between nuclear energy levels
(iii) Gamma rays	R. Pair production
(iv)Thermal neutrons	S. Crystal structure determination
	T. Photoelectric effect

(a) (i) - T; (ii) - P,S,T; (iii) - Q,R; (iv) - S
(b) (i) - P,T; (ii) - S; (iii) - R,S; (iv) - S,T
(c) (i) - T; (ii) - R,S; (iii) - Q,R; (iv) - S
(d) (i) - S,T; (ii) - P,S; (iii) - R,T; (iv) - S

36. A slow moving π^-particle is captured by a deuteron (d) and this reaction produces two neutrons (n) in the final state, i.e., $\pi^- + d \rightarrow n + n$, Neutron and deuteron have even intrinsic parities, whereas π^-has odd intrinsic parity. L and S are the orbital and spin angular momenta, respectively of the system of two neutrons. Which of the following statements regarding the final two-neutron state is (are) CORRECT?
(a) It has odd parity
(b) $L + S$ is odd
(c) $L = 1, S = 1$
(d) $L = 2, S = 0$

Nuclear Physics

37. A ^{60}Co nucleus emits a β-particle and is converted to ^{60}Ni* with $J^P = 4^+$, which in turn decays to the ^{60}Ni ground state with $J^P = 0^+$by emitting two photons in succession, as shown in the figure. Which one of the following statements is CORRECT?



(a) $4^+ \rightarrow 2^+$is an electric octupole transition
(b) $4^+ \rightarrow 2^+$is a magnetic quadrupole transition
(c) $2^+ \rightarrow 0^+$is an electric quadrupole transition
(d) $2^+ \rightarrow 0^+$is a magnetic quadrupole transition

38. The Ξ^{0^*} particle is a member of the Baryon decuplet with isospin state $|I, I_3\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$ and strangeness quantum number -2 . In the quark model, which one of the following is the flavour part of the Ξ^{0^*} wavefunction?
(a) $\frac{1}{\sqrt{2}}(uss - ssu)$
(b) $\frac{1}{\sqrt{3}}(uss + sus + ssu)$

(c) $\frac{1}{\sqrt{2}}(uss + ssu)$

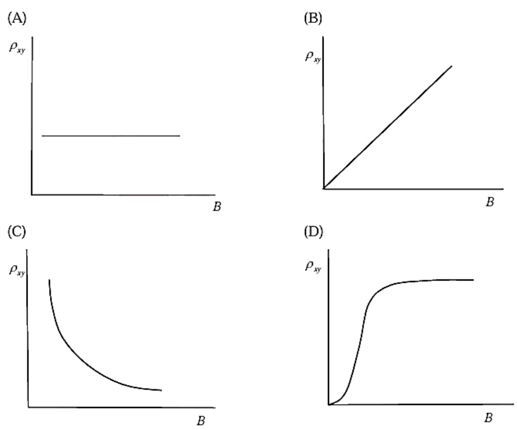
(d) $\frac{1}{\sqrt{3}}(uss - sus + ssu)$

39. In pion nucleon scattering, the pion and nucleon can combine to form a short lived bound state called the Δ particle ($\pi + N \rightarrow \Delta$). The masses of the pion, nucleon and the Δ particle are $140 \mathrm{MeV/c^2}$, $938 \mathrm{MeV/c^2}$ and $1230 \mathrm{MeV/c^2}$, respectively. In the lab frame, where the nucleon is at rest, what is the minimum energy (in $\mathrm{MeV/c^2}$, rounded off to one decimal place) of the pion to produce the Δ particle?

40. An α-particle is emitted from the decay of Americium (Am) at rest, i.e., ${}^{241}_{94}\mathrm{Am} \rightarrow {}^{237}_{92}U + \alpha$. The rest masses of ${}^{241}_{94}\mathrm{Am}$, ${}^{237}_{92}U$ and α are $224.544 \mathrm{GeV/c^2}$, $220.811 \mathrm{GeV/c^2}$ and $3.728 \mathrm{GeV/c^2}$ respectively. What is the kinetic energy (in $\mathrm{MeV/c^2}$, rounded off to two decimal places) of the α-particle?

Solid State Physics

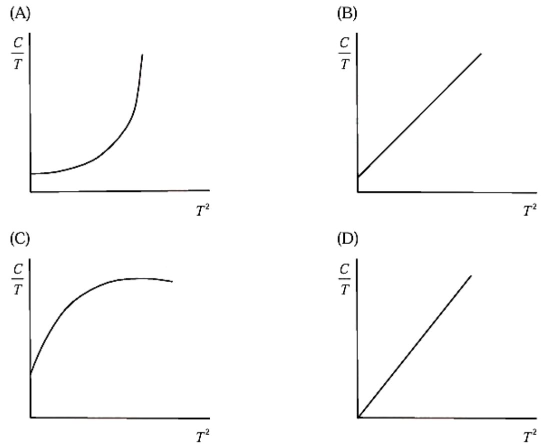
41. The Hall experiment is carried out with a non-magnetic semiconductor. The current I is along the x-axis and the magnetic field B is along the z-axis. Which one of the following is the CORRECT representation of the variation of the magnitude of the Hall resistivity ρ_{xy} as a function of the magnetic field?



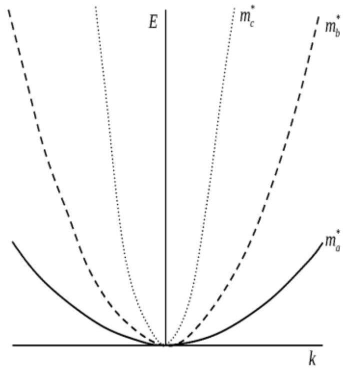
42. A compound consists of three ions X, Y and Z. The Z ions are arranged in an FCC arrangement. The X ions occupy $\frac{1}{6}$ of the tetrahedral voids and the Y ions occupy $\frac{1}{3}$ of the octahedral voids. Which one of the following is the CORRECT chemical formula of the compound?

(a) XY_2Z_4 (b) XYZ_3
(c) XYZ_2 (d) XYZ_4

43. For a non-magnetic metal, which one of the following graphs best represents the behaviour of $\frac{C}{T}$ vs. T^2, where C is the heat capacity and T is the temperature?



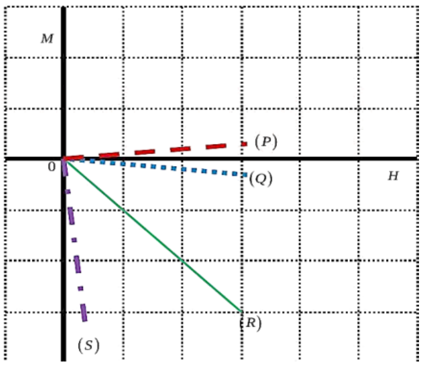
44. For non-relativistic electrons in a solid, different energy dispersion relations (with effective masses m_a^*, m_b^*, and m_c^*) are schematically shown in the plots. Which one of the following options is CORRECT?



(a) $m_a^* = m_b^* = m_c^*$
(b) $m_b^* > m_c^* > m_a^*$
(c) $m_c^* > m_b^* > m_a^*$

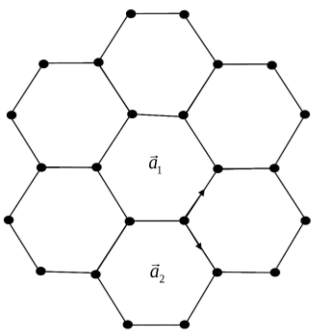
(d) $m_a^* > m_b^* > m_c^*$

45. The figure schematically shows the M (magnetization) - H (magnetic field) plots for certain types of materials. Here M and H are plotted in the same scale and units. Which one of the following is the most appropriate combination?



(a) (Q) - Paramagnet; (R) - Type-I Superconductor; (S) - Antiferromagnet
(b) (P) - Paramagnet; (Q) - Diamagnet; (R) - Type-I Superconductor
(c) (P) - Paramagnet; (Q) - Antiferromagnet; (R) - Type-I Superconductor
(d) (P) - Diamagnet; (R) - Paramagnet; (S) - Type-I Superconductor

46. Graphene is a two dimensional material, in which carbon atoms are arranged in a honeycomb lattice with lattice constant a. As shown in the figure, $\vec{a}_1$ and $\vec{a}_2$ are two lattice vectors. Which one of the following is the area of the first Brillouin zone for this lattice?



(a) $\frac{8\pi^2}{3\sqrt{3}a^2}$ (b) $\frac{4\pi^2}{3\sqrt{3}a^2}$
(c) $\frac{8\pi^2}{\sqrt{3}a^2}$ (d) $\frac{4\pi^2}{\sqrt{3}a^2}$

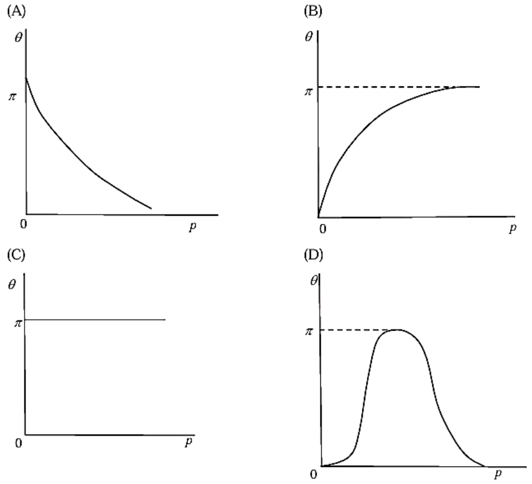
47. A neutron beam with a wave vector $\vec{k}$ and an energy 20.4meV diffracts from a crystal with an outgoing wave vector $\vec{k}'$. One of the diffraction peaks is observed for the reciprocal lattice vector $\vec{G}$ of magnitude $3 \cdot 14 Å^{-1}$. What is the diffraction angle in degrees (rounded off to the nearest integer) that $\vec{k}$ makes with the plane? (Use mass of neutron $= 1.67 \times 10^{-27}$Kg)
(a) 15 (b) 30
(c) 45 (d) 60

48. In the first Brillouin zone of a rectangular lattice (lattice constants $a = 6Å$ and $b = 4Å$), three incoming phonons with the same wave vector $\langle 1.2Å^{-1}, 0.6Å^{-1} \rangle$ interact to give one phonon. Which one of the following is the CORRECT wave vector of the resulting phonon?
(a) $\langle 2.56Å^{-1}, 0.23Å^{-1} \rangle$
(b) $\langle 3.60Å^{-1}, 1.80Å^{-1} \rangle$
(c) $\langle 0.48Å^{-1}, 0.23Å^{-1} \rangle$
(d) $\langle 3.60Å^{-1}, 0.80Å^{-1} \rangle$

49. For a covalently bonded solid consisting of ions of mass m, the binding potential can be assumed to be given by $U(r) = -\epsilon \left(\frac{r}{r_0}\right) e^{-\frac{r}{r_0}}$, where $\in$ and r_0 are positive constants. What is the Einstein frequency of the solid in Hz ?
(a) $\frac{1}{2\pi}\sqrt{\frac{\epsilon e}{m r_0^2}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{\epsilon}{m e r_0^2}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{2\epsilon}{m e r_0^2}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{\epsilon e}{2 m r_0^2}}$

50. In a hadronic interaction, π^0,s are produced with different momenta, and they immediately decay into two photons with an opening angle θ between them. Assuming that all these decays occur in one

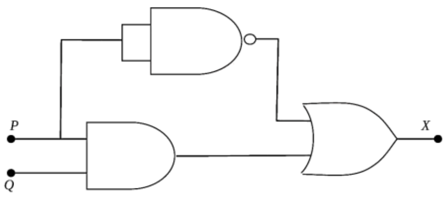
plane, which one of the following figures depicts the behaviour of θ as a function of the π^0 momentum p ?



51. Consider 6 identical, non-interacting, spin $\frac{1}{2}$ atoms arranged on a crystal lattice at absolute temperature T. The z-component of the magnetic moment of each of these atoms can be $\pm\mu_B$. If P and Q are the probabilities of the net magnetic moment of the solid being $2\mu_B$ and $6\mu_B$ respectively, what is the value of $\frac{P}{Q}$ (integer)?

Electronics

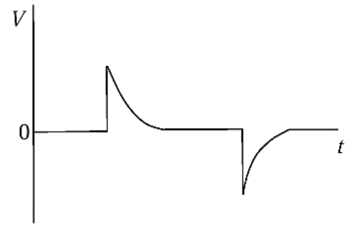
52. Which one of the following options is CORRECT for the given logic circuit?

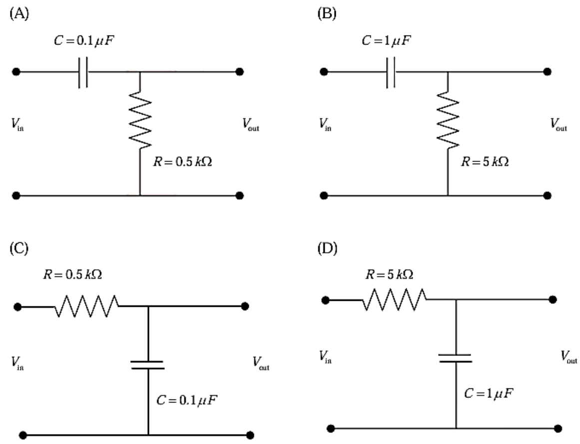


(a) $P = 1, Q = 1; X = 0$
(b) $P = 1, Q = 0; X = 1$
(c) $P = 0, Q = 1; X = 0$
(d) $P = 0, Q = 0; X = 1$

53. For a transistor amplifier, the frequency response is such that the mid band voltage gain is 200. The cutoff frequencies are 20 Hz and 20kHz. What is the ratio (rounded off to two decimal places) of the voltage gain at 10 Hz to that at 100kHz ?

54. An input voltage in the form of a square wave of frequency 1kHz is given to a circuit, which results in the output shown schematically below. Which one of the following options is the CORRECT representation of the circuit?





Answer Key					
1.	2. c	3. c	4. b	5. a	6. d
7. b	8. a,b,d	9. d	10. b	11. a,c,d	12. b
13. d	14. a,b,d	15. 0.38	16. d	17. c	18. a
19. 2	20. 13.5	21. c	22.	23. d	24. a
25.	26. a	27. 0.6	28. 200	29. a	30. b
31. a,c	32. a,b,c	33. 45	34. c	35. a	36. a,c
37. c	38. b	39. 340	40. 4.19	41. b	42. b
43. b	44. d	45. b	46. a	47. b	48. a
49. b	50. a	51. 15	52. d	53. 2.28	54. a
55.					

❖ Solutions

1. **Solution:**

2. **Solution:** Given wavefunction, $\psi(x, y, z) = Nze^{-\alpha(x^2+y^2+z^2)}, \alpha > 0$

z in spherical coordinates:$z = r\cos\,\theta$

So,$\psi = Nr\cos\,\theta e^{-\alpha r^2}, \psi(r, \theta, \phi) = \left(Nre^{-\alpha r^2}\right)\cos\,\theta$

Given:

$$Y_1^0(\theta, \phi) = \sqrt{\frac{3}{4\pi}}\cos\,\theta$$

Hence, $\cos\,\theta \propto Y_1^0$

Therefore, the state has: orbital quantum number $l = 1$

magnetic quantum number $m = 0$

Eigenvalues

Eigenvalue of L^2 :$L^2 = l(l+1)\hbar^2 = 1(1+1)\hbar^2 = 2\hbar^2$

Eigenvalue of L_z :$L_z = m\hbar = 0$

Correct option is (c)

3. **Solution:** Given

$$\psi(x) = \begin{cases} M, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

momentum-space wavefunction

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\psi(x)e^{-ipx/\hbar}dx$$

Since $\psi(x) = 0$ outside $[-a, a]$,

$$\phi(p) = \frac{M}{\sqrt{2\pi\hbar}}\int_{-a}^{a}e^{-ipx/\hbar}dx, \qquad \int e^{-ipx/\hbar}dx = \frac{-\hbar}{ip}e^{-ipx/\hbar}$$

$$\int_{-a}^{a}e^{-ipx/\hbar}dx = \frac{-\hbar}{ip}\left[e^{-ipa/\hbar} - e^{+ipa/\hbar}\right], \qquad e^{i\theta} - e^{-i\theta} = 2i\sin\,\theta$$

So$e^{-ipa/\hbar} - e^{+ipa/\hbar} = -2i\sin\,\left(\frac{pa}{\hbar}\right)$

$$\int_{-a}^{a}e^{-ipx/\hbar}dx = \frac{-\hbar}{ip}\left[-2i\sin\,\left(\frac{pa}{\hbar}\right)\right] = \frac{2\hbar}{p}\sin\,\left(\frac{pa}{\hbar}\right)$$

form of $\phi(p)$

$$\phi(p) = \frac{M}{\sqrt{2\pi\hbar}}\frac{2\hbar}{p}\sin\,\left(\frac{pa}{\hbar}\right) \; or \; \phi(p) \propto \frac{\sin\,(pa/\hbar)}{p}$$

Probability density

$$|\phi(p)|^2 \propto \left(\frac{\sin\,(pa/\hbar)}{p}\right)^2$$

Physical shape of the plot

Maximum at $p = 0$

Zeros at $p = \pm n\pi\hbar/a$

Oscillatory side lobes

Symmetric about $p = 0$

Correct option is (c)

4. **Solution:** For a 2D infinite square well of side $L(0 \le x \le L, 0 \le y \le L)$, the normalized stationary states are

$$\psi_{n_x,n_y}(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$$

with energies

$$E_{n_x,n_y} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2\right)$$

The wavefunction is zero only along the line $y = \frac{L}{2}$ (besides the boundaries).

The y-dependence is $\sin\left(\frac{n_y\pi y}{L}\right)$.

At $y = \frac{L}{2}$,

$$\sin\left(\frac{n_y\pi}{2}\right) = 0 \Rightarrow n_y = 2,4,\ldots$$

To have only one internal nodal line (just $y = L/2$), take the smallest such value: $n_y = 2$

No internal node in x so take, $n_x = 1$

the given state is $\left(n_x, n_y\right) = (1,2)$.

Energy of the given state

$$E = E_{1,2} = \frac{\pi^2\hbar^2}{2mL^2}(1^2 + 2^2) = 5\frac{\pi^2\hbar^2}{2mL^2}$$

Ground-state energy

The ground state is $\left(n_x, n_y\right) = (1,1)$:

$$E_0 = E_{1,1} = \frac{\pi^2\hbar^2}{2mL^2}(1^2 + 1^2) = 2\frac{\pi^2\hbar^2}{2mL^2}, \qquad \frac{E_0}{E} = \frac{2}{5} \Rightarrow E_0 = \frac{2}{5}E.$$

Correct option is (b)

5. **Solution:**

$$H = \frac{4\lambda}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2 = \frac{2\lambda}{\hbar^2}\left(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2\right)$$

$$\Rightarrow E = 2\lambda[S(S+1) - S_1(S_1+1) - S_2(S_2+1)]$$

$$S_1 = \frac{1}{2}, S_2 = \frac{1}{2} \Rightarrow S = 1,0$$

Case (1): $S = 1$

$$E_1 = 2\lambda\left[2 - \frac{3}{4} - \frac{3}{4}\right] = 2\lambda\left(\frac{1}{2}\right) = \lambda$$

Case (2): $S = 0$

$$E_0 = 2\lambda\left[0 - \frac{3}{4} - \frac{3}{4}\right] = -3\lambda$$

Given state: $|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle)$

Hence, $\langle E\rangle = \frac{1}{2}(\lambda) + \frac{1}{2}(-3\lambda) = -\lambda$

Correct option is (a)

6. **Solution:** Spin operator along x-axis:

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Eigenvalues of S_x are $+\frac{\hbar}{2}, -\frac{\hbar}{2}$ with corresponding eigenstates

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \left|\frac{1}{2},-\frac{1}{2}\right\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The spin operator along direction $\hat{x}'$, which makes an angle θ with the x-axis in the $x - y$ plane, is

$$\vec{S}\cdot\hat{x}' = S_x\cos\,\theta + S_y\sin\,\theta, \hat{x}' = \cos\,\theta\hat{\imath} + \sin\,\theta\hat{\jmath}$$

where

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Thus the operator

$$A = \vec{S}\cdot\hat{x}' = \frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$$

The eigenvalues of A are $+\frac{\hbar}{2}, -\frac{\hbar}{2}$ with eigenstates

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle_{x'} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta/2} \\ e^{i\theta/2} \end{pmatrix}, \left|\frac{1}{2},-\frac{1}{2}\right\rangle_{x'} = \frac{1}{\sqrt{2}}\begin{pmatrix} -e^{-i\theta/2} \\ e^{i\theta/2} \end{pmatrix}$$

Probability that the particle initially in

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle_x$$

is found in

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle_{x'} : P = \left|\left\langle\frac{1}{2},\frac{1}{2}\middle|\,\frac{1}{2},\frac{1}{2}\right\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}(1 \quad 1)\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta/2} \\ e^{i\theta/2} \end{pmatrix}\right|^2 = \left|\frac{e^{-i\theta/2} + e^{i\theta/2}}{2}\right|^2 = \cos^2\frac{\theta}{2}$$

Correct option is (d)

7. **Solution:** It is a skew Hermitian matrix. So, Eigen values are imaginary or zero. Trace should be zero.

Also $\sum_i \lambda_i^4 = 4$

Correct option is (b)

8. **Solution:** For two functions $f(x), g(x)$, define

$$W(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = fg' - f'g$$

If $W \neq 0$ for some x, the functions are linearly independent

(a) e^{ix} and e^{-ix}

$$f = e^{ix}, g = e^{-ix}, \; f' = ie^{ix}, g' = -ie^{-ix}$$

$$W = \begin{vmatrix} e^{ix} & e^{-ix} \\ ie^{ix} & -ie^{-ix} \end{vmatrix} = -2i \neq 0$$

Linearly independent

(b) x and e^x

$$f = x, g = e^x, f' = 1, g' = e^x, W = \begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix} = xe^x - e^x = e^x(x-1)$$

This is not zero.

Linearly independent

(c) 2^x and 2^{-3+x}

$$g = 2^{x-3} = \frac{1}{8}2^x$$

So one function is a constant multiple of the other.

$W = 0$

Linearly dependent

(d) e^{ix} and $\sin x$

$$f = e^{ix}, g = \sin x, \qquad f' = ie^{ix}, g' = \cos x$$

$$W = \begin{vmatrix} e^{ix} & \sin x \\ ie^{ix} & \cos x \end{vmatrix} = e^{ix}(\cos x - i\sin x) = e^{ix}e^{-ix} = 1, \qquad W \neq 0$$

Linearly independent

Correct option is (a,b,d)

9. **Solution:** Given $U(x,y) = xy(x^2 - y^2) = x^3y - xy^3$, $V(x,y) = ax^4 + by^4 + cx^2y^2 + k$

For $f(x+iy) = U + iV$ to be analytic, the Cauchy-Riemann (CR) equations must hold

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}, \frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}, \quad \frac{\partial U}{\partial x} = 3x^2y - y^3, \quad \frac{\partial U}{\partial y} = x^3 - 3xy^2$$

$$\frac{\partial V}{\partial x} = 4ax^3 + 2cxy^2, \qquad \frac{\partial V}{\partial y} = 4by^3 + 2cx^2y$$

Apply CR equations

$$\text{(i)} \frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}, \qquad 3x^2y - y^3 = 2cx^2y + 4by^3$$

Compare coefficients:

$$x^2y: 3 = 2c \Rightarrow c = \frac{3}{2}, \qquad y^3: -1 = 4b \Rightarrow b = -\frac{1}{4}$$

(ii) $\frac{\partial U}{\partial y}=-\frac{\partial V}{\partial x}$, $\quad x^3-3xy^2=-(4ax^3+2cxy^2)$

Compare coefficients:

$$x^3: 1=-4a \Rightarrow a=-\frac{1}{4}, \qquad xy^2: -3=-2c \Rightarrow c=\frac{3}{2}$$

$$a=-\frac{1}{4}, b=-\frac{1}{4}, c=\frac{3}{2}, \qquad abc=\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\left(\frac{3}{2}\right)=\frac{3}{32}$$

Correct option is (d)

10.**Solution:** Near $z=\pi$, write $z=\pi+t$.

$$\sin(\pi+t)=-t+\frac{t^3}{6}+\cdots$$

So $\sin z$ has a simple zero at $z=\pi$.

Denominator has a zero of order 4 .

Hence net order of the pole: $4-1=3$

Pole of order 3

Residue at $z=\pi$

Since the pole is of order 3,

$$\text{Res}_{z=\pi}f(z)=\frac{1}{2!}\lim_{z\to\pi}\frac{d^2}{dz^2}[(z-\pi)^3 f(z)], \qquad (z-\pi)^3 f(z)=\frac{z^2\sin z}{z-\pi}$$

Using $\sin(\pi+t)/t=-1+\frac{t^2}{6}+\cdots$,

$$\frac{z^2\sin z}{z-\pi}=-z^2+\frac{z^2}{6}(z-\pi)^2+\cdots$$

Second derivative at $z=\pi$:

$$\frac{d^2}{dz^2}(-z^2)=-2, \frac{d^2}{dz^2}\left[\frac{z^2}{6}(z-\pi)^2\right]_{z=\pi}=\frac{\pi^2}{3}$$

So, $\text{Res}_{z=\pi}f(z)=\frac{1}{2}\left(-2+\frac{\pi^2}{3}\right)=-1+\frac{\pi^2}{6}$

Correct option is (b)

11.**Solution:** A thin horizontal disc of radius $R=2$ m rotates anticlockwise with uniform angular speed $\omega=2\text{rads}^{-1}$ about the z-axis.

For rigid rotation, the velocity field in plane polar (cylindrical) coordinates is $\vec{V}=\omega r\hat{\theta}$ with components $V_r=0, V_\theta=\omega r, V_z=0$ and magnitude $V=|\vec{V}|=\omega r=2r$

(a) $\nabla V=2\hat{r}$

Since $V=2r$,

$$\nabla V=\frac{\partial V}{\partial r}\hat{r}=2\hat{r}$$

Correct

(b) $\nabla\cdot\vec{V}=2$

Divergence in polar coordinates:

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

Here, $V_r = 0, V_\theta = 2r, \quad \nabla \cdot \vec{V} = 0 + 0 = 0$

Incorrect

(c) $\nabla \times \vec{V} = 4\hat{z}$

Curl (only z-component survives):

$$(\nabla \times \vec{V})_z = \frac{1}{r}\frac{\partial}{\partial r}(rV_\theta) - \frac{1}{r}\frac{\partial V_r}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial r}(2r^2) = \frac{4r}{r} = 4, \qquad \nabla \times \vec{V} = 4\hat{z}$$

Correct

Laplacian of scalar $V = 2r$:

$$\nabla^2 V = \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = \frac{1}{r}\frac{d}{dr}(2r) = \frac{2}{r}$$

At $r = 1.5$:

$$\nabla^2 V = \frac{2}{1.5} = \frac{4}{3}$$

correct

Correct option is (a,c,d)

12.**Solution:**Rank-2 contravariant tensor in 2D Cartesian coordinates:

$$T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Coordinate system is rotated anticlockwise by angle θ.

Rotation matrix:

$$A = \begin{pmatrix} \cos\,\theta & \sin\,\theta \\ -\sin\,\theta & \cos\,\theta \end{pmatrix}, \det A = 1$$

Transformation rule (contravariant rank-2 tensor)

$$T' = ATA^T$$

$$AT = \begin{pmatrix} \cos\,\theta & \sin\,\theta \\ -\sin\,\theta & \cos\,\theta \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sin\,\theta & \cos\,\theta \\ \cos\,\theta & -\sin\,\theta \end{pmatrix}$$

Multiply by A^T

$$A^T = \begin{pmatrix} \cos\,\theta & -\sin\,\theta \\ \sin\,\theta & \cos\,\theta \end{pmatrix}, \qquad T' = \begin{pmatrix} \sin\,\theta & \cos\,\theta \\ \cos\,\theta & -\sin\,\theta \end{pmatrix}\begin{pmatrix} \cos\,\theta & -\sin\,\theta \\ \sin\,\theta & \cos\,\theta \end{pmatrix}$$

$$T' = \begin{pmatrix} \sin\,2\theta & \cos\,2\theta \\ \cos\,2\theta & -\sin\,2\theta \end{pmatrix}, \begin{pmatrix} \sin\,2\theta & \cos\,2\theta \\ \cos\,2\theta & -\sin\,2\theta \end{pmatrix}$$

Correct option is (b)

13.**Solution:** (A) Permittivity of free space ε_0

$$[\varepsilon_0] = \frac{C^2}{Nm^2} = M^{-1}L^{-3}T^4A^2$$

Not dimensionless

(B) Permeability of free space μ_0

$$[\mu_0] = \frac{N}{A^2} = MLT^{-2}A^{-2}$$

Not dimensionless

(C) Bohr magneton μ_B

$$\mu_B = \frac{e\hbar}{2m_e} \Rightarrow [\mu_B] = Am^2$$

Not dimensionless

(D) Fine structure constant α

Standard definition:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

Check dimensions: $e^2/\varepsilon_0 \sim ML^3T^{-2},\ \hbar c \sim ML^3T^{-2},\quad [\alpha] = 1$

Dimensionless

Correct option is (d)

14.**Solution:** Uniformly magnetized solid sphere

Magnetization: $\vec{M} = M\hat{z}$ (constant)

Magnetic field inside:

$$\vec{B} = \frac{2\mu_0}{3}\vec{M}$$

(A) Bound volume current density, $\vec{J_b} = \nabla \times \vec{M}$

Since $\vec{M}$ is uniform: $\nabla \times \vec{M} = 0$

Correct

(B) Bound surface current density, $\vec{K_b} = \vec{M} \times \hat{n}$

On the surface of the sphere: $|\vec{K_b}| = M\sin\ \theta$

Maximum at equator ($\theta = \pi/2$)

Maximum value: $|\vec{K_b}|_{\max} = M$

Correct

(C) Auxiliary field $\vec{H}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}, \qquad \vec{H} = \frac{2}{3}\vec{M} - \vec{M} = -\frac{1}{3}\vec{M}$$

But option (C) says:

$$\vec{H} = -\frac{2}{3}\vec{M}$$

Incorrect

(D) Field far from the sphere

A uniformly magnetized sphere behaves like a magnetic dipole at large distances.

Magnetic dipole moment:

$$\vec{m} = \vec{M} \times \text{ Volume } = \vec{M}\frac{4\pi R^3}{3}$$

Dipole field: $\vec{B}(\text{ far }) = \frac{\mu_0}{4\pi r^3}(2m\cos\ \theta\hat{r} + m\sin\ \theta\hat{\theta})$

Using field on axis to relate m : $\frac{\vec{m}}{4\pi R^3} = \frac{B}{2\mu_0}\hat{z}$

So statement is correct.

Correct option is (a,b,d)

15.**Solution:** Given

$$\vec{E} = \alpha\frac{e^{-r^2}}{r}\hat{r}$$

Sphere radius: $R = \sqrt{2}$

Electric flux through the sphere

Since $\vec{E}$ is radial and depends only on r, $\Phi = \oint\ \vec{E} \cdot d\vec{a}$

On the spherical surface:

$$|\vec{E}| = \alpha\frac{e^{-R^2}}{R}, da = 4\pi R^2$$

Hence, $\Phi = \alpha\frac{e^{-R^2}}{R}(4\pi R^2) = 4\pi\alpha Re^{-R^2}$

Required quantity

$$\frac{\Phi}{2\pi\alpha} = \frac{4\pi\alpha Re^{-R^2}}{2\pi\alpha} = 2Re^{-R^2}$$

$R = \sqrt{2}$:

$$\frac{\Phi}{2\pi\alpha} = 2\sqrt{2}e^{-2}$$

Numerical value, $2\sqrt{2}e^{-2} \approx 0.38$

Correct Answer:0.38

16.**Solution:** Young's double slit experiment (YDSE)

For fringe position: $y_n = \frac{n\lambda D}{d}$

Kinetic energies of both beams are same ⇒

$$\lambda = \frac{h}{\sqrt{2mk}}$$

YDSE with C_{60} beam

Mass: $m_1 = 60 \times 12\text{amu}$

De Broglie wavelength: $\lambda_1 = \frac{h}{\sqrt{2(60 \times 12\text{amu})k}}$

Slit separation:$d_1 = 50$ nm

Fringe width: $\beta_1 = \frac{D\lambda_1}{d_1} = \frac{1}{50}\frac{h}{\sqrt{2(60 \times 12\text{amu})k}}$

YDSE with C_{70} beam

Mass:$m_2 = 70 \times 12$amu

De Broglie wavelength$\lambda_2 = \frac{h}{\sqrt{2(70 \times 12\text{amu})k}}$

Slit separation:$d_2 = 92.5$ nm

Fringe width: $\beta_2 = \frac{D\lambda_2}{d_2} = \frac{1}{92.5}\frac{h}{\sqrt{2(70 \times 12\text{amu})k}}$

Given condition

Position of 4 th bright fringe of C_{60} equals position of n^{th} bright fringe of C_{70} :$4\beta_1 = n\beta_2$

Substitute:

$$4\left(\frac{1}{50\sqrt{60}}\right) = n\left(\frac{1}{92.5\sqrt{70}}\right) \Rightarrow \frac{n}{92.5\sqrt{70}} = \frac{4}{50\sqrt{60}}$$

$$n = \frac{92.5\sqrt{70} \times 4}{50\sqrt{60}} \approx 8$$

Correct option is (d)

17.**Solution:** Balmer series: transitions, $n \rightarrow 2$ $(n = 3,4,5, \dots)$

Young's double slit experiment:

Fringe width:

$$\beta = \frac{D\lambda}{d}$$

Screen distance D is constant.

To keep fringe width constant, we must have:

$$\frac{\lambda}{d} = \text{constant} \Rightarrow d \propto \lambda$$

Wavelength of Balmer series

Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{R(n^2-4)}{4n^2} \Rightarrow \lambda = \frac{4n^2}{R(n^2-4)} \propto \frac{4n^2}{n^2-4}$$

Condition for constant fringe width

Since

$$\beta = \frac{D\lambda}{d} = \text{constant} \Rightarrow d' = d \times \frac{4n^2}{n^2-4}$$

Correct option is (c)

18.**Solution:** Electric dipole moment $\vec{p}$

$$\vec{p} = \sum_i q_i \vec{r}_i$$

Under parity (P): $\vec{r}_i \xrightarrow{P} -\vec{r}_i$

$$\Rightarrow \vec{p} \xrightarrow{P} \sum_i q_i(-\vec{r}_i) = -\vec{p}$$

So,$\vec{p}$ is odd under parity

Magnetic dipole moment $\vec{\mu}$

$$\vec{\mu} = \frac{e}{2m}\vec{L}$$

Under time reversal (T): Momentum $\vec{p} \rightarrow -\vec{p}$

Hence angular momentum $\vec{L} = \vec{r} \times \vec{p} \rightarrow -\vec{L}$

$$\Rightarrow \vec{\mu} \xrightarrow{T} \frac{e}{2m}(-\vec{L}) = -\vec{\mu}$$

So,$\vec{\mu}$ is odd under time reversal

Correct option is (a)

19.**Solution:** Two configurations:(I) Isolated point charge, $q = 1\text{C}$

(II) Same charge q at the centre of a thick conducting spherical shell, $R_1 = 1$ m, $R_2 = 2$ m

Electrostatic energy:

$$W = \frac{\varepsilon_0}{2}\int E^2 dV$$

Given: $8\pi\varepsilon_0|W_I - W_{II}| = \frac{1}{n}$

Electric field of point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, \qquad W_I = \frac{\varepsilon_0}{2}\int_0^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 4\pi r^2 dr, \qquad W_I = \frac{1}{8\pi\varepsilon_0}\int_0^\infty \frac{dr}{r^2}$$

Energy for configuration (II)

Regions:$0 < r < R_1$: field due to point charge

$R_1 < r < R_2$: inside conductor, $E = 0$

$r > R_2$: field due to total charge q

$$W_{II} = \frac{\varepsilon_0}{2}\left[\int_0^{R_1} \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 4\pi r^2 dr + 0 + \int_{R_2}^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 4\pi r^2 dr\right]$$

Difference $W_I - W_{II}$

Only region contributing is $R_1 < r < R_2$:

$$W_I - W_{II} = \frac{1}{8\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{1}{8\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \;\; R_1 = 1, R_2 = 2$$

$$W_I - W_{II} = \frac{1}{8\pi\varepsilon_0} \times \frac{1}{2}, \qquad 8\pi\varepsilon_0(W_I - W_{II}) = \frac{1}{2}$$

Comparing with:

$$\frac{1}{n}, \qquad n = 2$$

Correct Answer: 2

20.**Solution:** Electromagnetic wave propagating in vacuum, $\vec{B} = \vec{B}_0 e^{i(kz-\omega t)}, B_0 = 10^{-8}$ T

Radius of circular area: $r = 1$ m

$$\varepsilon_0 = 10^{-11}\frac{C^2}{Nm^2}$$

Intensity of EM wave

Average energy density:

$$\langle u \rangle = \frac{B_0^2}{2\mu_0}$$

Intensity: $I = \frac{P}{A} = \langle u \rangle c = \frac{B_0^2 c}{2\mu_0}$

Using: $\frac{1}{\mu_0} = \varepsilon_0 c^2, \qquad I = \frac{B_0^2 c^3 \varepsilon_0}{2}$

Power through circular area

$$P = I \times A = \frac{B_0^2 c^3 \varepsilon_0}{2} \times \pi(1)^2 \Rightarrow \frac{10^3 P}{\pi} = \frac{10^3 B_0^2 c^3 \varepsilon_0}{2}$$

$$\frac{10^3 P}{\pi} = \frac{10^3}{2} \times (10^{-8})^2 \times (3 \times 10^8)^3 \times 10^{-11} = \frac{10^3}{2} \times 10^{-16} \times 27 \times 10^{24} \times 10^{-11}$$

$$= \frac{27}{2} = 13.5$$

Correct Answer: 13.5

21.**Solution:** Proper length of rod $= L$

Velocity of rod:

$$v = \frac{3c}{5}$$

Ground observer measures positions of P and Q at times t_p and t_q

Given: $x_Q - x_P = \frac{9L}{10}$

Lorentz transformation (inverse)

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v\Delta x'}{c^2\sqrt{1 - \frac{v^2}{c^2}}}, \qquad \Delta x = \frac{\Delta x'}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Here, $\Delta x' = L$

Use given condition

$$\Delta x = x_Q - x_P = \frac{9L}{10}\frac{9L}{10} = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute $v = \frac{3c}{5}$:

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}, \qquad \frac{9L}{10} = \frac{5L}{4} + \frac{3c}{5}\frac{\Delta t'}{4/5} \Rightarrow \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{7L}{12c}$$

Time difference in ground frame

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v\Delta x'}{c^2\sqrt{1-\frac{v^2}{c^2}}}, \qquad \Delta t = -\frac{7L}{12c} + \frac{3c}{5}\frac{L}{c^2}\frac{5}{4}, \qquad \Delta t = -\frac{7L}{12c} + \frac{5L}{20c} = \frac{L}{6c}$$

Correct option is (c)

22.**Solution:** Symmetric top:

$$I_1 = I_2 = \frac{2\alpha}{3}, I_3 = 2\alpha$$

Angular speed about symmetry axis:

$$\omega_3 = \frac{1}{8}\text{rad s}^{-1}$$

For a torque-free symmetric top, the angular velocity vector $\vec{\omega}$ precesses about the symmetry (3-) axis with angular frequency

$$\Omega = \frac{I_3 - I_1}{I_1}\omega_3, \qquad \frac{I_3 - I_1}{I_1} = \frac{2\alpha - \frac{2\alpha}{3}}{\frac{2\alpha}{3}} = \frac{\frac{4\alpha}{3}}{\frac{2\alpha}{3}} = 2$$

$$Hence, \Omega = 2\omega_3 = 2 \times \frac{1}{8} = \frac{1}{4}\text{rad s}^{-1}$$

Correct answer:

$$\Omega = \frac{1}{4}\text{rad s}^{-1}$$

But none of the options (2, 3, 5, 7) match this value.

Therefore the question is incorrectly framed / options wrong, and hence:

Marks to All

Correct option is(*)

23.**Solution:** Potential energy

$$\vec{F} = -\frac{\partial V}{\partial r}\hat{r} \Rightarrow \frac{\partial V}{\partial r} = \frac{k}{2r^3}$$

So $V = V(r)$ with

$$\frac{\partial V}{\partial r} = \frac{k}{2r^3}$$

Hamiltonian in polar coordinates

For motion in a 2D plane:

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + V(r)$$

Hamilton's equation for p_r

$$\dot{p}_r = -\frac{\partial H}{\partial r}, \qquad \frac{\partial H}{\partial r} = -\frac{p_\theta^2}{mr^3} + \frac{\partial V}{\partial r}$$

Substitute $\frac{\partial V}{\partial r} = \frac{k}{2r^3}$

$$\dot{p}_r = -\left(-\frac{p_\theta^2}{mr^3} + \frac{k}{2r^3}\right) \Rightarrow \frac{dp_r}{dt} = \frac{p_\theta^2}{mr^3} - \frac{k}{2r^3}, \qquad \frac{dp_r}{dt} = \frac{2p_\theta^2 - mk}{2mr^3}$$

Correct option is (d)

24.**Solution:** 1) $a \to b$: Isothermal expansion at higher temperature $T_2 \to \ln\ S - T$ diagram: horizontal line at T_2

2) $b \to c$: Adiabatic expansion

Entropy constant

$\ln\ S - T$ diagram: vertical line

3) $c \to d$: Isothermal compression at lower temperature T_1

4)$\ln\ S - T$ diagram: horizontal line at T_1

$d \to a$: Adiabatic compression

Entropy constant

$\ln\ S - T$ diagram: vertical line

Option (A)

$a \to b$: horizontal at T_2

$b \to c$: vertical (adiabatic)

$c \to d$: horizontal at T_1

$d \to a$: vertical (adiabatic)

Perfectly matches Carnot cycle

Option (B)

Sequence of processes is incorrect

Option (C)

$b \to c$ is not vertical → entropy not constant

Option (D)

Shape does not correspond to isothermal + adiabatic combination

Correct option is (a)

25.**Solution:**

26.**Solution:** Simple harmonic oscillator in thermal equilibrium at temperature T

$$\omega = \frac{2k_B T}{\hbar}$$

Energy levels

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Partition function

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n} = e^{-\beta\hbar\omega/2}\sum_{n=0}^{\infty} e^{-n\beta\hbar\omega}, \qquad Z = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}}$$

$$\beta\hbar\omega = \frac{1}{k_B T}\hbar\left(\frac{2k_B T}{\hbar}\right) = 2 \Rightarrow Z = \frac{e^{-1}}{1 - e^{-2}}, \qquad Z = \frac{e^{-1}}{1 - e^{-2}} = \frac{1}{e - e^{-1}} = \frac{e}{e^2 - 1}$$

Correct option is (a)

27.**Solution:** Two identical, non-interacting $^4He_2$ atoms are distributed among 4 different non-degenerate energy levels.

Since ^4He atoms are bosons, multiple occupancy of the same energy level is allowed.

The total number of possible arrangements of two identical bosons in 4 non-degenerate levels is:

$$\binom{4+2-1}{2} = \binom{5}{2} = 10$$

Out of these, the number of arrangements in which the two atoms occupy different energy levels is:

$$\binom{4}{2} = 6$$

Hence, the probability that the two ^4He atoms occupy different levels is:

$$p = \frac{6}{10}$$

Now consider two identical, non-interacting $^3He_2$ atoms.

Since ^3He atoms are fermions, the Pauli exclusion principle forbids both atoms from occupying the same energy level.

Thus, all allowed configurations already have the atoms in different energy levels.

The total number of possible arrangements is:

$$\binom{4}{2} = 6$$

Hence, the probability that the two ^3He atoms occupy different levels is:

$$q = \frac{6}{6} = 1$$

$$Ratio, \frac{p}{q} = \frac{6/10}{1} = 0.6$$

Correct Answer:0.6

28.**Solution:** Two identical bodies at, $T_1 = 800$ K, $T_2 = 200$ K

Let the final equilibrium temperature be T'.

Heat absorbed from hot body: $Q_1 = C(T_1 - T')$

Heat rejected to cold body, $Q_2 = C(T' - T_2)$

Work done $W = Q_1 - Q_2$

$$W = C[(T_1 - T') - (T' - T_2)] = C[T_1 + T_2 - 2T']$$

Condition for maximum work

For a reversible (maximum work) process: $\Delta S = 0,\ \Delta S_1 + \Delta S_2 = 0$

$$\int_{T_1}^{T'} \frac{CdT}{T} + \int_{T_2}^{T'} \frac{CdT}{T} = 0, \qquad C\ln\frac{T'}{T_1} + C\ln\frac{T'}{T_2} = 0 \Rightarrow \ln\frac{T'^2}{T_1T_2} = 0 \Rightarrow T' = \sqrt{T_1T_2}$$

Substitute values, $T' = \sqrt{800 \times 200} = \sqrt{160000} = 400\ \mathrm{K}$

Maximum work, $W = C[800 + 200 - 2(400)] = C(1000 - 800) = 200C, n = 200$

Correct Answer:200

29.**Solution:** Atomic number $Z = 6 \Rightarrow$ element is Carbon.

Electronic configuration:$1s^2 2s^2 2p^2$

The optically active (valence) electrons are the two electrons in the $2p$ subshell. Since the p-subshell can accommodate 6 electrons, $2p^2$ is less than half-filled.

For $2p^2$:

$$l_1 = l_2 = 1, s_1 = s_2 = \frac{1}{2}$$

According to Hund's first rule, maximize total spin:

$$S = |m_{s1}| + |m_{s2}| = \frac{1}{2} + \frac{1}{2} = 1$$

According to Hund's second rule, maximize total orbital angular momentum:

$$L = |m_{l1}| + |m_{l2}| = 1 + 0 = 1$$

Since the subshell is less than half-filled, Hund's third rule gives: $J = L - S = 1 - 1 = 0$

Thus, the ground state term is: ${}^{2S+1}L_J = {}^3P_0$

Correct option is (a)

30.**Solution:** Given that the atom has a non-zero magnetic moment, it must possess angular momentum $\vec{J}$.Magnitude of angular momentum is:$|\vec{J}| = \sqrt{j(j+1)}\hbar$

Given: $\sqrt{j(j+1)}\hbar = \sqrt{12}\hbar \Rightarrow j(j+1) = 12,\ j^2 + j - 12 = 0 \Rightarrow j = 3$

In a Stern-Gerlach experiment, the number of beams obtained equals the number of possible magnetic quantum states: $2j + 1,\ 2(3) + 1 = 7$

Correct option is (b)

31.**Solution:** The deuteron has a finite electric quadrupole moment, which implies that its charge distribution is not spherically symmetric.

For a spherical distribution, quadrupole moment would be zero, but experimentally:

$$Q = \frac{2}{5}Z(b^2 - a^2) \neq 0 \text{ (since } b \neq a)$$

Hence, the deuteron charge distribution is non-spherical.

Correct (B)

The magnetic moment of the deuteron is not exactly equal to the sum of the magnetic moments of the neutron and proton.

This deviation arises due to orbital motion and state mixing.

Incorrect(C)

The deuteron has spin $S = 1$ and even parity.

The observed quadrupole moment requires a D-state admixture in addition to the S-state.

Thus, the ground state is an admixture of: ${}^3S_1$ and ${}^3D_1$

Correct(D)

A ${}^3P_1$ state has odd parity, which is inconsistent with the observed even parity of the deuteron ground state.

Incorrect

Correct option is (a,c)

32.**Solution:** (A)Argon is used as the primary filling gas so that ambient light from the surroundings does not produce any signal in the detector.

Correct(B)Ethyl alcohol (a polyatomic gas) is used as a quenching gas.

It absorbs the energy of positive ions and prevents continuous discharge.

Correct(C)For a cylindrical GM tube, the electric field at a distance r from the axis is:

$$E = \frac{V}{r\ln(b/a)}$$

where a is the radius of the inner electrode and b is the radius of the tube.

Thus:

$$E \propto \frac{1}{r}$$

Field strength decreases from the axis to the edge

Direction of the electric field is radially outward

Correct (D)

This statement is opposite to the actual behaviour of the electric field in a GM tube.

Incorrect

Correct option is (a,b,c)

33.**Solution:** In the vector model of angular momentum, the angle θ between the orbital angular momentum vector $\vec{L}$ and the positive Z-axis is given by:

$$\cos\,\theta = \frac{m_l}{\sqrt{l(l+1)}}$$

For a $2p$ electron: $l = 1, m_l = 1, 0, -1$

Hence, $\cos\,\theta = \frac{m_l}{\sqrt{1(1+1)}} = \frac{m_l}{\sqrt{2}}$

So the possible values are:

$$\cos\,\theta = \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$$

Corresponding angles: $\theta = 45°, 90°, 135°$

The minimum angle is therefore:45°

Correct Answer:45

34.**Solution:** Given a homonuclear diatomic molecule X_2

Electronic ground state has even parity

Nuclear spin of atom X is $I = 0$

For a homonuclear diatomic molecule, the total wavefunction must be symmetric under exchange of the two identical nuclei.

The total wavefunction is a product of: (electronic) × (vibrational) × (rotational) × (nuclear spin)

Electronic wavefunction is even (symmetric)

Vibrational ground state is symmetric

Nuclear spin wavefunction for $I = 0$ is symmetric

Hence, the rotational wavefunction must also be symmetric to keep the total wavefunction symmetric.

For a diatomic molecule:

Rotational wavefunction is symmetric for even J

Rotational wavefunction is antisymmetric for odd J

Since antisymmetric rotational states are not allowed here, odd J states are absent.

Thus, only even J rotational levels contribute to the rotational Raman spectrum.

Correct option is (c)

35.**Solution:**(i) Visible light

Visible light has comparatively low energy. It can cause the photoelectric effect, but it does not have enough energy to produce core-level electronic transitions, nuclear transitions, or pair production. Hence, visible light corresponds only to photoelectric effect (T).

(ii) X-rays

X-rays have sufficiently high energy to cause transitions between inner (core) electronic energy levels of atoms. They are also widely used for crystal structure determination through X-ray diffraction and can produce the photoelectric effect.

Therefore, X-rays correspond to P, S, and T.

(iii) Gamma rays

Gamma rays originate from transitions between nuclear energy levels and have very high energy. Due to this high energy, they can also lead to pair production.

Thus, gamma rays correspond to Q and R.

(iv) Thermal neutrons

Thermal neutrons have wavelengths comparable to interatomic spacings in solids and are commonly used for crystal structure determination.

Hence, thermal neutrons correspond to S.

$$(i) - T, (ii) - P, S, T, (iii) - Q, R, (iv) - S$$

Correct option is (a)

36.**Solution:** The reaction is: $\pi^- + d \longrightarrow n + n$

The pion is slow moving, so its orbital angular momentum is:$L_\pi = 0$

The deuteron ground state is a mixture of $^3S_1$ and $^3D_1$ states, both of which have even orbital angular momentum.

Also, the intrinsic parity of: neutron $= +1$

deuteron $= +1$

$\pi^- = -1$

Parity conservation. Parity is conserved in strong interactions. Therefore,

$$(-1)^{L_\pi}\pi_\pi\pi_d = \pi_n\pi_n(-1)^L, \qquad (-1)^0(-)(+) = (+)(+)(-1)^L, \qquad -= (-1)^L$$

This implies that L must be odd ($L = 1,3, \dots$).

Hence, the final two-neutron state has odd parity.

So, option (A) is correct.

Spin conservation, Spin values involved are:

$$\pi^-: S = 0, d: S = 1, n: S = \frac{1}{2}$$

For the final state:

$$\frac{1}{2} + \frac{1}{2} \Rightarrow S = 1 \text{ or } 0$$

From parity requirement, the lowest allowed odd orbital angular momentum is:$L = 1$

For two identical fermions (neutrons), the total wavefunction must be antisymmetric.

With odd L, the spin part must be symmetric, hence:$S = 1$

Thus, the final state corresponds to:$L = 1, S = 1$

So, option (C) is correct, and option (D) is wrong.

Also, since $L + S = 1 + 1 = 2$, it is even, hence option (B) is incorrect.

(A) and (C)

Correct option is (a,c)

37.**Solution:** The reaction proceeds as $^{60}\text{Co} \xrightarrow{\beta} {}^{60}\text{Ni}^*(4^+) \longrightarrow {}^{60}\text{Ni}(2^+) \longrightarrow {}^{60}\text{Ni}(0^+)$

with two photons emitted in succession.

For electromagnetic transitions, the multipole order L must satisfy:$|J_i - J_f| \le L \le J_i + J_f$

and the parity selection rule: Electric transition EL : parity changes if L is odd, no change if L is even

Magnetic transition ML : parity changes if L is even, no change if L is odd

$4^+ \to 2^+$transition

Here,$|4-2| \leq L \leq 4+2 \Rightarrow 2 \leq L \leq 6$

Parity does not change ($+\to +$), so allowed electric multipoles have even L.

Thus, $L = 2,4,6$ are possible, and the electric quadrupole (E2) transition is allowed (though higher multipoles are weaker).

$2^+ \to 0^+$transition

Here,$|2-0| \leq L \leq 2+0 \Rightarrow L = 2$

Parity again does not change, so the transition must be electric with even L.

Hence, this transition is purely electric quadrupole (E2) and is strongly allowed.

(C) $2^+ \to 0^+$is an electric quadrupole transition

Correct option is (c)

38.**Solution:** The particle Ξ^{0*} belongs to the baryon decuplet.

It has isospin:

$$|I, I_3\rangle = \left|\frac{1}{2}, \frac{1}{2}\right|$$

and strangeness $S = -2$.

From the quark model, strangeness -2 implies that the particle contains two strange quarks.

The charge and isospin assignment of Ξ^{0*} then fixes the quark composition to be: $\Xi^{0*} = uss$

For baryons belonging to the decuplet, the spin-flavour wavefunction is totally symmetric.

Since the colour part of the wavefunction for baryons is antisymmetric, the flavour part must be symmetric under exchange of quarks.

Hence, the flavour wavefunction is the symmetric combination of all permutations of uss :

$$\frac{1}{\sqrt{3}}(uss + sus + ssu)$$

Correct option is (b)

39.**Solution:** The reaction is:$\pi + N \to \Delta$

Given masses:$m_\pi = 140\text{MeV}/c^2, m_N = 938\text{MeV}/c^2, m_\Delta = 1230\text{MeV}/c^2$

In the lab frame, the nucleon is at rest.

The minimum energy of the pion required to produce the Δ particle corresponds to threshold production.

The threshold energy of the pion is given by:

$$E_\pi = \frac{m_\Delta^2 - m_N^2 - m_\pi^2}{2m_N}$$

Substituting the given values:

$$E_\pi = \frac{1230^2 - 938^2 - 140^2}{2 \times 938}, \qquad E_\pi = \frac{1512900 - 879844 - 19600}{1876} \approx 340\text{MeV}$$

Correct Answer:340

40.**Solution:** The decay process is: ${}^{241}_{94}\mathrm{Am} \rightarrow {}^{237}_{92}\mathrm{U} + \alpha$

Given rest masses, $M_{\mathrm{Am}} = 224.544\mathrm{GeV}/c^2, M_{\mathrm{U}} = 220.811\mathrm{GeV}/c^2, M_\alpha = 3.728\mathrm{GeV}/c^2$

The Q-value of the reaction is: $Q_\alpha = (M_{\mathrm{Am}} - M_{\mathrm{U}} - M_\alpha)c^2,\ \ Q_\alpha = (224.544 - 220.811 - 3.728)\mathrm{GeV}$

$$= 0.005\mathrm{GeV} = 5\mathrm{MeV}$$

Since the parent nucleus is initially at rest, the kinetic energy of the emitted α-particle is given by:

$$K_\alpha = \frac{A-4}{A} Q_\alpha$$

where $A = 241$.

$$K_\alpha = \frac{241-4}{241} \times 5\mathrm{MeV} = \frac{237}{241} \times 5 = 4.917\mathrm{MeV}$$

Correct Answer:4.19

41.**Solution:** The Hall experiment is performed using a non-magnetic semiconductor.

The current I flows along the X -axis, and the magnetic field B is applied along the Z -axis.

The Hall resistivity is defined as:

$$\rho_{xy} = \frac{E_y}{J_x}$$

where E_y is the Hall electric field and J_x is the current density.

For the Hall effect:$E_y = R_H J_x B_z$ where R_H is the Hall coefficient.

Substituting:

$$\rho_{xy} = \frac{R_H J_x B_z}{J_x} = R_H B_z$$

Thus, the magnitude of the Hall resistivity is directly proportional to the magnetic field: $\rho_{xy} \propto B$

Hence, the correct graphical representation is a straight line passing through the origin, showing linear dependence on B.

Correct option is (b)

42.**Solution:** The compound consists of three ions X, Y, and Z.The Z ions form an FCC lattice.

An FCC unit cell contains 4 effective lattice points, hence .Number of Z ions per unit cell $= 4$

In an FCC lattice:Number of tetrahedral voids $= 8$

Number of octahedral voids $= 4$

The X ions occupy $\frac{1}{6}$ of the tetrahedral voids:

Number of X ions $= \frac{1}{6} \times 8 = \frac{4}{3}$

The Y ions occupy $\frac{1}{3}$ of the octahedral voids:

Number of Y ions $= \frac{1}{3} \times 4 = \frac{4}{3}$

Thus, the number of ions per unit cell is:

$$X:Y:Z = \frac{4}{3}:\frac{4}{3}:4$$

Dividing throughout by $\frac{4}{3}$:$X:Y:Z = 1:1:3, XYZ_3$

Correct option is (b)

43.**Solution:** For a non-magnetic metal, the heat capacity at low temperatures has two contributions: an electronic term proportional to Ta lattice (phonon) term proportional to T^3

Hence, the heat capacity is:$C = AT + BT^3$

where A and B are constants.

Dividing both sides by T

$$\frac{C}{T} = A + BT^2$$

Thus, $\frac{C}{T}$ varies linearly with T^2, with a non-zero intercept on the $\frac{C}{T}$ axis.

Therefore, the correct graph is a straight line with positive slope, starting from a finite value at $T^2 = 0$.

Correct option is (b)

44.**Solution:** For non-relativistic electrons in a solid, the energy dispersion near the band minimum is parabolic:

$$E(k) \approx E_0 + \frac{\hbar^2 k^2}{2m^*}$$

The effective mass is related to the curvature of the $E - k$ relation by:

$$\frac{1}{m^*} \propto \frac{d^2E}{dk^2}$$

Hence: Larger curvature ⇒ smaller effective mass

Smaller curvature ⇒ larger effective mass

From the given plot:Cuve c has the largest curvature

Curve b has intermediate curvature

Curve a has the smallest curvature

Thus:

$$\left(\frac{d^2E}{dk^2}\right)_c > \left(\frac{d^2E}{dk^2}\right)_b > \left(\frac{d^2E}{dk^2}\right)_a$$

This implies:$m_a^* > m_b^* > m_c^*$

Correct option is (d)

45.**Solution:** The graph shows magnetization M versus magnetic field H, plotted in the same scale and units.

Hence, the slope of each line directly represents the magnetic susceptibility:

$$\chi = \frac{M}{H}$$

Curve (P)This curve has a small positive slope, meaning M increases slightly with H.

Thus, the susceptibility is small and positive:$\chi > 0$

This is characteristic of a paramagnetic material.

Curve (Q)

This curve has a small negative slope, meaning M is opposite to H and small in magnitude.

Thus, the susceptibility is small and negative:$\chi < 0$

This corresponds to a diamagnetic material.

Curve (R)

This curve has a slope of approximately -1 , i.e.

$$\frac{M}{H} = -1$$

This implies complete expulsion of magnetic field (Meissner effect), which is the defining property of a TypeI superconductor.

(P) - Paramagnet, (Q) - Diamagnet, (R) - Type-I Superconductor

Correct option is (b)

46.**Solution:** Graphene has a two-dimensional honeycomb lattice. The primitive unit cell can be viewed as consisting of six equilateral triangles of side a.

The area of one equilateral triangle is:

$$\frac{\sqrt{3}}{4}a^2$$

Hence, the area of the real-space unit cell is:

$$A = 6 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2$$

The area of the first Brillouin zone in two dimensions is given by:

$$A^* = \frac{(2\pi)^2}{A}$$

Substituting the value of A :

$$A^* = \frac{4\pi^2}{\frac{3\sqrt{3}}{2}a^2} = \frac{8\pi^2}{3\sqrt{3}a^2}$$

Correct option is (a)

47.**Solution:** The energy of the neutron beam is:$E = 20.4\text{meV}$

The de Broglie wavelength of the neutron is:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Substituting the given values:

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 20.4 \times 10^{-3} \times 1.6 \times 10^{-19}}} \approx 2\text{Å}$$

For diffraction from a crystal, the Bragg condition can be written as:

$2d\sin\ \theta = \lambda$and the interplanar spacing d is related to the reciprocal lattice vector by:

$$d = \frac{2\pi}{|\vec{G}|}$$

Given$|\vec{G}| = 3.14\text{Å}^{-1}$

Hence, $\sin\ \theta = \dfrac{\lambda|\vec{G}|}{4\pi}$

Substituting values:

$$\sin\ \theta = \frac{2 \times 3.14}{4\pi} = \frac{1}{2} \Rightarrow \theta = 30°$$

Correct option is (b)

48.**Solution:** The interaction of three phonons follows crystal momentum conservation:$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = \vec{k} + \vec{G}$ or $\vec{k} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{G}$

Each incoming phonon has the same wave vector: $\vec{k}_1 = \vec{k}_2 = \vec{k}_3 = \left(1.2\text{Å}^{-1}, 0.6\text{Å}^{-1}\right)$

Thus, $k_x = 1.2 + 1.2 + 1.2 - G_x, k_y = 0.6 + 0.6 + 0.6 - G_y$

For a rectangular lattice, the reciprocal lattice vectors are:

$$G_x = \frac{2\pi}{a}, G_y = \frac{2\pi}{b}$$

Given: $a = 6\text{Å}, b = 4\text{Å}$

$$G_x = \frac{2\pi}{6} = \frac{\pi}{3} \approx 1.047\text{Å}^{-1}, \qquad G_y = \frac{2\pi}{4} = \frac{\pi}{2} \approx 1.57\text{Å}^{-1}$$

$$k_x = 3.6 - 1.047 = 2.56\text{Å}^{-1}, \qquad k_y = 1.8 - 1.57 = 0.23\text{Å}^{-1}$$

Correct option is (a)

49.**Solution:** The binding potential is given by

$$U(r) = -\varepsilon\left(\frac{r}{r_0}\right)e^{-r/r_0},$$

where ε and r_0 are positive constants.

For small oscillations about the equilibrium position, the Einstein frequency is obtained from the curvature of the potential at the equilibrium separation.

The equilibrium position is determined from

$$\frac{dU}{dr} = 0.$$

Differentiating,

$$\frac{dU}{dr} = -\varepsilon\left[\frac{1}{r_0}e^{-r/r_0} - \frac{r}{r_0^2}e^{-r/r_0}\right] = -\varepsilon e^{-r/r_0}\left(\frac{1}{r_0} - \frac{r}{r_0^2}\right).$$

Setting this equal to zero gives, $r = r_0$.

The second derivative of the potential is

$$\frac{d^2U}{dr^2} = -\frac{\varepsilon}{r_0} e^{-r/r_0} \left(-\frac{1}{r_0} + \frac{r}{r_0^2}\right).$$

Evaluating at $r = r_0$,

$$\left.\frac{d^2U}{dr^2}\right|_{r=r_0} = \frac{\varepsilon}{e r_0^2}.$$

The Einstein frequency is therefore

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{m}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}} = \frac{1}{2\pi}\sqrt{\frac{\varepsilon}{m e r_0^2}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{\varepsilon}{m e r_0^2}}$$

Correct option is (b)

50. **Solution:** In a hadronic interaction, a neutral pion π^0 decays immediately into two photons. The opening angle between the two photons is θ, and the pion can be produced with different momenta p.

If the π^0 is produced at rest, conservation of linear momentum requires the two photons to be emitted in opposite directions. Hence, the opening angle is: $\theta = 180^\circ = \pi$

When the π^0 is produced with non-zero momentum, the decay occurs in a moving frame. Due to momentum conservation, the two photons are no longer emitted back-to-back in the laboratory frame. As the momentum p of the π^0 increases, the photons become more forward-collimated, and the opening angle θ decreases continuously from π.

Thus: At $p = 0, \theta = \pi$

As p increases, θ decreases monotonically

This behaviour is correctly represented only by option (A).

Correct option is (a)

51. **Solution:** For total moment to be $2\mu_B$ the arrangement is like $\underbrace{\uparrow\uparrow\uparrow\uparrow}\downarrow\downarrow \rightarrow 2\mu_B \Rightarrow P = \frac{^6C_4}{2^6}$ For total moment to be $6\mu_B$ the arrangement is like

$$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \Rightarrow Q = \frac{^6C_6}{2^6}\frac{P}{Q} = \frac{6!}{2!\cdot 4!} = \frac{6\times 5}{2} = 15$$

Correct Answer:15

52. **Solution:** From the given logic circuit, the output X can be written using binary (Boolean) algebra. The upper gate gives the complemented input $\bar{P}$, while the lower gate produces the product PQ. These two outputs are then combined through an OR gate.

Hence, $X = \bar{P} + PQ$

Using Boolean identity, $\bar{P} + PQ = \bar{P} + Q$

Now evaluate X for the given options.

For $P = 0, Q = 0,\ X = \overline{0} + 0 = 1 + 0 = 1$

Correct option is (d)

53.**Solution:** The mid-band voltage gain of the amplifier is$A = 200$

The lower and upper cutoff frequencies are $f_L = 20$ Hz, $f_H = 20$kHz

Gain at 10 Hz

For frequencies below the lower cutoff,

$$A_1(f) = \frac{A}{\sqrt{1 + \left(\frac{f_L}{f}\right)^2}}$$

At $f = 10$ Hz,

$$A_1 = \frac{200}{\sqrt{1 + \left(\frac{20}{10}\right)^2}} = \frac{200}{\sqrt{5}} = 89.44$$

Gain at 100 kHz

For frequencies above the upper cutoff,

$$A_2(f) = \frac{A}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

At $f = 100$kHz,

$$A_2 = \frac{200}{\sqrt{1 + \left(\frac{100}{20}\right)^2}} = 39.22$$

Required ratio

$$\frac{A(10\text{ Hz})}{A(100\text{kHz})} = \frac{89.44}{39.22} = 2.28$$

Correct Answer:2.28

54.**Solution:** For an RC circuit to act as a differentiator, the time constant must be much larger than the time period of the input signal, i.e. $RC \gg T$

The input frequency is

$$f = 1\text{kHz} \Rightarrow T = \frac{1}{f} = 1\text{ ms}$$

For option (A), $R = 0.5\text{k}\Omega, C = 0.1\mu$ F, $RC = (0.5 \times 10^3)(0.1 \times 10^{-6}) = 5 \times 10^{-3}$ s $= 5$ ms

Since, $RC = 5T$ the condition for differentiation is satisfied.

The circuit in option (A) therefore produces a differentiated version of the square-wave input, which matches the given output waveform.

Correct option is (a)

55.**Solution:**