



**CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR**

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## **Question Paper**

**CSIR-NET DEC 2025**

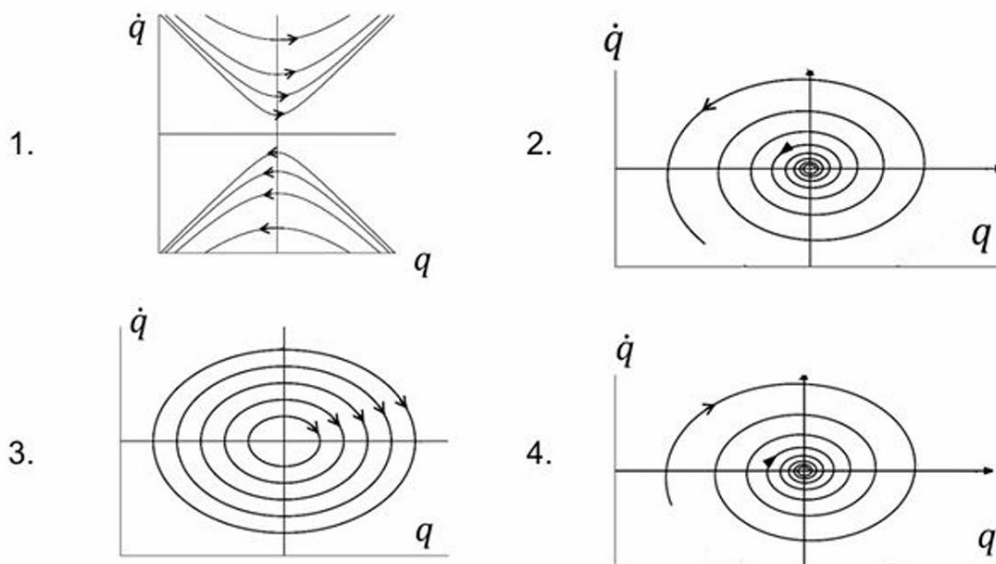
With Answer Key

**PHYSICAL SCIENCE**

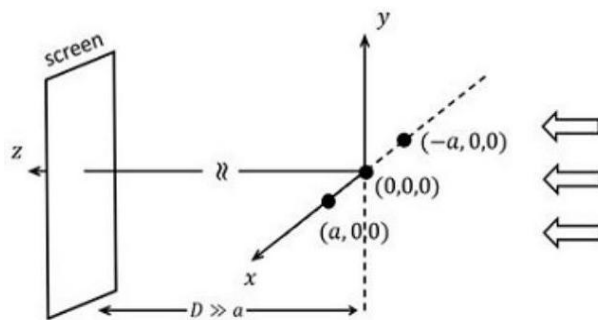
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**Section: PART-B**

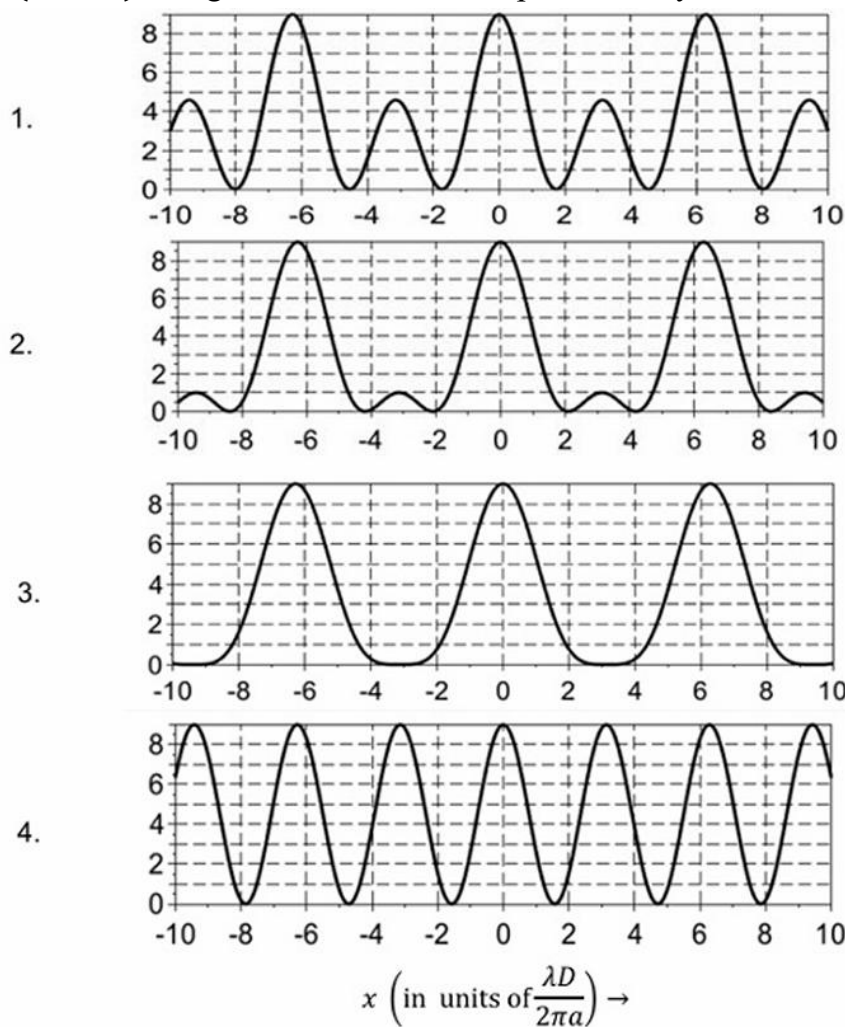
- Let  $p(x)$  be the probability density function for a positive real variable  $x$ , and  $g(\alpha) = \int_0^\infty p(x)e^{-\alpha x}dx$ . If  $g'(\alpha)$  and  $g''(\alpha)$  are respectively first and second derivatives of  $g(\alpha)$  with respect to  $\alpha$ , which of the following gives the variance of  $x$  ?
  - $g''(0) - [g'(0)]^2$
  - $g''(0) + [g'(0)]^2$
  - $[g''(0) - g'(0)]^2$
  - $\frac{g''(0)}{g'(0)g(0)}$
- $B, C$  and  $F$  are three systems which have particles of same mass and same number density kept at the same low temperature  $T$ . Here  $C$  is a classical ideal gas,  $F$  is a free Fermi gas and  $B$  is a free Bose gas, with pressures  $P_C, P_F$  and  $P_B$  respectively. Then
  - $P_B > P_C > P_F$
  - $P_F > P_C > P_B$
  - $P_C > P_F > P_B$
  - $P_C > P_B > P_F$
- Which of the following figures best represents the motion of an oscillator described by the differential equation  $\ddot{q} + \dot{q} + q = 0$  in  $q - \dot{q}$  plane?



- Three identical pinholes separated by distance  $a$  along the  $x$ -axis are illuminated by a collimated monochromatic coherent beam of light (wavelength  $\lambda$ ) as shown in the figure below.



The intensity (in arbitrary units) pattern of fringes obtained on a screen kept at distance  $D$  ( $D \gg a$ ) along the  $z$ -axis is best represented by



5. An isolated two-electron quantum state is described by the orbital angular momentum quantum number  $l$  and the total spin  $S$ . An allowed value of  $l$  and  $S$  is

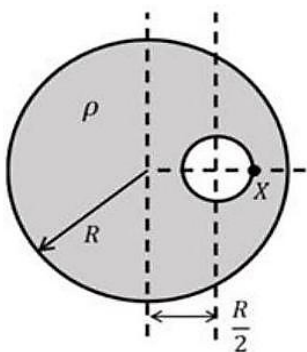
(a)  $S = 1, l = 0$

(b)  $S = 0, l = 1$

(c)  $S = 1, l = 1$

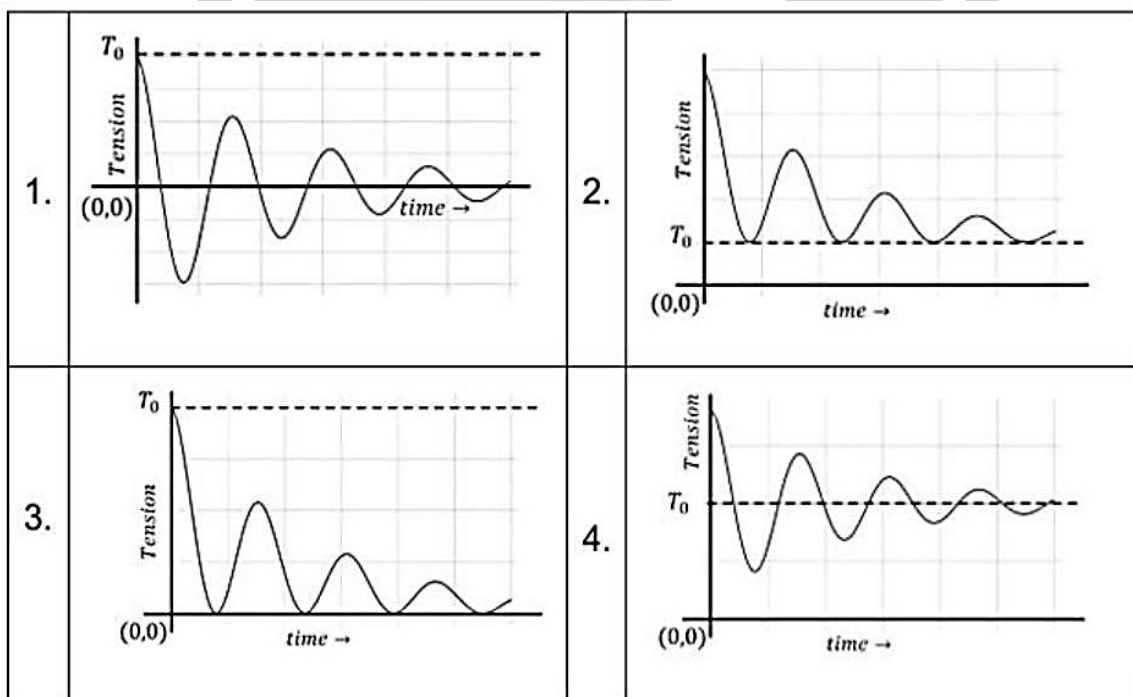
(d)  $S = 1, l = 2$

6. A solid sphere of radius  $R$  has uniform charge density  $\rho$ . A spherical volume of radius  $\frac{R}{4}$  is scooped out from the sphere as shown. The electric field at the point marked  $X$  is ( $\hat{r}$  denotes the unit vector along the radially outward direction)



- (a)  $\frac{2\rho R}{9\epsilon_0} \hat{r}$  (b)  $\frac{\rho R}{6\epsilon_0} \hat{r}$   
 (c)  $\frac{\rho R}{3\epsilon_0} \hat{r}$  (d)  $\frac{\rho R}{9\epsilon_0} \hat{r}$

7. A bow has a taut string of tension  $T_0$  (when it is at rest). The string is pulled and released at time  $t = 0$ . Which plot best represents the tension in the bow string as a function of time?



8. If  $C$  be the unit circle traversed clockwise, then the integral  $\oint_C dz |1 + 2z|^2$  equals

- (a)  $-4\pi i$  (b)  $-\pi i$   
 (c) 0 (d)  $-2\pi i$

9. A 1-dimensional random walker's displacement is always positive and is equally likely to be anywhere in the range  $[L, L + b]$ . After  $N$  statistically independent steps the mean distance covered by the walker is

- (a)  $NL$  (b)  $N\sqrt{L^2 + b^2}$   
 (c)  $N\left(L + \frac{b}{2}\right)$  (d)  $NL + b\sqrt{N}$

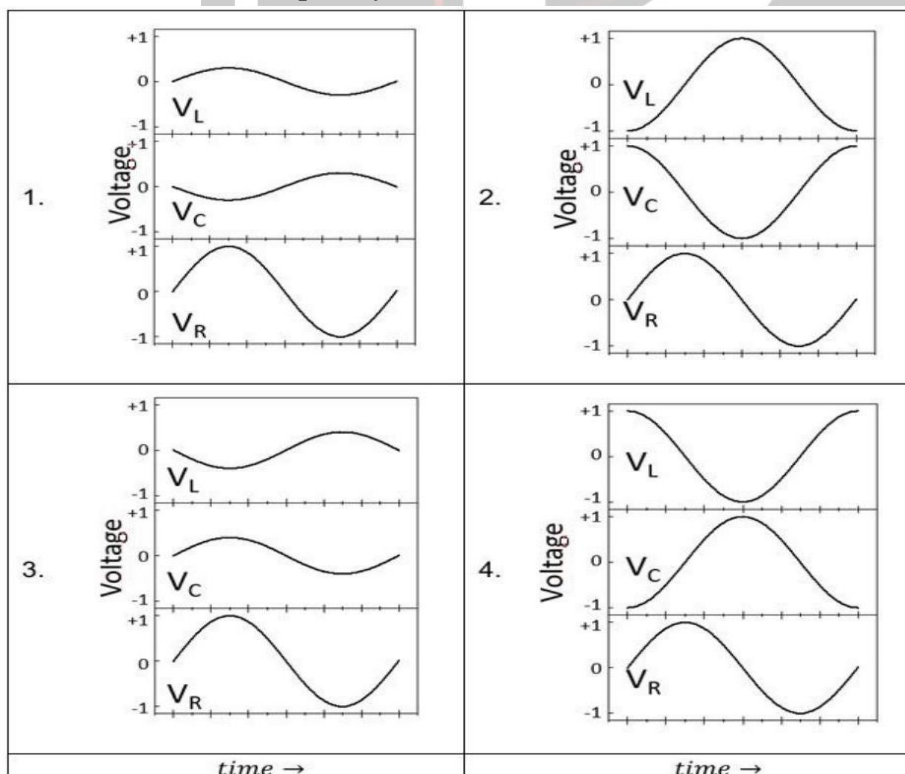
10. If  $\hat{L}$  is the angular momentum operator for a quantum particle, then

- $\hat{L} \times \hat{L}$  is  
 (a)  $\hbar^2$  (b)  $-i\hbar\hat{L}$   
 (c) 0 (d)  $i\hbar\hat{L}$

11. Commutator of two matrices  $A$  and  $B$  is defined as  $[A, B] = AB - BA$  and the anti-commutator as  $\{A, B\} = AB + BA$ . If  $\{A, B\} = 0$ . Then we can express  $[A, BC]$  as

- (a)  $B\{A, C\}$  (b)  $-B[A, C]$   
 (c)  $-B\{A, C\}$  (d)  $[A, B]C$

12. In an ideal series  $LCR$  circuit, which one of the following best represents the steady-state voltage waveforms  $V_L, V_C, V_R$  (only one cycle is shown) across  $L, C$  and  $R$  as a function of time at resonance frequency



13. Five indistinguishable atoms are sitting on the distinguishable vertices of a pentagon. The atoms can be in one of the two states:  $g$  with energy 0, and  $e$  with energy  $E$ . However neighbouring atoms cannot both be in the  $e$  state. The partition function of this system at temperature  $T$ , is

(a)  $1 + 5e^{-\frac{E}{k_B T}} + 2e^{-\frac{2E}{k_B T}}$  (b)  $1 + 5e^{-\frac{E}{k_B T}} + 3e^{-\frac{2E}{k_B T}}$   
 (c)  $1 + 5e^{-\frac{E}{k_B T}} + 10e^{-\frac{2E}{k_B T}}$  (d)  $1 + 5e^{-\frac{E}{k_B T}} + 5e^{-\frac{2E}{k_B T}}$

14. The position and velocity vector of a particle changes from  $\vec{R}_1$  to  $\vec{R}_2$  and  $\vec{V}_1$  to  $\vec{V}_2$  as time changes from  $t_1$  to  $t_2$ . If  $\vec{r}(t)$ ,  $\vec{a}(t)$  are instantaneous position and acceleration vectors of the particle then the integral

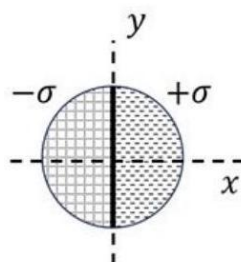
$I = \int_{t_1}^{t_2} dt(\vec{r}(t) \times \vec{a}(t))$  is

(a)  $\vec{R}_2 \times \vec{V}_1 - \vec{R}_1 \times \vec{V}_2$  (b)  $\vec{R}_2 \times \vec{V}_2 - \vec{R}_1 \times \vec{V}_1$   
 (c)  $\vec{R}_1 \times \vec{V}_1 - \vec{R}_2 \times \vec{V}_2$  (d)  $\vec{R}_1 \times \vec{V}_2 - \vec{R}_2 \times \vec{V}_1$

15. A fly of mass  $m$  rests on the edge of a uniform horizontal disc of radius  $R$  and mass  $M$ . The disc is free to rotate about the vertical axis through its centre. Initially the disc is stationary. The fly starts to walk around the circumference of the disc with speed  $v$  relative to the disc. The speed of the fly for a stationary observer is

(a)  $\frac{mv}{M + 2m}$  (b)  $\frac{Mv}{M - 2m}$   
 (c)  $\frac{Mv}{M + 2m}$  (d)  $\frac{mv}{M - 2m}$

16. A circular disc of radius  $R$  is made of 2 halves (as shown in the figure), separated by a dielectric of negligible thickness (along the  $y$  axis.)



If the surface charge density on the right half is  $+\sigma$  and that on the left half is  $-\sigma$ , the dipole moment of the disc is

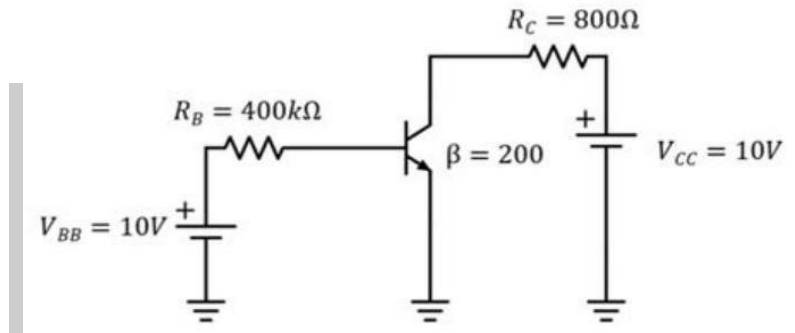
(a)  $P_x = 0, P_y = \frac{1}{3}\sigma R^3$

(b)  $P_x = 0, P_y = \frac{4}{3}\sigma R^3$

(c)  $P_x = \frac{1}{3}\sigma R^3, P_y = 0$

(d)  $P_x = \frac{4}{3}\sigma R^3, P_y = 0$

17. In the transistor circuit given below the voltage  $V_{CC}$  fluctuates by 5%. Then the fluctuation in  $V_{CE}$  would be closest to (take  $V_{BE} = 0.7 \text{ V}$ )



(a) 8%

(b) 7%

(c) 6%

(d) 5%

18. A spin- $\frac{1}{2}$  particle is in a magnetic field  $\vec{B} = B_x\hat{x} + B_y\hat{y}$  for which the spin-independent Hamiltonian is  $\hat{H} = -A\hat{S} \cdot \vec{B}$  ( $A$  is a positive constant and  $\hat{S}$  is the spin-operator). The eigenvalues of the Hamiltonian are

(a)  $\pm A\frac{\hbar}{2}(B_x + B_y)$

(b)  $\pm A\frac{\hbar}{2}\sqrt{B_x B_y}$

(c)  $\pm A\frac{\hbar}{2}(B_x^2 + B_y^2)^{\frac{1}{2}}$

(d) 0

19. A classical mono-atomic ideal gas is in thermal equilibrium at temperature  $T$ . The velocity of a molecule of this gas, of mass  $m$ , is  $\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$ . The value of the ensemble average  $\langle v_x^2 v_y^2 \rangle$  is

(a)  $\left(\frac{k_B T}{2m}\right)^2$

(b)  $\left(\frac{k_B T}{m}\right)^2$

(c)  $\left(\frac{3k_B T}{2m}\right)^2$

(d)  $\left(\frac{2k_B T}{m}\right)^2$

20. Two well separated conducting spheres (  $A$  and  $B$  ) of radii 10 cm and 20 cm carry charges +30 C and -20 C respectively. When they are connected by a thin conducting wire, the final charge on  $A$  is  $Q_A$  and that on  $B$  is  $Q_B$ . The values of  $Q_A$  and  $Q_B$  respectively, are closest to

(a) 6.7 C and 3.3 C

(b) 2.0 C and 8.0 C

(c) 3.3 C and 6.7 C

(d) 8.0 C and 2.0 C

21. The residue of  $f(z) = \frac{\cos \pi z}{(1-z^2)^3}$  at  $z = 1$  is

(a)  $\frac{\pi^2}{16}$

(b)  $\frac{3}{16}$

(c)  $\frac{3 + \pi^2}{16}$

(d)  $\frac{3 - \pi^2}{16}$

22. A quantum particle of mass  $m$  is moving in a potential

$$V(x, y) = \frac{m\omega^2}{8} [5(x^2 + y^2) + 8xy]$$

The lowest energy eigenstate with degeneracy has an energy

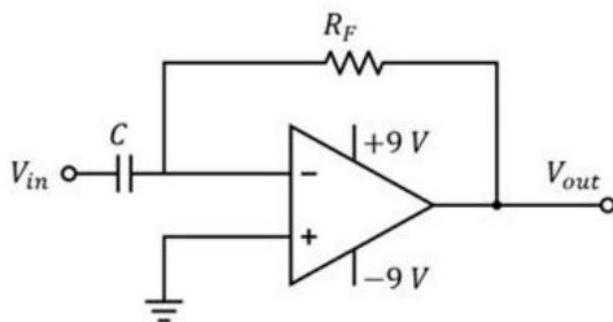
(a)  $\frac{7}{2} \hbar \omega$

(b)  $\frac{3}{2} \hbar \omega$

(c)  $4 \hbar \omega$

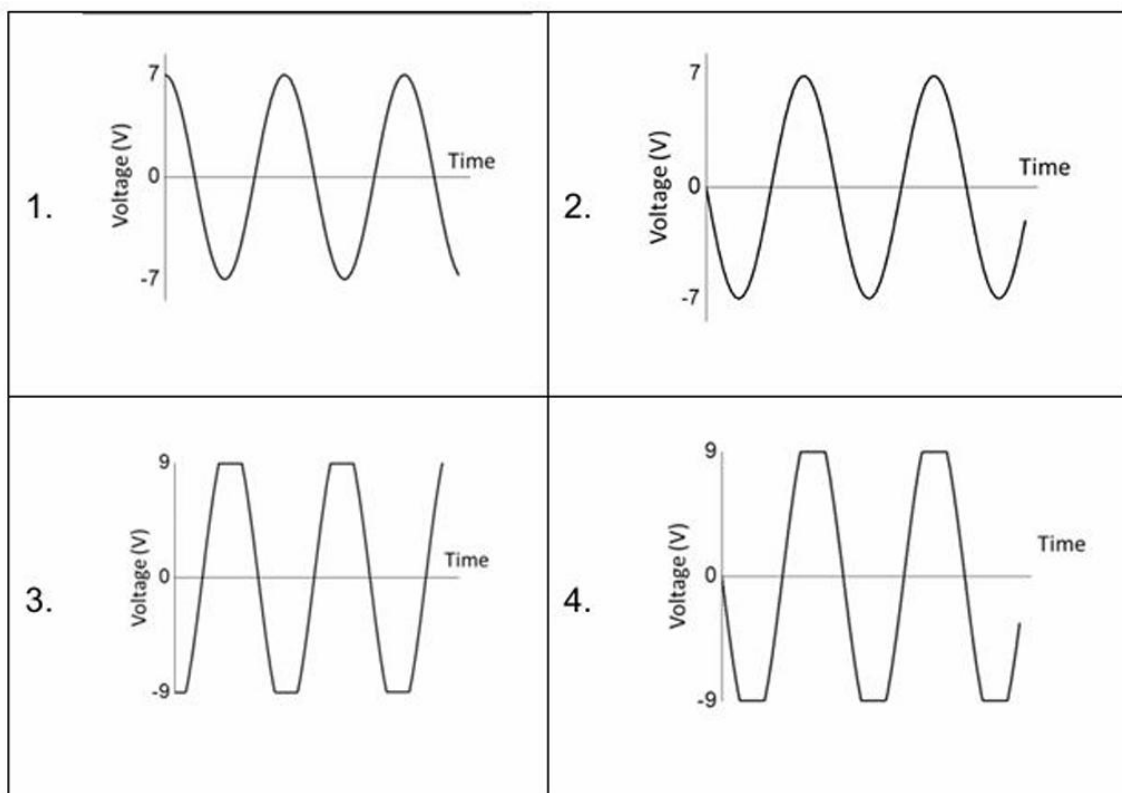
(d)  $\frac{5}{2} \hbar \omega$

23. In the circuit shown below, the input voltage is  $V_{in}(t) = 0.3 \sin 50t$  (Volts) and  $C = 100 \mu F$ ,  $R_F = 10 k\Omega$ .





Considering the opamp to be ideal and neglecting the transients, the best representation of the output voltage  $V_{\text{out}}(t)$  is



24. A fraction  $\frac{2}{3}$  of the volume of a parallel plate capacitor is filled with dielectric of relative permittivity  $\kappa = 1.5$  (as shown in the figure).

$$\kappa = 1.5$$

When the filled volume is reduced to  $\frac{1}{3}$  of the total volume, the capacitance is smaller by a factor of

(a)  $\frac{7}{8}$

(b)  $\frac{5}{6}$

(c)  $\frac{3}{4}$

(d)  $\frac{2}{3}$

25. A quantum mechanical particle in a harmonic potential has the wave function

$$\frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$$

at  $t = 0$ , where  $\psi_0(x)$  and  $\psi_1(x)$  are the wave functions of the ground state and the first excited state respectively. If the frequency of the oscillator is  $\omega$ , the probability density of finding the particle at  $x$  after time  $t = \frac{\pi}{\omega}$  is

(a)  $\frac{1}{2} |\psi_1(x) - \psi_0(x)|^2$

(b)  $\frac{1}{2} |\psi_1(x) + \psi_0(x)|^2$

(c)  $\frac{1}{2} |\psi_1(x) - i\psi_0(x)|^2$

(d)  $\frac{1}{2} |\psi_1(x)|^2 + \frac{1}{2} |\psi_0(x)|^2$

**Section: PART-C**

26. The bond dissociation energy of OH molecule is 4.18 eV with rotational constant  $18.8 \text{ cm}^{-1}$ . For rotational induced dissociation, the minimum value of rotational quantum number is closest to

(a) 114

(b) 454

(c) 45

(d) 90

27. A cubic sample of edge length  $L$  is maintained at a temperature of 4 K. The speed of sound in the material of the sample is  $5 \times 10^3 \text{ m/s}$ . The minimum value of  $L$  required to excite the lowest frequency phonon mode is closest to

(a) 10 nm

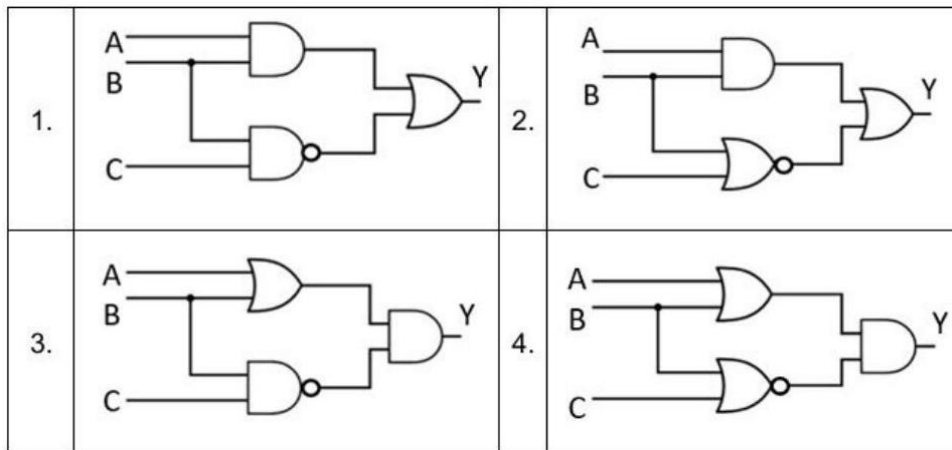
(b) 30 nm

(c) 20 nm

(d) 5 nm

28. The digital logic circuit that would give the following truth table

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



29. In a one-dimensional chain of atoms, the phonon energy dispersion is given by  $E = A|\sin ka|$ . Here,  $A$  is a constant,  $k$  is a vector in the reciprocal space and  $a$  is lattice spacing. The density of states is proportional to

(a)  $\frac{1}{\sqrt{A^2 - E^2}}$

(b)  $\frac{1}{\sqrt{A^2 + E^2}}$

(c)  $\frac{1}{\sqrt{A - E}}$

(d)  $\frac{1}{\sqrt{A + E}}$

30. Consider a one-dimensional lattice (with lattice spacing  $a$ ) along X-axis with sites labelled by  $x = 0, 1, 2, \dots, L$ . A particle carrying a charge  $-q$  can occupy any one of these sites. An electric field of strength  $E$  is applied in the positive x-direction. The average energy of the particle at a temperature  $T$  (in the limit  $L \rightarrow \infty$ ) is  $\left(\beta = \frac{1}{k_B T}\right)$

(a)  $\frac{Eq a}{e^{\beta Eq a} - 1}$

(b)  $\frac{Eq a}{1 + e^{\beta Eq a}}$

(c)  $\frac{Eq a}{2}$

(d)  $-Eq a$

31. A one-dimensional quantum harmonic oscillator with frequency  $\omega$  is in its ground state. Its normalised wave function is given by

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{2\hbar}x^2\right]$$

The frequency is suddenly increased to  $2\omega$ . The probability of finding the particle in its new ground state is

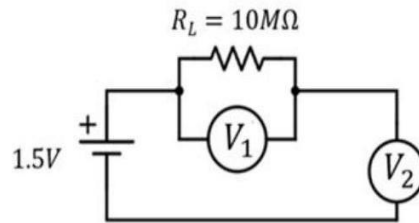
(a)  $\frac{2\sqrt{2}}{3}$

(b)  $\left(\frac{2\sqrt{2}}{3}\right)^{\frac{1}{2}}$

(c)  $\frac{2}{3}$

(d)  $\left(\frac{3}{2\sqrt{2}}\right)^{\frac{1}{2}}$

32. In the circuit shown below, the input impedance of voltmeters  $V_1$  and  $V_2$  are  $10\text{M}\Omega$ . If  $R_L = 10\text{M}\Omega$  and  $V_{\text{in}} = 1.5\text{ V}$ .



The measured voltages by  $V_1$  and  $V_2$  are closest to

- (a) 0.5 V and 1.0 V, respectively      (b) 0 V and 1.5 V, respectively  
(c) 1.5 V and 0 V, respectively      (d) 1.0 V and 0.5 V, respectively

33. In a heap of 20 biased coins, 17 have a 60% probability of showing heads while the other three special coins have a 90% probability of doing so. A coin is selected at random and tossed. If the result is a head, the probability that it was one of the three special coins is best approximated by

- (a) 0.18      (b) 0.14  
(c) 0.21      (d) 0.26

34. A thermistor measures an object's temperature  $T$ , by measuring its resistance  $R$  according to  $R = AT^{-n}$ , where  $A$  and  $n$  are positive constants. The observed resistances for different values of temperature (including environmental and instrumental sources of error) are

$T(\text{K})$	$R(\Omega)$
250	140
300	110
350	90

The estimated value of the exponent  $n$ , from the above data, is closest to

- (a) 2.0      (b) 0.8  
(c) 1.3      (d) 2.7

35. A monochromatic plane wave is incident normally from a dielectric medium  $A$  onto another dielectric medium  $B$ . The indices of refraction satisfy  $n_A < n_B$ . One-fourth of the incident energy is reflected back into medium  $A$ . Let  $\vec{E}$  be the resultant electric field due to the superposition of the incident wave and the reflected wave. Then, the ratio of the two timeaverages  $\langle \vec{E}^2 \rangle_{\min} / \langle \vec{E}^2 \rangle_{\max}$  is
- (a)  $\frac{1}{8}$  (b)  $\frac{1}{9}$   
 (c)  $\frac{4}{9}$  (d)  $\frac{1}{4}$
36. Consider an emission line of wave length  $\lambda = 550$  nm of Argon ( $A = 40, Z = 18$ ) at a temperature 400 K. The full Doppler width of the emission line will be closest to
- (a)  $10^{-2}$  nm (b)  $10^{-1}$  nm  
 (c)  $10^{-3}$  nm (d)  $10^{-5}$  nm
37. A spherical gaseous ball of radius 15 m was formed with a temperature  $T = 3 \times 10^5$  K. The gas expands adiabatically and its temperature drops to  $5 \times 10^3$  K. Given  $\gamma = \frac{5}{3}$  for this gas, the radius of the ball becomes approximately
- (a) 212 m (b) 86 m  
 (c) 137 m (d) 116 m
38. Consider the cross-sections  $\sigma_1 = \sigma(p + n \rightarrow \Delta^+ + n)$  and  $\sigma_2 = \sigma(p + n \rightarrow \Delta^0 + p)$  where the ( $\Delta^+, \Delta^0$ ) are part of the baryon decuplet. Then
- (a) one of the  $\sigma_{1,2}$  vanishes identically. (b)  $\sigma_1 \gg \sigma_2$ , with both being non-zero.  
 (c)  $\sigma_1 \ll \sigma_2$ , with both being non-zero. (d)  $\sigma_1 \approx \sigma_2$ .
39. A hydrogen atom is in a weak external magnetic field  $\vec{B}$ . Consider an electron of this atom with ( $l = 1, s = \frac{1}{2}$ ) and total  $j = \frac{3}{2}$ . There are multiple energy levels for this electron due to the magnetic field. The energy spacing between any two adjacent levels (in units of  $\mu_B B$ ) is

(a)  $\frac{1}{2}$

(b)  $\frac{1}{3}$

(c)  $\frac{3}{4}$

(d)  $\frac{4}{3}$

40. The excitations of a three-dimensional solid are bosonic in nature and their energy dispersion is given by  $\epsilon_k \propto k^2$ , in the long wavelength limit. If the chemical potential of the system is zero, the temperature dependence of specific heat of the system at low temperature is proportional to

(a)  $T^3$

(b)  $T^{\frac{3}{2}}$

(c)  $T^{\frac{5}{2}}$

(d)  $T^{\frac{1}{2}}$

41. An optical cavity of a laser, formed by two plane mirrors, is filled up with an active medium. The medium emits radiation at wavelengths 450 nm, 600 nm, and 750 nm. If the medium is continuously pumped, at which cavity length among the following, will all three wavelengths be amplified?

(a)  $750\mu\text{ m}$

(b)  $1500\mu\text{ m}$

(c)  $600\mu\text{ m}$

(d)  $450\mu\text{ m}$

42. Suppose that the volume and the surface terms are the most dominant ones in the semi-empirical formula for the binding energy of a nucleus. Let  $C_s$  and  $C_v$  be the coefficients of the surface and volume terms. Which of the following is a criterion for stability of the nucleus?

(a)  $A > \left(\frac{C_s}{C_v}\right)^3$

(b)  $A < \left(\frac{C_s}{C_v}\right)^3$

(c)  $A > \left(\frac{2C_s}{3C_v}\right)^3$

(d)  $A < \left(\frac{2C_s}{3C_v}\right)^3$

43. The Lagrangian of a two-particle system is given by

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2) - \frac{1}{2}m\omega^2\left(q_1^2 + q_2^2 + \frac{1}{2}q_1q_2\right).$$

The normal mode frequencies (in units of  $\omega$ ) are

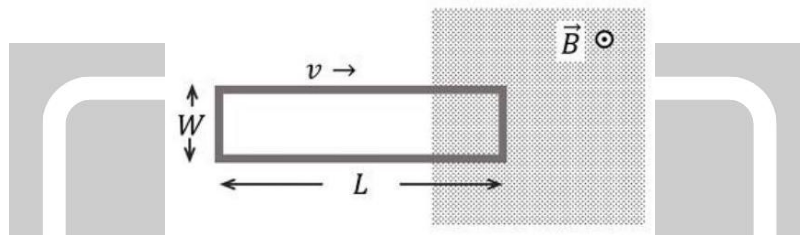
(a)  $\sqrt{\frac{5}{3}}, \frac{1}{2}$

(b)  $\sqrt{\frac{5}{6}}, \sqrt{\frac{3}{2}}$

(c)  $\sqrt{\frac{6}{5}}, \sqrt{2}$

(d)  $\sqrt{\frac{5}{6}}, \sqrt{2}$

44. A long rectangular metallic loop of width  $W$  and length  $L (\gg W)$  starts entering a region, where there is a uniform magnetic field  $B$  perpendicular to the plane of the loop. The resistance of the loop is  $R$  and its mass is  $M$ . If  $v_0$  is the velocity of the loop just before entering the region, then neglecting the self-inductance effect, the velocity at a later time  $t$  is



(a)  $v(t) = \frac{v_0}{1 + \frac{B^2 W^2}{MR} t}$

(b)  $v(t) = \frac{v_0}{1 + \left(\frac{B^2 W^2}{MR} t\right)^2}$

(c)  $v(t) = v_0 e^{-\frac{B^2 W^2}{MR} t}$

(d)  $v(t) = \frac{v_0}{1 + \ln \left(1 + \frac{B^2 W^2}{MR} t\right)}$

45. Consider the one-dimensional motion of a particle of positive charge  $q$  confined to an infinite potential well

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \pi \\ \infty & \text{otherwise} \end{cases}$$

which is subjected to a perturbing electric field  $\vec{E} = E_0 \hat{x}$ . The shift in the ground state energy, to the first order in  $q$ , is

(a)  $\frac{q\pi E_0}{2}$

(b)  $-\frac{q\pi E_0}{2}$

(c)  $q\pi E_0$

(d)  $-q\pi E_0$

46. A sequence of polynomial  $Q_n(x) [n = 0, 1, 2, \dots]$  satisfies the recursion relation

$$Q_{n+1}(x) - 2xQ_n(x) + 2nQ_{n-1}(x) = 0, \text{ for all } n \geq 0 [ \text{here } Q_{-1}(x) = 0 ]$$

The generating function for the polynomials,  $g(x, t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} Q_n(x)$ , satisfies

(a)  $\frac{\partial g}{\partial t} = 2(t + x)g$

(b)  $\frac{\partial g}{\partial t} = 2(x - t)g$

(c)  $\frac{\partial g}{\partial t} = \frac{2(x - t)}{t}g$

(d)  $\frac{\partial g}{\partial t} = 2 + (x + t)g$

47. For a particle in the angular momentum state  $|l = 4, m_l = 2\rangle$ , the expectation value of the operator  $L_x L_y$  is

(a)  $-\hbar^2$

(b)  $\hbar^2$

(c)  $-i\hbar^2$

(d)  $i\hbar^2$

48. A binary alloy consists of  $N_A$  number of  $A$ -type and  $N_B$  number of  $B$ -type atoms. The atoms sit on the sites of a simple cubic lattice and the nearest neighbours interact with each other. Assume an attractive interaction energy  $-J$  ( $J > 0$ ) between two like neighbours ( $AA$  or  $BB$  pair) and a repulsive interaction energy  $+J$  between two unlike neighbours ( $AB$  pair). If  $N$  is the total number of sites, then the average energy of the system at a very high temperature ( $k_B T \gg J$ ) is

(a)  $-3J \frac{(N_A - N_B)^2}{N}$

(b)  $3JN$

(c)  $3J \frac{(N_A + N_B)^2}{N}$

(d)  $-3J(N_A - N_B)$

49. The Lagrangian  $L = L(x, y, \dot{x}, \dot{y})$  is invariant under the transformation  $x \rightarrow x + \epsilon y$  and  $y \rightarrow y + \epsilon x$ , for any infinitesimal real parameter  $\epsilon$ . If  $P_x, P_y$  denote canonically conjugate momenta corresponding to  $x, y$  respectively, then the corresponding conserved quantity is

(a)  $yP_x - xP_y$

(b)  $yP_x + xP_y$

(c)  $xP_x + yP_y$

(d)  $xP_x - yP_y$

50. Electromagnetic waves of frequency  $\omega$  are incident on an electron gas, whose relaxation time is  $\tau$ . Let  $\sigma_{\text{low}}$  and  $\sigma_{\text{high}}$  represent the respective electrical conductivities of the gas in low frequency ( $\omega\tau \ll 1$ ) and high frequency ( $\omega\tau \gg 1$ ) limits. The ratio ( $\sigma_{\text{low}}/\sigma_{\text{high}}$ ) is

(a) inversely proportional to  $\omega^2$ .

(b) directly proportional to  $\omega^2$ .



(c) independent of  $\omega$ .

(d) directly proportional to  $\omega$ .

51. In a high energy scattering experiment involving two identical particles, each of rest mass  $m_0$ , one particle is initially at rest, while the other one is incident upon it with energy  $E$  and momentum  $p$ . The total energy of the two-particle system in the centre-of-mass frame, in the limit  $E \gg m_0 c^2$ , is approximately given by

(a)  $E$

(b)  $2E$

(c)  $\sqrt{\frac{Em_0 c^2}{2}}$

(d)  $\sqrt{2Em_0 c^2}$

52. The Hamiltonian of a simple pendulum consisting of mass  $m$  attached to a massless string of length  $l$  is  $H = \frac{p_\theta^2}{2ml^2} + mgl(1 - \cos \theta)$ . If  $L$  denotes the Lagrangian, then  $\frac{dL}{dt}$  is

(a)  $\frac{g}{l} P_\theta \cos \theta$

(b)  $\frac{-g}{l} P_\theta \sin \theta$

(c)  $\frac{-2g}{l} P_\theta \sin \theta$

(d)  $\frac{g}{l} P_\theta \cos (2\theta)$

53. Consider a one-dimensional chain of atoms with lattice constant  $a$ . The energy of an electron with wave-vector  $k$  is  $\epsilon(k) = \mu - 2\gamma \cos ka$ , where  $\mu$  and  $\gamma$  are constants. If an electric field  $\vec{E}$  is applied along the chain, the time dependent velocity of the electron is proportional to (assume initial wave vector  $k = k_0$  at  $t = 0$ )

(a)  $\sin^2 \left( k_0 a - \frac{eEa}{\hbar} t \right)$ .

(b)  $\cos \left( k_0 a - \frac{eEa}{\hbar} t \right)$ .

(c)  $\sin \left( k_0 a - \frac{eEa}{\hbar} t \right)$ .

(d)  $\cos^2 \left( k_0 a - \frac{eEa}{\hbar} t \right)$ .

54. Find the curve that extremizes the functional

$$I(y) = \int_0^1 \left[ \left( \frac{dy}{dx} \right)^2 + 12xy \right] dx$$

for the given boundary conditions  $y(0) = 0$  and  $y(1) = 1$

(a)  $y = x^3$

(b)  $y = x^2$

(c)  $y = 2x^2 - x$

(d)  $y = 3x^3 - 2x^2$

55. For a spherical nucleus, consider the interior charge distribution to be

$$\rho(r) = \frac{\rho_0}{1 + \exp [(r - R)/a]}$$

where  $\rho_0$ ,  $R$  and  $a$  are constants of appropriate dimensions. In the limit  $a \rightarrow 0^+$ , the number of protons (charge  $e$ ) inside a sphere of radius  $2R$  is given by

(a)  $\frac{2\rho_0}{e} \left( \frac{4}{3} \pi R^3 \right)$

(b)  $\frac{\rho_0}{e} \left( \frac{4}{3} \pi R^3 \right)$

(c)  $\frac{8\rho_0}{e} \left( \frac{4}{3} \pi R^3 \right)$

(d)  $\frac{4\rho_0}{e} \left( \frac{4}{3} \pi R^3 \right)$

❖ Answer Key

1. a	2. b	3. d	4. b	5. c	6. b	7. b	8. 1	9. c	10. d
11. c	12. d	13. d	14. b	15. c	16. d	17. a	18. c	19. b	20. c
21. d	22. d	23. c	24. a	25. a	26. c	27. b	28. b	29. a	30. a
31. a	32. a	33. c	34. c	35. b	36. c	37. d	38. d	39. d	40. b
41. d	42. a	43. b	44. c	45. b	46. b	47. d	48. a	49. b	50. d
51. d	52. c	53. c	54. a	55. b					