GATE 2022

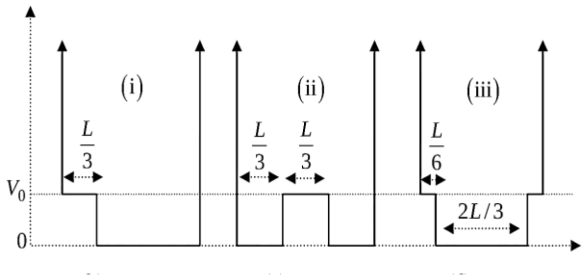
Quantum Mechanics

1. From the pairs of operators given below, identify the ones which commute. Here l and j correspond to the orbital angular momentum and the total angular momentum, respectively.
(a) l^2, j^2 (b) j^2, j_z
(c) j^2, l_z (d) l_z, j_z

2. Pauli spin matrices satisfy
(a) $\sigma_\alpha\sigma_\beta - \sigma_\beta\sigma_\alpha = i \in_{\alpha\beta\gamma} \sigma_\gamma$
(b) $\sigma_\alpha\sigma_\beta - \sigma_\beta\sigma_\alpha = 2i \in_{\alpha\beta\gamma} \sigma_\gamma$
(c) $\sigma_\alpha\sigma_\beta + \sigma_\beta\sigma_\alpha = \in_{\alpha\beta\gamma} \sigma_\gamma$
(d) $\sigma_\alpha\sigma_\beta + \sigma_\beta\sigma_\alpha = 2\delta_{\alpha\beta}$

3. The wave function of a particle in a one-dimensional infinite well of size $2a$ at a certain time is $\psi(x) = \frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\left(\frac{\pi x}{a}\right) + \sqrt{3}\cos\left(\frac{\pi x}{2a}\right) + \cos\left(\frac{3\pi x}{2a}\right)\right]$. Probability of finding the particle in $n = 2$ state at that time is % (Round off to the nearest integer)

4. Consider a particle in three different boxes of width L. The potential inside the boxes vary as shown in figures (i), (ii) and (iii) with $V_0 < \frac{\hbar^2\pi^2}{2mL^2}$. The corresponding ground-state energies of the particle are E_1, E_2 and E_3, respectively. Then



(a) $E_2 > E_1 > E_3$ (b) $E_3 > E_1 > E_2$
(c) $E_2 > E_3 > E_1$ (d) $E_3 > E_2 > E_1$

5. In cylindrical coordinates (s, φ, z) which of the following is a Hermitian operator?
(a) $\frac{1}{i}\frac{\partial}{\partial s}$ (b) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$
(c) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$ (d) $\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$

6. For a one-dimensional harmonic oscillator, the creation operator $(a^\dagger)$ acting on the n^{th} state $|\psi_n\rangle$ where $n = 0,1,2, \ldots$, gives $a^\dagger|\psi_n\rangle = \sqrt{n+1}|\psi_{n+1}\rangle$. The matrix representation of the position operator $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$ for the first three rows and columns is
(a) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \end{pmatrix}$
(b) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
(c) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$
(d) $\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 0 & 0 \\ \sqrt{3} & 0 & 1 \end{pmatrix}$

7. A particle of mass m is moving inside a hollow spherical shell of radius a so that the potential is
$$V(r) = \begin{cases} 0 \text{ for } r < a \\ \infty \text{ for } r \geq a \end{cases}$$
The ground state energy and wave function of the particle are E_0 and $R(r)$, respectively. Then which of the following options are correct?
(a) $E_0 = \frac{\hbar^2\pi}{2ma^2}$
(b) $-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = E_0 R(r < a)$
(c) $-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d^2R}{dr^2} = E_0 R\ (r < a)$
(d) $R(r) = \frac{1}{r}\sin\left(\frac{\pi r}{a}\right)\ (r < a)$

8. A particle of mass m in the $x - y$ plane is confined in an infinite two-dimensional well with vertices (0,0), $(0, L)$, (L, L), $(L, 0)$. The eigen-functions of this particle are $\psi_{n_x n_y} =$

$\frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$. If perturbation of the form $V = Cxy$, where C is a real constant, is applied, then which of the following statements are correct for the first excited state?
(a) The unperturbed energy is $\frac{3\pi^2\hbar^2}{2mL^2}$
(b) The unperturbed energy is $\frac{5\pi^2\hbar^2}{2mL^2}$
(c) First order energy shift due to the applied perturbation is zero
(d) The shift (δ) in energy due to the applied perturbation is determined by an equation of the form $\begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$, where a and b are real, non-zero constants

Mathematical Physics

9. What is the maximum number of free independent real parameters specifying an n dimensional orthogonal matrix?
(a) $n(n-2)$ (b) $(n-1)^2$
(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

10. Consider the ordinary differential equation
$$y'' - 2xy' + 4y = 0$$
and its solution $y(x) = a + bx + cx^2$. Then
(a) $a = 0, c = -2b \neq 0$
(b) $c = -2a \neq 0, b = 0$
(c) $b = -2a \neq 0, c = 0$
(d) $c = 2a \neq 0, b = 0$

11. Complex function $f(z) = z + |z-a|^2$ (a is a real number) is
(a) continuous at (a, a)
(b) complex-differentiable at (a, a)
(c) complex-differentiable at $(a, 0)$
(d) analytic at $(a, 0)$

12. If $g(k)$ is the Fourier transform of $f(x)$ then which of the following are true?
(a) $g(-k) = +g*(k)$ implies $f(x)$ is real
(b) $g(-k) = -g*(k)$ implies $f(x)$ is purely imaginary
(c) $g(-k) = +g*(k)$ implies $f(x)$ is purely imaginary
(d) $g(-k) = -g*(k)$ implies $f(x)$ is real

13. The ordinary differential equation
$$(1-x^2)y'' - xy' + 9y = 0$$
has a regular singularity at
(a) -1 (b) 0
(c) +1 (d) no finite value of x

Electromagnetic Theory

14. On the surface of a spherical shell enclosing a charge free region, the electrostatic potential values are as follows: One quarter of the area has potential ϕ_0, another quarter has potential $2\phi_0$ and the rest has potential $4\phi_0$. The potential at the centre of the shell is
(You can use a property of the solution of Laplace's equation.)
(a) $\frac{11}{4}\phi_0$ (b) $\frac{11}{2}\phi_0$
(c) $\frac{7}{3}\phi_0$ (d) $\frac{7}{4}\phi_0$

15. For the refractive index $n = n_r(\omega) + in_{im}(\omega)$ of a material, which of the following statements are correct?
(a) n_r can be obtained from n_{im} and vice versa
(b) n_{im} could be zero
(c) n is an analytic function in the upper half of the complex ω plane
(d) n is independent of ω for some materials

16. Electric field is measured along the axis of a uniformly charged disc of radius 25 cm. At a distance d from the centre, the field differs by 10% from that of an infinite plane having the same charge density. The value of d is cm.
(Round off to one decimal place)

17. An electromagnetic pulse has a pulse width of 10^{-3} s. The uncertainty in the momentum

of the corresponding photon is of the order of 10^{-N} kg m s^{-1}, where N is an integer. The value of N is (speed of light $= 3 \times 10^8$ ms^{-1}, $h = 6.6 \times 10^{-34}$Js)

18. A student sets up Young's double slit experiment with electrons of momentum p incident normally on the slits of width w separated by distance d. In order to observe interference fringes on a screen at a distance D from the slits, which of the following conditions should be satisfied?
(a) $\frac{\hbar}{p} > \frac{Dw}{d}$ (b) $\frac{\hbar}{p} > \frac{dw}{D}$
(c) $\frac{\hbar}{p} > \frac{d^2}{D}$ (d) $\frac{\hbar}{p} > \frac{d^2}{\sqrt{Dw}}$

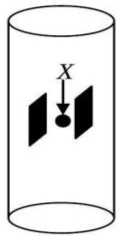
19. A parallel plate capacitor with spacing d and area of cross-section A is connected to a source of voltage V. If the plates are pulled apart quasistatically to a spacing of $2d$, then which of the following statements are correct?
(a) The force between the plates at spacing $2d$ is $\frac{1}{8}\left(\frac{\epsilon_0 AV^2}{d^2}\right)$
(b) The work done in moving the plates is $\frac{1}{8}\left(\frac{\epsilon_0 AV^2}{d}\right)$
(c) The energy transferred to the voltage source is $\frac{1}{2}\left(\frac{\epsilon_0 AV^2}{d}\right)$
(d) The energy of the capacitor reduces by $\frac{1}{4}\left(\frac{\epsilon_0 AV^2}{d}\right)$

20. . A plane polarized electromagnetic wave propagating in $y-z$ plane is incident at the interface of two media at Brewster's angle. Taking $z = 0$ as the boundary between the two media, the electric field of the reflected wave is given by

$$\vec{E}_R = A_R \cos\left[k_0\left\{\frac{\sqrt{3}}{2}y - \frac{1}{2}z\right\} - \omega t\right]\hat{x}$$

then which among the following statements are correct?
(a) The angle of refraction is $\frac{\pi}{6}$
(b) Ratio of permittivity of the medium of refraction (ϵ_2) with respect to the medium on incidence (ϵ_1), $\frac{\epsilon_2}{\epsilon_1} = 3$
(c) The incident wave can have components of its electric field in $y-z$ plane
(d) The angle of reflection is $\frac{\pi}{6}$

21. A current of $1A$ is flowing through a very long solenoid made of winding density 3000 turns $/m$. As shown in the figure, a parallel plate capacitor, with plates oriented parallel to the solenoid axis and carrying surface charge density $6 \epsilon_0$ Cm^{-2}, is placed at the middle of the solenoid. The momentum density of the electromagnetic field at the



midpoint X of the capacitor is $n \times 10^{-13}$Nsm^{-3}. The value of n is (Round off to the nearest integer) (speed of light $c = 3 \times 10^8$ ms^{-1})

Classical Mechanics

22. A particle of mass 1 kg is released from a height of 1 m above the ground. When it reaches the ground, what is the value of Hamilton's action for this motion in Js ? (g is the acceleration due to gravity; take gravitation potential to be zero on the ground)
(a) $-\frac{2}{3}\sqrt{2g}$ (b) $\frac{5}{3}\sqrt{2g}$
(c) $3\sqrt{2g}$ (d) $-\frac{1}{3}\sqrt{2g}$

23. If $(\dot{x}\dot{y} + axy)$ is a constant of motion of a two-dimensional isotropic harmonic oscillator with Lagrangian

$$L = \frac{m(\dot{x}^2 + \dot{y}^2)}{2} - \frac{k(x^2 + y^2)}{2}$$

then α is

(a) $+\frac{k}{m}$ (b) $-\frac{k}{m}$

(c) $-\frac{2k}{m}$ (d) 0

24. A particle of unit mass moves in a potential $V(r) = -V_0 e^{-r^2}$. If the angular momentum of the particle is $L = 0.5\sqrt{V_0}$, then which of the following statements are true?

(a) There are two equilibrium points along the radial coordinate

(b) There is one stable equilibrium point at r_1 and one unstable equilibrium point at $r_2 > r_1$

(c) There are two stable equilibrium points along the radial coordinate

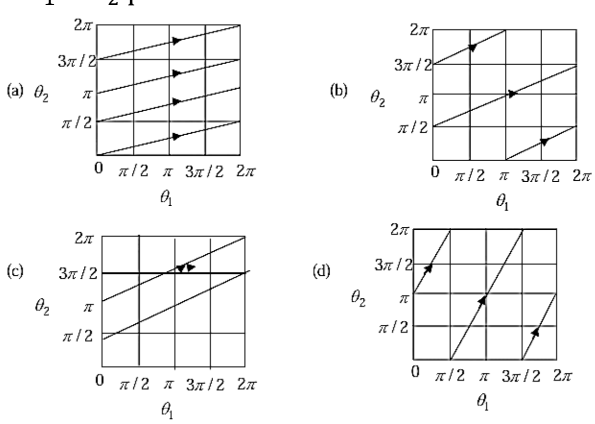
(d) There is only one equilibrium point along the radial coordinate

25. A system with time independent Hamiltonian $H(q,p)$ has two constants of motion $f(q,p)$ and $g(q,p)$. Then which of the following Poisson brackets are always zero?

(a) $\{H, f+g\}$ (b) $\{H,\{f,g\}\}$

(c) $\{H+f, g\}$ (d) $\{H, H+fg\}$

26. In the action-angle variables $(I_1, I_2, \theta_1, \theta_2)$ consider the Hamiltonian $H = 4I_1I_2$ and $0 \leq \theta_1, \theta_2 < 2\pi$. Let $\frac{I_1}{I_2} = \frac{1}{2}$. Which of the following are possible plots of the trajectories with different initial conditions in $\theta_1 - \theta_2$ plane?



27. Two identical particles of rest mass m_0 approach each other with equal and opposite velocity $v = 0.5c$, where c is the speed of light. The total energy of one particle as measured in the rest frame of the other is $E = \alpha m_0 c^2$. The value of α is (Round off to two decimal places)

Statistical Mechanics

28. Which of the following relationship between the internal energy U and the Helmholtz's free energy F is true?

(a)$U = -T^2\left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$

(b) $U = +T^2\left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$

(c)$U = +T\left[\frac{\partial F}{\partial T}\right]_V$

(d)$U = -T\left[\frac{\partial F}{\partial T}\right]_V$

29. Consider a non-interacting gas of spin 1 particles, each with magnetic moment μ, placed in a weak magnetic field B, such that $\frac{\mu B}{k_B T} << 1$. The average magnetic moment of a particle is

(a) $\frac{2\mu}{3}\left(\frac{\mu B}{k_B T}\right)$ (b) $\frac{\mu}{2}\left(\frac{\mu B}{k_B T}\right)$

(c) $\frac{\mu}{3}\left(\frac{\mu B}{k_B T}\right)$ (d) $\frac{3\mu}{4}\left(\frac{\mu B}{k_B T}\right)$

30. Water at $300K$ can be brought to $320K$ using one of the following processes.
Process 1: Water is brought in equilibrium with a reservoir at 320 K directly.
Process 2: Water is first brought in equilibrium with a reservoir at 310 K and then with the reservoir at 320 K.
Process 3: Water is first brought in equilibrium with a reservoir at 350 K and then with the reservoir at 320 K.
The corresponding changes in the entropy of the universe for these processes are $\Delta S_1, \Delta S_2$ and ΔS_3, respectively. Then

(a) $\Delta S_2 > \Delta S_1 > \Delta S_3$

(b) $\Delta S_3 > \Delta S_1 > \Delta S_2$
(c) $\Delta S_3 > \Delta S_2 > \Delta S_1$
(d) $\Delta S_1 > \Delta S_2 > \Delta S_3$

31. A piston of mass m is fitted to an airtight horizontal cylindrical jar. The cylinder and piston have identical unit area of cross-section. The gas inside the jar has volume V and is held at pressure $P = P_{\text{atmosphere}}$. The piston is pushed inside the jar very slowly over a small distance. On releasing, the piston performs an undraped simple harmonic motion of low frequency. Assuming that the gas is ideal and no heat is exchanged with the atmosphere, the frequency of the small oscillations is proportional to

(a) $\sqrt{\frac{P}{\gamma m V}}$ (b) $\sqrt{\frac{P\gamma}{Vm}}$
(c) $\sqrt{\frac{P}{mV^{\gamma-1}}}$ (d) $\sqrt{\frac{\gamma P}{mV^{\gamma-1}}}$

32. A paramagnetic salt of mass m is held at temperature T in a magnetic field H. If S is the entropy of the salt and M is its magnetization, then $dG = -SdT - MdH$, where G is the Gibbs free energy. If the magnetic field is changed adiabatically by $\Delta H \to 0$ and the corresponding infinitesimal changes in entropy and temperature are ΔS and ΔT, then which of the following statements are correct

(a)$\Delta S = -\frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_H \Delta T$
(b) $\Delta S = 0$
(c)$\Delta T = -\frac{\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H}\Delta H$
(d) $\Delta T = 0$

Atomic Molecular Physics

33. An excited state of Ca atom is $[Mg]3p^5 4s^2 3d^1$. The spectroscopic terms corresponding to the total orbital angular momentum are

(a) S, P, and D (b) P, D and F
(c) P and D (d) S and P

34. A point charge q is performing simple harmonic oscillations of amplitude A at angular frequency ω. Using Larmor's formula, the power radiated by the charge is proportional to

(a) $q\omega^2 A^2$ (b) $q\omega^4 A^2$
(c) $q^2\omega^2 A^2$ (d) $q^2\omega^4 A^2$

35. For normal Zeeman lines observed ∥ and ⊥ to the magnetic field applied to an atom, which of the following statements are true?

(a) Only π-lines are observed ∥ to the field
(b) σ-lines ⊥ to the field are plane polarized
(c) π-lines ⊥ to the field are plane polarized
(d) Only σ-lines are observed ∥ to the field

36. In a solid, a Raman line observed at 300 cm^{-1} has intensity of Stokes line four times that of the anti-Stokes line. The temperature of the sample is K.
(Round off to the nearest integer)
($1\ \text{cm}^{-1} \equiv 1.44$ K)

37. A spectrometer is used to detect plasma oscillations in a sample. The spectrometer can work in the range of $3 \times 10^{12}\text{rads}^{-1}$ to $30 \times 10^{12}\text{rads}^{-1}$. The minimum carrier concentration that can be detected by using this spectrometer is $n \times 10^{21}\ \text{m}^{-3}$. The value of n is
(Round off to two decimal places)
(Charge of an electron = -1.6×10^{-19}C, mass of an electron = 9.1×10^{-31} kg and $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-1}$)

38. In a diatomic molecule of mass M, electronic, rotational and vibrational energy scales are of magnitude E_e, E_R and E_V, respectively. The spring constant for the vibrational energy is determined by E_e. If the electron mass is m then

(a) $E_R \sim \frac{m}{M} E_e$ (b) $E_R \sim \sqrt{\frac{m}{M}} E_e$

(c) $E_V \sim \sqrt{\frac{m}{M}} E_e$ (d) $E_V \sim \left(\frac{m}{M}\right)^{1/4} E_e$

39. In a nucleus, the interaction $V_{so}\,\vec{l}\cdot\vec{s}$ is responsible for creating spin-orbit doublets. The energy difference between $p_{1/2}$ and $p_{3/2}$ states in units of $V_{so}\frac{\hbar^2}{2}$ is
(Round off to the nearest integer)

40. For 1 mole of Nitrogen gas, the ratio $\left(\frac{\Delta S_I}{\Delta S_{II}}\right)$ of entropy change of the gas in processes (I) and (II) mentioned below is (Round off to one decimal place)
(I) The gas is held at 1 atm and is cooled from 300 K to 77 K.
(II) The gas is liquefied at 77 K.
(Take $C_p = 7.0 \text{calmol}^{-1} K^{-1}$, Latent heat $L = 1293.6 \text{calmol}^{-1}$)

41. Frequency bandwidth Δv of a gas laser of frequency v Hz is

$$\Delta v = \frac{2v}{c}\sqrt{\frac{\alpha}{A}}$$

where $\alpha = 3.44 \times 10^6\ \text{m}^2\ \text{s}^{-2}$ at room temperature and A is the atomic mass of the lasing atom. For $^4\text{He} -\ ^{20}\text{Ne}$ laser (wavelength $= 633$ nm), $\Delta v = n \times 10^9$ Hz. The value of n is (Round off to one decimal place)

Nuclear Physics

42. Match the order of β-decays given in the left column to appropriate clause in the right column. Here $X(I^\pi)$ and $Y(I^\pi)$ are nuclei with intrinsic spin I and parity π.
1. $X\left(\frac{1^+}{2}\right) \to Y\left(\frac{1^+}{2}\right)$
2. $X\left(\frac{1^-}{2}\right) \to Y\left(\frac{5^+}{2}\right)$
3. $X(3^+) \to Y(0^+)$
4. $X(4^-) \to Y(0^+)$
(i) First forbidden β-decay
(ii) Second forbidden β-decay
(iii) Third forbidden β-decay
(iv) Allowed β-decay
(a) 1 - i, 2 - ii, 3 - iii, 4 - iv
(b) 1 - iv, 2 - i, 3 - ii, 4 - iii
(c) 1 - i, 2 - iii, 3 - ii, 4 - iv
(d) 1 - iv, 2 - ii, 3 - iii, 4 – i

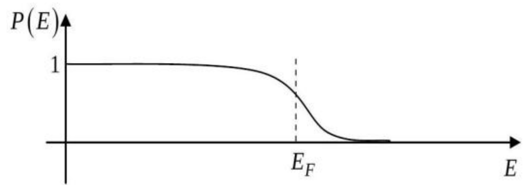
43. If nucleons in a nucleus are considered to be confined in a three-dimensional cubical box, then the first four magic numbers are
(a) 2,8,20,28 (b) 2,8,16,24
(c) 2,8,14,20 (d) 2,10,16,28

Solid State Physics

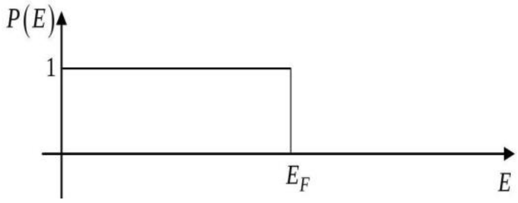
44. Potassium metal has electron concentration of $1.4 \times 10^{28}\ \text{m}^{-3}$ and the corresponding density of states at Fermi level is $6.2 \times 10^{46} \text{Joule}^{-1}\ \text{m}^{-3}$. If the Pauli paramagnetic susceptibility of Potassium is $n \times 10^{-k}$ in standard scientific form, then the value of k (an integer) is
(Magnetic moment of electron is 9.3×10^{-24} Joule T^{-1}; permeability of free space is $4\pi \times 10^{-7} \text{TmA}^{-1}$)

45. In a two-dimensional square lattice, frequency ω of phonons in the long wavelength limit changes linearly with the wave vector k. Then the density of states of phonons is proportional to
(a) ω (b) ω^2
(c) $\sqrt{\omega}$ (d) $\frac{1}{\sqrt{\omega}}$

46. At $T = 0K$, which of the following diagram represents the occupation probability $P(E)$ of energy states of electrons in a BCS type superconductor?
(a)



(b)



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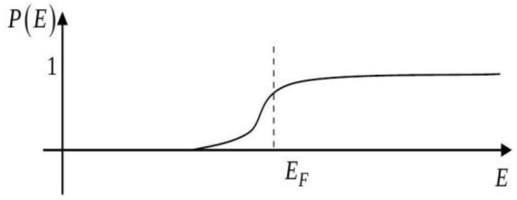
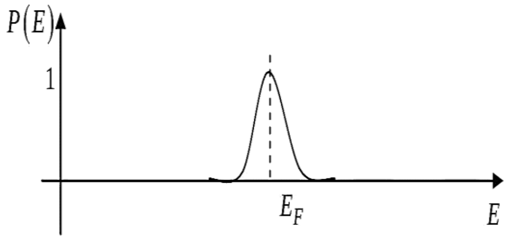


(d)

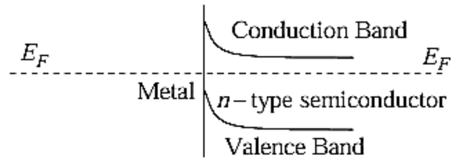


47. Electronic specific heat of a solid at temperature T is $C = \gamma T$, where γ is a constant related to the thermal effective mass (m_{eff}) of the electrons. Then which of the following statements are correct?
(a) $\gamma \propto m_{eff}$
(b) m_{eff} is greater than free electron mass for all solids
(c) Temperature dependence of C depends on the dimensionality of the solid
(d) The linear temperature dependence of C is observed at $T \ll$ Debye temperature

48. In a Hall effect experiment on an intrinsic semiconductor, which of the following statements are correct?
(a) Hall voltage is always zero
(b) Hall voltage is negative if the effective mass of holes is larger than those of electrons
(c) Hall coefficient can be used to estimate the carrier concentration in the semiconductor
(d) Hall voltage depends on the mobility of the carriers

49. A junction is formed between a metal on the left and an n-type semiconductor on the right. Before forming the junction, the Fermi level E_F of the metal lies below that of the semiconductor. Then which of the following schematics are correct for the bands and the $I - V$ characteristics of the junction?

(a)



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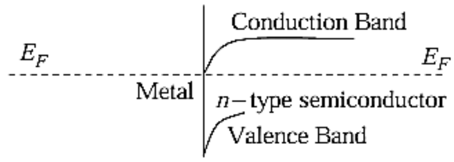
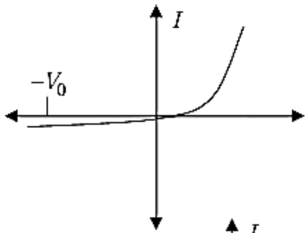
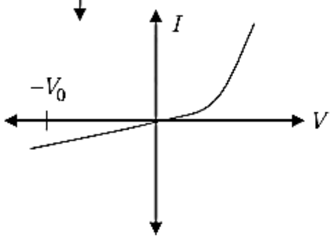


(c)



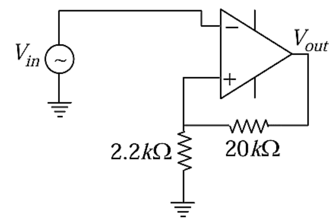
(d)



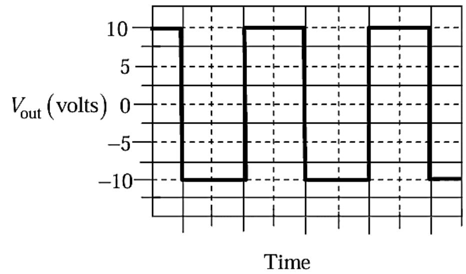
50. In an X-Ray diffraction experiment on a solid with FCC structure, five diffraction peaks corresponding to (111), (200), (220), (311) and (222) planes are observed using 1.54ÅX rays. On using 3Å°X-rays on the same solid, the number of observed peaks will be

Electronics

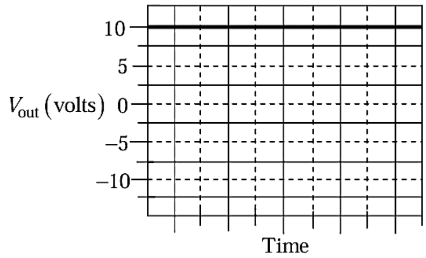
51. For the Op-Amp circuit shown below, choose the correct output waveform corresponding to the input $V_{in} = 1.5\sin\ 20\pi t$ (in Volts). The saturation voltage for this circuit is $V_{\text{sat}} = \pm 10$ V.



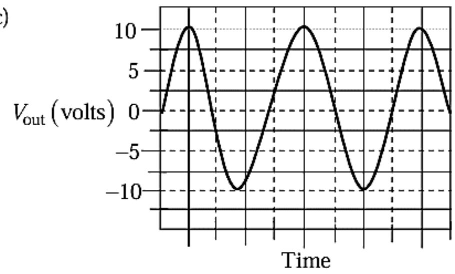
(a)



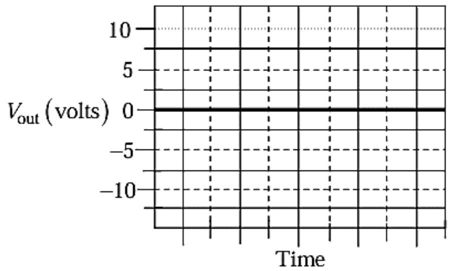
(b)



(c)



(d)



52. For an Op-Amp based negative feedback, non-inverting amplifier, which of the following statements are true?
    (a) Closed loop gain < Open loop gain
    (b) Closed loop bandwidth < Open loop bandwidth
    (c) Closed loop input impedance > Open loop input impedance
    (d) Closed loop output impedance < Open loop output impedance

53. For a bipolar junction transistor, which of the following statements are true?
    (a) Doping concentration of emitter region is more than that in collector and base region
    (b) Only electrons participate in current conduction
    (c) The current gain β depends on temperature
    (d) Collector current is less than the emitter current

54. A power supply has internal resistance R_S and open load voltage $V_S = 5$ V. When a load resistance R_L is connected to the power supply, a voltage drop of $V_L = 4V$ is measured across the load. The value of $\frac{R_L}{R_S}$ is (Round off to the nearest integer)

55. The minimum number of two-input NAND gates required to implement the following Boolean expression is
$$Y = [A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

Answer Key					
1. a,b,d	2. b,d	3. 33-34	4. a	5. c	6. c
7. a,b,d	8. b,d	9. c	10. b	11. a,c	12. a,b
13. a,c	14. a	15. a,c	16. 2.4-2.6	17. 39-40	18. b
19. a,c,d	20. a,b,c	21. 2-2	22. d	23. a	24. a,b
25. a,b,d	26. b,c	27. 1.65-1.70	28.	29. a	30. b
31. b	32. b,c	33. b	34. d	35. b,c,d	36. 311-312
37. 2.70-2.996	38. a,c	39. 3-3	40. 0.5-0.7	41. 1.2-1.4	42. b
43.	44. 6-6	45. a	46. a	47. a,d	48. d
49. a,c	50. 1-1	51. a	52. a,c,d	53. a,c,d	54. 4-4
55. 3-3					

❖ Solutions

1. **Solution:** The commutator relation between $J_1^2, L^2, S^2,\ J_z, \vec{L}.\vec{S}, \vec{S}_z, \vec{L}_z$ are as follows,

   (i) J^2, L^2, S^2, Jz commutes with $\vec{L}\cdot\vec{S}$ but not L_z and S_z

   Thus option (c) is incorrect.

   Correct option is (a), (b), (d)

2. **Solution:** General anti commutator relation.

   Let us verify option (b)

$$\sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_x\sigma_y - \sigma_y\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i\sigma_z$$

   Let us verify the relation in option (a).

$$\sigma_x\sigma_y + \sigma_y\sigma_x = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

   Thus, option (b) and (d) are correct option.

   Correct option is (b), (d)

3. **Solution:** We have

$$\psi(x) = \frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\frac{\pi x}{a} + \sqrt{3}\cos\frac{\pi x}{2a} + \cos\frac{3\pi x}{2a}\right]$$

$$= \frac{1}{\sqrt{6}}\sqrt{\frac{2}{a}}\sin\frac{\pi x}{a} + \frac{\sqrt{3}}{\sqrt{12}}\frac{\sqrt{2}}{\sqrt{a}}\cos\frac{\pi x}{2a} + \frac{1}{\sqrt{12}}\sqrt{\frac{2}{a}}\cos\frac{3\pi x}{2a}\Bigg] = \frac{1}{\sqrt{6}}|\phi_2\rangle + \frac{\sqrt{3}}{\sqrt{12}}|\phi_1\rangle + \frac{1}{\sqrt{12}}|\phi_3\rangle$$

   The wave function of the particle in such a potential is given by.

$$|\psi_1\rangle = \sqrt{\frac{2}{a}}\cos\frac{\pi x}{2a}; |\psi_2\rangle = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}, \qquad |\psi_3\rangle = \sqrt{\frac{2}{a}}\cos\frac{3\pi x}{2a}$$

   The normalization constant is obtained as follows.

$$|\psi\rangle = A\left(\frac{1}{\sqrt{6}}|\phi_2\rangle + \frac{1}{\sqrt{4}}|\phi_1\rangle + \frac{1}{\sqrt{12}}|\phi_3\rangle\right)$$

$$\langle\psi \mid \psi\rangle = A^2\left(\frac{1}{6}\langle\phi_2 \mid \phi_2\rangle + \frac{1}{4}\langle\phi_1 \mid \phi_1\rangle + \frac{1}{12}\langle\phi_3 \mid \phi_3\rangle\right) = 1$$

$$A^2\left(\frac{1}{6} + \frac{1}{4} + \frac{1}{12}\right) = 1 \Rightarrow A^2\left(\frac{2+3+1}{12}\right) = 1 \Rightarrow A = \sqrt{2}$$

   Thus the normalized wave function is given by

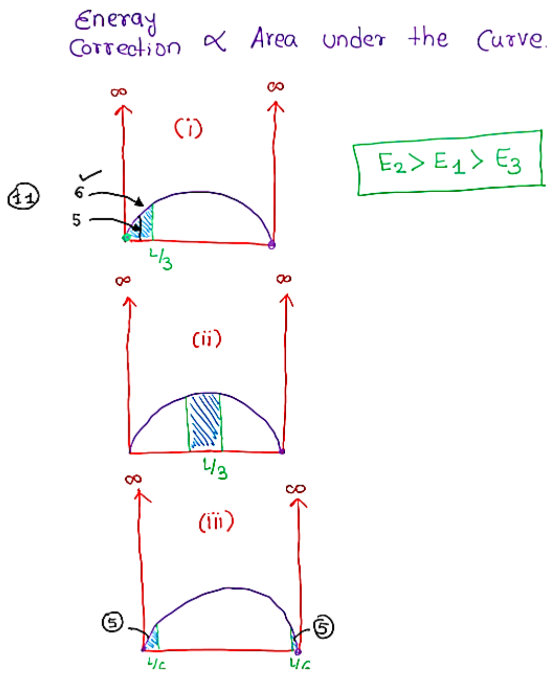
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{3}}|\phi_2\rangle + \frac{1}{\sqrt{6}}|\phi_3\rangle$$

The probability of finding the particle in state $n = 2$ is

$$(\phi_2) = |\langle \phi_2 \mid \psi \rangle|^2 = \left| \frac{1}{\sqrt{3}} \langle \phi_2 \mid \phi_2 \rangle \right|^2 = \frac{1}{3} = 33.33\%$$

Correct Answer: 33 to 34

4. **Solution:**



Correct option is (a)

5. **Solution:** Let us choose operator given is option (c).

$$A = \frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$$

$$\langle A\phi \mid \varphi \rangle = \int \left(-\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)\right)\phi^*(s)\varphi(s)ds$$

$$= -\frac{1}{i}\left(\int_0^\infty \frac{\partial}{\partial s}\phi^*(s)\varphi(s)sds + \frac{1}{2s}\int_0^\infty \phi^*(s)\varphi(s)sds\right.$$

$$= -\frac{1}{i}\left[[\phi^*(s)\varphi(s)s]_0^\infty \int \phi^*(s)\frac{d\varphi}{ds}(s)sds - \int \phi^*(s)\varphi(s)ds + \frac{1}{2}\int \phi^*(s)\varphi(s)ds\right]$$

$$= -\frac{1}{i}\left[-\int \phi^*(s)\frac{d\varphi}{ds}(s)sds - \frac{1}{2s}\int \phi^*(s)\varphi(s)sds\right]$$

$$= \int \phi^*(s)\left[\frac{1}{i}\left[\frac{\partial}{\partial s} + \frac{1}{2s}\right]\varphi(s)sds = \langle \phi(s) \mid A\varphi(s) \rangle\right.$$

Thus, operator A is Hermitian.

Correct option is (c)

6. **Solution:** The position operator is given by

$$\langle x\rangle = \left\langle m\left|\sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)\right|n\right\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\delta_{\mathrm{m,n+1}} + \sqrt{n}\delta_{\mathrm{m,n-1}}\right)$$

For $m = n$,i.e., all diagonal elements in the matrix must be zero.

For $m = 0, n = 1$ the value of $\langle x\rangle$ is

$$\langle x\rangle_{01} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{2}\delta_{0,2} + \sqrt{1}\delta_{1,0}\right] = \sqrt{\frac{\hbar}{2m\omega}}$$

For $m = 1, n = 0$, the value of $\langle x\rangle$ is

$$\langle x\rangle_{10} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{0+1}\delta_{1,1} + \sqrt{0}\delta_{1,-1}\right] = \sqrt{\frac{\hbar}{2m\omega}}$$

For $m = 0, n = 2$, the value of $\langle x\rangle$ is

$$\langle x\rangle_{0,2} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{0+2}\delta_{0,3} + \sqrt{2}\delta_{0,1}\right] = 0$$

For $m = 1, n = 2$, the value of $\langle x\rangle_{12}$ is

$$\langle x\rangle_{12} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{2+1}\delta_{1,3} + \sqrt{2}\delta_{1,1}\right] = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{2}$$

For $m = 2, n = 1$, the value of $\langle x\rangle_{21}$ is

$$\langle x\rangle_{21} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{1+1}\delta_{212} + \sqrt{1}\delta_{2,0}\right] = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{2}$$

For $m = 2, n = 0$, the value of $\langle x\rangle_{20}$ is

$$\langle x\rangle_{20} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{1+0}\delta_{2,1} + \sqrt{0}\delta_{2,-1}\right] = 0$$

The matrix representation of position vector is

$$\langle x\rangle = \begin{bmatrix} \langle 0|x|0\rangle & \langle 0|x|1\rangle & \langle 0|x|2\rangle \\ \langle 1|x|0\rangle & \langle 1|x|1\rangle & \langle 1|x|2\rangle \\ \langle 2|x|0\rangle & \langle 2|x|1\rangle & \langle 2|x|2\rangle \end{bmatrix}, \qquad \langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

and general from is

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} \\ 0 & 0 & 0 & \sqrt{4} & 0 \end{bmatrix}$$

Correct option is (c)

7. **Solution:** The Schrödinger equation for a particle moving in radial potential is given by

$$H\varphi = E\varphi \Rightarrow \frac{-\hbar^2}{2m}\frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2\frac{dR}{dr}\right) + V_{eff}R(r) = E_0R(r)$$

where $V_{\text{eff}} = V + \frac{\ell(\ell+1)}{2mr^2}\hbar^2 = 0$

as $V = 0, \ell = 0$.

Thus Schrödinger equation to

$$\frac{-\hbar^2}{2m}\frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2\frac{dR}{dr}\right) = E_0 R(r)$$

or $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}ER(r) = 0$

Defining $R(r) = \frac{U(r)}{r}$ and substituting in above equation, we get

$$\frac{d^2U(r)}{dr^2} + k^2R(r) = 0,\ \ r^2 = \frac{2mE}{\hbar^2}r < a$$

The solution of above equation is $\mathrm{U}(r) = A\sin\ kr + B\cos\ kr$

Applying Boundary condition, $U(r=0) = 0; U(r=a) = 0$

$$U(r=0) = A\sin\ 0 + B\cos\ 0 = 0 \Rightarrow B = 0$$

Thus the wave function is given by, $U(r) = A\sin\ kr$

Applying Boundary condition $U(r=a) = 0$

$$U(r=a) = A\sin\ ka = 0 \Rightarrow ka = n\pi \Rightarrow k\frac{n\pi}{a}$$

or

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

and the Radial wave function is given by

$$R(r) = \frac{U(r)}{r} = \frac{A}{r}\sin\ \frac{\pi nr}{a}$$

For ground state the energy and wave function of the particle are

$$E = \frac{\pi^2\hbar^2}{2ma^2}; R(r) = \frac{1}{r}\sin\ \frac{\pi r}{a}$$

Correct option is (a), (b), (d)

8. **Solution:** We have,

$$\psi_{n_x,n_y} = \frac{2}{L}\sin\ \left(\frac{n_x\pi x}{L}\right)\sin\ \left(\frac{n_y\pi y}{L}\right)$$

and its corresponding energies are,

$$E_{n_x,n_y} = \left(n_x^2 + n_y^2\right)\frac{\pi^2\hbar^2}{2mL^2}$$

For ground state $n_x = n_y = 1$, the ground state energy is given by

$$E_{11} = (1^2 + 1^2)\frac{\pi^2\hbar^2}{2mL^2} = 2\frac{\pi^2\hbar^2}{2mL^2}$$

The first order correction in ground state energy is

$$E_{11}^{(1)} = \langle\psi_{11}(x,y)|H'|\psi_{11}(x,y)\rangle$$

$$= \frac{2}{L}\frac{2}{L} C\int_0^L x\sin^2 \frac{\pi x}{L} dx \int_0^L y\sin^2 \frac{\pi y}{L} dy$$

$= \left(\frac{2}{L}\right)^2 \left(\frac{L}{4}\right)^2 \left(\frac{L^2}{4}\right) L = \frac{CL^2}{4}$ as the energy state energy is odd function .

The first excited state has energy,

$$(n_x, n_y) = \begin{cases}(2,1)\\(1,2)\end{cases}, \qquad E_{21} = E_{12} = (2^2 + 1^2)\frac{\pi^2\hbar^2}{2ma^2} = 5\frac{\pi^2\hbar^2}{2ma^2}$$

The wave function of the particles are

$$n_x = 1, n_y = 2, E_{12} = \frac{5\pi^2\hbar^2}{2ma^2}; \psi_{12}^0(x, y) = \frac{2}{a}\sin\frac{\pi x}{a}\sin\frac{2\pi y}{a}$$

$$n_x = 2, n_y = 1, E_{21} = \frac{5\pi^2\hbar^2}{2ma^2}; \psi_{21}^0(x, y) = \frac{2}{a}\sin\frac{2\pi x}{a}\sin\frac{\pi y}{a}$$

The perturbed matrix for this Hamiltonian is given by

$$H_p = \begin{bmatrix}\langle 1,2|H'|1,2\rangle & \langle 1,2|H'|2,1\rangle\\ \langle 2,1|H'|1,2\rangle & \langle 2,1|H'|2,1\rangle\end{bmatrix} = \begin{bmatrix}a & b\\ b & a\end{bmatrix}$$

The values of inner product are

$$\langle 1,2|H'|1,2\rangle = \frac{4}{L^2}C\int_0^L x\sin^2\frac{\pi x}{L}dx\int_0^L y\sin^2\frac{2\pi y}{L}dy = \frac{CL^2}{4}$$

$$\langle 1,1|H'|2,1\rangle = \frac{4}{L^2}C\int_0^L x\sin^2\frac{2\pi x}{L}dx\int_0^L y\sin^2\frac{\pi y}{L}dy = \frac{CL^2}{4}$$

$$\langle 2,1|H'|1,2\rangle = \frac{4}{L^2}C\int_0^L x\sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}dx\int_0^L y\sin\frac{\pi y}{L}\sin\frac{2\pi y}{L}dL = \frac{25bCL^2}{81\pi^4}$$

$$\langle 2|H'|2,1\rangle = \frac{4}{L^2}C\int_0^L x\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}dx\int_0^L 9\sin\frac{2\pi y}{L}\sin\frac{\pi y}{L}dy = \frac{25bCL^2}{81\pi^4}$$

Thus the eigen value of perturbed matrix is determined ground secular equation.

$$|H - \delta I| = \begin{vmatrix}a-\delta & b\\ b & a-\delta\end{vmatrix} = 0$$

Correct option is (b), (d)

9. **Solution:** Consider a 2×2 orthogonal matrix $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$. As the matrix is orthogonal

$$\Rightarrow A^T A = I$$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^T \begin{bmatrix}a & b\\ c & d\end{bmatrix} = I$$

$$\begin{bmatrix}a & c\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = I$$

$$\begin{bmatrix}a^2 + c^2 & ab + cd\\ ab + cd & b^2 + d^2\end{bmatrix} = I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Thus, we have

$a^2 + c^2 = 1$ and $b^2 + d^2 = 1$, Two constraints on diagonal elements.

This will make two parameters' components dependent.

And $ab + cd = 1$, one constraint ($1/2C$ No. of diagonal elements) on the value of diagonal elements. This will make one more component dependent.

Thus total no. of m dependent

Components $= 4 - 3 = 1$

$$= \left[\frac{n(n-1)}{2}\right] \{n = 2 \text{ for } 2 \times 2 \text{ matrix }\}.$$

Generalizing for $n \times n$ matrix.

Total components $= n^2$

n-diagonal elements are dependent because of n-constrain

$\frac{1}{2}(n^2 - n)$ - diagonal elements are dependent.

$$\Rightarrow \frac{n^2 - n}{2} + n = \frac{n(n+1)}{2} \text{elements are dependent}$$

Hence

$$n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$$

elements ax independent.

Correct option is (c)

10.**Solution:** $y'' - 2xy' + 4y = 0$

Given solution $y(x) = a + bx + cx^2$

$$y' = b + 2cx$$

$$y'' = 2c$$

Put the value of y, y' and $y''m(1)$, we get

$$2c - 2x[b + 2cx] + 4[a + bx + cx^2] = 0$$

$$2c - 2bx - 4cx^2 + 4a + 4bx + 4cx^2 = 0$$

$$2bx + (2c + 4a) = 0$$

$$\Rightarrow b = 0 \text{ and } 2c + 4a = 0$$

$$\Rightarrow c = -2a$$

Thus, $b = 0$ and $c = -2a$

Correct option is (b)

11.**Solution:** Given $f(z) = z + |z - a|^2, a \in \mathbb{R}, z = x + iy$

$$|z - a|^2 = (x - a)^2 + y^2,\ f(z) = (x + iy) + (x - a)^2 + y^2$$

So, $u(x, y) = x + (x - a)^2 + y^2, v(x, y) = y$

(a) continuous at (a, a)

Both u and v are polynomials in $x, y \Rightarrow$ continuous everywhere.

TRUE

(b) complex-differentiable at (a, a)

Check Cauchy-Riemann equations: $u_x = 1 + 2(x - a), u_y = 2y,\ v_x = 0, v_y = 1$

CR conditions: $u_x = v_y \Rightarrow 1 + 2(x - a) = 1 \Rightarrow x = a,\ u_y = -v_x \Rightarrow 2y = 0 \Rightarrow y = 0$

At (a, a): $y = a \neq 0$ (in general) $\Rightarrow$ CR not satisfied

FALSE

(c) complex-differentiable at $(a, 0)$

At $(x, y) = (a, 0)$, both CR equations hold.

Partial derivatives are continuous $\Rightarrow$ differentiable.

TRUE

(d) analytic at $(a, 0)$

Analytic $\Leftrightarrow$ differentiable in a neighborhood, not just at a point.

CR holds only at the single point $(a, 0)$, not around it.

FALSE

Correct option is (a), (c)

12.**Solution:** As per statements

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

taking complex conj of (1)

$$g*(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f*(x) e^{ikx} dx$$

Replacing k by $-km(1)$

$$g(-k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

Now if $g(-k) = g*(k)$ {condition 'a'}

$$\Rightarrow (2) = (3)$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f*(x) e^{ikx} dx = \frac{1}{\sqrt{2\pi}} \int f(x) e^{ikx} dx$$

$$\Rightarrow f(x) = f*(x)$$

Hence $f(x)$ must be real and not purely imaginary {Condition 'b'}

$$g(-k) = -g*(k)$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f*(x) e^{ikx} dx$$

$$\Rightarrow f(x) = -f*(x).$$

Thus $f(x)$ must be purely imaginary and not real.

Thus ' a ' and ' b ' are correct options.

Correct option is (a), (b)

13.**Solution:** $(1 - x^2)y'' - xy' + 9y = 0$

Dividing by $1 - x^2$

$$y'' - \frac{x}{1-x^2}y' + \frac{9}{1-x^2}y = 0$$

Compare with $y'' + p(x)y' + Q(x)y = 0$

$$P(x) = \frac{-x}{1-x^2}, Q(x) = \frac{9}{1-x^2}$$

At 1 and -1 both $P(x)$ and $Q(x)$ diverge first condition satisfied.

At $x = 1$

$$(x-1)P(x) = (x-1)\frac{-x}{1-x^2} = (x-1)\cdot\frac{x}{x^2-1} = \frac{x}{x+1} = \frac{1}{2} \text{ finite.}$$

At $x = -1$

$$(x+1)P(x) = (x+1)\cdot\frac{x}{x^2-1} = \frac{x}{x-1} = \frac{-1}{-1-1} = \frac{1}{2}$$

Thus $(x - x_0)P(x)$ remains finite .

At $x = 1$

$$(x-1)^2Q(x) = (x-1)^2\frac{9}{(1-x)(1+x)} = -9\cdot\frac{x-1}{x+1} = -9\times\frac{0}{2} = 0$$

At $x = -1$

$$(x+1)^2Q(x) = (x+1)^2\cdot\frac{-9}{(1+x)(1-x)} = -9\cdot\frac{(x+1)}{(x-1)} = -9\times\frac{0}{-2} = 0$$

Thus $(x - x_0)^2Q(x)$ remains finite.

Thus both 1 and -1 are regular singular points.

Correct option is (a), (c)

14.**Solution:** $V_{\text{in}}(r,\theta) = \sum A_\alpha r^\alpha P_\alpha(\cos\,\theta) = A_0 r^0 P_0(\cos\,\theta) + A_1 r^1 P_1(\cos\,\theta) + \cdots$

$V_{\text{in}}(r,\theta) = A_0 + A_1 r P_1(\cos\,\theta) + \cdots$.

$V_{\text{in}}(0,\theta) = A_0 + A_1 \times 0 + \cdots$ All other terms are zero.

Thus potential at centre is only decided by A_0; $A_0 = ?$

Also, it can be shown that surface area of sphere from $0 = 0$ to $\frac{\pi}{3}$ and $\frac{\pi}{3} = \frac{\pi}{2}$ and $\frac{\pi}{2}$ to π is respectively $\pi R^2, \pi R^2$ and $2\pi R^2$

Let's apply the boundary condition, which is $V(R,\theta) = \begin{cases} \phi_0, & 0 < \theta < \frac{\pi}{3} \\ 2\phi_0, & \frac{\pi}{3} < \theta < \frac{\pi}{2} \\ 4\phi_0, & \frac{\pi}{2} < \theta < \pi \end{cases}$

Thus $V(R,\theta) = \sum A_\alpha R^\alpha P_\alpha(\cos\,\theta)$

Now $A_\alpha = \frac{2\alpha+1}{2R^\alpha}\int_0^\pi V(R,\theta)P_\alpha(\cos\,\theta)\sin\,\theta d\theta$ {Griffith page 140 3rd edition}

As we only want A_0

$$\Rightarrow A_0 = \frac{2\times 0+1}{2R^0}\int_0^\pi V(R,\theta)P_0(\cos\,\theta)\sin\,\theta d\theta$$

$P_0(\cos\,\theta) = 1$

$$A_0 = \frac{1}{2}\left[\int_0^{\pi/3} \phi_0(\sin\,\theta)d\theta + \int_{\pi/3}^{\pi/2} 2\phi_0(\sin\,\theta)d\theta + \int_{\pi/2}^{\pi} (\phi_0)\sin\,\theta d\theta\right]$$

$$A_0 = \left[\phi_0 \times -|\cos\,\theta|_0^{[\pi/3} + 2\phi_0 \times -|\cos\,\theta|_{\pi/3}^{[\pi/2} + 4\phi_0 \times -|\cos\,\theta|_{\pi/2}^{\pi}\right]$$

$$\Rightarrow A_0 = \frac{1}{2}\left[\phi_0\left(\frac{1}{2} - 1\right) - 2\phi_0\left(-\frac{1}{2} + 0\right) + 4\phi_0\right]$$

$$\Rightarrow A_0 = \frac{1}{2}\left[+\frac{\phi_0}{2} + \phi_0 + 4\phi_0\right] = \frac{1}{2}\left[\frac{(1+2+8)\phi_0}{2}\right] = \frac{11}{4}\phi_0 = \frac{11}{4}\phi_0$$

As $V_{in}(0,\theta) = A_0$, Potential at centre $= \frac{11}{4}\phi_0$

Correct option is (a)

15.**Solution:** (a) n_r can be obtained from n_{im} and vice versa - TRUE

Because of Kramers-Kronig relations, the real and imaginary parts of any causal response function are Hilbert transforms of each other.

Knowing one over all frequencies determines the other.

(b) If $n_{im}(\omega) = 0$ for all $\omega \rightarrow$ then

$\Rightarrow n_r(\omega)$ must be constant

$\Rightarrow$ violates causality.

Hence n_{im} cannot be identically zero for a real material.

So (b) is taken as FALSE.

(c) n is an analytic function in the upper half of the complex ω-plane - TRUE

(d) n is independent of ω for some materials - FALSE

A frequency-independent refractive index would violate causality and dispersion relations.

In reality, all materials are dispersive, even if weakly so over limited ranges.

Correct option is (a), (c)

16.**Solution:**

$$E_{\text{disc}} = \frac{\sigma}{2\,\epsilon_0}\left[1 - \frac{d}{\sqrt{R^2+d^2}}\right], E_{\text{infinitesheet}} = \frac{\sigma}{2\epsilon_0}$$

$$E_{\text{disc}} = 10\% E_{\text{inf.}} \Rightarrow \frac{\sigma}{2\,\epsilon_0}\left[1 - \frac{d}{\sqrt{R^2+d^2}}\right] = \frac{90}{100} \times \frac{\sigma}{2\epsilon_0} \Rightarrow 1 - \frac{d}{\sqrt{R^2+d^2}} = \frac{9}{10}$$

$$\Rightarrow \frac{d}{\sqrt{R^2+d^2}} = 1 - \frac{9}{10} = \frac{1}{10} \Rightarrow 100d^2 = R^2 + d^2$$

$$\Rightarrow 99d^2 = R^2 \Rightarrow d = \frac{R}{\sqrt{99}} = \frac{25}{\sqrt{99}}\text{ cm} = 2.5\text{ cm}$$

Correct Answer: 2.4 to 2.6

17.**Solution:** We have $\Delta t = 10^{-3}$sec, $h = 6.6 \times 10^{-34}$, $c = 3 \times 10^8$ m/s

The uncertainty in energy is

$$\Delta E \cdot \Delta t = \frac{\hbar}{2} \Rightarrow \Delta E = \frac{\hbar}{2\Delta t}$$

The uncertainty in the momentum is given by.

$$\Delta p = \frac{\Delta E}{c} = \frac{\hbar}{2c\Delta t} = \frac{1.05 \times 10^{-34}}{2 \times 3 \times 10^{8} \times 10^{-3}} = 0.175 \times 10^{-39} = 1.75 \times 10^{-40} \text{ kg m/s}$$

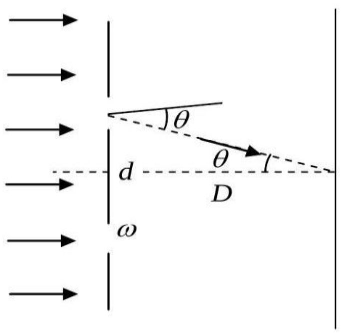
Correct Answer: 39 to 40

18.**Solution:** $\Delta x = \omega$, $\Delta p = 2p\sin\theta$, $\Delta x \Delta p \sim \hbar$, $\omega \times 2p\sin\theta \sim \hbar$

$$\sin\theta \sim \theta \sim \frac{d/2}{D}, \qquad \omega \times 2p\frac{d}{2D} \sim \hbar, \qquad \frac{d\omega}{D} \sim \frac{\hbar}{p}$$

More accurately

$$\frac{d\omega}{D} < \frac{\hbar}{p}$$



Correct Option is (b)

19.**Solution:**

(a) $F = Q_0 E = \frac{Q_0^2}{2 \epsilon_0 A} = \frac{C_0^2 V^2}{2 \epsilon_0 A} = \left(\frac{\epsilon_0 A}{2d}\right)^2 \times \frac{V^2}{2 \epsilon_0 A} = \frac{\epsilon_0 AV^2}{8d^2}$

(b) $W = \int \vec{F} \cdot d\vec{l} = \int_d^{2d} \frac{\epsilon_0 AV^2}{2x^2} dx = \frac{\epsilon_0 AV^2}{4d}$

(c) Energy transferred to source must be equal to energy decrease of the capacitor.

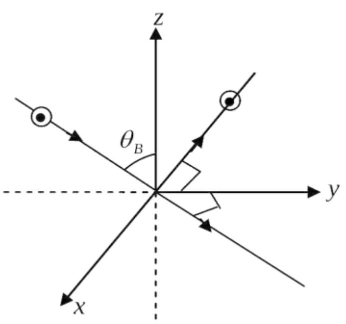
(d) Initial energy $= \frac{1}{2}\frac{\epsilon_0 AV^2}{d}$, final energy $= \frac{1}{2}\frac{\epsilon_0 AV^2}{2d}$

change $= \frac{1}{2}\frac{\epsilon_0 A}{d}V^2 - \frac{1}{2}\frac{\epsilon_0 A}{2d}V^2 = -\frac{1}{4}\frac{\epsilon_0 AV^2}{d^2}$

Correct Option is (a), (c), (d)

20.**Solution:**

$$\vec{k} = \frac{\sqrt{3}}{2}k_0\hat{y} - \frac{k_0}{2}\hat{z}, \qquad \tan\theta_B = \frac{k_y}{k_z} = \frac{\sqrt{3}k_0/2}{2k_0} = \sqrt{3}, \qquad \tan_B = \tan\left(\frac{\pi}{3}\right) \Rightarrow \theta_B = \frac{\pi}{3}$$



$$\theta_R + \theta_B + \frac{\pi}{2} = \pi, \qquad \theta_R + \frac{\pi}{3} + \frac{\pi}{2} = \pi \Rightarrow \theta_R = \pi - \frac{\pi}{2} - \frac{\pi}{3} = +\frac{\pi}{6}$$

$$\tan\ \theta_B = \frac{n_2}{n_1} = \frac{\sqrt{\epsilon_{r_2}}}{\sqrt{\epsilon_{r_1}}} = \sqrt{3} \Rightarrow \frac{\epsilon_{r_2}}{\epsilon_{r_1}} = \sqrt{3} \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 3$$

Correct option is (a), (b), (c)

21.**Solution:**

Solution:

$n' = 3000$ turns /m, $I = 1A, B_{\text{inside}} = \mu_0 n' I$

Electric field inside capacitor $E = \frac{\sigma}{\epsilon_0}$

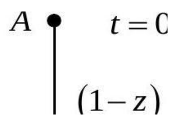
$$P_d = \frac{S}{c^2} = \frac{1}{c^2} \times \frac{1}{\mu_0} EB = \frac{1}{c^2 \mu_0} \times \frac{\sigma}{\epsilon_0} \times \mu_o n' I$$

$$P_d = \frac{\sigma}{c^2 \in_0} n' I = \frac{\sigma \in_0}{(3 \times 10^8)^2 \times \in_0} \times 3000 \times A = \frac{2000}{10^{16}} = 2 \times 10^{-13} \text{Nsm}^{-3} \Rightarrow n = 2$$

Correct Answer: 2 to 2

22.**Solution:** At point B

$$L = \frac{1}{2} m\dot{z}^2 - mgz, \qquad u = 0 \to \dot{z} = 0 + gt = gt, \qquad (1 - z) = 0 + \frac{1}{2} gt^2$$



$$z = 1 - \frac{1}{2} gt^2$$

Time taken to reach the point C

$$0 = 1 - \frac{1}{2} gT^2 \Rightarrow T = \sqrt{\frac{2}{g}}$$

Action $A = \int_0^T L dt$

$$= \int_0^{\sqrt{2/g}} \left[\frac{1}{2} mg^2 t^2 - mg\left(1 - \frac{1}{2} gt^2\right)\right] dt = \int_0^{\sqrt{2/g}} [mg^2 t^2 - mg] dt$$

$$= \left[\frac{1}{3} \times 1 \times g^2 t^3 - 1 \times gt\right]_0^{\sqrt{2/g}} = \frac{2}{3}\sqrt{2g} - \sqrt{2g} = -\frac{1}{3}\sqrt{2g}$$

Correct option is (d)

23.**Solution:**

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{k}{2}(x^2 + y^2), \qquad H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{k}{2}(x^2 + y^2), \qquad \dot{x} = \frac{\partial H}{\partial p_x} = \frac{p_x}{m}$$

$$p_x = m\dot{x};\ p_y = m\dot{y}, \qquad A = \dot{x}\dot{y} + \alpha xy = \frac{p_x p_y}{m^2} + \alpha xy$$

$[A, H] = 0$ if A is constant of motion

$$\frac{\partial A}{\partial x}\frac{\partial H}{\partial p_x}-\frac{\partial A}{\partial p_x}\frac{\partial H}{\partial x}+\frac{\partial A}{\partial y}\frac{\partial H}{\partial p_y}-\frac{\partial A}{\partial p_y}\frac{\partial H}{\partial y}=0$$

$$(\alpha y)\left(\frac{p_x}{m}\right)-\frac{p_y}{m^2}(kx)+(\alpha x)\frac{p_y}{m}-\frac{p_x}{m^2}(ky)=0$$

$$\frac{\alpha}{m}(yp_x+xp_y)-\frac{k}{m^2}(xp_y+yp_x)=0$$

$$(xp_y+yp_x)\left(\frac{\alpha}{m}-\frac{k}{m^2}\right)=0\Rightarrow\alpha=+\frac{k}{m}$$

Second method

$$\frac{d}{dt}(\dot{x}\dot{y}+\alpha xy)=0,\qquad \ddot{x}\dot{y}+\dot{x}\ddot{y}+\alpha(\dot{x}y+x\dot{y})=0$$

Equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=0,\qquad m\ddot{x}+kx=0$$

Similarly,

$$\left.\begin{aligned}\ddot{x}&=-\frac{k}{m}x\\ \ddot{y}&=-\frac{k}{m}y\end{aligned}\right]$$

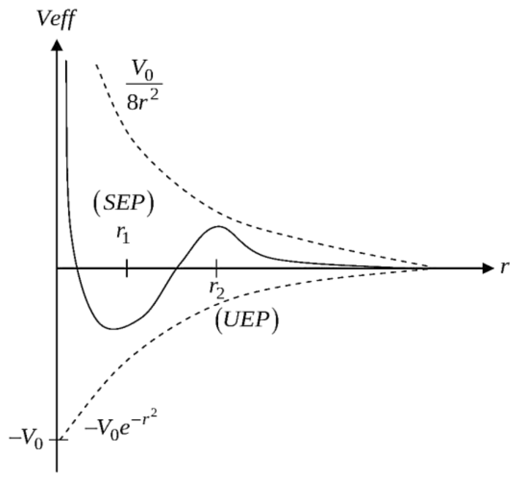
Substitute these values into Equation (1)

$$-\frac{k}{m}(x\dot{y}+y\dot{x})+\alpha(\dot{x}y+x\dot{y})=0,\qquad (x\dot{y}+y\dot{x})\left(-\frac{k}{m}+\alpha\right)=0,\qquad \alpha=\frac{k}{m}$$

Correct option is (a)

24.**Solution:**

$$V_{\text{eff}}=\frac{l^2}{2mr^2}-V_0e^{-r^2},\qquad l=0.5\sqrt{V_0},m=1,\qquad V_{\text{eff}}=\frac{V_0}{\theta r^2}-V_0e^{-r^2}$$



Here $r_1 < r_2$

These equilibrium points are corresponding to circular orbits of radius r_1 and r_2 respectively.

Correct option is (a), (b)

25.**Solution:** $\{H,f\}=0$, $\{H,g\}=0$

(a) $\{H, f+g\} = \{H, f\} + \{H, g\} = 0$

(b) $\{H, \{f, g\}\} = -\{f, \{g, H\}\} - \{g, \{H, f\}\} = -\{f, 0\} - \{g, 0\} = 0$

Here, Jacobi Identity is used.

(c) $\{H+f, g\} = \{H, g\} + \{f, g\} = \{f, g\}$

(d) $\{H, H+fg\} = \{H, H\} + \{H, fg\} = \{H, f\}g + f\{H, g\} = 0$

Correct option is(a), (b), (d)

26.**Solution:** $H = 4I_1I_2$

$$\dot{\theta}_1 = \frac{\partial H}{\partial I_1} = 4I_2, \qquad \dot{\theta}_2 = \frac{\partial H}{\partial I_2} = 4I_1, \qquad \frac{d\theta_1/dt}{d\theta_2/dt} = \frac{4I_2}{4I_1} = 2, \qquad \frac{d\theta_2}{d\theta_1} = \frac{1}{2}$$

Slope of $\theta_1 - \theta_2$ curve $= \frac{1}{2}$

Correct option (b), (c)

27.**Solution:** $v_{AE} = 0.5c,\ \ v_{BE} = -0.5c$

$$v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE}v_{BE}}{c^2}} = \frac{0.5c - (-0.5c)}{1 - \frac{(0.5c)(-0.5c)}{c^2}}, \qquad v_{AB} = \frac{c}{1 + 0.25} = \frac{4c}{5}$$

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v_{AB}^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1 - \frac{16}{25}}}, \qquad E = \frac{5}{3}m_0c^2 = \alpha m_0c^2, \qquad \alpha = \frac{5}{3} = 1.67$$

Correct Answer:1.65 to 1.70

28.**Solution:** $F = U - TS,\ dF = -SdT - PdV$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V, \qquad F = U + T\left(\frac{\partial F}{\partial T}\right)_V, \qquad U = F - T\left(\frac{\partial F}{\partial T}\right)_V$$

Now $\left[\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right]_V = \frac{1}{T}\left(\frac{\partial F}{\partial T}\right)_V - \frac{F}{T^2} = -\frac{1}{T^2}\left[F - T\left(\frac{\partial F}{\partial T}\right)_V\right] U = F - T\left(\frac{\partial F}{\partial T}\right)_V = -T^2\left[\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right]_V$

29.**Solution:** In quantum mechanical treatment, single-dipole partition function is (see RK Patharia, Article 3.9)

$$Q_1(\beta) = \frac{\sinh\left\{\left(1 + \frac{1}{2J}\right)x\right\}}{\sinh\left(\frac{1}{2J}x\right)}$$

Where $x = \beta(g\mu_B J)B$. The mean magnetic moment of the system is then given by

$$M_z = N\langle\mu_z\rangle = \frac{N}{\beta}\frac{\partial \ln\ Q_1(\beta)}{\partial\beta} = Ng\mu_B J B_j(x), \qquad \frac{M_z}{N} = \langle\mu_z\rangle = g\mu_B J B_j(x)$$

Where $B_J(x)$ is the Brillouin function. For

$x <<< 1$, i.e., $\frac{\mu_B B}{k_B T} <<< 1$

$$B_j(x) \approx \frac{1}{3}\left(1 + \frac{1}{J}\right)x + \cdots$$

Therefore,

$$\langle \mu_z \rangle = \frac{(g\mu_B J)^2}{3k_B T}\left(1+\frac{1}{J}\right)B = g^2\frac{2}{3}\mu_B\left(\frac{\mu_B B}{k_B T}\right)$$

Where, last term is written for J = 1

Correct option is (a)

30.**Solution:** Here initial temperature (T_i) of water is 300 K& final temperature (T_f) is 320 K.

Process1: $\Delta S_{\text{Water}} = C_W \ln\left(\frac{T_f}{T_i}\right) = C_W \ln\left(\frac{320}{300}\right) = 0.06454 C_W$

$$\Delta S_{\text{reseservoir}} = \frac{-C_W \Delta T}{320} = -\frac{C_W(320-300)}{320} = -0.0625 C_W$$

$$\Delta S_{\text{Uniucesse}} = \Delta S_1 = \Delta S_{\text{water}} + \Delta S_{\text{reservoir}} = 0.06454 C_W - 0.0625 C_W = 0.00204 C_W$$

Process-2: ΔS_{water} will be same as initial and final equilibrium states are same.

$$\therefore \Delta S_{\text{water}} = C_W \ln\frac{320}{300} = 0.06454 C_W$$

$$\Delta S_{\text{reservoir}} = -C_W\left[\frac{10}{310}+\frac{10}{320}\right] = -C_W[0.03226+0.03125] = -0.06351 C_W$$

$$\Delta S_2 = \Delta S_{\text{water}} + \Delta S_{\text{reservoir}} = 0.00103 C_W$$

Process 3: $\Delta S_{\text{water}} = 0.06454 C_W$

$$\Delta S_{\text{reseruor}} = -C_W\left[\frac{50}{350}\right] + C_W\left(\frac{30}{320}\right) = -0.142857 C_W + 0.09375 C_W$$

$$= -0.04911 C_W$$

$$\Delta S_3 = 0.06454 C_W - 0.04911 C_W = 0.01543 C_W$$

$$\therefore \Delta S_3 > \Delta S_1 > \Delta S_2$$

Correct option (b)

31.**Solution:** Let initial pressure is $P_1 = P$

Initial volume is V

When pressure changes slightly by ΔP, i.e piston is pushed in side, the volume is reduced by ΔV.

Further given that no heat exchange is there

$$PV^\gamma = (P+\Delta P)(V-\Delta V)^\gamma = P\left[1+\frac{\Delta P}{P}\right]V^\gamma\left[1-\frac{\Delta V}{V}\right]^\gamma$$

$$= PV^\gamma\left[1+\frac{\Delta P}{P}\right]\left[1-\gamma\frac{\Delta V}{V}+\cdots.\right] = PV^\gamma\left[1+\frac{\Delta P}{P}-\gamma\frac{\Delta V}{V}-\gamma\frac{\Delta P\Delta V}{PV}\right]$$

As $\Delta P \& \Delta V$ are using small, $\Delta P\Delta V$ can be neglected.

$$\therefore 1 = \left[1+\frac{\Delta P}{P}-\gamma\frac{\Delta V}{V}\right], \qquad \Delta P = \gamma P\frac{\Delta V}{V}$$

Applied external force F, that caused a displacement x (a volume change of gas by $\Delta V = Ax$) is given by

$$F = -A\Delta P = -\gamma AP\frac{\Delta V}{V} = \frac{-\gamma A^2 P}{V}x$$

Acceleration produced in piston is

$$a = \frac{F}{m} = -\frac{\gamma A^2 P}{mV} x$$

$$a \propto x$$

Therefore $\omega = \sqrt{\frac{\gamma A^2 P}{mV}}$

$$\omega \propto \sqrt{\frac{\mathrm{P}\gamma}{mV}}$$

Correct option (b)

32.**Solution:** The magnetic interaction energy $= -M.dH$

$$dU = TdS - M \cdot dH, \quad dG - SdT - M.dH$$

$$S = -\left(\frac{\partial G}{\partial T}\right)_H, M = -\left(\frac{\partial G}{\partial H}\right), \qquad \left(\frac{\partial M}{\partial H}\right)_H = \left(\frac{\partial S}{\partial H}\right)_T$$

Now $\left(\frac{\partial T}{\partial H}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial S}{\partial H}\right)_T = -\left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial M}{\partial T}\right)_H = \frac{-\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H}$

∴ (5) implies

$$\Delta T = -\frac{\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H} \Delta H$$

This is indeed the process of adiabatic unitization.

Where negative ΔH implies negative $\Delta T \& \Delta S = 0$.

Correct option is (b), (c)

33.**Solution:**

Solution: We ignore the electrons in the [Mg] core and the electrons in the 4 s block as well. We have to consider only the p electron, ($l_1 = 1$) and d electron ($l_2 = 2$). Thus, total orbital angular momentum $\mathrm{L} = |l_1 + l_2| \ldots |l_1 - l_2| = 3,2,1$ i.e P, D and F

Correct option is (b)

34.**Solution:**$p(t) = qx(t) = qA\omega \& \omega t$, $\dot{p} = -qA\omega \sin\ \omega t \Rightarrow \ddot{p} = -qA\omega^2 \cos^2\ \omega t$, $\langle P \rangle \propto (\ddot{p})^2 \propto q^2 A^2 \omega^4$

Correct option is (d)

35.**Solution:** For the normal Zeeman effect, π-lines:

No change in magnetic quantum number → $\Delta m = 0$

Plane polarized

Observed parallel (II) to the magnetic field σ-lines (σ^+, σ^-):$\Delta m = \pm 1$

Circularly polarized when seen || to the field

Plane polarized when seen perpendicular (⊥) to the field

Now check each statement:

(a) Only π-lines are observed || to the field - FALSE

Along the field direction, σ-lines are observed, π-lines are not seen.

(b) σ-lines ⊥ to the field are plane polarized - TRUE

When observed perpendicular to the field, σ-lines are linearly (plane) polarized.

(c) π-lines ⊥ to the field are plane polarized - TRUE

π-lines are always plane polarized and are visible perpendicular to the field.

(d) Only σ-lines are observed || to the field - TRUE

Along the field direction, only $\sigma^+$and σ^-appear.

Correct option is (b), (c) (d)

36.**Solution:** Temperature dependence of intensity of Stoke and Anti-stoke lines are given by:

$$I_S \propto \frac{1}{1-e^{-\frac{h\nu_j}{k_BT}}}, \qquad I_{AS} \propto \frac{1}{e^{\frac{h\nu_j}{k_BT}-1}}$$

where, ν_j is the frequency shift of the Raman line, taking the ratio of the two intensities,

$$\frac{I_S}{I_{AS}} = \frac{\frac{1}{1-e^{-\frac{h\nu_j}{k_BT}}}}{\frac{1}{\frac{h\nu_j}{k_BT}}-1} = \frac{\frac{e^{h\nu}}{k_BT}-1}{1-e^{-\frac{h\nu_j}{k_BT}}} \Rightarrow \frac{h\nu_j}{k_BT} = \ln\left[\frac{I_S}{I_{AS}}\right]$$

$$T = \frac{hc\bar{\nu}}{k_B\ln\left[\frac{I_S}{I_{AS}}\right]} = \frac{6.626\times10^{-34}\text{JS}\times3\times10^8\ \text{m/s}\times300\times100\ \text{m}^{-1}}{1.38\times10^{-23}\ \text{J/K}\times\ln\ 4}$$

$$= 31.17\times10^1\ \text{K} = 311.7\ \text{K}$$

Hence, T = 311 to 312

Correct Answer: 311 to 312

37.**Solution:**

$$\omega_p = \sqrt{\frac{n_0e^2}{\epsilon_0 m}} \Rightarrow 3\times10^{12} = \sqrt{\frac{n_0\times(1.6\times10^{-19})^2}{8.85\times10^{-12}\times9.1\times10^{-31}}}$$

$$n_0 = \frac{9\times10^{24}\times8.85\times10^{-12}\times9.1\times10^{-31}}{2.56\times10^{-38}} = 283.1\times10^{19}\ \text{m}^{-3}$$

$$n_0 = 2.83\times10^{21}\ \text{m}^{-3}, \quad \simeq 2.83\times10^{21}\ \text{m}^{-3}$$

Correct Answer: 2.70 to 2.96

38.**Solution:** Electronic energy E_e

Set by electron mass m and electronic motion → atomic scale.

Vibrational energy E_v

$$E_v \sim \hbar\omega \sim \hbar\sqrt{\frac{k}{M}}$$

The spring constant k comes from electronic binding, so

$$k \sim \frac{E_e}{a^2}$$

Hence, $E_v \sim \sqrt{\frac{m}{M}} E_e$

Rotational energy E_R

$$E_R \sim \frac{\hbar^2}{2I}, I \sim Ma^2$$

Using a set by electronic scale,

$$E_R \sim \frac{m}{M} E_e$$

Check each option

(a)$E_R \sim \frac{m}{M} E_e$

TRUE

(b)$E_R \sim \sqrt{\frac{m}{M}} E_e$

X FALSE (this scaling is for vibration, not rotation)

(c)$E_v \sim \sqrt{\frac{m}{M}} E_e$

TRUE

(d)$E_v \sim \left(\frac{m}{M}\right)^{1/4} E_e$

FALSE

Correct option is (a), (c)

39.**Solution:** We know, $\vec{j} = \vec{l} + \vec{s} \Rightarrow j^2 + s^2 + 2(\vec{l} \cdot \vec{s}) \therefore \vec{l} \cdot \vec{s} = \frac{1}{2}(j^2 - l^2 - s^2)$

Thus, $H_{so}|\psi\rangle = E|\psi\rangle = V_{so}\frac{\hbar^2}{2}[j(j+1) - l(l+1) - s(s+1)]$

For $p_{1/2}$: $s = 1/2; l = 1; j = 1/2$

$$E_1 = V_{so}\frac{\hbar^2}{2} \times \left[\frac{3}{4} - 2 - \frac{3}{4}\right] = V_{so}\frac{\hbar^2}{2} \times -2 = -V_{so}\hbar^2$$

For $p_{3/2}$: $s = 1/2; l = 1; j = 3/2$

$$E_2 = V_{so}\frac{\hbar^2}{2} \times \left[\frac{15}{4} - 2 - \frac{3}{4}\right] = V_{so}\frac{\hbar^2}{2} \times \frac{4}{4} = V_{so}\frac{\hbar^2}{2}$$

Thus, the energy difference

$$\Delta E = E_2 - E_1 = V_{so}\frac{3\hbar^2}{2}$$

Correct Answer: 3 to 3

40.**Solution:**

$$S_1 = \int_{300}^{77} \frac{d\theta}{T} = \int_{300}^{77} \frac{C_P dT}{T} = C_p \ln\left(\frac{77}{300}\right) = 7\ln\frac{77}{300} = -9.519839\text{CalK}^{-1}$$

$$\Delta S_2 = \frac{d\theta}{T} = \frac{L}{T} = \frac{1293.6}{77} = -16.8\text{Cal K}^{-1}$$

$$\frac{\Delta S_I}{\Delta S_{II}} = \frac{9.519839}{16.8} = 0.5666$$

$$\approx 0.6$$

Correct Answer: 0.5 to 0.7

41.**Solution:** Frequency bandwidth Δv of a He-Ne laser is given by,

$$\Delta v = \frac{2v}{c}\sqrt{\frac{\alpha}{A}} = \frac{2}{\lambda}\sqrt{\frac{\alpha}{A}} = \frac{2}{633\times10^{-9}}\sqrt{\frac{3.44\times10^6}{20}} = \frac{2\times414.73}{633}\times10^9 \sim 1.3\times10^9\text{ Hz}$$

The lasing atom is Ne for which atomic mass is 20amu.

Correct Answer; 1.2 to 1.4

42.**Solution:** Check by options (1) $\Delta I = 0, \Delta\pi = \text{No}$

Allowed β-decay

(2) $\Delta I = 2, \Delta\pi = \text{YES}$

First forbidden β - decay

(3) $\Delta I = 3, \Delta\pi = \text{No}$

Second forbidden β - decay

(4) $\Delta I = 4, \Delta\pi = \text{YES}$

Third forbidden β – decay

Correct option is (b)

43.**Solution:**

$$E = \left(n_x^2 + n_y^2 + n_z^2\right)\frac{\pi^2\hbar^2}{2mL^2}$$

n_x	n_y	n_z
(1	1	1)
(1	1	2)
(2	1	1)

44.**Solution:** Given, $n_e = 1.4\times10^{28}\text{ m}^{-3}, D(E_F) = 6.2\times10^{46}\text{ J m}^{-3}, \mu_B = 9.3\times10^{-24}$ Joule T^{-1},

$$\mu_0 = 4\pi\times10^{-7}\text{Tm } A^{-1}$$

We know that, $\chi_{\text{Pauli}} = \mu_0\mu_B^2 D(E_F)$

$$\chi_{\text{Pauli}} = \mu_0\mu_B^2 D(E_F) = 4\pi\times10^{-7}\times(9.3\times10^{-24})^2\times6.2\times10^{46}$$

$$\chi_{\text{Pauli}} = 4\pi\times(9.3)^2\times6.2\times10^{-7-48+46} = 6.735\times10^{-6}$$

$$k = 6$$

Correct Answer: 6 to 6

45.**Solution:** Density of states $D(E) \propto E^{\left(\frac{d}{s}-1\right)}$

From question $d = 2$ and $s = 1$

So, $D(E) \propto E^{\left(\frac{2}{1}-1\right)} \Rightarrow D(E) \propto E = \hbar\omega \Rightarrow D(E) \propto \omega$

Correct option is (a)

46.**Solution:** Correct option is (a)

47.**Solution:** (a) $\gamma \propto m_{\text{eff}}$

TRUE

Larger effective mass $\Rightarrow$ larger density of states at $E_F \Rightarrow$ largerγ.

(b) m_{eff} is greater than free electron mass for all solids

FALSE

m_{eff} depends on band curvature

Can be greater than, less than, or even negative

Example: semiconductors often have $m_{\text{eff}} < m$

(c) Temperature dependence of C depends on the dimensionality of the solid

FALSE (for electronic part)

Electronic specific heat:$C_{\text{el}} \propto T$ in 1D, 2D, and 3D (only prefactor changes, not power of T)

Dimensionality affects phonons, not electronic T-dependence

(d) Linear temperature dependence of C is observed at $T \ll \Theta_D$

TRUE

At low T :

Electronic: $C_{\text{el}} \sim T$

Phonon: $C_{\text{ph}} \sim T^3$

Hence linear term is observable only when $T \ll \Theta_D$

Correct option is (a), (d)

48.**Solution:** Hall voltage depends on the mobility of the carriers

Correct option is (d)

49.**Solution:** Correct option is (a), (c)

50.**Solution:** Bragg's Law

$$2d_{hkl}\sin\,\theta = n\lambda \Rightarrow \sin\,\theta = \frac{\lambda}{2d_{hkl}} = \frac{\lambda}{2a}\sqrt{h^2+k^2+l^2}$$

Corresponding to maximum value of sin $\theta(=1)$, the expression of $\sqrt{h^2+k^2+l^2}$ has maximum values for the sin θ. From this condition we can find out the value of lattice parameter (a) from the peak corresponding to (222) plane. So

$$1 = \frac{1.54}{2a}\sqrt{2^2+2^2+2^2} \Rightarrow a = 2.66\text{Å}$$

For $\lambda = 3\mathring{A}$, Bragg's Law

$$\text{Sin } \theta = \frac{\lambda}{2a}\sqrt{h^2+k^2+l^2} = \frac{3}{2\times 2.66}\sqrt{h^2+k^2+l^2}$$

For Peak (111) $\text{Sin } \theta = \frac{3}{2\times 2.66}\sqrt{1^2+1^2+1^2} = \frac{3}{2\times 2.66}\sqrt{3} = 0.976$

Peak (200) $\text{Sin } \theta = \frac{3}{2\times 2.66}\sqrt{2^2+0+0} = \frac{3}{2\times 2.66}\sqrt{4} = 1.27$

Peak (220) $\text{Sin } \theta = \frac{3}{2\times 2.66}\sqrt{2^2+2^2+0} = \frac{3}{2\times 2.66}\sqrt{8} = 1.657$

Peak (310) $\text{Sin } \theta = \frac{3}{2\times 2.66}\sqrt{3^2+1^2+0} = \frac{3}{2\times 2.66}\sqrt{10} = 1.85$

Peak (222) $\text{Sin } \theta = \frac{3}{2\times 2.66}\sqrt{2^2+2^2+2^2} = \frac{3}{2\times 2.66}\sqrt{12} = 2.029$

The maximum value of sin θ will be 1 . So for wavelength $\lambda = 3\mathring{A}$ only (111) peak observed.

Correct Answer is: 1 to 1

51.**Solution:** Given circuit is an Schmitt Trigger circuit. In this output will be always saturated

Correct option is (a)

52.**Solution:** (a) Closed loop gain < Open loop gain - TRUE

Negative feedback always reduces gain. The open-loop gain A is very large, while the closed-loop gain is fixed by resistors and is much smaller.

(b) Closed loop bandwidth < Open loop bandwidth - FALSE

Negative feedback increases bandwidth.

So, closed-loop bandwidth > open-loop bandwidth.

(c) Closed loop input impedance > Open loop input impedance - TRUE

For a non-inverting amplifier, negative feedback increases input impedance by a factor $(1 + A\beta)$.

(d) Closed loop output impedance < Open loop output impedance - TRUE

Negative feedback reduces output impedance by the same factor $(1 + A\beta)$.

Correct option is (a), (c), (d)

53.**Solution:**(a) Doping concentration of emitter region is more than that in collector and base region

TRUE

Emitter: heavily doped → injects maximum carriers

Base: lightly doped → reduces recombination

Collector: moderately doped → withstands high reverse voltage

Order:

Emitter > Collector > Base

(b) Only electrons participate in current conduction

FALSE

BJT is bipolar → both electrons and holes participate

Even in NPN, holes contribute to base current

This is the key difference from FETs (unipolar)

(c) The current gain β depends on temperature

TRUE

$$\beta = \frac{I_C}{I_B}$$

Carrier mobility, recombination, and leakage currents are temperature dependent

In general, β increases with temperature

(d) Collector current is less than the emitter current

TRUE

$$I_E = I_C + I_B \Rightarrow I_C < I_E$$

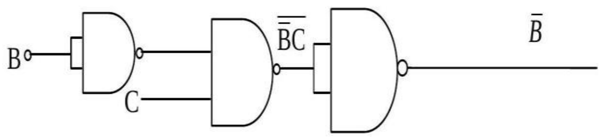
Correct option is (a), (c), (d)

54.**Solution:**

$$V_L = \frac{R_L}{R_L + R_S} \times 5V = 4V \Rightarrow 5R_L = 4R_L + 4R_S \Rightarrow \frac{R_L}{R_S} = 4$$

Correct Answer: 4 to 4

55.**Solution:** $Y = A\bar{B}(C + BD)C + \bar{A}\bar{B}C + A\bar{B}BDC + \bar{A}\bar{B}C = A\bar{B}C + \bar{A}\bar{B}C = (A + \bar{A})\bar{B}C = \bar{B}$



Correct Answer: 3 to 3