D PHYSICS

CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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PHYSICAL SCIENCE

THERMODYNAMICS STATISTICAL MECHANICS

Previous Year Questions [Topic-Wise]

With Answer Key

CSIR-NET/JRF, GATE, JEST, TIFR

❖ **CSIR-NET PYQ**

1. Let ΔW be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about ΔW is correct?

[CSIR: DEC-2012]

(a) ΔW is a perfect differential if the process is isothermal

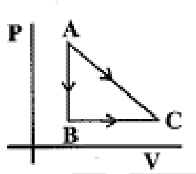
(b) ΔW is a perfect differential if the process is adiabatic

(c) ΔW is always a perfect differential

(d) ΔW cannot be a perfect differential.

2. A given quantity of gas is taken from the state A → C reversibly, by two paths, A → C directly and A → B → C as shown in the figure below. During the A → C the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process A → B → C the work done by the gas is 30 J, the heat absorbed is:

[CSIR: DEC-2012]



(a) 20 J (b) 80

(c) 220 J (d)280 J

3. The pressure P of a fluid is related to its number density ρ by the equation of state $P = a\rho + b\rho^2$ where a and b are constants. If the initial volume of the fluid is V_0, the work done on the system when it is compressed so as to increase the number density from an initial value of ρ_0 to $2\rho_0$ is **[CSIR: DEC-2014]**

(a) $a\rho_0 V_0$

(b) $(a + b\rho_0)\rho_0 V_0$

(c) $\left(\frac{3a}{2} + \frac{7\rho_0 b}{3}\right)\rho_0 V_0$

(d) $(a\ln\ 2 + b\rho_0)\rho_0 V_0$

4. When a gas expands adiabatically from volume V_1 to V_2 by a quasi-static reversible process, it cools from temperature T_1 to T_2. If now the same process is carried out adiabatically and irreversibly, and T_2' is the temperature of the gas when it has equilibrated, then

[CSIR: DEC-2014]

(a) $T_2'' = T_2$ (b) $T_2' > T_2$

(c)$T_2' = T_2\left(\frac{V_2 - V_1}{V_2}\right)$ (d) $T_2' = \frac{T_2 V_1}{V_2}$

5. A silica particle of radius 0.1μm is put in a container of water at $T = 300$ K. The densities of silica and water are 2000 kg/m^3 and 1000 kg/m^3, respectively. Due to thermal fluctuations, the particle is not always at the bottom of the container. The average height of the particle above the base of the container is approximately

[CSIR: DEC-2016]

(a) 10^{-3} m (b) 3×10^{-4} m

(c) 10^{-4} m (d) 5×10^{-5} m

6. The internal energy $E(T)$ of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$. Then the entropy $S(T)$, as a function of temperature, is

(a) $\frac{1}{2}aT^2 + \frac{1}{4}bT^4$ (b) $2aT^2 + 4bT^4$

(c)$2aT + \frac{4}{3}bT^3$ (d) $2aT + 2bT^3$

7. When an ideal monoatomic gas is expanded adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio T/T_0 is

[CSIR: JUNE-2016]

(a) $\frac{1}{3}$ (b) $\left(\frac{1}{3}\right)^{2/3}$

(c) $\left(\frac{1}{3}\right)^{1/3}$ (d) 3

8. A box, separated by a movable wall, has two compartment filled by a monoatomic gas of

$$\frac{C_P}{C_\gamma} = \gamma.$$

Initially the volumes of the two compartments are equal, but the pressure are $3P_0$ and P_0, respectively. When the wall is allowed to move, the final pressure in the two compartments become equal. The final pressure is

[CSIR: JUNE-2017]

(a) $\left(\frac{2}{3}\right)^{\gamma} P_0$ (b) $3\left(\frac{2}{3}\right)^{7} P_0$

(c) $\frac{1}{2}\left(1+3^{1/\gamma}\right)^{\gamma} P_0$ (d) $\left(\frac{3^{1/\gamma}}{1+3^{1/\gamma}}\right)^{\gamma} P_0$

9. Which of the following statements concerning the coefficient of volume expansion α and the isothermal compressibility κ of a solid is true ?

[CSIR: JUNE-2018]

(a) α and κ are both intensive variables.

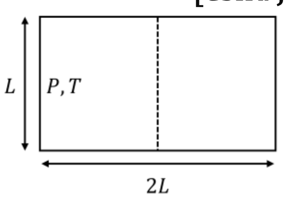
(b) α is an intensive and k is an extensive variable.

(c) α is an extensive and κ is an intensive variable.

(d) α and k are both extensive variables.

10. A thermally insulated chamber of dimensions $(L, L, 2L)$ is partitioned in the middle. One side of the chamber is filled with n moles of an ideal gas at a pressure P and temperature T, while the other side is empty. At $t = 0$, the partition is removed and the gas is allowed to expand freely. The time to reach equilibrium varies as

[CSIR: JUNE-2018]



(a) $n^{1/3}L^{-1}T^{1/2}$ (b) $n^{2/3}LT^{-1/2}$

(c) $n^0LT^{-1/2}$ (d) $nL^{-1}T^{1/2}$

11. The van der Waals equation for one mole of a gas is $\left(p+\frac{a}{V^2}\right)(V-b) = RT$. The corresponding equation of state for n moles of this gas at pressure p, volume V and temperature T, is **[CSIR: JUNE-2018]**

(a) $\left(p+\frac{an^2}{V^2}\right)(V-nb) = nRT$

(b) $\left(p+\frac{a}{V^2}\right)(V-nb) = nRT$

(c) $\left(p+\frac{an^2}{V^2}\right)(V-nb) = RT$

(d) $\left(p+\frac{a}{V^2}\right)(V-nb) = RT$

12. The equation of state of an ideal gas is $pV = RT$. At very low temperatures, the volume expansion coefficient $\frac{1}{V}\frac{\partial V}{\partial T}$ at constant pressure

[CSIR: JUNE-2019]

(a) diverges as $1/T^2$

(b) diverges as $1/T$

(c) vanishes as T

(d) is independent of the temperature

13. A mole of gas at initial temperature T_i comes into contact with a heat reservoir at temperature T_f and the system is allowed to reach equilibrium at constant volume. If the specific heat of the gas is $C_V = \alpha T$, where α is a constant, the total change in entropy is

[CSIR: DEC-2019]

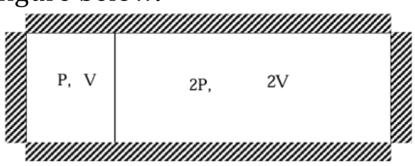
(a) Zero

(b) $\alpha(T_f - T_i) + \frac{\alpha}{2T_f}(T_f - T_i)^2$

(c) $\alpha(T_f - T_i)$

(d) $\alpha(T_f - T_i) + \frac{\alpha}{2T_f}(T_f^2 - T_i^2)$

14. A thermally isolated container, filled with an ideal gas at temperature T, is divided by a partition, which is clamped initially, as shown in the figure below.



The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction. The final pressure at equilibrium is

[CSIR: JUNE-2022]

(a)5P/3 (b)5P/4

(c)3P/5 (d)4P/5

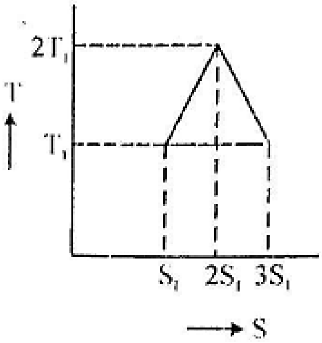
15. A classical ideal gas is subjected to a reversible process in which its molar specific heat changes with temperature T as $C(T) = C_V + R\frac{T}{T_0}$. If the initial temperature and volume are T_0 and V_0, respectively, and the final volume is $2V_0$, then the final temperature is **[CSIR: DEC-2023]**

(a)$T_0/\ln 2$ (b)$2T_0$

(c)$T_0/[1-\ln 2]$ (d)$T_0[1+\ln 2]$

❖ **GATE PYQ**

1. A revisable engine cycle is shown in the following T-S diagram. The efficiency of the engine is: **[GATE 1999]**



(a) 1/3 (b) ½

(c) 1/5 (d) 1/4

2. An amount of heat Q is transferred from a heat reservoir at temperature T to another heat reservoir at temperature T_B. What is the change in the entropy ΔS of the combined system?

[GATE 2000]

(a)$Q\left(\frac{1}{T_B}-\frac{1}{T_A}\right)$ (b) $Q\left(\frac{1}{T_B}+\frac{1}{T_A}\right)$

(c)$\frac{Q}{\sqrt{T_BT_A}}\ln\frac{T_B}{T_A}$ (d) $O\left(\frac{1}{T_A}-\frac{1}{T_B}\right)$

3. An engine absorbs heat at a temperature of 1000 K and rejects heat at 600 K . If the engine operates at maximum possible efficiency, the amount of work performed by the engine, for 2000 J heat input is: **[GATE]**

(a) 1600 J (b) 1200 J

(c) 800 J (d) 400 J

4. Which of the following is not an exact differential? **[GATE 1997]**

(a) dQ(Q = heat absorbed)

(b) $dU(U=$ internal energy)

(c) dS(S = entropy)

(d) dF(F = free energy)

5. Boyle's law can be expressed in different form as **[GATE 1999]**

(a) dV/dP = 1 (b) dV/dP = V/P

(c) dV/dP = P/V (d) dV/dP = −V′P

6. An ideal gas in a cylinder is compressed adiabatically to one-third of its initial volume. During this process 20 J work is done on the gas by compressing agent. Which of the following statements is true for this case? **[GATE 1997]**

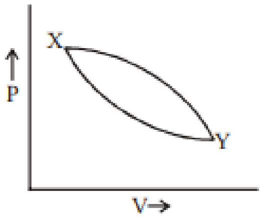
(a) Change in the internal energy in this process is zero.

(b) The internal energy increases by 20 J

(c) The internal energy decreases by 20 J

(d) Temperature of the gas decreases.

7. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle X → Y → X as shown. The work done by the gas is positive if the direction of the thermal cycle is **[GATE: 2003]**



(a) clockwise

(b) counter-clockwise

(c) neither clockwise nor counter-clockwise

(d) clockwise from X → Y and counter-clockwise from Y → X

8. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the work done by the gas is given by

[GATE: 2003]

(a) $(PV\Delta S)/(nR)$ (b) $nR\Delta S$

(c) PV (d) $(P\Delta S)/(nRV)$

9. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is

[GATE: 2005]

(a) $pV^{1/3} = \text{constant}$ (b) $pV^{2/3} = \text{constant}$

(c) $pV^{4/3} = \text{constant}$ (d) $pV^{3/5} = \text{constant}$

10. The internal energy of n moles of a gas is given

$$E = \frac{3}{2}n\text{RT} - \frac{a}{V},$$

where V is the volume of the gas at temperature T and a is a positive constant. One mole of the gas in state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is **[GATE: 2006]**

(a) $\text{T}_1 + \text{R}a\left(\frac{1}{\text{V}_2} + \frac{1}{\text{V}_1}\right)$

(b) $\text{T}_1 - \frac{2}{3}\text{R}a\left(\frac{1}{\text{V}_2} - \frac{1}{\text{V}_1}\right)$

(c) $\text{T}_1 + \frac{2}{3}\text{R}a\left(\frac{1}{\text{V}_2} - \frac{1}{\text{V}_1}\right)$

(d) $\text{T}_1 - \frac{1}{3}\text{R}a\left(\frac{1}{\text{V}_2} - \frac{1}{\text{V}_1}\right)$

11. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}(\text{ Pa})^{-1}$, the pressure required to increase its density by 1% is approximately

[GATE: 2011]

(a) 10^4 Pa (b) 10^6 Pa

(c) 10^8 Pa (d) 10^{10} Pa

12. The isothermal compressibility, κ of an ideal gas at temperature T_0 and volume V_0 is given by

[GATE: 2012]

(a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$

(c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial V}\Big|_{T_0}$

13. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

[GATE: 2013]

(a) pressure and temperature

(b) volume and temperature

(c) pressure and volume

(d) volume and energy

14. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to

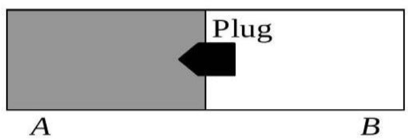
[GATE: 2014]

(a) $V^{-1/2}$ (b) $V^{-2/3}$

(c) $V^{-3/5}$ (d) $V^{-5/3}$

15. As shown in the figure, an ideal gas is confined to chamber A of an insulated container, with vacuum in chamber B. When the plug in the wall separating the chambers A and B is removed, the gas fills both the chambers. Which one of the following statements is true?

[GATE: 2020]



(a) The temperature of the gas remains unchanged

(b) Internal energy of the gas decreases

(c) Temperature of the gas decreases as it expands to fill the space in chamber B

(d) Internal energy of the gas increases as its atoms have more space to move around

16. A paramagnetic salt of mass m is held at temperature T in a magnetic field H. If S is the entropy of the salt and M is its magnetization, then $dG = -SdT - MdH$, where G is the Gibbs free energy. If the magnetic field is changed adiabatically by $\Delta H \to 0$ and the corresponding infinitesimal changes in entropy and temperature are ΔS and ΔT, then which of the following statements are correct

[GATE: 2022]

(a) $\Delta S = -\frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_H \Delta T$ (b) $\Delta S = 0$

(c) $\Delta T = -\frac{\left(\frac{\partial M}{\partial T}\right)_H}{\left(\frac{\partial S}{\partial T}\right)_H}\Delta H$ (d) $\Delta T = 0$

17. A piston of mass m is fitted to an airtight horizontal cylindrical jar. The cylinder and piston have identical unit area of cross-section. The gas inside the jar has volume V and is held at pressure $P = P_{\text{atmosphere}}$. The piston is pushed inside the jar very slowly over a small distance. On releasing, the piston performs an undraped simple harmonic motion of low frequency. Assuming that the gas is ideal and no heat is exchanged with the atmosphere, the frequency of the small oscillations is proportional to

[GATE: 2022]

(a) $\sqrt{\frac{P}{\gamma m V}}$ (b) $\sqrt{\frac{P\gamma}{Vm}}$

(c) $\sqrt{\frac{P}{mV^{\gamma-1}}}$ (d) $\sqrt{\frac{\gamma P}{mV^{\gamma-1}}}$

❖ **JEST PYQ**

1. A monoatomic ideal gas at 17°C is adiabatically compressed to 1/8 of its original volume. The temperature after compression is

[JEST-2012]

(a) 2.1°C (b) 17°C

(c) −200.5°C (d) 887°C

2. A thermally insulated ideal gas of volume V1 and temperature T expands to another enclosure of volume V2 through a prous plug. What is the change in the temperature of the gas?

[JEST-2012]

(a) 0 (b) $T\ln(V_1/V_2)$

(c) $T\ln(V_2/V_1)$ (d) $T\ln[(V_2/V_1)/V_2)]$

3. For a diatomic ideal gas near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at constant pressure?

[JEST-2013]

(a) 1/7 (b) 5/7

(c) 3/4 (d) 2/7

4. An ideal gas is compressed adiabatically from an initial volume V to a final volume αV and a work W is done on the system in doing so. The final pressure of the gas will be $\left(\gamma = \frac{C_p}{C_v}\right)$

[JEST-2015]

(a) $\frac{W}{V^\gamma}\frac{1-\gamma}{\alpha-\alpha^\gamma}$ (b) $\frac{W}{V^\gamma}\frac{\gamma-1}{\alpha-\alpha^\gamma}$

(c) $\frac{W}{V}\frac{1-\gamma}{\alpha-\alpha^\gamma}$ (d) $\frac{W}{V}\frac{\gamma-1}{\alpha-\alpha^\gamma}$

5. An ideal gas has a specific heat ratio $\frac{C_p}{C_v} = 2$.. Starting at a temperature T_1 the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

[JEST-2016]

(a) $\frac{T_1}{2}$ (b) $\sqrt{2}T_1$

(c) $2T_1$ (d) $\frac{T_1}{\sqrt{2}}$

6. After the detonation of an atom bomb, the spherical ball of gas was found to be 15 meter radius at a temperature of 3×10^5 K. Given the adiabatic expansion coefficient $\gamma = 5/3$, what will be the radius of the ball when its temperature reduces to 3×10^3 K ?

[JEST-2017]

(a) 156 m (b) 50 m

(c) 150 m (d) 100 m

7. In a thermodynamic process, the volume of one mole of an ideal is varied as $V = \alpha T^{-1}$, where α is a constant. The adiabatic exponent of the gas is γ. What is the amount of heat received by the gas is the temperature of the gas increases by ΔT in the process?

[JEST-2018]

(a) $R\Delta T$ (b) $\frac{R\Delta T}{1-\gamma}$

(c) $\frac{R\Delta T}{2-\gamma}$ (d) $R\Delta T\frac{2-\gamma}{\gamma-1}$

8. A theoretical model for a real (non-ideal) gas gives the following expression for the internal energy (U) and the pressure (P), and $U(T,V) = {}_aV^{-2/3} + {}_bV^{2/3}\,T^2$

$$P(T,V) = \frac{2}{3}a^{-5/3} + \frac{2}{3}bV^{-1/3}\,T^2,$$

Where a and b are constants. Let V_0 and T_0 be the initial volume and initial temperature respectively. If the gas expands adiabatically, the volume of the gas is proportional to

[JEST-2018]

(a) T (b) $T^{3/2}$

(c) $T^{-3/2}$ (d) $T^{-2}n$

9. A frictionless, heat conducting piston of negligible mass and heat capacity divides a vertical, insulated cylinder of height 2H and cross-sectional area A into two halves. Each half contains one mole of an ideal gas at temperature T_0 and pressure P_0 corresponding to STP. The heat capacity ratio $\gamma = C_p/C_u$ is given. A load of weight W is tied to the piston and suddenly released. After the system comes to equilibrium, the piston is at rest and the temperatures of the gases in the two compartments are equal. What is the final displacement y of the piston from its initial position, assuming $yW >> T_0C_0$?

[JEST-2018]

(a) $\frac{2H}{\sqrt{\gamma}}$ (b) $H\gamma$

(c) $\frac{H}{\sqrt{\gamma}}$ (d) $\frac{2H}{\gamma}$

10. In an experiment, certain quantity of an ideal gas at temperature T_0, pressure P_0 and volume V_0 is heated by a current flowing through a wire for a duration of t seconds. The volume is kept constant and the pressure changes to P_1. If the experiment is performed at constant pressure starting with the same initial conditions, the volume changes from V_0 to V_1. The ratio of the specific heats at constant pressure and constant volume is **[JEST-2018]**

(a) $\frac{P_1 - P_0V_0}{V_1 - V_0P_0}$ (b) $\frac{P_1 - P_0V_1}{V_1 - V_0P_1}$

(c) $\frac{P_1V_1}{P_0V_0}$ (d) $\frac{P_0V_0}{P_1V_1}$

11. An ideal fluid is subjected to a thermodynamics process described by $\rho = CV^{-a}$ and $P = n\rho^{\Gamma}$, where ρ is energy density and P is pressure. For what values of n and Γ, the process is adiabatic if the volume is changed slowly? **[JEST-2018]**

(a) $\Gamma = \alpha - 1, n = 1$ (b) $\Gamma = 1 - \alpha, n = \alpha$

(c) $\Gamma = 1, n = \alpha - 1$ (d) $\Gamma = \alpha, n = 1 - \alpha$

12. In an experiment, certain quantity of an ideal gas at temperature T_0, pressure P_0 and volume V_0 is heated by a current flowing through a wire for a duration of t seconds. The volume is kept constant and the pressure changes to P_1. If the experiment is performed at constant pressure starting with the same initial conditions, the volume changes from V_0 to V_1. The ratio of the specific heats at constant pressure and constant volume is

[JEST-2018]

(a) $\frac{P_1 - P_0V_0}{V_1 - V_0P_0}$ (b) $\frac{P_1 - P_0V_1}{V_1 - V_0P_1}$

(c) $\frac{P_1V_1}{P_0V_0}$ (d) $\frac{P_0V_0}{P_1V_1}$

13. A diatomic ideal gas at room temperature, is expanded at a constant pressure P_0. If the heat absorbed by the gas is $Q = 14$ Joules, what is the maximum work in Joules that can be extracted from the system? **[JEST-2019]**

14. Consider a classical harmonic oscillator in thermal equilibrium at a temperature T. If the spring constant is changed to twice its value isothermally, then the amount of work done on the system is

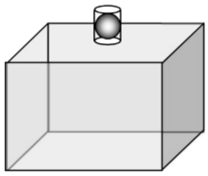
[JEST-2020]

(a) $k_B T \ln 2$ (b) $k_B T \frac{\ln 2}{2}$

(c) $2k_B T \ln 2$ (d) $-k_B T \ln 2$

15. A large box, of volume V is fitted with a vertical glass tube of cross-sectional area A, in which a metal ball of mass m fits exactly. The box contain an ideal gas at a pressure slightly higher than atmospheric pressure P because of the weight of the ball. If the ball is displaced slightly from equilibrium, find the angular frequency ω of simple harmonic oscillations. Assume adiabatic behaviour, with ratio of specific heats $\gamma = C_P/C_V$.

[JEST-2021]



(a) $\omega = \sqrt{\frac{A^2(P + mg/A)}{2\gamma V m}}$

(b) $\omega = \sqrt{\frac{2\gamma A^2(P + mg/A)}{Vm}}$

(c) $\omega = \sqrt{\frac{A^2(P + mg/A)}{\gamma V m}}$

(d) $\omega = \sqrt{\frac{\gamma A^2(P + mg/A)}{Vm}}$

16. A thin tube of length 1080 mm and uniform cross-section is sealed at both ends, and placed horizontally on a table. At the exact center of the tube is a mercury (Hg) pellet of length 180 mm. The pressure of the air on both sides of the mercury pellet is P_0. When the tube is held at an angle of 60 degrees with the vertical, the length of the air column above and below the Hg become 480 mm and 420 mm, respectively. Assuming the temperature of the system to be constant, calculate the pressure P_0 in mm of Hg.

[JEST-2021]

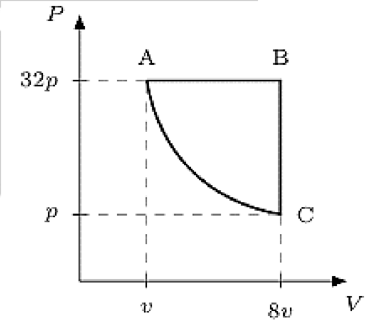
17. An ideal diatomic gas at pressure P is adiabatically compressed so that its volume becomes $\frac{1}{n}$ times the initial value. The final pressure of the gas will be

[JEST-2022]

(a) $n^{\frac{7}{5}}P$ (b) $n^{\frac{7}{2}}P$

(c) $n^{-\frac{7}{5}}P$ (d) $n^{\frac{5}{3}}P$

18. One mole of an ideal gas undergoes a thermodynamic cycle formed by an isobaric process, an isochoric process, and an adiabatic process (see figure). At A, the temperature of the gas is T. What is the change in the internal energy of the gas, in the units of RT (R is the universal gas constant) as the system goes from A to B



[JEST-2022]

19. An ideal gas initially at pressure P_i undergoes the following sequence of processes:
1.A reversible adiabatic expansion that doubles its volume.
2.A reversible isothermal compression that restores its original volume.
3.A reversible isothermal expansion that doubles its volume.
4.A reversible adiabatic compression that restores its original volume.
If the final pressure of the gas is P_f, which of the following is true?

[JEST-2024]

(a) $P_f = P_i$.
(b) $P_f > P_i$.
(c) $P_f < P_i$.
(d) The relation between P_f and P_i depends on theinitial conditions.

20. The ratio of the molar specific heats of an ideal gas is

$$\gamma = \frac{c_p}{c_v} = \frac{3}{2}$$

. It undergoes a reversible isothermal expansion in which its volume doubles. Next, it undergoes a reversible isochoric process such that the change in entropy of the second process is equal to the change in entropy of the first process. What is the ratio of the final temperature to the initial temperature?

[JEST-2024]

(a) 3 (b) 2

(c) $\sqrt{2}$ (d) $\frac{3}{2}$

21. One mole of ideal gas with a constant C_V undergoes a reversible adiabatic expansion. Which one of the following equations is valid? [$\gamma = \frac{C_P}{C_V}$ for the gas]

[JEST-2025]

(a) VT^{γ} = constant

(b) $V^{\gamma}T$ = constant

(c) $P^{1-\gamma}T^{\gamma}$ = constant

(d) $P^{\gamma-1}T^{\gamma}$ = constant

❖ **TIFR PYQ**

1. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15°C. At this point, it bursts. Assuming the sudden expansion of the air (a mixture of O_2 and N_2) that was inside the tyre to be adiabatic, its temperature after the burst is

[TIFR-2009]

(a) −55°C (b) −37°C

(c) −26°C (d) +9°C

2. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15°C. At this point, it bursts. Assuming the sudden expansion of the air (a mixture of O_2 and N_2) that was inside the tyre to be adiabatic, its temperature after the burst is

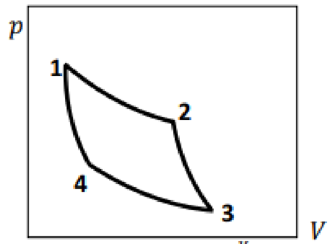
[TIFR-2010]

(a) −55°C (b) −37°C

(c) −26°C (d) +9°C

3. The pV-diagram for a Carnot cycle executed by an ideal gas with $C_P/C_V = \gamma > 1$ is shown below. Note that 1,2,3 and 4 label the changeover points in the cycle.

[TIFR-2013]



$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^X$$

If, for this cycle,
then $X =$

(a) $1 - 1/\gamma$ (b) 0

(c) 1 (d) $-1/\gamma$

4. The entropy S of a black hole is known to be of the form

$$S = \alpha k_B A$$

where A is the surface area of the black hole and α is a constant, which can be written in terms of c (velocity of light in vacuum), $\hbar$ (reduced Planck's constant) and G_N (Newton's constant of gravitation). Taking the radius of the black hole as

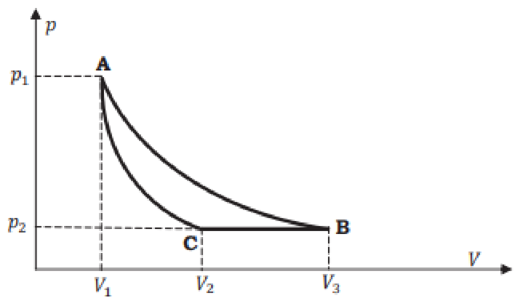
$$R = \frac{2G_N M}{c^2}$$

it follows that the entropy S is
[λ is a numerical constant] **[TIFR-2013]**

(a) $\frac{G_N^2 M^2 k_B}{\lambda(\hbar c)^4}$ (b) $\frac{\hbar c k_B}{\lambda G_N M}$

(c) $\frac{G_N^2 M^2 k_B}{\lambda \hbar c^4}$ (d) $\frac{G_N M^2 k_B}{\lambda \hbar c}$

5. One mole of an ideal gas undergoes the cycle ACBA shown in the pV diagram below.



One of the curved lines in the cycle represents an isothermal change at temperature T, while the other represents an adiabatic change. The net heat gained by the gas in this cycle is

[TIFR-2014]

(a) $-p_2(V_3 - V_2) + RT\ln\dfrac{V_2}{V_1}$

(b) $-p_2(V_3 - V_2) + RT\ln\dfrac{V_3}{V_1}$

(c) $-p_2(V_3 - V_2) + \gamma RT\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$

(d) $(p_1V_1 - p_2V_2) - RT\ln\dfrac{V_3}{V_1}$

6. A thermally-insulated container of volume V_0 is divided into two equal halves by a non-permeable partition. A real gas with equation of state

$$b^3\left(p + \frac{a^2}{V^3}\right) = nRT$$

where a and b are constants, is confined to one of these halves at a temperature T_0. The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat C_V, will be

[TIFR-2014]

(a) $T_0 - \dfrac{3a^2}{2C_VV_0^2}$ (b) $T_0 - \dfrac{2a^2}{3C_VV_0^2}$

(c) $T_0 + \dfrac{3a^2}{2C_VV_0^2}$ (d) $T_0 + \dfrac{2a^2}{3C_VV_0^2}$

7. The equation of state of a gas is given by

$$V = \frac{RT}{P} - \frac{b}{T}$$

where R is the gas constant and b is another constant parameter. The specific heat at constant pressure C_P and the specific heat at constant volume C_V for this gas is related by $C_P - C_V =$

[TIFR-2015]

(a) R (b) $R\left(1 + \dfrac{RT^2}{bP}\right)^2$

(c) $R\left(1 + \dfrac{bP}{RT^2}\right)^2$ (d) $R\left(1 - \dfrac{bP}{RT^2}\right)^2$

8. An ideal diatomic gas is initially at a temperature $T = 0°C$. Then it expands reversibly and adiabatically to 5 times its volume. Its final temperature will be approximately

[TIFR-2015]

(a) $-180°C$ (b) $-150°C$

(c) $-130°C$ (d) $0°C$

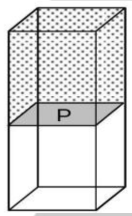
9. A thermally isolated container stores N_2 gas at $27.24°C$ at one atmospheric pressure. Suddenly the pressure of the gas is increased to two atmospheric pressures. Assuming N_2 to behave as an ideal gas, estimate the change in temperature of the gas, in Celsius degrees ($°C$)

[TIFR-2017]

10. The volume V of a rectangular box is divided into two equal parts by a solid non-permeable partition P. On one side of the partition P there is a vacuum, while the other side is filled with a real gas having equation of state

$$pVe^{a/RTV} = nRT$$

where a and b are constants, The gas was initially at a uniform temperature T_0. Then the partition P was removed instantaneously, and the gas was allowed to expand to fill the full



volume of the box and come to equilibrium.
The final temperature of the gas, in terms of its specific heat C_V will be

[TIFR-2020]

(a) $T - \left(\dfrac{na}{C_V}\right)\ln 2$ (b) $T + \left(\dfrac{na}{C_V}\right)\ln 2$

(c) $T - 2n\left(\dfrac{RTa}{C_V}\right)^{3/2}$ (d) $T + 2n\left(\dfrac{RTa}{C_V}\right)^{3/2}$

11. A spherical balloon of radius R is made of a material with surface tension γ and filled with N particles of an ideal gas. If the outside air pressure is P, the pressure P_b inside the balloon is given by **[TIFR-2021]**

(a) $P_b = P + 2\gamma/R$ (b) $P_b = P$

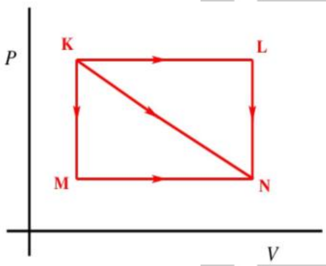
(c) $P_b = P - 2\gamma/R$ (d) $P_b = P + 3\gamma/R$

12. A bicycle tyre is pumped with air to an internal pressure of 6 atm at $20°C$, at which point it suddenly bursts. Assuming the external pressure to be 1 atmosphere and the subsequent sudden expansion to be adiabatic, the temperature immediately after the burst is approximately **[TIFR-2022]**

(a) $-97.5^{\circ}\mathrm{C}$ (b) $-108.5^{\circ}\mathrm{C}$

(c) $45.5^{\circ}\mathrm{C}$ (d) $216.0^{\circ}\mathrm{C}$

13. [In the previous version, the arrow on the segment MN was incorrectly marked. This small typographical error has been fixed. The question and the correct answer were clear from the text.] An ideal gas on the Pressure (P)-Volume (V) diagram can be taken from point K to point N along three different paths, as shown below. $\mathrm{K} \rightarrow \mathrm{L} \rightarrow \mathrm{N}, \mathrm{K} \rightarrow \mathrm{N}$, and $\mathrm{K} \rightarrow \mathrm{M} \rightarrow \mathrm{N}$. Which of the following options is a true statement? **[TIFR-2024]**



(a)There is no work done along the path $\mathrm{K} \rightarrow \mathrm{N}$

(b) The same work is done along each path

(c) The same amount of heat is added to each the gas along each path

(d) The change in internal energy is the same along each path

❖ Answer Key

CSIR-NET				
1. b	2. b	3. d	4. b	5. c
6. d	7. b	8. c	9. a	10. c
11.	12. b	13. b	14. a	15. d
GATE				
1. a	2. a	3. c	4. a	5. d
6. b	7. a	8. a	9. c	10. c
11. c	12. a	13.	14. d	15. a
16. b,c	17. b			
JEST				
1. d	2.	3. d	4. c	5. b
6. c	7. d	8. a	9.	10. a
11. a	12. a	13. 4	14. b	15. d
16. 0672	17. a	18. 17.5	19. a	20. c
21. c				
TIFR				
1.	2. c	3. a	4. d	5. b
6. a	7. c	8. c	9. 066	10. a
11. a	12. a	13. d		

- **Second Low of Thermodynamics**

❖ CSIR-NET PYQ

1. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2, respectively, and $T_1 > T_2$ then their common final temperature is **[CSIR: DEC-2013]**

(a) T_1^2/T_2 (b) T_2^2/T_1

(c) $\sqrt{T_1 T_2}$ (d) $\frac{1}{2}(T_1+T_2)$

2. The heat capacity of (the interior of) a refrigerator is 4.2 kJ/K. The minimum work that must be done to lower the internal temperature from $18^{\circ}\mathrm{C}$ to $17^{\circ}\mathrm{C}$ when the outside temperature is $27^{\circ}\mathrm{C}$ will be **[CSIR: DEC-2015]**

(a) 2.20 kJ (b) 0.80 kJ

(c) 0.30 kJ (d) 0.14 kJ

3. The heat capacity C_V at constant volume of a metal, as a function of temperature, is $\alpha T+\beta T^3$, where α and β are constants. The temperature dependence of the entropy at constant volume is **[CSIR: DEC-2018]**

(a)$\alpha T+\frac{1}{3}\beta T^3$ (b) $\alpha T+\beta T^3$

(c) $\frac{1}{2}\alpha T+\frac{1}{3}\beta T^3$ (d) $\frac{1}{2}\alpha T+\frac{1}{4}\beta T^3$

4. An ideal Carnot engine extracts 100 J from a heat source and dumps 40 J to a heat sink at 300 K. The temperature of the heat source is

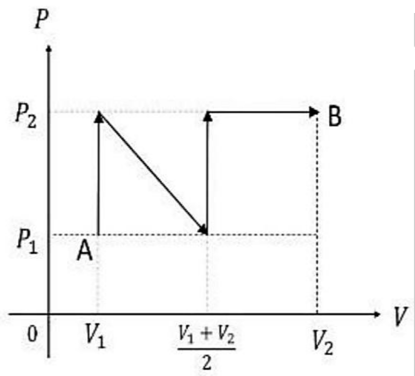
[CSIR: DEC-2019]

(a) 600 K (b) 700 K

(c) 750 K (d) 650 K

5. The following $P-V$ diagram shows a process, where an ideal gas is taken quasi-statically from A to B along the path as shown in the figure.

[CSIR JUNE-2024]



The work done W in this process is

(a) $\frac{1}{4}(V_2-V_1)(3P_2+P_1)$

(b) $\frac{1}{4}(V_2-V_1)(3P_2-P_1)$

(c) $\frac{1}{2}(V_2-V_1)(P_1+P_2)$

(d) $\frac{1}{2}(V_2+V_1)(P_2-P_1)$

❖ **GATE PYQ**

1. For any process, the second law of thermodynamics requires that the change of entropy of the universe be

[GATE: 2004]

(a) positive only (b) positive or zero

(c) zero only (d) negative or zero

2. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperatures of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

[GATE: 2006]

(a) $\frac{3}{2}\,Nk_B\ln\left[\frac{T_1^2+T_2^2}{4\,T_1\,T_2}\right]$ (b) $Nk_B\ln\left[\frac{T_2}{T_1}\right]$

(c) $\frac{3}{2}\,Nk_B\ln\left[\frac{(T_1+T_2)^2}{4\,T_1\,T_2}\right]$ (d) $2\,Nk_B$

3. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22°C by supplying $450\,\mathrm{kJ\,s^{-1}}$. If the outside temperature is 0°C, the heat taken, in $\mathrm{kJs^{-1}}$, from the outside air is approximately

[GATE: 2007]

(a) 487 (b) 470

(c) 467 (d) 417

4. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,

[GATE: 2008]

(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.

(b) the entropy of mixing is $2Nk_B\ln\,2$

(c) the entropy of the final system is less than of sum of the initial entropies of the two gases

(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

5. A Carnot cycle operates on a working substance between two reservoirs at temperatures T_1 and T_2, with $T_1>T_2$. During each cycle, an amount of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is incorrect?

[GATE: 2011]

(a) work done in one cycle is Q_1-Q_2

(b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

(c) entropy of the hotter reservoir decreases

(d) entropy of the universe (consisting of the working substance and the two reservoirs) increases

6. A reversible Carnot engine is operated between temperatures T_1 and $T_2 (T_2 > T_1)$ with a photon gas as the working substance. The efficiency of the engine is **[GATE: 2017]**

(a) $1 - \frac{3T_1}{4T_2}$ (b) $1 - \frac{T_1}{T_2}$

(c) $1 - \left(\frac{T_1}{T_2}\right)^{3/4}$ (d) $1 - \left(\frac{T_1}{T_2}\right)^{4/3}$

7. An air-conditioner maintains the room temperature at 27°C while the outside temperature is 47°C. The heat conducted through the walls of the room from outside to inside due to temperature difference is 7000 W. The minimum work done by the compressor of the air-conditioner per unit time is W.

[GATE: 2018]

8. In a thermally insulated container, 0.01 kg of ice at 273 K is mixed with 0.1 kg of water at 300 K. Neglecting the specific heat of the container, the change in the entropy of the system in J/K on attaining thermal equilibrium (rounded off to two decimal places) is
(Specific heat of water is 4.2 kJ/kg − K and the latent heat of ice is 335 kJ/kg).

[GATE: 2019]

9. Water at $300K$ can be brought to $320K$ using one of the following processes.
Process 1: Water is brought in equilibrium with a reservoir at 320 K directly.
Process 2: Water is first brought in equilibrium with a reservoir at 310 K and then with the reservoir at 320 K.
Process 3: Water is first brought in equilibrium with a reservoir at 350 K and then with the reservoir at 320 K.
The corresponding changes in the entropy of the universe for these processes are $\Delta S_1, \Delta S_2$ and ΔS_3, respectively. Then

[GATE: 2022]

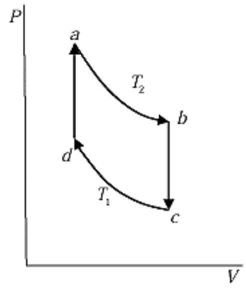
(a) $\Delta S_2 > \Delta S_1 > \Delta S_3$ (b) $\Delta S_3 > \Delta S_1 > \Delta S_2$

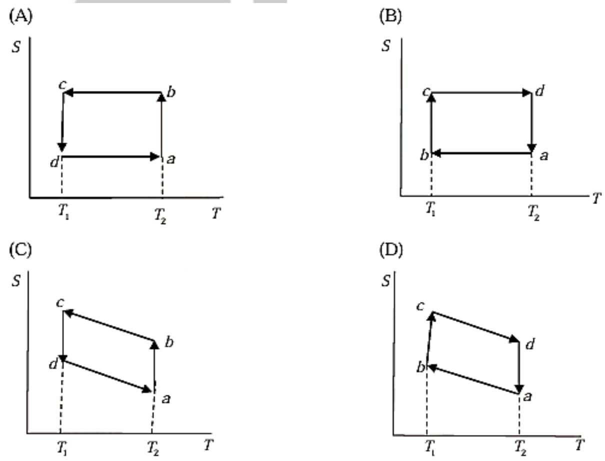
(c) $\Delta S_3 > \Delta S_2 > \Delta S_1$ (d) $\Delta S_1 > \Delta S_2 > \Delta S_3$

10. For 1 mole of Nitrogen gas, the ratio $\left(\frac{\Delta S_I}{\Delta S_{II}}\right)$ of entropy change of the gas in processes (I) and (II) mentioned below is (Round off to one decimal place)
(I) The gas is held at 1 atm and is cooled from 300 K to 77 K.
(II) The gas is liquefied at 77 K.
(Take $C_p = 7.0 \text{calmol}^{-1} K^{-1}$, Latent heat $L = 1293.6 \text{calmol}^{-1}$) **[GATE: 2022]**

11. Which one of the following entropy (S) - temperature (T) diagrams CORRECTLY represents the Carnot cycle $abcda$ shown in the $P - V$ diagram?'

[GATE: 2023]





12. Two identical bodies kept at temperatures 800 K and 200 K act as the hot and the cold reservoirs of an ideal heat engine, respectively. Assume that their heat capacity (c) in Joules/K is independent of temperature and that they do not undergo any phase change. Then, the maximum work that can be obtained from the heat engine is $n \times C$ Joules. What is the value of n (in integer)? **[GAET 2023]**

13. An infinite one dimensional lattice extends along x-axis. At each lattice site there exits an ion with spin $\frac{1}{2}$. The spin can point either in $+z$ or $-z$ direction only. Let S_P, S_F and S_A denote the entropies of paramagnetic, ferromagnetic and antiferromagnetic configurations, respectively. Which of the following relation is/are true? **[GAET 2024]**

(A) $S_P > S_F$ (B) $S_A > S_F$
(C) $S_A = 4S_F$ (D) $S_P > S_A$

14. One mole of an ideal monatomic gas at absolute temperature T undergoes free expansion to double its original volume, so that the entropy change is ΔS_1. An identical amount of the same gas at absolute temperature $2T$ undergoes isothermal expansion to double its original volume, so that the entropy change is ΔS_2. The value of $\frac{\Delta S_1}{\Delta S_2}$ (in integer) is

[GAET 2025]

15. One mole of an ideal monatomic gas at absolute temperature T undergoes free expansion to double its original volume, so that the entropy change is ΔS_1. An identical amount of the same gas at absolute temperature $2T$ undergoes isothermal expansion to double its original volume, so that the entropy change is ΔS_2. The value of $\frac{\Delta S_1}{\Delta S_2}$ (in integer) is

[GAET 2025]

❖ **JEST PYQ**

1. Consider a ideal gas of mass ' m ' at temperature T_1 which is mixed isobarically (i.e. at constant pressure) with an equal mass of same gas at temperature T_2 in a thermally insusulated container. What is the change of entropy of the universe?

[JEST-2012]

(a) $2mC_\rho l\,n\left(\dfrac{T_1+T_2}{2\sqrt{T_1T_2}}\right)$

(b) $2mC_\rho l\,n\left(\dfrac{T_1+T_2}{2T_1T_2}\right)$

(c) $2mC_\rho l\,n\left(\dfrac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

(d) $2mC_\rho \ln\left(\dfrac{T_1-T_2}{2T_1T_2}\right)$

2. Efficiency of a perfectly reversible (Carnot) heat engine operating between absolute temperature T and zero is equal to

[JEST-2012]

(a) 0 (b) 0.5

(c) 0.75 (d) 1

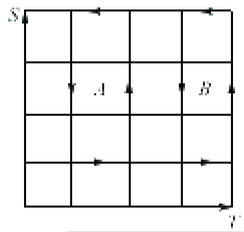
3. What is the contribution of the conduction electrons in the molar entropy of a metal with electronic coefficient of specific heat?

[JEST-2014]

(a) γT (b) γT^2

(c) γT^3 (d) T^4

4. The entropy temperature diagram of two Carnot engines, A and B, are shown in the figure 5 . The efficiencies of the engines are η_A and η_B respectively. Which one of the following equalities is correct? **[JEST-2015]**



(a) $\eta_A = \frac{\eta_B}{2}$ (b) $\eta_A = \eta_B$

(c) $\eta_A = 3\eta_B$ (d) $\eta_A = 2\eta_B$

5. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P = P_0 - \alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to P_0/α. At what value of the volume during the process does the gas have maximum entropy?

[JEST-2016]

(a) $\dfrac{P_0}{\alpha(1+\gamma)}$ (b) $\dfrac{\gamma P_0}{\alpha(1-\gamma)}$

(c) $\frac{\gamma P_0}{\alpha(1+\gamma)}$ (d) $\frac{P_0}{\alpha(1-\gamma)}$

6. A diatomic ideal gas at room temperature, is expanded at a constant pressure P_0. If the heat absorbed by the gas is $Q = 14$ Joules, what is the maximum work in Joules that can be extracted from the system? **[JEST-2019]**

7. M grams of water at temperature T_a is adiabatically mixed with an equal mass of water at temperature T_b, keeping the pressure constant. Find the change in entropy of the system (specific heat of water is C_p).

[JEST-2021]

(a)$\Delta S = MC_p \ln\left[1 - \frac{(T_a - T_b)^2}{4T_aT_b}\right]$

(b)$\Delta S = MC_p \ln\left[1 + \frac{(T_a + T_b)^2}{4T_aT_b}\right]$

(c)$\Delta S = MC_p \ln\left[1 + \frac{(T_a - T_b)^2}{4T_aT_b}\right]$

(d)$\Delta S = MC_p \ln\left[\frac{T_a + T_b}{(4T_aT_b)^{1/2}}\right]$

8. An ideal diatomic gas at pressure P is adiabatically compressed so that its volume becomes $\frac{1}{n}$ times the initial value. The final pressure of the gas will be **[JEST-2022]**

(a) $n^{\frac{7}{5}}P$ (b) $n^{\frac{7}{2}}P$

(c) $n^{-\frac{7}{5}}P$ (d) $n^{\frac{5}{3}}P$

9. A steam engine takes steam from a boiler at 200°C (pressure 1.5×10^6 Pa) and exhausts directly into the air at 100°C (pressure 10^5 Pa). The maximum possible efficiency is closest to:

(a) 78% (b) 21%

(c) 50% (d) 93%

10. One mole of an isolated ideal gas in equilibrium at pressure P_1, volume V_1, and temperature T_1 undergoes a process that changes its state. In the final state the gas is in equilibrium at pressure P_2, volume V_2, and temperature T_2. Suppose the ratios of the final and initial temperatures and pressures are:

$$\eta_T = \frac{T_2}{T_1},\ \eta_P = \frac{P_2}{P_1}.$$

Work out the change in entropy ΔS in the process. Take the heat capacities of the gas at constant pressure and constant volume to be C_p and C_v respectively, and the ideal gas constant to be R.

[JEST-2023]

(a) $C_p \ln(\eta_T) - R\ln(\eta_P)$

(c) $C_v \ln(\eta_T) - R\ln(\eta_P)$

(b) $C_p \ln(\eta_T) + R\ln(\eta_P)$

(d) $C_v \ln(\eta_T) + R\ln(\eta_P)$

11. Given that the latent heat of liquefaction is 80Cal/g, what is the change in entropy when 10 g of ice at 0°C is converted into water at the same temperature?

[JEST-2024]

(a) 4.5CalK^{-1} (b) 3.42CalK^{-1}

(c) 2.0CalK^{-1} (d) 2.93CalK^{-1}

12. The ratio of the molar specific heats of an ideal gas is $\gamma = \frac{c_p}{c_v} = \frac{3}{2}$. It undergoes a reversible isothermal expansion in which its volume doubles. Next, it undergoes a reversible isochoric process such that the change in entropy of the second process is equal to the change in entropy of the first process. What is the ratio of the final temperature to the initial temperature?

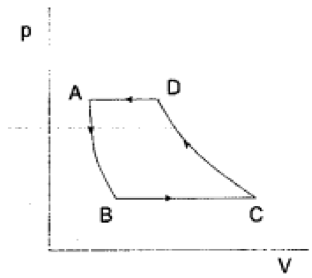
[JEST-2024]

(a) 3 (b) 2

(c) $\sqrt{2}$ (d) $\frac{3}{2}$

❖ **TIFR PYQ**

1. The pV diagram given below represents a



[TIFR 2010]

(a) Carnot refrigerator

(b) Carnot engine

(c) gas turbine refrigerator

(d) gas turbine engine

2. A certain amount of fluid with heat capacity C_F Joules / °C is initially at a temperature 0°C. It is then brought into contact with a heat bath at a temperature of 100°C, and the system is allowed to come into equilibrium. In this process, the entropy (in Joules / °C) of the Universe changes by

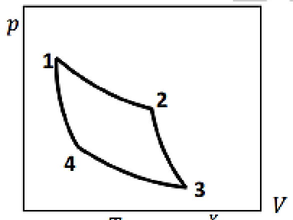
[TIFR-2013]

(a) $100C_F$ (b) 0

(c) $0.055C_F$ (d) $0.044C_F$ ∨

3. The pV-diagram for a Carnot cycle executed by an ideal gas with $C_P/C_V = \gamma > 1$ is shown below. Note that 1,2,3 and 4 label the changeover points

in the cycle.



$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^X$$

If, for this cycle,

[TIFR-2013]

then $X =$

(a) $1 - 1/\gamma$ (b) 0

(c) 1 (d) $-1/\gamma$

4. An system at temperature T has three energy states $0, \pm\varepsilon$. The entropy of the system in the low temperature $(T \to 0)$ and high temperature $(T \to \infty)$ limits are, respectively,

[TIFR-2013]

(a) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_{\mathrm{B}}\exp(-3)$

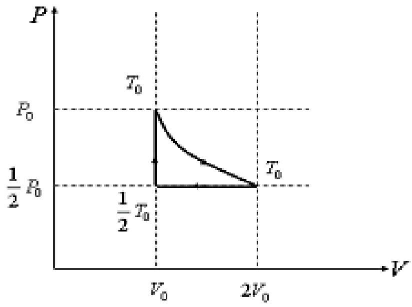
(b) $S_{T\to 0} = S_{T\to\infty} = k_{\mathrm{B}}\ln 3$

(c) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_{\mathrm{B}}\ln 3$

(d) $S_{T\to 0} = 0$ and $S_{T\to\infty} = 3k_{\mathrm{B}}/2$

5. One mole of monoatomic ideal gas is initially at pressure P_0 and volume V_0. The gas then undergoes a three-stage cycle consisting of the following processes:

[TIFR-2017]



An isothermal expansion till it reaches volume $2V_0$, and heat Q flows into the gas
ii) An isobaric compression back to the original volume V_0
iii) An isochoric increase in pressure till The corresponding P − V diagram is shown above. the original pressure P_0 is regained. The efficiency of this cycle can be expressed as

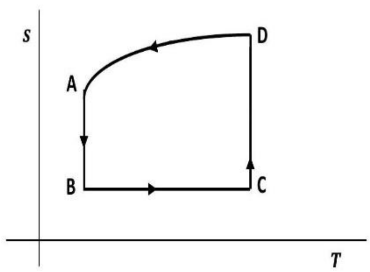
(a) $\epsilon = \dfrac{4Q + 2RT_0}{4Q + RT_0}$ (b) $\epsilon = \dfrac{4Q + 2RT_0}{4Q - 3RT_0}$

(c) $\epsilon = \dfrac{4Q - 2RT_0}{4Q + RT_0}$ (d) $\epsilon = \dfrac{4Q - 2RT_0}{4Q + 3RT_0}$

6. A heat engine is operated between two bodies that are kept at constant pressure. The constant pressure heat capacity C_p of the reservoirs is independent of temperature. Initially the reservoirs are at temperatures 300 K and 402 K. If, after some time, they come to a common final temperature T_f, the process remaining adiabatic, what is the value of T_f (in Kelvin)?

[TIFR-2018]

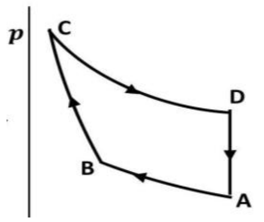
7. An ideal gas engine is run according to the cycle shown in the $s - T$ diagram below, where the process from D to A is known to be isochoric (i.e. maintaining V = constant).



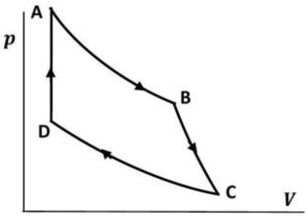
The corresponding cycle in the p - V diagram will most closely resemble

[TIFR-2019]

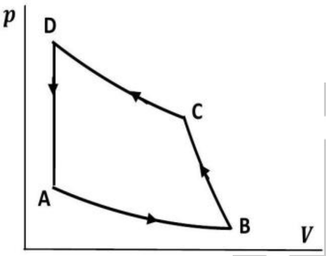
(a)



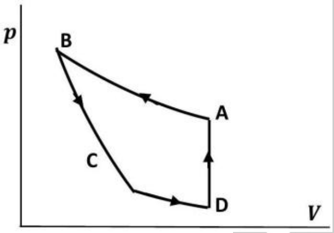
(b)



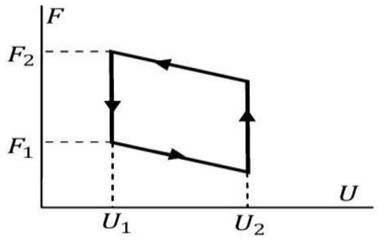
(c)



(d)



8. An ideal gas is passed through a cyclic process where the corresponding changes in the thermodynamic potentials are plotted on the adjoining graph. Here U is the internal energy and F is the Helmholtz free energy.



The efficiency of this cycle is given by

(a) $1-\frac{U_1}{U_2}$

(b) $1-\exp\left(-\frac{F_2}{F_1}\right)$

(c) $1-\frac{U_1}{U_2}\exp\left(-\frac{F_2}{F_1}\right)$

(d) $\exp\left(\frac{U_1}{U_2}\right)-\exp\left(-\frac{F_2}{F_1}\right)$

9. Which of the following is the entropy generated when two identical blocks at temperatures $2T$ and T are brought into thermal contact and allowed to reach equilibrium?
[Assume that the heat capacity of each block is C]

[TIFR-2021]

(a) $C(2\ln 3-3\ln 2)$ (b) zero

(c) $2C\ln\frac{3}{2}$ (d) $C(2\ln 2-3\ln 3)$

10. A mass of M kg of water at temperature T_a is isobarically and adiabatically mixed with an equal mass of water at temperature T_b. The specific heat of water at constant pressure is C_p. What is the entropy change (ΔS) of the system?

[TIFR-2024]

(a) $\Delta S=MC_p\ln\left\{1+\frac{(T_a-T_b)^2}{4T_aT_b}\right\}$

(b) $\Delta S=MC_p\ln\left\{1-\frac{(T_a+T_b)^2}{4T_aT_b}\right\}$

(c) $\Delta S=MC_p\ln\left\{1+\frac{4T_aT_b}{(T_a-T_b)^2}\right\}$

(d) $\Delta S=MC_p\ln\left\{\frac{T_a+T_b}{\sqrt{T_aT_b}}\right\}$

11. Two types of particles A and B have the same mass, but are distinguished by an internal degree of freedom. A classical ideal gas in a volume V at temperature T contains $(X)2N$ particles of A-type and $(Y)N$ particles of A-type and N particles of B-type. Which of the following is true? **[TIFR-2025]**

(a) Pressure of (X) and (Y) are same; (Y) has more entropy than (X)
(b) Pressure of (X) and (Y) are same; (X) has more entropy than (Y)
(c) Pressure of (X) is greater than pressure of (Y); (X) has more entropy than (Y)
(d) Pressure of (X) is greater than pressure of (Y); (Y) has more entropy than (X)

❖ Answer Key				
CSIR-NET				
1. c	2. d	3. a	4. c	5. a
GATE				
1. b	2. c	3. d	4. b	5. d
6. b	7. 466.67	8. 1.03	9. b	10. 0.5
11. a	12. 200	13. a	14. 1	15.
JEST				
1. a	2. d	3. a	4. d	5. c
6. 4	7. c	8. a	9. b	10. a
11. d	12. c			
TIFR				
1. a	2. d	3. a	4. c	5. d
6. 347	7. a	8. a	9. a	10. a
11. a				

Kinetic Theory Of Gases

❖ **CSIR-NET PYQ**

1. A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $u(x) = u_0 x$. The mean position of the particle at temperature T is: **[CSIR: JUNE-2011]**

(a) $\frac{k_B T}{u_0}$ (b) $(k_B T)^2/u_0$

(c) $\sqrt{\frac{k_B T}{u_0}}$ (d) $u_0 k_B T$

2. Consider a Maxwellian distribution of the velocity of the molecules of an ideal gas. Let V_{mp} and V_{mmz} denote the most probable velocity and the root mean square velocity, respectively. The magnitude of the ratio $V_{\text{mp}}/V_{\text{rms}}$ is: **[CSIR: DEC-2011]**

(a) 1 (b) 2/3

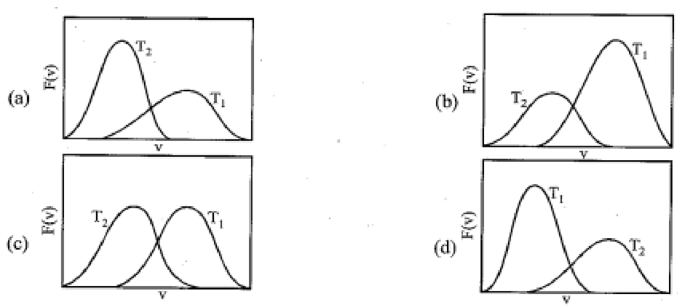
(c) $\sqrt{2/3}$ (d) 3/2

3. Gas molecules of mass ' m ' are confined in a cylinder of radius R and height L (with $R > L$) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that mgL >> k_BT) is given by **[CSIR: DEC-2011]**

(a) $Nk_BT/2$ (b) $3Nk_BT/2$

(c) $2Nk_BT$ (d) $5Nk_{\bar{n}}T/2$

4. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution F(v) of the speed v of a molecule in three dimensions is correctly represented by the following figure: **[CSIR: JUNE-2013]**



5. The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is **[CSIR: DEC-2013]**

(a) $(\alpha^2 - \beta^2)k_\beta T/m$ (b) $(\alpha^2 + \beta^2)k_\beta T/m$

(c) $(\alpha + \beta)^2 \cdot k_\beta T/m$ (d) $(\alpha - \beta)^2 k_\beta T/m$

6. A system of N classical non-interacting particles, each of mass m, is at n temperature T and is confined by the external potential $V(r) = \frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is **[CSIR: DEC-2013]**

(a) $3Nk_BT$ (b) $\frac{3}{2}Nk_{\bar{B}}T$

(c) $N(2mA)^{3/2}k_nT$ (d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_nT}{m}\right)$

7. Two different thermodynamic systems are described by the following equations of state:

$$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}}$$

and

$$\frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$$

where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures; the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{sat} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{\text{tot}}}$ is **[CSIR: DEC-2013]**

(a) $\frac{5N^{(2)}}{3N^{(1)} + 5N^{(2)}}$ (b) $\frac{3N^{(1)}}{3N^{(1)} + 5N^{(2)}}$

(c) $\frac{N^{(1)}}{N^{(1)} + N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)} + N^{(2)}}$

8. In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is

[CSIR: DEC-2014]

(a) $\frac{1}{2}k_B$ (b) k_B

(c) $\frac{3}{2}k_B$ (d) $\frac{5}{2}k_B$

9. Consider a gas of Cs atoms at a number density of 10^{12} atoms/cc. When the typical inter-particle distance is equal to the thermal de-Broglie wavelength of the particles, the temperature of the gas is nearest to (Take the mass of a Cs atom to be 22.7×10^{-26} kg).

[CSIR: JUNE-2016]

(a) 1×10^{-9} K (b) 7×10^{-5} K

(c) 1×10^{-3} K (d) 2×10^{-8} K

10. The specific heat per molecule of a gas of diatomic molecules at high temperatures is

[CSIR: JUNE-2016]

(a) $8k_B$ (b) $3.5k_B$

(c) $4.5k_B$ (d) $3k_B$

11. The Hamiltonian of a classical nonlinear one-dimensional oscillator is

$$H = \frac{1}{2m}p^2 + \lambda x^4,$$

where $\lambda > 0$ is a constant. The specific heat of a collection of N independent such oscillators is

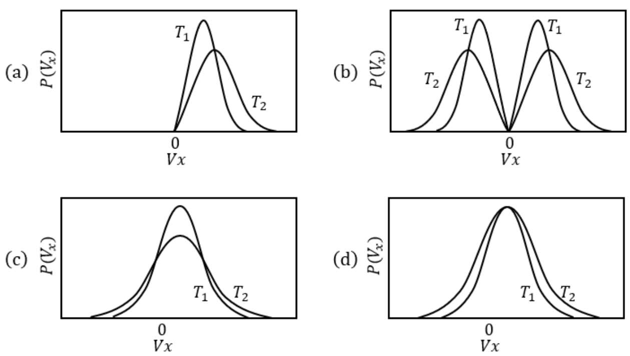
[CSIR: JUNE-2019]

(a) $\frac{3Nk_B}{2}$ (b) $\frac{3Nk_p}{4}$

(c) Nk_B (d) $\frac{Nk_B}{2}$

12. Consider a classical gas in thermal equilibrium at temperatures T_1 and T_2. Which of the following graphs correctly represents the qualitative behaviour of the probability density function of the x component of the velocity?

[CSIR: JUNE-2019]



13. The Hamiltonian of a system of N non-interacting particles, each of mass m, in one dimension is

$$H = \sum_{i=1}^{N}\left(\frac{p_i^2}{2m} + \frac{\lambda}{4}x_i^4\right)$$

where $\lambda > 0$ is a constant and p_i and x_i are the momentum and position respectively of the i th particle. The average internal energy of the system is

[CSIR: JUNE-2020]

(a) $\frac{4}{3}k_BT$ (b) $\frac{3}{4}k_BT$

(c) $\frac{3}{2}k_BT$ (d) $\frac{1}{3}k_BT$

14. The ratio $\frac{C_P}{C_V}$ of the specific heats at constant pressure and volume of a monoatomic ideal gas in two dimensions is

[CSIR: JUNE -2021]

(a) $\frac{3}{2}$ (b) 2

(c) $\frac{5}{3}$ (d) $\frac{5}{2}$

15. A system of N non-interacting particles in one-dimension, each of which is in a potential $V(x) = gx^6$ where $g > 0$ is a constant and x denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

[CSIR: JUNE-2O22]

(a) $\frac{7}{6}Nk_B$ (b) $\frac{4}{3}Nk_B$

(c) $\frac{3}{2}Nk_B$, (d) $\frac{2}{3}Nk_B$**y**

16. Consider two independently diffusing non-interacting particles in 3-dimensional space, both placed at the origin at time $t = 0$. These particles have different diffusion constants D_1 and D_2. The quantity $\left\langle\left[\vec{R}_1(t) - \vec{R}_2(t)\right]^2\right\rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

[CSIR JUNE 2011]

(a) $6t(D_1 + D_2)$ (b) $6t(D_1 - D_2)$

(c) $6t\sqrt{D_1^2 + D_2^2}$ (d) $6t\sqrt{D_1D_2}$

17. A large number N of Brownian particles in one-dimension start their diffusive motion from the origin at time $t=0$. The diffusion coefficients is D. The number of particles crossing a point at a distance L from the origin, per unit time, depends on L and time t as

[CSIR JUNE 2015]

(a) $\frac{N}{\sqrt{4\pi Dt}} e^{-L^2/(4Dt)}$ (b) $\frac{NL}{\sqrt{4\pi Dt}} e^{-4DtI^t}$

(c) $\frac{N}{\sqrt{16\pi Dt^3}} e^{-l^2/(40r)}$ (d) $\mathrm{Ne}^{-4Dti\,L^2}$

18. Consider a particle diffusing in a liquid contained in a large box. The diffusion constant of the particle in the liquid is $1.0\times10^{-2}\ \mathrm{cm^2/s}$. The minimum time after which the root-mean-squared displacement becomes more than 6 cm is

[CSIR JUNE 2018]

(a) 10 min (b) 6 min

(c) 30 min (d) $\sqrt{6}$ min

19. Consider one mole of an ideal diatomic gas molecule at temperature T such that $k_BT \gg hv$, where v is the frequency of its vibrational mode. If C_p and C_v are specific heats of this gas at constant pressure and volume respectively, then the ratio $\gamma=\frac{C_p}{C_v}$, is

[CSIR JUNE 2025]

(a) 2 (b) $\frac{7}{5}$

(c) $\frac{5}{3}$ (d) $\frac{9}{7}$

❖ **GATE PYQ**

1. The ratio $\gamma = \mathrm{C_p/C_V}$ of H_2 gas is 7/5 at room temperature, and 5/3 at low temperatures (around 50 K).
The reason for this change is

[GATE 1992]

(a) Dissociation of H_2 molecules

(b) Suppression of rotational degrees of freedom

(c) Formation fo random clusters of molecules.

(d) Liquefaction of the gas.

2. If $\bar{v}, v_p$ and v_{rms} denote the average, most probable, and root mean square values respectively of the molecular speeds of a gas at room temperature being Maxwellian velocity distribution, then **[GATE 1994]**

(a) $\mathrm{v}_{rms} < \overline{\mathrm{V}} < \mathrm{v}_p$ (b) $\mathrm{v}_{rms} < \mathrm{v}_p < \bar{\mathrm{v}}$

(c) $\bar{\mathrm{v}} < \mathrm{v}_{rms} < \mathrm{v}_p$ (d) $\mathrm{v}_p < \overline{\mathrm{V}} < \mathrm{V}_{rms}$

3. Mean total energy of a classical three-dimensional harmonical oscillator in equilibrium with a heat reservoir at temperature T is: **[GATE 1997]**

(a) $\mathrm{k_BT}$ (b) $\frac{3}{2}\mathrm{k_BT}$

(c) $2\mathrm{k_BT}$ (d) $3\mathrm{k_BT}$

4. The root-mean square speed of a particle of mass m in the kinetic theory is given by

[GATE 1997]

(a) $\sqrt{\frac{k_BT}{m}}$ (b) $\sqrt{\frac{2\mathrm{k_pT}}{\mathrm{m}}}$

(c) $\sqrt{\frac{3\mathrm{k_BT}}{\mathrm{m}}}$ (d) $\sqrt{\frac{8k_BT}{m}}$

5. The mean free path of the particles of a gas at temperature $\mathrm{T_0}$ and pressure $\mathrm{p_0}$ has a value λ_0. If the pressure is increased to $1.5P_0$ and the temperanure is reduced to $0.75T_0$, the mean fiee path **[GATE 2002]**

(a) remains unchanged.

(b) is reduced to half

(c) is doubled

(d) is equal to 1.125λ.

6. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is

[GATE: 2006]

(a) $\frac{1}{2}k_BT$ (b) k_BT

(c) $\frac{3}{2}k_BT$ (d) $3k_BT$

7. A system of N non-interacting classical point particles is constrained to move on the two dimensional surface of a sphere. The internal energy of the system is

[GATE: 2010]

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$

(c) Nk_BT (d) $\frac{5}{2}Nk_BT$

8. A classical gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $\left(v_x+v_y\right)^2$ is

[GATE: 2012]

(a) $\frac{k_BT}{m}$ (b) $\frac{3}{2}\frac{k_BT}{m}$

(c) $\frac{1}{2}\frac{k_BT}{m}$ (d) $\frac{2k_BT}{m}$

9. At a certain temperature T, the average speed of nitrogen molecules in air is found to be $400\ \mathrm{m/s}$. The most probable and the root mean square speeds of the molecules are, respectively

[GATE: 2012]

(a) $355\ \mathrm{m/s}, 434\ \mathrm{m/s}$

(b) $820\ \mathrm{m/s}, 917\ \mathrm{m/s}$

(c) $152\ \mathrm{m/s}, 301\ \mathrm{m/s}$

(d) $422\ \mathrm{m/s}, 600\ \mathrm{m/s}$

10. Consider a gas of atoms obeying Maxwell Boltzmann statistics. The average value of $e^{i\vec{a}\cdot\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is

[GATE: 2013]

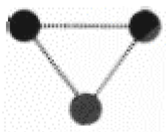
(a) one (b) zero

(c) $e^{-\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

11. At a given temperature T the average energy per particle of a non-interacting gas of two dimensional classical harmonic oscillator is.......... k_BT (k_B is the Boltzmann constant)

12. Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is:

[GATE: 2017]



(a) $\frac{3}{2}k_B$ (b) $3k_B$

(c) $\frac{9}{2}k_B$ (d) $6k_B$

13. Hydrogen molecules (mass m) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is

[GATE: 2003]

(a) $(k_BT)/m$ (b) $2k_BT/m$

(c) $\left(\sqrt{2k_BT/m}\right)$ (d) $m/\left(\sqrt{2}k_BT\right)$

14. A classical gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $\left(v_x+v_y\right)^2$ is

[GATE: 2012]

(a) $\frac{k_BT}{m}$ (b) $\frac{3}{2}\frac{k_BT}{m}$

(c) $\frac{1}{2}\frac{k_BT}{m}$ (d) $\frac{2k_BT}{m}$

15. At a certain temperature T, the average speed of nitrogen molecules in air is found to be $400\ \mathrm{m/s}$. The most probable and the root mean square speeds of the molecules are, respectively

[GATE: 2012]

(a) $355\ \mathrm{m/s}, 434\ \mathrm{m/s}$

(b) 820 m/s, 917 m/s

(c) 152 m/s, 301 m/s

(d) 422 m/s, 600 m/s

16. The mean free path of the particles of a gas at a temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to $1.5p_0$ and the temperature is reduced to 0.75 T_0, the mean free path

[GATE 2002]

(a) remains unchanged
(b) is reduced to half

(c) is doubled

(d) is equal to $1.125\lambda_0$

17. The quantum effects in an ideal gas become important below a certain temperature T_Q when de Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass 2×10^{-26} kg and number density $6.4 \times 10^{25}\ \text{m}^{-3}$, $T_Q = \ \times 10^{-3}$ K (up to one decimal place).

[GATE 2018]

18. For a gas of non-interacting particles, the probability that a particle has a speed v in the internal v to $v + dv$ is given by

$$f(v)dv = 4\pi v^2 dv \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT}$$

If E is the energy of a particle, then the maximum in the corresponding energy distribution in units of E/k_BT occurs at (rounded off to one decimal place).

[GATE 2020]

❖ **JEST PYQ**

1. For which gas the ratio of specific heats (C_p/C_v) will be largest?

[JEST-2014]

(a) mono-atomic (b) di-atomic

(c) tri-atomic (d) hexa-atomic

2. Electrons of mass m in a thin, long wire at a temperature T follow a one-dimensional Maxwellian velocity distribution. The most probable speed of these electrons is,

[JEST-2015]

(a) $\sqrt{\left(\frac{kT}{2\pi m}\right)}$ (b) $\sqrt{\left(\frac{kT}{m}\right)}$

(c) 0 (d) $\sqrt{\left(\frac{8kT}{\pi m}\right)}$

3. If the Hamiltonian of classical particle is

$$H = \frac{p_x^2 + p_y^2}{2m} + xy$$

then $\langle x^2 + xy + y^2 \rangle$ at temperature T is equal to

[JEST-2017]

(a) k_BT (b) $\frac{1}{2}k_BT$

(c) $2k_BT$ (d) $\frac{3}{2}k_BT$

4. If the mean square fluctuations in energy of a system in equilibrium at temperature T is proportional to T^α, then the energy of the system is proportional to **[JEST-2017]**

(a) $T^{\alpha-2}$ (b) $T^{\frac{\alpha}{2}}$

(c) $T^{\alpha-1}$ (d) T^α

5. Let a particle of mass 1×10^{-9} kg, constrained to have one dimensional motion, be initially at the origin ($x = 0$ m). The particle is in equilibrium with a thermal bath ($kBT = 10^{-8} J$). What is $\langle x^2 \rangle$ of the particle after a time t = 5 s ? **[JEST-2017]**

6. For a classical system of non-interacting particles in the presence of a spherically symmetric potential V (r) $= \gamma r^3$, what is the mean energy per particle? γ is a constant.

[JEST-2018]

(a) $\frac{3}{2}k_BT$ (b) $\frac{5}{2}k_BT$

(c) $\frac{3}{2}\gamma k_BT$ (d) $\frac{3}{2}\gamma k_BT$

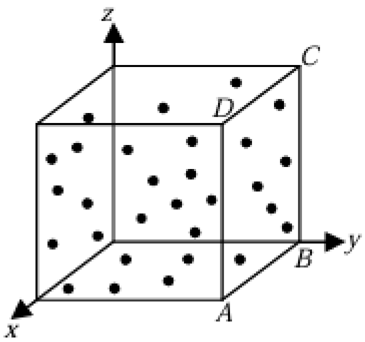
7. A particle of mass m is placed in a potential well $U(x) = cX^n$, where c is a positive constant and n is an even positive integer. If the particle is in equilibrium at constant temperature, which one of the following relations between average kinetic energy $\langle K\rangle$ and average potential energy $\langle U\rangle$ is correct? **[JEST-2020]**

(a) $\langle K\rangle=\frac{2}{n}\langle U\rangle$ (b) $\langle K\rangle=\langle U\rangle$

(c) $\langle K\rangle=\frac{n}{2}\langle U\rangle$ (d) $\langle K\rangle=2\langle U\rangle$

8. Consider a cube (see figure) of volume V containing N molecules each of mass m with uniform density $n=\frac{N}{V}$. Suppose this system is equivalent to a system of M non-interacting gases such that molecules of the ith gas are $N_i=n_iV$ in number, each with an identical y component of velocity v_i. What is the pressure P on the surface $\square\, ABCD$ of area A ?

[JEST-2020]



(a) $P=m\sum_{i=1}^{M}n_iv_i^2$ (b) $P=\frac{m\sum_{i=1}^{M}n_iv_i^2}{\sum_{i=1}^{M}n_i}$

(c) $P=\frac{m\sum_{i=1}^{M}n_iV_i^2}{2}$ (d) $P=2m\sum_{i=1}^{M}n_iV_i^2$

9. If mean and standard deviation of the energy distribution of an equilibrium system vary with temperature T as T^{ν} and T^{α} respectively, then ν and α must satisfy

[JEST-2022]

(a) $2\nu+1=\alpha$ (b) $\nu+1=2\alpha$

(c) $\nu=1+2\alpha$ (d) $2\nu=1+\alpha$

10. A system of N classical non-identical particles moving in one dimensional space is governed by the Hamiltonian

$$H=\sum_{i=1}^{N}\left(A_ip_i^2+B_i|q_i|^{\alpha}\right),$$

where p_i and q_i are momentum and position of the i-th particle, respectively, and the constant parameters A_i and B_i characterize the individual particles. When the system is in equilibrium at temperature T, then the internal energy is found to be

$$E=\langle H\rangle=\frac{2}{3}Nk_BT,$$

where k_B is the Boltzmann constant. What is the value of α ? **[JEST-2022]**

11. The ratio of specific heat of electrons in a heated copper wire at two temperatures 200°C and 100°C is **[JEST-2022]**

(a) 1.61 (b) 2

(c) 1.41 (d) 1.27

12. Consider a system of classical non-interacting particles constrained to be in the XY plane subject to the potential:

$$V(x,y)=\frac{1}{2}\alpha(x-y)^2.$$

If they are in equilibrium with a thermal bath at temperature T, what is the average energy per particle? The Boltzmann constant is k_B.

[JEST-2023]

(a) $\frac{5}{2}k_BT$ (b) $\frac{1}{2}k_BT$

(c) $2k_BT$ (d) $\frac{3}{2}k_BT$

13. A gas is in equilibrium at temperature T. Using the kinetic theory of gasses, compute the following quantity:

$$\frac{\langle|\vec{v}|\rangle^2}{\langle|v_x|\rangle^2+\langle|v_y|\rangle^2+\langle|v_z|\rangle^2}$$

where $\vec{v}$ represents the velocity vector with the components v_x,v_y,v_z and $\langle\ldots\rangle$ represents the thermal average of the quantity.

[JEST-2023]

(a) 0 (b) $\frac{4}{3}$

(c) 1 (d) $\frac{1}{3}$

14. The speed distribution of the molecules of an ideal gas in equilibrium at inverse temperature $\beta\left(=\frac{1}{k_BT}\right)$ is found to obey the Maxwell distribution:

$$P(v) = Cv^2 \exp\left(-\frac{1}{2}\beta m v^2\right)$$

where m is the mass of a molecule and C is a normalization constant. Compute $(\langle v^4\rangle)^{1/4}$.

[JEST-2024]

(a) $\sqrt{\frac{4k_BT}{m}}$ (b) $\sqrt{\frac{\sqrt{15}k_BT}{m}}$

(c) $\sqrt{\frac{3k_BT}{m}}$ (d) $\sqrt{\frac{11k_BT}{\pi m}}$

❖ **TIFR PYQ**

1. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat C_V of CO gas is given in terms of the Boltzmann constant k_B by

[TIFR-2014]

(a) $\frac{5}{2}k_B$ (b) $2k_B$

(c) $\frac{3}{2}k_B$ (d) $\frac{1}{2}k_B$

2. A gas of non-interacting particles, each of rest mass 1MeV, is at a temperature $T = 2.0 \times 10^7$ K and has an average particle density $n = 2.7 \times 10^{34}\ \text{cm}^{-3}$. We can obtain a reasonably correct treatment of this system

[TIFR 2016]

(a) only by using special relativity as well as quantum mechanics.

(b) by neglecting quantum mechanics but not special theory of relativity.

(c) by neglecting special relativity but not quantum mechanics.

(d) by neglecting both special relativity and quantum mechanics.

3. Two containers are maintained at the same temperature and are filled with ideal gases whose molecules have mass m_1 and m_2 respectively. The mean speed of molecules of the second gas is 10 times the r.m.s. speed of the molecules of the first gas. Find the ratio of m_1/m_2, to the nearest integer.

[TIFR-2016]

4. A classical ideal gas of atoms with masses m is confined in a three-dimensional potential

$$V(x,y,z) = \frac{\lambda}{2}(x^2 + y^2 + z^2)$$

at a temperature T. If k_B is the Boltzmann constant, the root mean square (r.m.s.) distance of the atoms from the origin is

[TIFR-2018]

(a) $\left(\frac{3k_BT}{\lambda}\right)^{1/2}$ (b) $\left(\frac{3k_BT}{2\lambda}\right)^{1/2}$

(c) $\left(\frac{2k_BT}{3\lambda}\right)^{1/2}$ (d) $\left(\frac{k_BT}{\lambda}\right)^{1/2}$

5. The mean free path λ of molecules of a gas at room temperatures is given approximately by

$$\lambda = \frac{1}{n\sigma}$$

where n is the number density of the molecules and σ is the collision cross-section of two molecules. It follows that the mean free path of air molecules at normal temperature and pressure is of order

[TIFR 2020]

(a) 500μm (b) 500 nm

(c) 0.5 nm (d) 500fm

6. Consider two ideal gases A and B with atomic masses m_A and m_B respectively such that $m_A > m_B$. The two gases with same number of moles are kept at the same temperature and confined in containers with the same volume. Which of the gases will exert more pressure and molecules of which gas will have a higher RMS momentum?

[TIFR 2025]

(a) Both will exert the same pressure but molecules of Gas A will have more RMS momentum

(b) Gas A will exert more pressure and molecules of Gas B will have more RMS momentum

(c) Gas B will exert more pressure but molecules of Gas A will have more RMS momentum

(d) Both will exert the same pressure and molecules of both gases have the same RMS momentum

❖ Answer Key				
CSIR-NET				
1. a	2. c	3. d	4. a	5. b
6. a	7. b	8. b	9. c	10. bc
11. b	12. c	13. b	14. b	15. d
16. a	17. a	18. a		
GATE				
1. b	2. d	3. d	4. c	5. b
6. b	7. c	8. d	9. a	10. c
11. c	12. d	13. c	14. d	15. a
16. b	17. 84.2	18. 0.5		
JEST				
1. a	2. c	3. a	4. c	5. 250
6. b	7. c	8. a	9. b	10. c
11. d	12. d	13. b	14. b	
TIFR				
1. a	2. c	3. 118	4. a	5. b
6. a				

❖ **CSIR-NET PYQ**

1. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?

[CSIR: JUNE-2011]

(a) The Gibbs free energy

(c) The entropy

(b) The internal energy

(d) The specific volume

2. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?

[CSIR: JUNE-2013]

(a) internal energy

(b) Helmholtz free energy

(c) Gibbs free energy

(d) entropy

3. The van der Waals equation of state for a gas is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)-RT$$

where, P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

[CSIR: JUNE-2014]

(a) $\frac{a}{9b}$ (b) $\frac{a}{27b^2}$

(c) $\frac{8a}{27bR}$ (d) $3b$

4. The free energy F of a system depends on a thermodynamics variable ψ as

$$F=-\alpha\psi^2+b\psi^6$$

with a, b > 0. The value of ψ, when the system is in thermodynamic equilibrium, is

[CSIR: JUNE-2014]

(a) zero (b) $\pm(a/6b)^{1/4}$

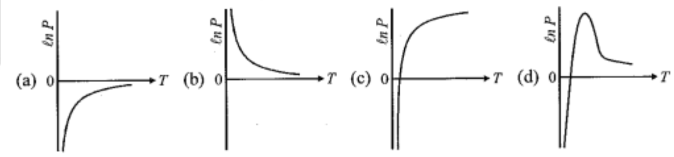
(c) $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

5. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation

$$\frac{dP}{dT}\approx\frac{Q_l}{Tv_{\text{vap}}}$$

(Clausius-Clayperon equation), where v_{vp} is the volume per particle in the vapour phase, and Q_l is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct?

[CSIR: JUNE-2015]



6. The pressure P of a system of N particles contained in a volume V at a temperature T is given by $P=nk_BT-\frac{1}{2}an^2+\frac{1}{6}bn^3$ where n is the number density and a and b are temperature independent constants. If the system exhibits a gas-liquid transition, the critical temperature is

[CSIR: JUNE-2018]

(a) $\frac{a}{bk_B}$ (b) $\frac{a}{2b^2k_B}$

(c) $\frac{a^2}{2bk_B}$ (d) $\frac{a^2}{b^2k_B}$

7. The pressure p of a gas depends on the number density ρ of particles and the temperature T as $p=k_BT\rho-B_2\rho^2+B_3\rho^3$, where B_2 and B_3 are positive constants. Let T_c, ρ_c and p_c denote the critical temperature, critical number density and critical pressure, rec spectively. The ratio $\rho_ck_BT_c/p_c$ is equal to

[CSIR: DEC-2019]

(a) 1/3 (b) 3

(c) 8/3 (d) 4

❖ **GATE PYQ**

1. A quantity of water is completely converted into steam by boiling.

[GATE 1991]

(a) This is a second order phase transition.

(b) At the boiling point, the chemical potential in the vapour phase is less than that in the liquid phase.

(c) The boiling temperature would increase if the boiling were done under increased pressure.

(d) The entropy of the steam is greater than that of the water.

2. The sublimation curve of solid ammonia is given by $\ln\ p = 23 - 3750/\mathrm{T}$ and the vaporization curve of the liquid ammonia is given by $\ln\ \mathrm{p} = 19.5 - 3050/\mathrm{T}$, where p is in mm of Hg and T is in K . The temperature of the triple point of ammonia is:

[GATE 1998]

(a) 3750 K (b) 3050 K

(c) 700 K (d) 200 K

3. The boiling point of a liquid at pressure P_0 is T_0. Its molar latent heat of vaporisation is L and molar volume of the liquid phase is negligible as compared to the vapour phase. The vapour phase obeys perfect gas equation. The boiling point T at pressure P is given by

[GATE 1999]

(a) $\mathrm{P}/\mathrm{P}_0 = (L/RT_0)(1 + \mathrm{T}_0, \mathrm{T})$

(b) $\ln\ (P/P_0) = (L/RT_0)(1 - T_0/T)$

(c) $\ln\ (\mathrm{P}/\mathrm{P}_0) = (\mathrm{L}/\mathrm{RT}_0)(1 - \mathrm{T}/\mathrm{T}_0)$

(d) $\mathrm{P}/\mathrm{P}_0 = 1 + (\mathrm{L}/\mathrm{RT}_0)\ln\ (\mathrm{T}_0/\mathrm{T})$

4. Which of the following is an example of a first order phase transition?

[GATE 2000]

(a) A liquid - gas phase transition at the critical point.

(b) A paramagnet - ferromagnet phase transition

(c) A normal metal-superconductor phase transition.

(d) A liquid - gas phase transition away from the critical! point.

5. A second order phase transition is one in which

[GATE: 2003]

(a) the plot of entropy as a function of temperature shows a discontinuity

(b) the plot of specific heat as a function of temperature shows a discontinuity

(c) the plot of volume as a function of pressure shows a discontinuity

(d) the plot of comprehensibility as a function of temperature is continuous

6. Which one of the following is a first order phase transition? **[GATE: 2004]**

(a) Vaporization of a liquid at its boiling point

(b) Ferromagnetic to paramagnetic

(c) Normal liquid He to superfluid He

(d) Superconducting to normal state

7. In the region of co-existence of a liquid and vapor phases of a material

[GATE: 2004]

(a) C_p and C_v are both infinite

(b) C_V and $\beta \left[= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right]$ are both finite

(c) C_v and $K \left[= -\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_T\right]$ are both finite

(d) C_p, β and K are all infinite

8. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ln\ p = 23 - 3863/T$ and that of its liquid phase by $\ln\ p = 19 - 3063/T$. The triple point (in Kelvin) of the material is

[GATE: 2007]

(a) 185 (b) 190

(c) 195 (d) 200

9. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid to solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from above is **[GATE: 2008]**

(a) the entropy of solid is zero in this temperature region

(b) the entropy increases when the system goes from liquid to solid phase in this temperature region

(c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature

(d) the change in entropy is zero in the liquid to solid transition region

10. Identify which one is a first order phase transition? **[GATE: 2009]**

(a) A liquid to gas transition at its critical temperature

(b) A liquid to gas transition close to its triple point

(c) A paramagnetic to ferromagnetic transition in the absence of a magnetic field

(d) A metal to superconductor transition in the absence of a magnetic field

11. In a first order phase transition at the transition temperature specific heat of the system **[GATE: 2011]**

(a) diverges and its entropy remains the same

(b) diverges and its entropy has finite discontinuity

(c) remains unchanged and its entropy has finite discontinuity

(d) has finite discontinuity and its entropy diverges

12. Across a first order phase transition, the free energy is **[GATE: 2013]**

(a) proportional to the temperature

(b) a discontinuous function of the temperature

(c) a continuous function of the temperature but its first derivative is discontinuous

(d) such that the first derivative with respect to temperature is continuous

13. Water freezes at 0°C atmospheric pressure (1.01×10^5 Pa). The densities of water and ice at this temperature and pressure are 1000 kg/m^3 and 934 kg/m^3 respectively. The latent heat of fusion is 3.34×10^5 J/kg. The pressure required for depressing the melting temperature of ice by 10°C is______ GPa. (up to two decimal places) **[GATE: 2017]**

14. Which of the following is (are) the CORRECT option(s) for the Joule-Thomson effect? **[GATE: 2023]**

(a) It is an isentropic process

(b) It is an isenthalpic process

(c) It can result in cooling as well as heating

(d) For an ideal gas it always results in cooling

15. The vapor pressure (P) of solid ammonia is given by $\ln(P) = 23.03 - \frac{3754}{T}$, while that of liquid ammonia is given by $\ln(P) = 19.49 - \frac{3063}{T}$, where T is the temperature in K .
The temperature of the triple point of ammonia is K (rounded off to two decimal places). **[GATE: 2024]**

❖ **JEST PYQ**

1. Ice of density ρ_1 melts at pressure P and absolute temperature T to form water of density ρ_2. The latent heat of melting of 1 gram of ice is L. What is the change in the internal energy ΔU

resulting from the melting point of 1 gram of ice?

[JEST-2014]

(a) $L+P\left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)$ (b) $L-P\left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)$

(c) $L-P\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)$ (d) $L+P\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)$

2. Given that the latent heat of liquefaction is 80Cal/g, what is the change in entropy when 10 g of ice at $0^{\circ}\mathrm{C}$ is converted into water at the same temperature?

[JEST-2024]

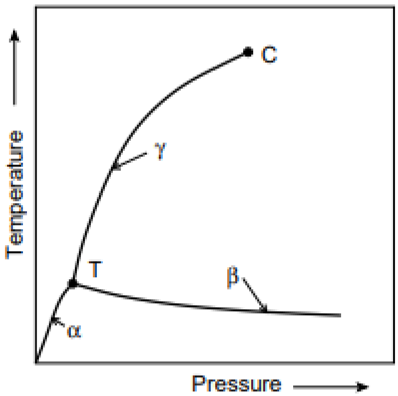
(a) $4.5\mathrm{CalK}^{-1}$ (b) $3.42\mathrm{CalK}^{-1}$

(c) $2.0\mathrm{CalK}^{-1}$ (d) $2.93\mathrm{CalK}^{-1}$

❖ **TIFR PYQ**

1. The phase diagram of a pure substance is given in the figure below, where ' T ' denotes the triple point and ' C ' denotes the critical point.

[TIFR-2011]



The phase transitions occurring along the lines marked α,β and γ are

(a) $\alpha=$ melting; $\beta=$ condensation; $\gamma=$ sublimation

(b) $\alpha=$ sublimation; $\beta=$ vaporisation; $\gamma=$ melting;

(c) $\alpha=$ melting; $\beta=$ vaporisation; $\gamma=$ condensation

(d) $\alpha=$ sublimation; $\beta=$ melting; $\gamma=$ vaporisation

2. A monoatomic gas is described by the equation of state

$$p(V-bn)=nRT$$

where b and R are constants and other quantities have their usual meanings. The maximum density (in moles per unit volume) to which this gas can be compressed is

[TIFR-2013]

(a) $1/bn$ (b) b

(c) $1/b$ (d) infinity

3. The equation of state for a gas is given by

$$\left[p+\left(\frac{\alpha N}{V}\right)^2\right](V-\beta N)=Nk_BT$$

where P,V,T,N, and k_B represent pressure, volume, temperature, number of atoms and the Boltzmann constant, respectively, while α and β are constants specific to the gas.

If the critical point C corresponds to a point of inflexion of the $p-V$ curve, then the critical volume V_c and critical pressure p_c for this gas are given by

[TIFR-2016]

(a) $V_c=3\beta N, p_c=\alpha^2/3\beta^2$

(b) $V_c=3\beta N, p_c=\alpha/27\beta^2$

(c) $V_c=3\beta N, p_c=8\alpha^2/27\beta$

(d) $V_c=3\beta N, p_c=\alpha^2/27\beta^2$

4. A many-body system undergoes a phase transition between two phases A and B at a temperature T_c. The temperature-dependent specific heat at constant volume C_V of the two phases are given by $C_V^{(A)}=aT^3+bT$ and $C_V^{(B)}=cT^3$. Assuming negligible volume change of the system, and no latent heat generated in the phase transition, T_c is

[TIFR-2018]

(a) $\sqrt{\frac{2b}{c}}$ (b) $\sqrt{\frac{b}{c-a}}$

(c) $\sqrt{\frac{3b}{c-a}}$ (d) $\sqrt{\frac{4b}{c-a}}$

❖ Answer Key

CSIR-NET				
1. a	2. c	3. d	4. c	5. c
6. c	7. b			
GATE				
1. cd	2. d	3. b	4. d	5. b
6. a	7. a	8. d	9. d	10. a
11. b	12. c	13. 0.173	14. bc	15.
JEST				
1. bd	2. d			
TIFR				
1. d	2. c	3. d	4. c	

❖ **CSIR-NET PYQ**

1. The internal energy E of a system is given by $E=\frac{bS^3}{VN}$, where b is a constant and other symbols have their usual meaning. The temperature of this system is equal to **[CSIR DEC 2011]**

(a) $\frac{bS^2}{VN}$ (b) $\frac{3bS^2}{VN}$

(c) $\frac{bS^3}{V^3N}$ (d) $\left(\frac{S}{N}\right)^2$

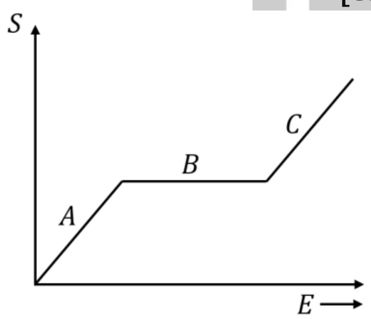
2. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V=\gamma T^3$ where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is: **[CSIR JUNE 2011]**

(a) $4\gamma T^4$ (b) $2\gamma T^4$

(c) $\frac{\gamma T^4}{4}$ (d) γT^4

3. The entropy S of a thermodynamic system as a function of energy E is given by the following graph

The temperatures of the phases A, B and C, denoted by $T_A \times T_B$ and T_C, respectively, satisfy the following inequalities: **[CSIR JUNE 2013]**



(a) $T_C > T_B > T_A$ (b) $T_A > T_C > T_B$

(c) $T_B > T_C > T_A$ (d) $T_B > T_A > T_C$

4. For a particular thermodynamics system the entropy S is related to the internal energy U and volume V by

$$S=cU^{3/4}V^{1/4}$$

where c is a constant. The Gibbs potential $G=U-TS+pV$ for this system is **[CSIR JUNE 2014]**

(a) $\frac{3pU}{4T}$ (b) $\frac{cU}{3}$

(c) zero (d) $\frac{US}{4V}$

5. The free energy of a gas N particled in a volume V and at a temperature T is

$$F=NK_BT\ln\left[a_0V(k_BT)^{5/2}/N\right]$$

where a_0 is a constanf and k_B denotes the Boltzmann constant. The internal energy of gas is **[CSIR JUNE 2012]**

(a) $\frac{3}{2}Nk_pT$

(b) $\frac{5}{2}Nk_sT$

(c) $Nk_BT\ln\left[a_0V(k_BT)^{5/2}/N\right]-\frac{3}{2}Nk_BT$

(d) $Nk_BT\ln\left[a_0V/(k_BT)^{5/2}\right]$

6. A thermodynamic function $G(T,P,N)=U-TS+PV$ is given in terms of the internal energy U, temperature T, entropy S, pressure P, volume V and the number of particles N. Which of the following relations is true? (In the following μ is the chemical potential). **[CSIR: JUNE-2017]**

(a) $S=-\left.\frac{\partial G}{\partial T}\right|_{N,p}$ (b) $S=\left.\frac{\partial G}{\partial T}\right|_{N,P}$

(c) $V=-\left.\frac{\partial G}{\partial P}\right|_{N,T}$ (d) $\mu=-\left.\frac{\partial G}{\partial N}\right|_{P,T}$

7. The relation between the internal energy U, entropy S, temperature T, pressure p, volume V, chemical potential μ and number of particles N of a thermodynamic system is $dU=TdS-pdV+\mu dN$. Thal U is an exact differential implies that **[CSIR: DEC-2017]**

(a) $-\left.\frac{\partial p}{\partial S}\right|_{V,N}=\left.\frac{\partial T}{\partial V}\right|_{S,N}$

(b) $p\left.\frac{\partial U}{\partial T}\right|_{S,N}=S\left.\frac{\partial U}{\partial V}\right|_{s,n}$

(c) $p\left.\frac{\partial U}{\partial T}\right|_{S,N}=-\frac{1}{T}\left.\frac{\partial U}{\partial V}\right|_{S,\mu}$

(d) $\left.\frac{\partial p}{\partial S}\right|_{V;N}=\left.\frac{\partial T}{\partial V}\right|_{S,N}$

8. Consider a particle diffusing in a liquid contained in a large box. The diffusion constant of the particle in the liquid is $1.0\times10^{-2}\ \mathrm{cm}^2/\mathrm{s}$. The minimum time after which the root-mean-squared displacement becomes more than 6 cm is

[CSIR: JUNE-2018]

(a) 10 min (b) 6 min

(c) 30 min (d) $\sqrt{6}$ min

9. The internal energy of a system is given by

$$U=g(N)V^{-\frac{2}{3}}\exp\left[\frac{2S}{3NR}\right]$$

where V is the volume, S is the entropy, N is the number of molecules and R is a constant. The function $g(N)$ is proportional to

[CSIR JUNE 2025]

(a) $N^{5/3}$ (b) $N^{1/3}$

(c) $N^{2/3}$ (d) N

10. A thermodynamic system (at temperature T and volume V), is described by its internal energy $U=AT^4V$ and pressure $p=\frac{1}{3}AT^4$, where A is a constant of appropriate dimension. The Helmholtz free energy of the system is

[CSIR JUNE 2025]

(a) $\frac{4}{3}AT^4V$ (b) $\frac{1}{3}AT^4V$

(c) $-\frac{1}{3}AT^4V$ (d) $-\frac{4}{3}AT^4V$

❖ GATE PYQ

1. For an isolated thermodynamical system p, V, T, U, S, and F represent the pressure, volume, temperature, internal energy, entropy, and free energy respectively. Then the following relation is true **[GATE 1994]**

(c) $\left(\frac{\partial U}{\partial T}\right)_P=T$ (d) $\left(\frac{\partial U}{\partial V}\right)_T=P$

2. If the entropy of a system remains constant in a thermodynamic process, the process is:

[GATE 1995]

(a) isobaric (b) Isochoric

(c) isothermal (d) adiabatic

3. The free energy of a photon gas enclosed in a volume V is given by

$$F=-\frac{1}{3}aVT^{-4}$$

where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

[GATE: 2006]

(a) 0 (b) $\frac{4}{3}aVT^3$

(c) $\frac{1}{3}a\,T^{-4}$ (d) aVT^{-4}

4. The free energy for a photon gas is given by

$$F=-\left(\frac{a}{3}\right)VT^4$$

, where a is a constant. The entropy S and the pressure P of the photon gas are

[GATE: 2007]

(a) $S=\frac{4}{3}aVT^3, P=\frac{a}{3}T^4$

(b) $S=\frac{1}{3}aVT^4, P=\frac{4a}{3}T^3$

(c) $S=\frac{4}{3}aVT^4, P=\frac{a}{3}T^3$

(d) $S=\frac{1}{3}aVT^3, P=\frac{4a}{3}T^4$

5. Which among the following sets of Maxwell relations is correct? (U internal energy H - enthalpy, A-Helmholtz free energy and G- Gibbs free energy)

[GATE: 2010]

(a) $T=\left(\frac{\partial U}{\partial V}\right)_S$ and $P=\left(\frac{\partial U}{\partial S}\right)_V$

(b) $V=\left(\frac{\partial H}{\partial P}\right)_S$ and $T=\left(\frac{\partial H}{\partial S}\right)_P$

(c) $P = -\left(\frac{\partial G}{\partial V}\right)_T$ and $V = \left(\frac{\partial G}{\partial P}\right)_S$

(d) $P = -\left(\frac{\partial A}{\partial S}\right)_T$ and $S = -\left(\frac{\partial A}{\partial P}\right)_V$

6. For a system at constant temperature and volume, which of the following statements is correct at equilibrium? **[GATE: 2016]**

(a) The Helmholtz free energy attains a local minimum

(b) The Helmholtz free energy attains a local maximum

(c) The Gibbs free energy attains a local minimum

(d) The Gibbs free energy attains a local maximum

7. Which of the following relationship between the internal energy U and the Helmholtz's free energy F is true? **[GATE: 2012]**

(a) $U = -T^2 \left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$

(b) $U = +T^2 \left[\frac{\partial\left(\frac{F}{T}\right)}{\partial T}\right]_V$

(c) $U = +T\left[\frac{\partial F}{\partial T}\right]_V$

(d) $U = -T\left[\frac{\partial F}{\partial T}\right]_V$

8. The internal energy U of a system is given by $U(S,V) = \lambda V^{-2/3}S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system? **[GATE: 2020]**

(a) $\frac{PV^{1/3}}{T^2} =$ constant (b) $\frac{PV}{T^{1/3}} =$ constant

(c) $\frac{P}{V^{1/3}T} =$ constant (d) $\frac{PV^{2/3}}{T} =$ constant

9. If a thermodynamical system is adiabatically isolated and experiences a change in volume under an externally applied constant pressure, then the thermodynamical potential minimized at equilibrium is the

[GATE: 2024]

(a) enthalpy

(b) Helmholtz free energy

(c) Gibbs free energy

(c) grand potential

10. The Joule-Thomson expansion of a gas is

[GATE: 2025]

(a) Isentropic (b) Isenthalpic

(c) Isobaric (d) Isochoric

❖ **JEST PYQ**

1. Consider an ideal gas whose entropy is given by

$$S = \frac{n}{2}\left[\sigma + 5R\ln\frac{U}{n} + 2R\ln\frac{V}{n}\right],$$

where n is the number of moles, σ is a constant, R is the universal gas constant, U is the internal energy and V is the volume of the gas. The specific heat at constant pressure is then given by

[JEST-2020]

(a) $\frac{5}{2}nR$ (b) $\frac{7}{2}nR$

(c) $\frac{3}{2}nR$ (d) nR

2. Using the first law of thermodynamics $dU = TdS - PdV$, and the definitions of the thermodynamic potentials $H = U + PV, F = U - TS, G = H - TS$, work out the four Maxwell relations. Using these compute:

$$\chi = \left(\frac{\partial T}{\partial V}\right)_S + \left(\frac{\partial T}{\partial P}\right)_S + \left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_P.$$

Which of the following does χ equal?

[JEST-2023]

(a) $\left(\frac{\partial S}{\partial V}\right)_T - \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P - \left(\frac{\partial P}{\partial S}\right)_V$

(b) $\left(\frac{\partial S}{\partial V}\right)_T - \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P + \left(\frac{\partial P}{\partial S}\right)_V$

(c) $\left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P - \left(\frac{\partial P}{\partial S}\right)_V$

(d) $\left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P + \left(\frac{\partial P}{\partial S}\right)_V$

3. Which of the following functions is not a valid thermodynamic function of internal energy U in terms of entropy S, volume V, and number of particles N ? Here U_0, α, β, A, B and C are constants. **[JEST-2024]**

(a) $\frac{BS^2V^2}{N^3}$

(b) $\left(\frac{AV^2}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right)$

(c) $U_0 \exp\left(\frac{\alpha V^2 N}{S^2}\right)$

(d) $\frac{CN^2}{\sqrt{SV}}$

❖ **TIFR PYQ**

1. A gas has the following equation of state

$$U = \frac{aS^5}{N^2V^2}$$

where U is the internal energy, V is the volume and N is the number of particles. Here a is a constant of the appropriate dimension. It follows that the equation of state of this gas relating its pressure P to its temperature T and its density $\rho = N/V$ is given by **[TIFR-2020]**

(a) $\frac{P^4}{T^5\rho^2} = \text{constant}$ (b) $\frac{P^5}{T^4\rho^3} = \text{constant}$

(c) $\frac{P}{T\rho} = \text{constant}$ (d) $\frac{P^3}{T^2\rho^3} = \text{constant}$

❖ **Answer Key**

CSIR-NET				
1. b	2. c	3. c	4. c	5. b
6. a	7. a	8. a	9. a	10. c
GATE				
1. a	2. d	3. a	4. a	5. b
6. a	7. a	8. a	9.	10. b
JEST				
1. b	2. a	3. c		
TIFR				
1. a				

❖ CSIR-NET PYQ

1. A layer of ice has formed on a very deep lake. The temperature of water, as well as that of ice at the ice-water interface, are $0°C$ whereas the temperature of the air above is $-10°C$. The thickness $L(t)$ of the ice increases with time t. Assuming that all physical properties of air and ice are independent of temperature, $L(t) \sim L_0 t^{\alpha}$ for large t. The value of α is

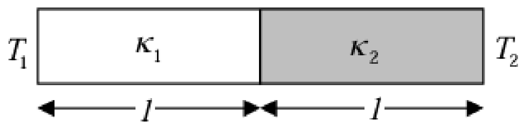
[CSIR: JUNE-2023]

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 4.1

❖ JEST PYQ

1. A rod consists of two equal sections of length 1 each with coefficient of thermal conductivity κ_1 and κ_2, respectively. One end of the rod is kept at a fixed temperature T_1 and the other end at a temperature $T_2 (T_1 > T_2)$. If $\kappa_2 = 2\kappa_1$ then the temperature at the interface is **[JEST-2020]**

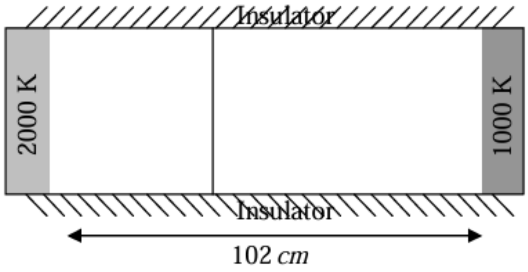


(a) $T = \dfrac{(2T_1 - T_2)}{3}$ (b) $T = \dfrac{(2T_1 + T_2)}{3}$

(c) $T = \dfrac{(T_1 + 2T_2)}{3}$ (d) $T = \dfrac{(T_1 - 2T_2)}{3}$

2. Two compartments in a cylinder with uniform cross section and total length 102 cm are separated by a sliding partition which can move but does not allow heat to pass across it. No molecules are present in either of the compartments. The radiation inside each compartment is in thermal equilibrium with its walls. The walls at the two ends of the cylinder are maintained at temperatures 2000 K and 4000 K, respectively. The sides are perfectly insulated. Find the location of the partition, measured from the left end of the container.

[JEST-2020]



3. Given an isolated thermodynamic system with a total energy E, total volume V and total number of particles N, the condition for stable thermal equilibrium, in terms of its entropy S under small changes ΔE and ΔV, is given by

[JEST-2025]

(a) $-S(E+\Delta E, V+\Delta V, N) + S(E-\Delta E, V-\Delta V, N) - 2S(E,V,N) < 0$

(b) $S(E+\Delta E, V+\Delta V, N) + S(E-\Delta E, V-\Delta V, N) - 2S(E,V,N) < 0$

(c) $S(E+\Delta E, V+\Delta V, N) + S(E-\Delta E, V-\Delta V, N) + 2S(E,V,N) < 0$

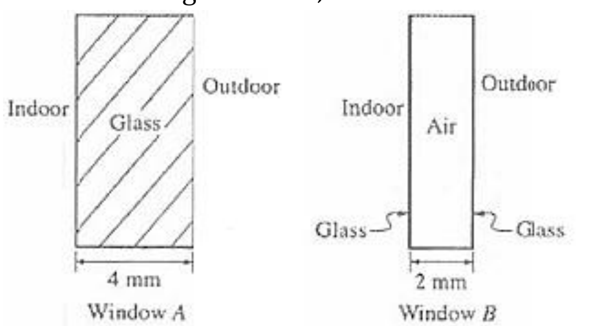
(d) $S(E+\Delta E, V+\Delta V, N) - S(E-\Delta E, V-\Delta V, N) - 2S(E,V,N) < 0$

4. A heat engine works between a high temperature source and a sink at $27°C$. If the maximum efficiency possible for it to achieve is 50%, what is the temperature of the source in $°C$?

[JEST-2025]

❖ TIFR PYQ

1. A manufacturer is able to offer two models of heat-conserving windows, as described below.



Window A is a simple pane of glass, 4 mm thick. Window B, on the other hand, consists of two extremely thin panes of glass, separated by an air gap of 2 mm, as shown in the figure above. If the thermal conductivity of glass is known to be $0.8 Wm^{-1} K^{-1}$ and that of air is $0.025 Wm^{-1} K^{-1}$, then the ratio of heat flow Q_A through Window A to the heat flow Q_B through Window B is given by $\frac{Q_A}{Q_B} =$

[TIFR-2014]

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$

(c) 4 (d) 16

2. In a cold country, in winter, a lake was freezing slowly. It was observed that it took 2 hours to form a layer of ice 2 cm thick on the water surface. Assuming a constant thermal conductivity throughout the layer, the thickness of ice would get doubled after

[TIFR-2015]

(a) 2 more hours. (b) 4 more hours.

(c) 6 more hours. (d) 8 more hours.

❖ Answer Key				
CSIR-NET				
1. c				
JEST				
1. c	2. 6	3. b	4. 327	
TIFR				
1. d	2. c			

❖ **CSIR-NET PYQ**

1. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C. What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is 1cal/gm/ °C and latent heat of melting of ice is 80cal/gm)

[CSIR: JUNE-2013]

(a) 0°C (b) 7.5°C

(c) 12.5°C (d) −1.25°C

2. A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressures p_1 and p_2, and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be

[CSIR: JUNE-2013]

(a) $\frac{p_1 T_1 + p_2 T_2}{p_1 + p_2}$ (b) $\frac{V_1 T_1 + V_2 T_2}{V_1 + V_2}$

(c) $\frac{p_1 V_1 + p_2 V_2}{(p_1 V_1/T_1) + (p_2 V_2/T_2)}$ (d) $(T_1 T_2)^{1/2}$

3. Two different thermodynamic systems are described by the following equations of state:

$$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}}$$

and

$$\frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$$

where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures; the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{sat} denote the total energy when

these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{\text{tot}}}$ is

[CSIR: DEC-2013]

(a) $\frac{5N^{(2)}}{3N^{(1)} + 5N^{(2)}}$ (b) $\frac{3N^{(1)}}{3N^{(1)} + 5N^{(2)}}$

(c) $\frac{N^{(1)}}{N^{(1)} + N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)} + N^{(2)}}$

4. Two ideal gases in a box are initially separated by a partition. Let N_1, V_1 and N_2, V_2 be the numbers of particles and volume occupied by the two systems. When the partition is removed, the pressure of the mixture at an equilibrium temperature T, is

[CSIR: NOV-2020]

(a) $k_B T\left(\frac{N_1 + N_2}{2(V_1 + V_2)}\right)$ (b) $k_B T\left(\frac{N_1 + N_2}{V_1 + V_2}\right)$

(c) $k_B T\left(\frac{N_1}{V_1} + \frac{N_2}{V_2}\right)$ (d) $\frac{1}{2} k_B T\left(\frac{N_1}{V_1} + \frac{N_2}{V_2}\right)$

5. A refrigerator can be thought to be a reversible engine operating between $T_2 = 20°C$ and $T_1 = -10°C$. The work needed to run this is supplied by another engine, that takes in energy at the rate of 500 W and runs with 50% efficiency. If the refrigerator freezes 5 kg of water at $0^0 C$ (latent heat $Q_L = 334$ kJ/kg for ice) in n hours, then n is closest to

[CSIR NOV 2025]

(a)0.4 (b)0.3

(c)0.1 (d)0.2

❖ **JEST PYQ**

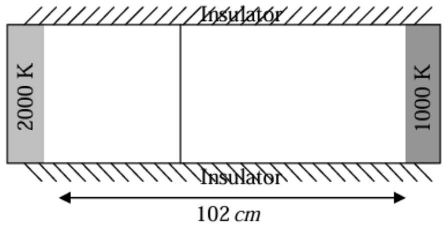
1. A metal bullet comes to rest after hitting its target with a velocity of 80 m/s. If 50% of the heat generated remains in the bullet, what is the increase in its temperature? (The specific heat of the bullet = 160 Joule per Kg per degree. C)

[JEST-2013]

(a) 14°C (b) 12.5°C

(c) 10°C (d) 8.2°C

2. Two compartments in a cylinder with uniform cross section and total length 102 cm are separated by a sliding partition which can move but does not allow heat to pass across it. No molecules are present in either of the compartments. The radiation inside each compartment is in thermal equilibrium with its walls. The walls at the two ends of the cylinder are maintained at temperatures 2000 K and 4000 K, respectively. The sides are perfectly insulated. Find the location of the partition, measured from the left end of the container.



[JEST-2020]

3. A container has two compartments. One compartment contains Oxygen gas at pressure P_1, volume V_1 and temperature T_1. The second compartment contains Nitrogen gas at pressure P_2, volume V_2, and temperature T_2. The partition separating two compartments is removed and the gases are allowed to mix. What is the temperature of the mixture when it comes to equilibrium?

[JEST-2022]

(a) $\frac{(V_1T_1 + V_2T_2)}{V_1 + V_2}$

(b) $\frac{(P_1V_1 + P_2V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$

(c) $\frac{(P_1V_1T_2 + P_2V_2T_1)}{P_1V_1 + P_2V_2}$

(d) $\frac{(P_1V_1T_1 + P_2V_2T_1)}{P_1V_1 + P_2V_2}$

❖ **TIFR PYQ**

1. An ideal gas at a temperature T is enclosed in a rigid container whose walls are initially at temperature T_1, where $T_1 < T$. The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container **[TIFR-2014]**

(a) remains constant throughout.

(b) is lower at the initial stage than at the final stage.

(c) is higher at the initial stage than at the final stage.

(d) is the same at the initial and final stages.

2. In the temperature range 100 – 1000C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula $C_p = (1 + T/5)$J-deg C^{-1} – mol^{-1}. If 0.2 kg of the metal at 600C is brought in thermal contact with 0.1 kg of the same metal at 300C, the final equilibrium temperature, in deg C, will be
[Assume that no heat is lost due to radiation and/or other effects.]

[TIFR-2016]

(a) 466 (b) 567

(c) 383 (d) 519

3. A thermally-insulated coffee mug contains 500 g of warm coffee at 80°C. Assuming that the heat capacity of this liquid is 1calg^{-1} °C^{-1} and the latent heat of fusion for ice is 80calg^{-1}, the amount of ice that must be dropped into the cup to convert it into cold coffee at 5°C is about

[TIFR-2019]

(a) 421 g (b) 441 g

(c) 469 g (d) 471 g

4. A boiler of volume 1.7 m^3, when filled with 1.0 kg of steam at 100°C, has a pressure of 1.0 atm. What will be the boiling point of water in this boiler when the pressure is 2.0 atm?

[The latent heat of vaporization of water is 2250×10^3 J/kg; 1 atm $= 10^5$ N/m^2]

[TIFR-2021]

(a) 128°C (b) 118°C

(c) 78°C (d) 88°C

Answer Key				
CSIR-NET				
1. a	2. c	3. b	4. b	5.
JEST				
1. c	2. 0006	3. b		
TIFR				
1. c	2. d	3.	4. a	

- **CSIR-NET PYQ**

1. Consider a system of non-interacting particles in d dimensions obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, ' s ' is an integer and A a constant. The density of states, $N(\varepsilon)$, is proportional to

[CSIR: JUNE-2012]

(a) $\varepsilon^{\frac{1}{d}-1}$ (b) $\varepsilon^{\frac{d}{s}+1}$

(c) $\varepsilon^{\frac{d}{s}}$ (d) $\varepsilon^{\frac{s}{d}-s}$

2. The entropy of a system, S, is related to the accessible phase space volume Γ by $S = k_i \ell n\Gamma(E,N,V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that Γ

[CSIR DEC-2012]

(a) does not change during evolution to equilibrium

(b) Oscillates during evolution to equilibrium

(c) Is a maximum in equilibrium

(d) Is a minimum in equilibrium

3. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states D(E) depends on the energy as

[CSIR: JUNE-2013]

(a) $1/\sqrt{E}$ (b) $\sqrt{E}$

(c) E (d) constant

4. The low-energy electronic excitations in a two-dimensional sheet of graphene is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to

[CSIR: JUNE-2015]

(a) E (b) $E^{1/2}$

(c) $E^{1/2}$ (d) E^2

5. For an ideal gas consisting of N distinguishable particles in a volume V, the probability of finding exactly 2 particles in a volume $\delta V \square V$, in the limit $N, V \to \infty$, is

[CSIR: JUNE-2020]

(a) $2N\delta V/V$ (b) $(N\delta V/V)^2$

(c) $\frac{(N\delta V)^2}{2V^2} e^{-N\delta V/V}$ (d) $\left(\frac{\delta V}{V}\right)^2 e^{-N\delta V/V}$

6. Four distinguishable particles fill up energy levels $0, \epsilon, 2\epsilon$. The number of available microstates for the total energy 4ϵ is

[CSIR: DEC-2023]

(a)20 (b)24

(c)11 (d)19

7. Two non-interacting classical particles having masses m_1 and m_2 are moving in a onedimensional box of length L. For total energy not exceeding a given value E, the phase space "volume" is given by

[CSIR JUNE 2024]

(a)$\pi L^2 E\left(\frac{m_1 m_2}{m_1+m_2}\right)$ (b)$\pi L^2 E\sqrt{m_1 m_2}$

(c)$2\pi L^2 E\left(\frac{m_1 m_2}{m_1+m_2}\right)$ (d)$2\pi L^2 E\sqrt{m_1 m_2}$

8. An isolated box of volume V contains 5 identical, but distinguishable and noninteracting particles. The particles can either be in the ground state of zero energy or in an excited state of energy ε. The ground state is non-degenerate while the excited state is doubly degenerate. There is no restriction on the number of particles that can be put in a given state. The number of accessible microstates corresponding to the microstate of the system with energy $E = 2\varepsilon$ are

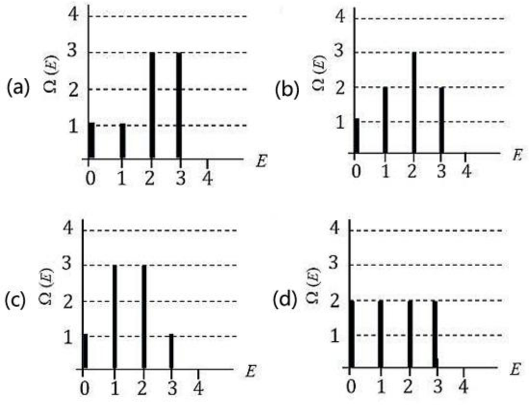
[CSIR MARCH 2024]

(a)10 (b)20

(c)40 (d)30

9. There are two boxes, one at the ground level, and the other at a fixed height $\boldsymbol{h}$.
There are three balls of different colours, each having mass m and radius $r \ll h$. There is no restriction on the number of balls that can be simultaneously put in a given box. For a given value of the total energy E (in units of mgh, g

being the acceleration due to gravity), the number of accessible microstates is $\Omega(E)$. The plot of $\Omega(E)$ vs E is



❖ **GATE PYQ**

Data for Q. No. 1 to 2

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value $+1/2$ and $-1/2$. The total spin of the system is $S_z = S_z(1) + S_z(2) + S_z(3)$.

1. The total number of possible micro-states of this system is

[GATE: 2003]

(a) 3 (b) 6

(c) 7 (d) 8

2. The total number of micro-states with $S_z = 1/2$ is

[GATE: 2003]

(a) 3 (b) 5

(c) 6 (d) 7

3. Consider an ensemble of systems where each microstate has equal probability. The ensemble average of S_z is

[GATE: 2003]

(a) $-1/2$ (b) 0

(c) $1/2$ (d) $3/2$

4. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E + \delta E (\delta E \ll E)$, is proportional to

[GATE: 2005]

(a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$ (b) $E^{N/2}\delta E$

(c) $NE^{1/2}\delta E$ (d) $E^N \delta E$

5. The total number of accessible states of N non-interacting particles of spin $-\frac{1}{2}$ is

[GATE: 2005]

(a) 2^N (b) N^2

(c) $2^{N/2}$ (d) N

6. The energy levels $E_0, 2E_0, 3E_0, \ldots$, where the excited states are triply degenerate. Four noninteracting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is

[GATE: 2007]

(a) 2 (b) 3

(c) 4 (d) 5

Statement for Linked Answer Questions 7 & 8 : Consider a two dimensional electron gas of N electrons of mass m each in a system of size $L \times L$.

7. The density of states between energy ε and $\varepsilon + d\varepsilon$ is

[GATE: 2008]

(a) $\frac{4\pi L^2 m}{h^2} d\varepsilon$ (b) $\frac{4\pi L^2 m}{h^2} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$

(c) $\frac{4\pi L^2 m}{h^2} \sqrt{\varepsilon} d\varepsilon$ (d) $\frac{4\pi L^2 m}{h^2} \varepsilon d\varepsilon$

8. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electron N is given by

(a) $\frac{1}{3} N E_F$ (b) $\frac{1}{2} N E_F$

(c) $\frac{2}{3} N E_F$ (d) $\frac{3}{5} N E_F$

9. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by $g(E) = CE^n$, where C is constant. The value of n is__________.

[GATE: 2015]

10. Consider a system having three energy levels with energies $0, 2\varepsilon$ and 3ε, with respective degeneracies of 2,2 and 3. Four bosons of spin zero have to be accommodated in these levels such that the total energy of the system is 10ε. The number of ways in which it can be done is____________.

[GATE: 2016]

11. Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contain a particle. Hence, what is the probability that level 2 also contain a particle for each of the two cases: (i) when the two particles are distinguishable and (ii) when the two particles are bosons?

[GATE: 2017]

(a) (i) 1/2 and (ii) 1/3

(b) (i) 1/2 and (ii) 1/2

(c) (i) 2/3 and (ii) ½

(d) (i) 1 and (ii) 0

12. Three particles are to be distributed in four non-degenerate energy levels. The possible number of ways of distribution: (i) for distinguishable particles, and (ii) for identical Boson, respectively, is

[GATE: 2018]

(a) (i) 24, (ii) 4
(b) (i) 24, (ii) 20
(c) (i) 64, (ii) 20
(d) (i) 64, (ii) 16

13. The energy-wavevector $(E-k)$ dispersion relation for a particle in two dimensions is $E = Ck$, where C is a constant. If its density of states $D(E)$ is proportional to E^p then the value of p is

[GATE: 2019]

14. For a finite system of Fermions where the density of states increases with energy, the chemical potential

[GATE: 2021]

(a) Decreases with temperature

(b)Increases with temperature

(c)Does not vary with temperature

(d)Corresponds to the energy where the occupation probability is 0.5

15. In a two-dimensional square lattice, frequency ω of phonons in the long wavelength limit changes linearly with the wave vector k. Then the density of states of phonons is proportional to

[GATE: 2022]

(a) ω
(b) ω^2
(c) $\sqrt{\omega}$
(d) $\frac{1}{\sqrt{\omega}}$

❖ JEST PYQ

1. For non-interacting Fermions in d-dimensions, the density of states $D(E)$ varies as $E^{(d/2-1)}$. The Fermi Energy E_F of an N particle system in 3-, 2- and 1-dimensions will scale respectively as,

[JEST-2015]

(a) $N^2, N^{2/3}, N$
(b) $N, N^{2/3}, N^2$
(c) $N, N^2, N^{2/3}$
(d) $N^{2/3}, N, N^2$

2. Adding 1eV of energy to a large system did not change its temperature (27°C) whereas it changed the number of micro-states by a factor r. r is of the order [Note: 1eV $\simeq$ 11600 K]

[JEST 2022]

(a) 10^{17}
(b) 10^{23}
(c) 10^4
(d) 10^{-19}

3. System A consists of 3 identical non-interacting bosons. System B consists of 2 identical non-interacting bosons. They both have identical energy spectra - three non-degenerate energy levels $0, \epsilon, 2\epsilon$. The particles of A and B are distributed in various energy levels in such a way that the total energy of the combined system is 4ϵ. The average energy of the system A in units of ϵ is

[JEST-2023]

(a) 2.2
(b) 2.3
(c) 2.1
(d) 2.4

4. The density of states of a system of N particles at energy E is

$$g(E,N) = \begin{cases} \dfrac{1}{(\hbar\omega)^N} \dfrac{E^{N-1}}{(N-1)!} & \text{for } E \geq 0 \\ 0 & \text{for } E < 0 \end{cases}$$

where $\hbar$ is the Planck's constant and ω is a natural frequency of the system. Taking k_B to be the Boltzmann constant, compute the temperature of the system at energy E.

[JEST-2024]

(a) $\frac{1}{k_B}\left(\frac{E}{N}+\frac{1}{2}\hbar\omega\right)$

(b) $\frac{E}{Nk_B}$

(c) $\frac{1}{k_B}\left(\frac{E}{N}+\hbar\omega\right)$

(d) $\frac{1}{k_B}\sqrt{\left(\frac{E}{N}\right)^2+(\hbar\omega)^2}$

❖ **TIFR PYQ**

1. Five identical bosons are distributed in energy levels E_1 and E_2 with degeneracy of 2 and 3, respectively. Find the number of microstates if there are three bosons in the energy level E_1 and two bosons in the energy level E_2.

[TIFR-2023]

(a) 24 (b) 1024

(c) 120 (d) 6

❖ **Answer Key**

CSIR-NET				
1. c	2. c	3. c	4. a	5. c
6. d	7. d	8. c	9. c	
GATE				
1. d	2. a	3. b	4. c	5. a
6. b	7. a	8. b	9. 0	10. 18
11. c	12. c	13. 1	14. a,b	15. a
JEST				
1. d	2. a	3. b	4. b	
TIFR				
1. a				

❖ **CSIR-NET PYQ**

1. Consider a system of N non-interacting spins, each of which has classical magnetic moment of magnitude μ. The Hamiltonian of this system in an external magnetic field $\vec{H}$ is $H=-\sum_{i=1}^{N}\vec{\mu}_i, \vec{H}$, where $\vec{\mu}_i$ is the magnetic moment of the i^{th} spin. The magnetization per spin at temperature T is:

[CSIR: JUNE 2011]

(a) $\frac{\mu^2 H}{k_B T}$

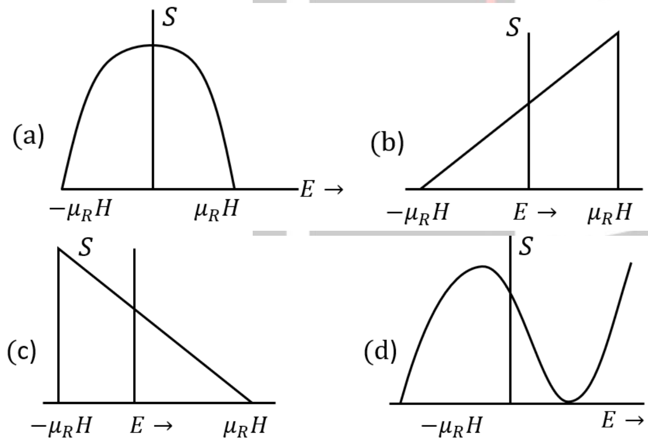
(b) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right)-\frac{k_B T}{\mu H}\right]$

(c) $\mu\sinh\left(\frac{\mu H}{k_B T}\right)$

(d) $\mu\tanh\left(\frac{\mu H}{k_B T}\right)$

2. A system of N non-interacting spin- $1/2$ particles is placed in an external magnetic field H. The behavior of the entropy of the system as a function of energy is given by

[CSIR: DEC 2011]



3. The energy levels of electrons of mass ' m ' and charge ' e ' confined in an area A in the xy-plane with a uniform magnetic field B applied in the z-direction are given by

$$E_n=\left(n+\frac{1}{2}\right)\frac{eBh}{mc}, n=0,1,2\ldots$$

The degeneracy of each level is $\frac{eBA}{\hbar c}$. The lowest level is completely filled the other are empty. The fermi energy $\frac{\hbar^2 N}{2\pi m A}$, where N is the total number of electrons, is:

[CSIR: DEC 2011]

(a) coincident with the $n=0$ level

(b) coincident with the n: 1 level

(c) midway between the $n=0$ and the $n=1$ levels

(d) midway between the $n=1$ and the $n=2$ levels.

4. A gas of N non-interacting particles is in thermal equilibrium at temperature T. Each particle can be in any of the possible non-degenerate states of energy $0, 2\varepsilon$ and 4ε. The average energy per particle of the gas, when $\beta\varepsilon \ll 1$, is:

[CSIR: DEC 2011]

(a) 2ε

(b) 3ε

(c) $2\varepsilon/3$

(d) ε

5. A one-dimensional chain consists of a set of N rods each of length a. When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is $-\varepsilon$ when aligned parallel to the length of the chain and is $+\varepsilon$ when perpendicular to it. When the chain is in thermal equilibrium at temperature T, its average length is:

[CSIR: DEC 2011]

(a) Na/2

(b) Na

(c) $\mathrm{Na}/\left(1+e^{-2s/\hbar_e T}\right)$

(d) $\mathrm{Na}/\left(1+e^{-2e/\epsilon_s T}\right)$

6. The free energy of a gas N particled in a volume V and at a temperature T is $F=NK_B T\ln\left[a_0 V(k_B T)^{5/2}/N\right]$, where a_0 is a constanf and k_B denotes the Boltzmann constant. The internal energy of gas is

[CSIR: JUNE-2012]

(a) $\frac{3}{2}Nk_p T$

(b) $\frac{5}{2}Nk_s T$

(c) $Nk_BT\ln\left[a_0V(k_BT)^{5/2}/N\right]-\frac{3}{2}Nk_BT$

(d) $Nk_BT\ln\left[a_0V/(k_BT)^{5/2}\right]$

7. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2=2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\pi\omega_1$? [In the following $x=e^{-\beta\,\text{no}}$ and Z is the partition function]

[CSIR: JUNE-2012]

(a) $x^{3/2}(x+2x^2)/Z$

(b) $x^{3/2}(1+x+x^2)/Z$

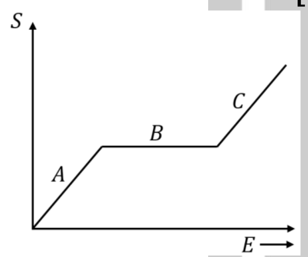
(c) $x^{3/2}(1+2x^2)/Z$

(d) $x^{3/2}(1+x+2x^2)/Z$

8. The entropy S of a thermodynamic system as a function of energy E is given by the following graph

The temperatures of the phases A, B and C, denoted by $T_A\times T_B$ and T_C, respectively, satisfy the following inequalities:

[CSIR: DEC-2013]



(a) $T_C>T_B>T_A$ (b) $T_A>T_C>T_B$

(c) $T_B>T_C>T_A$ (d) $T_B>T_A>T_C$

9. A system can have three energy levels: $E=0,\pm\varepsilon$. The level $E=0$ is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature β is

[CSIR: JUNE-2014]

(a) $-\varepsilon\tanh(\beta\varepsilon)$ (b) $\dfrac{\varepsilon\left(e^{\beta\varepsilon}-e^{-\beta\varepsilon}\right)}{(1+e^{\beta\varepsilon}+e^{-\beta\varepsilon})}$

(c) Zero (d) $-\varepsilon\tanh\left(\dfrac{\beta\varepsilon}{2}\right)$

10. The average local internal magnetic field acting on an Ising spin is $H_{\text{ins}}=\alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c, the magnetic susceptibility at zero extcrnal field is proportional to (k_B is the Boltzmann constant)

[CSIR JUNE 2014]

(a) $k_BT-\alpha$ (b) $(k_BT+\alpha)^{-1}$

(c) $(k_BT-\alpha)^{-1}$ (d) $\tanh(k_BT+\alpha)$

11. A system of N non-intersecting classical particles, each of mass m is in a two-dimensional harmonic potential of the form $V(r)=\alpha(x^2+y^2)$ where α is a positive constant. The canonical partition function of the system at temperature T is $\left(\beta=\frac{1}{k_BT}\right)$.

[CSIR: JUNE-2015]

(a) $\left[\left(\dfrac{\alpha}{2m}\right)^2\dfrac{\pi}{\beta}\right]^N$ (b) $\left(\dfrac{2m\pi}{\alpha\beta}\right)^{2N}$

(c) $\left(\dfrac{\alpha\pi}{2m\beta}\right)^N$ (d) $\left(\dfrac{2m\pi^2}{\alpha\beta^2}\right)^N$

12. A system of N distinguishable particles, each of which can be in one of the two energy levels 0 and ε, has a total energy $n\varepsilon$, where n is an integer. The entropy of the system is proportional to

[CSIR: JUNE-2015]

(a) $N\ell nn$ (b) $n\ln N$

(c) $\ln\left(\dfrac{N!}{n!}\right)$ (d) $\ln\left(\dfrac{N!}{n!\,(N-n)!}\right)$

13. For a system of independent non-interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T\to 0$, is

[CSIR: DEC-2015]

(a) $\dfrac{1}{2}\hbar\omega$ (b) $\hbar\omega$

(c) $\dfrac{3}{2}\hbar\omega$ (d) 0

14. The partition function of a system of N Ising spins is $Z=\lambda_1^N+\lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N. If $\lambda_1>\lambda_2$, the free energy per spin in the limit $N\to\infty$ is

[CSIR: DEC-2015]

(a) $-k_B T\ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (b) $-k_B T\ln\lambda_2$

(c) $-k_\beta T\ln(\lambda_1\lambda_2)$ (d) $-k_B T\ln\lambda_1$

15. A gas of non-relativistic classical particles in one-dimension is subjected to a potential $V(x)=\alpha|x|$, where α is a constant). The partition function is $\left(\beta=\frac{1}{k_B T}\right)$

[CSIR: JUNE-2016]

(a) $\sqrt{\frac{4m\pi}{\beta^3\alpha^2h^2}}$ (b) $\sqrt{\frac{2m\pi}{\beta^3\alpha^2h^2}}$

(c) $\sqrt{\frac{8m\pi}{\beta^3\alpha^2h^2}}$ (d) $\sqrt{\frac{3m\pi}{\beta^3\alpha^2h^2}}$

16. The partition function of a two-level system governed by the Hamiltonian $H=\begin{bmatrix}\gamma & -\delta\\ -\delta & -\gamma\end{bmatrix}$ is

[CSIR: DEC-2016]

(a) $2\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$

(b) $2\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$

(c) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)+\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$

(d) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)-\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$

17. An atom has a non-degenerate ground state and a doubly-degenerate ox cited state, The energy difference between the two states is ε. The specific heat at very low temperatures $(\beta\varepsilon\neq 1)$ is given by

[CSIR: DEC-2016]

(a) $k_B(\beta\varepsilon)$ (b) $k_B e^{-\beta\varepsilon}$

(c) $2k_B(\beta\varepsilon)^2e^{-\beta\varepsilon}$ (d) k_n

18. The electrons in graphene can be thought of as a two-dimensional gas with a linear energy-momentum relation $E=|\vec{p}|v$, where $\vec{p}=(p_x,p_y)$ and v is a constant. If ρ is the number of electrons per unit area, the energy per unit area is proportional to

[CSIR: DEC-2016]

(a) $\rho^{3/2}$ (b) ρ

(c) $\rho^{1/3}$ (d) p^2

19. Consider a gas of N classical particles in a two-dimensional square box of side L. If the total energy of the gas is E, the entropy (apart from an additive constant) is

[CSIR: DEC-2016]

(a) $Nk_B\ln\left(\frac{L^2E}{N}\right)$ (b) $Nk_B\ln\left(\frac{LE}{N}\right)$

(c) $2Nk_B\ln\left(\frac{L\sqrt{E}}{N}\right)$ (d) $L^2k_B\ln\left(\frac{E}{N}\right)$

20. The single particle energy levels of a non-interacting three-dimensional isotropic system, labelled by momentum k, are proportional to k^3. The ratio $\vec{P}/\varepsilon$ of the average pressure $\vec{P}$ to the energy density ε at a fixed temperature, is

[CSIR: JUNE-2017]

(a) 1/3 (b) 2/3

(c) 1 (d) 3

21. In a thermodynamic system in equilibrium, each molecule can exist in three possible states with probabilities $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{6}$ respectively. The entropy per molecule is [CSIR: JUNE-2017]

(a) $k_B\ln 3$

(b) $\frac{1}{2}k_B\ln 2+\frac{2}{3}k_B\ln 3$

(c) $\frac{2}{3}k_B\ln 2+\frac{1}{2}k_B\ln 3$

(d) $\frac{1}{2}k_{\bar{b}}\ln 2+\frac{1}{6}k_B\ln 3$

22. A closed system having three non-degenerate energy levels with energies $E=0,\pm\varepsilon$, is at temperature T. For $\varepsilon=2k_BT$, the probability of finding the system in the state with energy $E=0$, is

[CSIR: DEC-2017]

(a) $\frac{1}{(1+2\cosh 2)}$ (b) $\frac{1}{(2\cosh 2)}$

(c) $\frac{1}{2}\cosh\ 2$ (d) $\frac{1}{\cosh\ 2}$

23. Two non-degenerate energy levels with energies 0 and ε are occupied by N non-interacting particles at a temperatures T. Using classical statistics, the average internal energy of the system is

[CSIR DEC-2017]

(a) $\frac{N\varepsilon}{(1+e^{\varepsilon/k_bT})}$ (b) $\frac{N\varepsilon}{(1-e^{\varepsilon/k_nT})}$

(c) $N\varepsilon e^{-\varepsilon/k_BT}$ (d) $\frac{3}{2}Nk_BT$

24. The dispersion relation of a gas of spin $-\frac{1}{2}$ fermions in two dimensions is $E=\hbar v|\vec{k}|$, where E is the energy, $\vec{k}$ is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is ε_F, the number of particles per unit area is

[CSIR DEC-2017]

(a) $\frac{\varepsilon_F}{(4\pi v\hbar)}$ (b) $\frac{\varepsilon_F^3}{(6\pi^2v^3\hbar^2)}$

(c) $\frac{\pi\varepsilon_F^{3/2}}{(3v^3\hbar^3)}$ (d) $\frac{\varepsilon_F^2}{(2\pi v^2\hbar^2)}$

25. The number of microstates of a gas of N particles in a volume V and of internal energy U, is given by

$$\Omega(U,V,N)=(V-Nb)^N\left(\frac{aU}{N}\right)^{3N/2},$$

(where a and b are positive constants). Its pressure P, volume V and temperature T, are related by

[CSIR: DEC-2017]

(a) $\left(P+\frac{aN}{V}\right)(V-Nb)=Nk_BT$

(b) $\left(P-\frac{aN}{V^2}\right)(V-Nb)=Nk_BT$

(c) $PV=Nk_BT$

(d) $P(V-Nb)=Nk_BT$

26. The energy levels accessible to a molecule have energies $E_1=0, E_2=\Delta$ and $E_3=2\Delta$ (where Δ is a constant). A gas of these molecules is in thermal equilibrium at temperature T. The specific heat at constant volume in the high temperature limit $(k_BT\gg\Delta)$ varies with temperature as

[CSIR: DEC-2018]

(a) $1/T^{3/2}$ (b) $1/T^3$

(c) $1/T$ (d) $1/T^2$

27. The rotational energy levels of a molecule are

$$E_l=\frac{\hbar^2}{2I_0}l(l+1)$$

where $l=0,1,2,\ldots$ and I_0 is its mo. ment of inertia. The contribution of the rotational motion to the Helmholtz free energy per molecule at low temperature in a dilute gas of these molecules, is approximately

[CSIR: DEC-2018]

(a) $-k_BT\left(1+\frac{\hbar^2}{I_0k_BT}\right)$ (b) $-k_BTe^{-\frac{\hbar^2}{I_0k_BT}}$

(c) $-k_BT$ (d) $-3k_BTe^{-\frac{\hbar^2}{I_0k_BT}}$

28. The vibrational motion of a diatomic molecule may be considered to be that of a simple harmonic oscillator with angular frequency ω. If a gas of these molecules is at a temperature T, what is the probability that a randomly picked molecule will be found in its lowest vibrational state?

[CSIR: DEC-2018]

(a) $1-e^{-\frac{\hbar\omega}{k_BT}}$ (b) $e^{-\frac{\hbar\omega}{2k_BT}}$

(c) $\tanh\left(\frac{\hbar\omega}{k_BT}\right)$ (d) $\frac{1}{2}\text{cosech}\left(\frac{\hbar\omega}{2k_BT}\right)$

29. Consider an ideal Fermi gas in a grand canonical ensemble at a constant chemical potential. The variance of the occupation number of the single particle energy level with mean occupation number $\bar{n}$ is

[CSIR: DEC-2018]

(a) $\bar{n}(1-\bar{n})$ (b) $\sqrt{\bar{n}}$

(c) $\bar{n}$ (d) $1/\sqrt{\bar{n}}$

30. At low temperatures, in the Debye approximation, the contribution of the phonons to the heat capacity of a two-dimensional solid is proportional to

[CSIR DEC 2018]

(a) T^2 (b) T^3

(c) $T^{1/2}$ (d) $T^{3/2}$

31. In a system comprising of approximately 10^{23} distinguishable particles, each particle may occupy any of 20 distinct states. The maximum value of the entropy per particle is nearest to

[CSIR: JUNE-2019]

(a) $20k_B$ (b) $3k_B$

(c) $10(\ln\ 2)k_B$ (d) $20(\ln\ 2)k_B$

32. The free energy of a magnetic system, as a function of its magnetization m, is F

$$\frac{1}{2}am^2 - \frac{1}{4}bm^4 + \frac{1}{6}m^6$$

, where a and b are positive constants. At a fixed value of a, the critical value of b, above which the minimum of F will be at a non-zero value of magnetisation, is

[CSIR JUNE 2019]

(a) $\sqrt{10a/3}$ (b) $\sqrt{16a/3}$

(c) $\frac{10}{3}\sqrt{a}$ (d) $\frac{16}{3}\sqrt{a}$

33. The angular frequency of oscillation of a quantum harmonic oscillator in two dimensions is ω. It is contact with an external heat bath at temperature T, its partition function is (in the following

$$\beta = \frac{1}{k_B T})$$

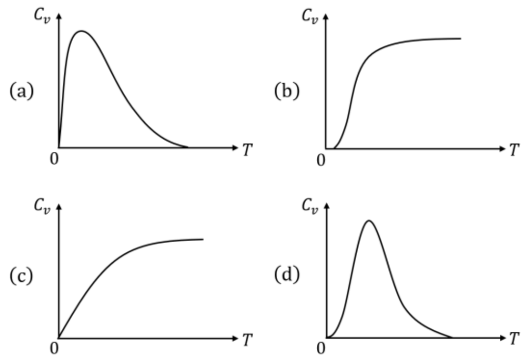
[CSIR DEC 2019]

(a) $\frac{e^{2\beta\hbar\omega}}{(e^{2\beta\hbar\omega}-1)^2}$ (b) $\frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$

(c) $\frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ (d) $\frac{e^{2\beta\hbar\omega}}{e^{2\beta\hbar\omega}-1}$

34. The energies available to a three-state system are $0, E$ and $2E$, where $E > 0$. Which of the following
graphs best represents the temperature dependence of the specific heat?

[CSIR: DEC-2019]



35. The Hamiltonian of two particles, each of mass m, is

$$H(q_1, p_1; q_2, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$

$$+k\left(q_1^2 + q_2^2 + \frac{1}{4}q_1 q_2\right)$$

, where $k > 0$ is a constant. The value of the partition function $Z(\beta) = \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dp_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} dp_2 e^{-\mu H(q_1,\mu_1;q_2,p_2)}$ is

[CSIR: DEC-2019]

(a) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{16}{15}}$ (b) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{15}{16}}$

(c) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{63}{64}}$ (d) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{64}{63}}$

36. For T much less than the Debye temperature of copper, the temperature dependence of the specific heat at constant volume of copper, is given by (in the following a and b are positive constants)

(a) aT^3 (b) $aT + bT^3$

(c)$aT^2 + bT^3$ (d)$\exp\left(-\frac{a}{k_n T}\right)$

37. An idealized atom has a non-degenerate ground state at zero energy and a g-fold degenerate excited state of energy E. In a non-interacting system of N such atoms, the population of the excited state may exceed that of the ground state above a temperature $T > \frac{E}{2k_B \ln\ 2}$. The minimum value of g for which this is possible is

[CSIR: JUNE-2020]

(a) 8 (b) 4

(c) 2 (d) 1

38. An idealized atom has a non-degenerate ground state at zero energy and a g-fold degenerate excited state of energy E. In a non-interacting system of N such atoms, the population of the excited state may exceed that of the ground state above a temperature $T > \frac{E}{2k_B \ln 2}$. The minimum value of g for which this is possible is

[CSIR NOV 2020]

(a) 8 (b) 4

(c) 2 (d) 1

39. The energy levels of a non-degenerate quantum system are $\epsilon_n = nE_0$, where E_0 is a constant and $n = 1,2,3, \ldots$. At a temperature T, the free energy F can be expressed in terms of the average energy E by

[CSIR: JUNE -2021]

(a) $E_0 + k_B T \ln \frac{E}{E_0}$ (b) $E_0 + 2k_B T \ln \frac{E}{E_0}$

(c) $E_0 - k_B T \ln \frac{E}{E_0}$ (d) $E_0 - 2k_B T \ln \frac{E}{E_0}$

40. A polymer made up of N monomers, is in thermal equilibrium at temperature T. Each monomer could be of length a or $2a$. The first contributes zero energy, while the second one contributes $\in$. The average length (in units of Na) of the polymer at temperature $T = \frac{\epsilon}{k_B}$ is

[CSIR: JUNE -2021]

(a) $\frac{5+e}{4+e}$ (b) $\frac{4+e}{3+e}$

(c) $\frac{3+e}{2+e}$ (d) $\frac{2+e}{1+e}$

41. Balls of ten different colours labeled by $a = 1,2,\ldots,10$ are to be distributed among different coloured boxes. A ball can only go in a box of the same colour, and each box can contain at most one ball. Let n_a and N_a denote, respectively the numbers of balls and boxes of colour a. Assuming that $N_a \gg n_a \ggg 1$, the total entropy (in units of the Boltzmann constant) can be best approximated by **[CSIR: JUNE -2021]**

(a) $\sum_a \left(N_a \ln N_a + n_a \ln n_a - (N_a - n_a)\ln(N_a - n_a)\right)$

(b) $\sum_a \left(N_a \ln N_a - n_a \ln n_a - (N_a - n_a)\ln(N_a - n_a)\right)$

(c) $\sum_a \left(N_a \ln N_a - n_a \ln n_a + (N_a - n_a)\ln(N_a - n_a)\right)$

(d) $\sum_a \left(N_a \ln N_a + n_a \ln n_a + (N_a - n_a)\ln(N_a - n_a)\right)$

42. If the average energy $\langle E\rangle_T$ of a quantum harmonic oscillator at a temperature T is such that $\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$, then T satisfies

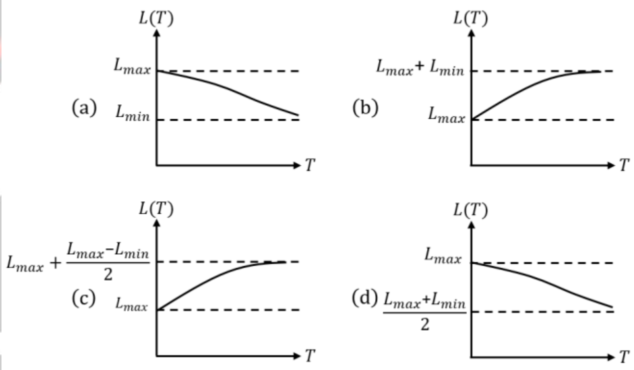
[CSIR: JUNE -2022]

(a) $\cot h\left(\frac{\hbar\omega}{k_B T}\right) = 2$ (b) $\cot h\left(\frac{\hbar\omega}{2k_B T}\right) = 2$

(c) $\cot h\left(\frac{\hbar\omega}{k_B T}\right) = 4$ (d) $\coth\left(\frac{\hbar\omega}{2k_B T}\right) = 4$

43. An elastic rod has a low energy state of length L_{max} and high energy state of length L_{min}. The best schematic representation of the temperature (T) dependence of the mean equilibrium length L(T) of the rod, is

[CSIR: JUNE-2022]



44. The energy levels of a system, which is in equilibrium at temperature $T = 1/(k_B\beta)$, are $0, \epsilon$ and 2ϵ. If two identical bosons occupy these energy levels, the probability of the total energy being 3ϵ, is

[CSIR: JUNE-2022]

(a) $\frac{e^{-3\beta\epsilon}}{1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta}}$

(b) $\frac{e^{-3\beta\epsilon}}{1+2e^{-\beta}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$

(c) $\frac{e^{-3\beta\epsilon}}{e^{-\beta}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$

(d) $\dfrac{e^{-3\beta}}{1+e^{-\beta\epsilon}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$

45. The energies of a two-level system are $\pm E$. Consider an ensemble of such non-interacting systems at a temperature T. At low temperatures, the leading term in the specific heat depends on T as

[CSIR: JUNE-2022]

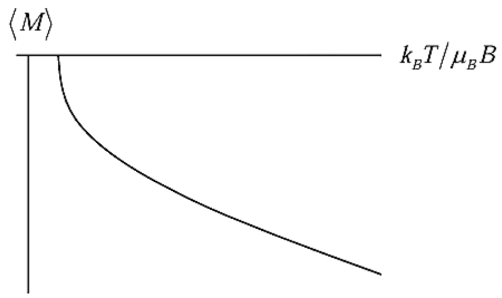
(a) $\dfrac{1}{T^2}e^{-E/k_BT}$ (b) $\dfrac{1}{T^2}e^{-\frac{2E}{k_BT}}$

(c) T^2e^{-E/k_BT} (d) $T^2e^{-\frac{2E}{k_BT}}$

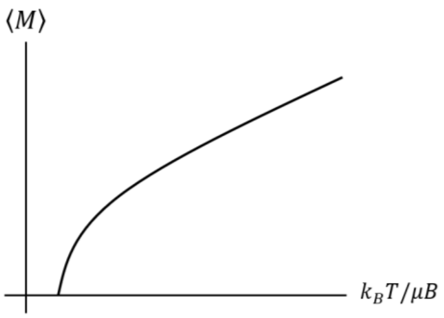
46. A paramagnetic salt with magnetic moment per ion $\mu_{\pm}=\pm\mu_B$ (where μ_B is the Bohr magneton) is in thermal equilibrium at temperature T in a constant magnetic field B. The average magnetic moment $\langle M\rangle$, as a function of $\frac{k_BT}{\mu_BB}$, is best represented by

[CSIR JUNE 2022]

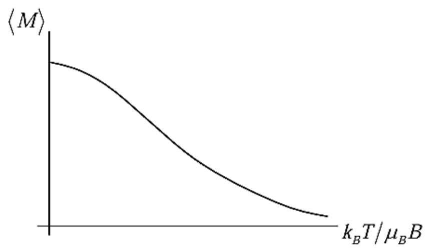
(a)



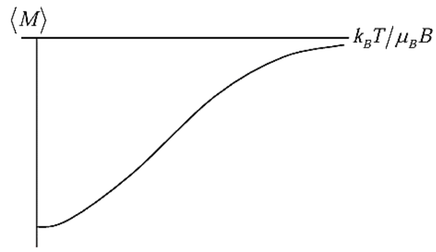
(b)



(c)



(d)



47. Two energy levels, 0 (non-degenerate) and ε (Doubly degenerate), are available to N noninteracting distinguishable particles. If U is the total energy of the system, for large values of N the entropy of the system is

$$k_B\left[N\ln N-\left(N-\frac{U}{\varepsilon}\right)\ln\left(N-\frac{U}{\varepsilon}\right)+X\right]$$

In this expression X is

[CSIR: JUNE-2023]

(a) $-\dfrac{U}{\varepsilon}\ln\left(\dfrac{U}{2\varepsilon}\right)$ (b) $-\dfrac{U}{\varepsilon}\ln\left(\dfrac{2U}{\varepsilon}\right)$

(c) $-\dfrac{2U}{\varepsilon}\ln\left(\dfrac{2U}{\varepsilon}\right)$ (d) $-\dfrac{U}{\varepsilon}\ln\left(\dfrac{U}{\varepsilon}\right)$

48. A system of N non-interacting classical spins, where each spin can take values $\sigma=-1,0,1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H=-\mu_Bh\sigma+\Delta(1-\sigma^2),$$

where μ_B,Δ are positive constants with appropriate dimensions.

If M is the magnetization, the zero-field magnetic susceptibility per spin

$$\frac{1}{N}\frac{\partial M}{\partial h}\bigg|_{h\to 0}$$

at a temperature $T=1/\beta k_B$ is given by

[CSIR: DEC-2023]

(a) $\beta\mu_B^2$ (b) $\dfrac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$

(c) $\beta\mu_B^2e^{-\beta\Delta}$ (d) $\dfrac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

49. A system of N non-interacting classical spins, where each spin can take values $\sigma=-1,0,1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H=-\mu_Bh\sigma+\Delta(1-\sigma^2),$$

where μ_B,Δ are positive constants with appropriate dimensions.

If M is the magnetization, the zero-field magnetic susceptibility per spin $\frac{1}{N}\frac{\partial M}{\partial h}\Big|_{h\to 0}$, at a temperature $T = 1/\beta k_B$ is given by

[CSIR DEC-2023]

(a) $\beta\mu_B^2$ (b) $\dfrac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$

(c) $\beta\mu_B^2 e^{-\beta\Delta}$ (d) $\dfrac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

50. A quantum system is described by the Hamiltonian

$$H = -J\sigma_z + \lambda(t)\sigma_x,$$

where $\sigma_i (i = x, y, z)$ are Pauli matrices, J and λ are positive constants ($J \gg \lambda$) and

$$\lambda(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \lambda & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$$

At $t < 0$, the system is in the ground state. The probability of finding the system in the excited state at $t \gg T$, in the leading order in λ is

[CSIR DEC-2023]

(a) $\frac{\lambda^2}{8J^2}\sin^2\frac{JT}{\hbar}$ (b) $\frac{\lambda^2}{J^2}\sin^2\frac{JT}{\hbar}$

(c) $\dfrac{\lambda^2}{4J^2}\sin^2\dfrac{JT}{\hbar}$ (d) $\dfrac{\lambda^2}{16J^2}\sin^2\dfrac{JT}{\hbar}$

51. A quantum system is described by the Hamiltonian

$$H = JS_z + \lambda S_x$$

where $S_i = \frac{\hbar}{2}\sigma_i$ and $\sigma_i (i = x, y, z)$ are the Pauli matrices. If $0 < \lambda \ll J$, then the leading correction in λ to the partition function of the system at temperature T is

[CSIR DEC-2023]

(a) $\dfrac{\hbar\lambda^2}{2Jk_BT}\coth\left(\dfrac{J\hbar}{2k_BT}\right)$

(b) $\dfrac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\dfrac{J\hbar}{2k_BT}\right)$

(c) $\dfrac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\dfrac{J\hbar}{2k_BT}\right)$

(d) $\dfrac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\dfrac{J\hbar}{2k_BT}\right)$

52. The work done on a material to change its magnetization M in an external field H is $dW = HdM$. Its Gibbs free energy is

$$G(T, H) = -\left(\gamma T + \frac{aH^2}{2T}\right),$$

where $\gamma, a > 0$ are constants. The material is in equilibrium at a temperature $T = T_0$ and in an external field $H = H_0$. If the field is decreased to $\frac{H_0}{2}$ adiabatically and reversibly, the temperature changes to

[CSIR DEC-2023]

(a) $2T_0$ (b) $\dfrac{T_0}{2}$

(c) $\left(\dfrac{a}{2\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$ (d) $\left(\dfrac{a}{\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$

53. A single particle can exist in two states with energies 0 and E respectively. At high temperatures ($k_BT \gg E$) the specific heat of the system (C_V) will be approximately

[CSIR JUNE 2024]

(a) proportional to $\frac{1}{T}$ (b) proportional to $\frac{1}{T^2}$

(c) proportional to $e^{\frac{E}{k_BT}}$ (c) constant

54. A system comprises of N distinguishable atoms ($N \gg 1$). Each atom has two energy levels ω and $3\omega (\omega > 0)$. Let ε_{eq} denote the average energy per particle when the system is in thermal equilibrium, the upper limit of ε_{eq} is

[CSIR MARCH 2025]

(a) $\dfrac{3\omega}{2}$ (b) 3ω

(c) $\dfrac{5\omega}{2}$ (d) 2ω

55. Eigenstates of a system are specified by two non negative integers n_1 and n_2. The energy of the system is given by

$$E_n = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)2\hbar\omega$$

If

$$\alpha \equiv \exp\left(-\frac{\hbar\omega}{k_BT}\right)$$

what is the probability that at temperature T the energy of the system will be less than $4\hbar\omega$?

(a) $(1-\alpha^2)(1-\alpha)(2+\alpha+2\alpha^2)$

(b) $(1-\alpha)^2(1-\alpha)(2+\alpha+\alpha^2)$

(c) $(1-\alpha^2)(1+\alpha)(1+\alpha+2\alpha^2)$

(d) $(1-\alpha)^2(1+\alpha)(1+\alpha+2\alpha^2)$

❖ **GATE PYQ**

1. A microcanonical ensemble represents

[GATE 1997]

(a) a system in contact with a heat reservoir
(b) an isolated system in equilibrium
(c) a system that can exchange particles with its surroundings.
(d) a system under constant external pressure.

2. In a canonical ensemble

[GATE 2000]

(a) the energy and the temperature are constants.
(b) the entropy and the energy are constants.
(c) the temperature and the density are constants.
(d) the density and the entropy are constants.

3. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by

[GATE:2001]

(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$

(c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$

4. Write down the partition function of a particle of mass m whose potential energy is given by $V(x,y,z)=ax^2+b(y^2+z^2)^{1/2}$, where a and b are positive constants of suitable dimensions. Also calculate the average energy of the particle.

$$\left[\text{Standard Integral}: \int_{-\infty}^{\infty} dx e^{-x^2/2\beta^2}=\sqrt{2\pi}\beta\right]$$

[GATE:2001]

5. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as "equivalent to one micro-state" is

[GATE: 2002]

(a) h^{3N} (b) h^{2N}

(c) h^N (d) h

where h is the Planck's constant

6. Which of the following statements is true?

[GATE: 2002]

(a) In a micro-canonical ensemble the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants

(b) In a micro-canonical ensemble the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants

(c) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary

(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies

Data for Q. No. 7 to 8

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N\ln\left\{(V-bN)(k_BT)^{3/2}\right\}$ where b is a constant with appropriate dimensions

7. If P is the pressure of the gas, the equation of state is given by

[GATE: 2003]

(a) $P(V-bN)=Nk_BT$

(b) $P(V-bN)=k_BT$

(c) $P(V-b)=Nk_BT$

(d) $P(V-b)=Nk_BT$

8. The interval energy of the gas is given by

[GATE: 2003]

(a) $U=(1/2)k_BT$ (b) $U=Nk_BT$

(c) $U=(3/2)Nk_BT$ (d) $U=2Nk_BT$

9. If $\beta\varepsilon_0=2$, the probability of finding the system in the level $\varepsilon=0$ is

[GATE: 2004]

(a) $\frac{(\cosh 2)}{2}$ (b) $(\cosh 2)^{-1}$

(c) $(2\cosh 2)^{-1}$ (d) $(1+2\cosh 2)^{-1}$

10. The free energy of the system at high temperature (i.e., $x \equiv \beta\varepsilon_0 \ll 1$) is approximately

[GATE: 2004]

(a) $-Nk_BTx^2$

(b) $-Nk_BT[\ln\ 2 + x^2/2]$

(c) $-Nk_BT[\ln\ 3 + x^2/3]$

(d) $-Nk_BT\ln\ 3$

11. The dimension of phase space of ten rigid diatomic molecules is

[GATE: 2004]

(a) 5 (b) 10

(c) 50 (d) 100

12. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, then the free energy of N such independent oscillator is

[GATE: 2005]

(a) $\frac{3}{2}NkT$ (b) $kT\ln\frac{\hbar\omega}{kT}$

(c) $NkT\ln\frac{\hbar\omega}{kT}$ (d) $NkT\ln\frac{\hbar\omega}{2kT}$

13. The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε is

[GATE: 2005]

(a) $1 + e^{-2\varepsilon/kT} + 2e^{-\varepsilon/kT}$

(b) $1 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

(c) $2 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

(d) $e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

14. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of steps is

[GATE: 2005]

(a) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$

(c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

15. The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}a\mathrm{V}\mathrm{T}^{-4}$, where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

[GATE: 2006]

(a) 0 (b) $\frac{4}{3}a\mathrm{V}\mathrm{T}^3$

(c) $\frac{1}{3}a\ \mathrm{T}^{-4}$ (d) $a\mathrm{V}\mathrm{T}^{-4}$

16. A system of N localized, non-interacting spin $-\frac{1}{2}$ ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is

[GATE: 2006]

(a) $Nk_BT\ln\left(\cosh\frac{\mu H}{k_BT}\right)$

(b) $-Nk_BT\ln\left(2\cosh\frac{\mu H}{k_BT}\right)$

(c) $Nk_BT\ln\left(2\cosh\frac{\mu H}{k_BT}\right)$

(d) $-Nk_BT\ln\left(2\sinh\frac{\mu H}{k_BT}\right)$

17. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperatures of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

[GATE: 2006]

(a) $\frac{3}{2}\ \mathrm{N}k_B\ln\left[\frac{\mathrm{T}_1^2+\mathrm{T}_2^2}{4\ \mathrm{T}_1\ \mathrm{T}_2}\right]$ (b) $\mathrm{N}k_B\ln\left[\frac{\mathrm{T}_2}{\mathrm{T}_1}\right]$

(c) $\frac{3}{2}\ \mathrm{N}k_B\ln\left[\frac{(\mathrm{T}_1+\mathrm{T}_2)^2}{4\ \mathrm{T}_1\ \mathrm{T}_2}\right]$ (d) $2\ \mathrm{N}k_B$

18. A monatomic crystalline solid comprises of N atms, out of which n atoms are in interstitial positions. If the available interstitial sites are N′, then number of possible microstates is

[GATE: 2006]

(a) $\dfrac{(\mathrm{N}'+n)!}{n!\,\mathrm{N}!}$

(b) $\dfrac{\mathrm{N}!}{n!\,(\mathrm{N}'+n)!}\dfrac{\mathrm{N}!}{n!\,(\mathrm{N}'+n)!}$

(c) $\dfrac{\mathrm{N}!}{n!\,(\mathrm{N}'-n)!}$

(d) $\dfrac{\mathrm{N}!}{n!\,(\mathrm{N}'-n)!}\dfrac{\mathrm{N}!}{n!\,(\mathrm{N}'-n)!}$

19. The partition function of a single gas molecule is Z_α. The partition function of N such non-interacting gas molecules is then given by

[GATE: 2007]

(a) $\dfrac{(Z_\alpha)^N}{N!}$ (b) $(Z_\alpha)^N$

(c) $N(Z_\alpha)$ (d) $\dfrac{(Z_\alpha)^N}{N}$

20. The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant. The entropy S and the pressure P of the photon gas are

[GATE: 2007]

(a) $S = \dfrac{4}{3}aVT^3, P = \dfrac{a}{3}T^4$

(b) $S = \dfrac{1}{3}aVT^4, P = \dfrac{4a}{3}T^3$

(c) $S = \dfrac{4}{3}aVT^4, P = \dfrac{a}{3}T^3$

(d) $S = \dfrac{1}{3}aVT^3, P = \dfrac{4a}{3}T^4$

Statement for Linked Answer Questions 21 & 22:
An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/k_\mathrm{B}\beta$

21. The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by

[GATE: 2007]

(a) $Z = \dfrac{e^{-\beta h\omega/2}}{1-e^{-\beta/h_0}}$ (b) $Z = \dfrac{e^{-\beta/k\omega/2}}{1+e^{-\beta/\hbar\omega}}$

(d) $Z = \dfrac{1}{1+e^{-\beta/\omega}}$

22. The average number of energy quanta of the oscillations is given by

[GATE: 2007]

(a) $\langle n\rangle = \dfrac{1}{e^{\beta\hbar\omega}-1}$

(b) $< n \geq \dfrac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$

(c) $\langle n\rangle = \dfrac{1}{e^{\beta\hbar\omega}+1}$

(d) $< n >= \dfrac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

23. A system containing N non-interacting localized particles of spin 1/2 and magnetic moment μ each is kept in constant external magnetic field B and in thermal equilibrium at temperature T. The magnetization of the system is

[GATE: 2008]

(a) $N\mu\coth\left(\dfrac{\mu B}{k_BT}\right)$ (b) $N\mu\tanh\left(\dfrac{\mu B}{k_BT}\right)$

(c) $N\mu\sinh\left(\dfrac{\mu B}{k_BT}\right)$ (d) $N\mu\cosh\left(\dfrac{\mu B}{k_BT}\right)$

24. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0, \varepsilon$ and 2ε having degeneracies $g(E) = 1,2$ and 4 respectively. The mean energy of the system is

[GATE: 2008]

(a) $\varepsilon\left(\dfrac{4exp\,(-\varepsilon/k_BT)+8exp\,(-2\varepsilon/k_BT)}{1+2exp\,(-\varepsilon/k_BT)+4exp\,(-2\varepsilon/k_BT)}\right)$

(b) $\varepsilon\left(\dfrac{2exp\,(-\varepsilon/k_BT)+8exp\,(-2\varepsilon/k_BT)}{1+2exp\,(-\varepsilon/k_BT)+4exp\,(-2\varepsilon/k_BT)}\right)$

(c) $\varepsilon\left(\dfrac{2exp\,(-\varepsilon/k_BT)+4exp\,(-2\varepsilon/k_BT)}{1+2exp\,(-\varepsilon/k_BT)+4exp\,(-2\varepsilon/k_BT)}\right)$

(d) $\varepsilon\left(\dfrac{exp\,(-\varepsilon/k_BT)+2exp\,(-2\varepsilon/k_BT)}{1+exp\,(-\varepsilon/k_BT)+exp\,(-2\varepsilon/k_BT)}\right)$

25. Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential μ etc. For a system to be specified by Microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles the parameters required for the respective ensemble are

[GATE: 2008]

(a) $MC: (N, V, T); CE: (E, V, N); GC: (V, T, \mu)$

(b) $MC: (E, V, N); CE: (N, V, T); GC: (V, T, \mu)$

(c) $MC: (V, T, \mu); CE: (N, V, T); GC: (E, V, N)$

(d) $MC: (F, V, N); CF: (V, T, \mu); GC: (N, V, T)$

26. Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts B, F and C correspond to whether the particles are bosons, fermions and classical particles, respectively then $r_B: r_{FT_C}$ is equal to

[GATE: 2008]

(a) $\frac{1}{2}: 0: 1$ (b) $1: \frac{1}{2}: 1$

(c) $1: \frac{1}{2}: \frac{1}{2}$ (d) $1: 0: \frac{1}{2}$

Common Data Questions

Common Data for Questions 27 and 28:

Consider a two level quantum system with energies $\varepsilon_1 = 0$ and $\varepsilon_1 = \varepsilon$.

27. The Helmholtz free energy of the system is given by

[GATE: 2009]

(a) $-k_B T \ln\left(1 + e^{-e/k_B T}\right)$

(b) $k_B T \ln\left(1 + e^{-\varepsilon/k_B T}\right)$

(c) $\frac{3}{2} k_B T$

(d) $\varepsilon - k_B T$

28. The specific heat of the system is given by

[GATE: 2009]

(a) $\frac{\varepsilon}{k_B T} \frac{\varepsilon^{-\varepsilon/k_B T}}{(1 + e^{-\varepsilon/k_B T})}$

(b) $\frac{\varepsilon^2}{k_B T^2} \frac{e^{-\varepsilon//k_B T}}{(1 + e^{-\varepsilon/k_B T})}$

(c) $-\frac{\varepsilon^2 e^{-\varepsilon/k_k T}}{(1 + e^{-\varepsilon/kk_B T})^2}$

(d) $\frac{\varepsilon^2}{k_B T^2} \frac{\varepsilon^{-\varepsilon/k_B T}}{(1 + e^{-\varepsilon/k_B T})^2}$

29. A system has two energy levels with energies ε and 2ε. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is

[GATE: 2011]

(a) $\frac{1}{1 + e^{-\varepsilon/k_B T}}$

(b) $\frac{1}{1 + 2e^{\varepsilon/k_B T}}$

(c) $\frac{1}{2e^{\varepsilon/k_B T} + 4e^{2\varepsilon/k_B T}}$

(d) $\frac{1}{2e^{\varepsilon/k_B T} - 4e^{2e/k_B T}}$

30. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

[GATE: 2011]

(a) $2k_B \ln N$ (b) $3k_B \ln N$

(c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

31. Consider a system whose three energy levels are given by $0, \varepsilon$ and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta = \frac{1}{k_B T}$ is given by

[GATE: 2012]

(a) $1 + 2e^{-\beta c}$ (b) $2e^{-\beta z} + e^{-2\beta c}$

(c) $\left(1 + e^{-\beta\varepsilon}\right)^2$ (d) $1 + e^{-\beta c} + e^{-2\beta c}$

32. A paramagnetic system consisting of N spin-half particles is placed in an external magnetic field.

It is found that N/2 spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is

[GATE: 2012]

(a) $2Nk_B\ln\ 2$ (b) $\frac{N}{2}k_B\ln\ 2$

(c) $\frac{3N}{2}k_B\ln\ 2$ (d) $Nk_B\ln\ 2$

Common Data Questions

Common Data for Questions 33 and 34: There are four energy levels $E, 2E, 3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are

33. Two identical fermions

[GATE: 2013]

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $\left(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}\right)^2$

(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

34. Two distinguishable particles

[GATE: 2013]

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $\left(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E}\right)^2$

(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

35. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is

[GATE: 2013]

(a) negative for some energies

(b) increases monotonically with energy

(c) decreases monotonically with energy

(d) zero

36. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

[GATE: 2013]

(a) pressure and temperature

(b) volume and temperature

(c) pressure and volume

(d) volume and energy

37. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

[GATE: 2013]

(a) zero (b) Nk

(c) $\frac{1}{2}Nk$ (d) $Nk\ln(2)$

38. For a system of two bosons, each of which can occupy any of the two-energy level 0 and ε, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_BT}$ is given by

[GATE: 2014]

(a) $\frac{\varepsilon e^{-\beta e} + 2\varepsilon e^{-2\beta e}}{1 + 2e^{-\beta e} + e^{-2\beta e}}$ (b) $\frac{1 + \varepsilon e^{-\beta z}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta c} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta c}}$

39. Consider a system of 3 fermions which can occupy any of the 4 available energy with equal probability. The energy of the system is

[GATE: 2014]

(a) $k_B\ln\ 2$ (b) $2k_B\ln\ 2$

(c) $2k_B\ln\ 4$ (d) $3k_B\ln\ 4$

40. The average energy U of a one-dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by

[GATE: 2015]

(a) $U = \frac{1}{2}h\omega\coth\left(\frac{1}{2}\beta h\omega\right)$

(b) $U = \frac{1}{2} h\omega \sinh \left(\frac{1}{2}\beta h\omega\right)$

(c) $U = \frac{1}{2} h\omega \tanh \left(\frac{1}{2}\beta h\omega\right)$

(d) $U = \frac{1}{2} h\omega \cosh \left(\frac{1}{2}\beta h\omega\right)$

41. Consider a system of N non-interacting spin-1/2 particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

[GATE: 2015]

(a) $\Omega = \dfrac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$

(b) $\Omega = \dfrac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$

(c) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$

(d) $\Omega = \dfrac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

42. The entropy of a gas containing N particles enclosed in a volume V is given by

$$S = Nk_B \ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right)$$

where E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by

[GATE: 2015]

(a) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{5}{2}\right]$

(b) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{3}{2}\right]$

(c) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{5}{2}\right]$

(d) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{3}{2}\right]$

43. A two-level system has energies zero and E. The level with zero energy is non-degenerate, while the level with energy E is triply degenerate. The mean energy of a classical particle in this system at a temperature T is

[GATE: 2016]

(a) $\dfrac{Ee^{-E/k_sT}}{1 + 3e^{-E/k_yT}}$ (b) $\dfrac{Ee^{-E/k_sT}}{1 + e^{-E/k_sT}}$

(c) $\dfrac{3Ee^{-E/k_sT}}{1 + e^{-E/k_sT}}$ (d) $\dfrac{3Ee^{-E/k_sT}}{1 + 3e^{-E/k_sT}}$

44. The entropy S of a system of N spins, which may align either in the upward or in the downward direction, is given by $S = -k_B N[p\ln p + (1-p)\ln(1-p)]$. Here k_B is the Boltzmann constant. The probability of alignment in the upward direction is p. The value of p, at which the entropy is maximum, is__________ (Give your answer upto one decimal place)

[GATE: 2016]

45. N atoms of an ideal gas are enclosed in a container of volume V. The volume of the container is changed to 4 V, while keeping the total energy constant. The change in the entropy of the gas, in units of $Nk_B \ln 2$, is, where k_B is the Boltzmann constant.

[GATE: 2016]

46. Consider N non-interacting, distinguishable particle in a two-level system at temperature T. The energies of the levels are 0 and ε, where $\varepsilon > 0$. In the high temperature limit ($k_B T \gg \varepsilon$), what is the population of particle in the level with energy ε ?

[GATE: 2017]

(a) $\frac{N}{2}$ (b) N

(c) $\frac{N}{4}$ (d) $\frac{3N}{4}$

47. The partition function of an ensemble at a temperature T is

$$Z = \left(2\cosh\frac{\varepsilon}{k_B T}\right)^N$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T = \frac{\varepsilon}{k_B}$ is XNk_B, where the value of X is (up to two decimal places).

[GATE: 2018]

48. A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state, or energy $\in$ state. Both states are non-degenerate. If the total energy of this ensemble is 4 $\in$, its entropy will be k_B (up to one decimal place), where k_B is the Boltzmann constant.

[GATE: 2018]

49. Consider two systems A and B each having two distinguishable particles. In both the systems, each particle can exist in states with energies 0,1,2 and 3 units with equal probability. The total energy of the combined system is 5 units. Assuming that the system A has energy 3 units and the system B has energy 2 units, the entropy of the combined system is $k_{\mathrm{B}}\ln\ \lambda$. The value of λ is

[GATE: 2019]

50. Consider a gas of hydrogen atoms in the atmosphere of the Sun where the temperature is 5800 K. If a sample from this atmosphere contains 6.023×10^{23} of hydrogen atoms in the ground state, the number of hydrogen atoms in the first excited state is approximately 8×10^{n}, where n is an integer. The value of n is
(Boltzmann constant: $8.617 \times 10^{-5}\mathrm{eV/K}$)

[GATE: 2020]

51. The internal energy U of a system is given by $U(S,V) = \lambda V^{-2/3}S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system? [GATE: 2020]

(a) $\dfrac{PV^{1/3}}{T^2} = \text{constant}$ (b) $\dfrac{PV}{T^{1/3}} = \text{constant}$

(c) $\dfrac{P}{V^{1/3}T} = \text{constant}$ (d) $\dfrac{PV^{2/3}}{T} = \text{constant}$

52. For a gas of non-interacting particles, the probability that a particle has a speed v in the internal v to $v + dv$ is given by

$$f(v)dv = 4\pi v^2 dv \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT}$$

If E is the energy of a particle, then the maximum in the corresponding energy distribution in units of E/k_BT occurs at (rounded off to one decimal place).

[GATE: 2020]

53. Consider a single one-dimensional harmonic oscillator of angular frequency ω, in equilibrium at temperature $T = (k_B\beta)^{-1}$. The states of the harmonic oscillator are all nondegenerate having energy $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ with equal probability, where n is the quantum number. The Helmholtz free energy of the oscillator is

[GATE: 2021]

(a) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp\left(\beta\hbar\omega\right)\right]$

(b) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp\left(-\beta\hbar\omega\right)\right]$

(c) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 + \exp\left(-\beta\hbar\omega\right)\right]$

(d) $\beta^{-1}\ln\left[1 - \exp\left(-\beta\hbar\omega\right)\right]$

54. A system of two atoms can be in three quantum states having energies 0, $\in$ and 2 $\in$. The system is in equilibrium at temperature $T = (k_B\beta)^{-1}$. Match the following Statistics with the Partition function.

[GATE: 2021]

Statistics	Partition function
CD: Classical (distinguishable particles)	Z1: $e^{-\beta\epsilon} + e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$
CI: Classical (indistinguishable particles)	Z2: $1 + e^{-\beta\epsilon} + 2e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
FD: Fermi-Dirac	Z3: $1 + 2e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} + 2e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
BE: Bose-Einstein	Z4: $\frac{1}{2} + e^{-\beta\epsilon} + \frac{3}{2}e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + \frac{1}{2}e^{-4\beta\epsilon}$

(a) CD: Z1, CI: Z2, FD: Z3, BE: Z4

(b) CD: Z2, CI: Z3, FD: Z4, BE: Z1

(c) CD: Z3, CI: Z4, FD: Z1, BE: Z2

(d) CD: Z4, CI: Z1, FD: Z2, BE: Z3

55. Consider a non-interacting gas of spin 1 particles, each with magnetic moment μ, placed in a weak magnetic field B, such that $\frac{\mu B}{k_B T} << 1$. The average magnetic moment of a particle is

[GATE 2022]

(a) $\frac{2\mu}{3}\left(\frac{\mu B}{k_B T}\right)$ (b) $\frac{\mu}{2}\left(\frac{\mu B}{k_B T}\right)$

(c) $\frac{\mu}{3}\left(\frac{\mu B}{k_B T}\right)$ (d) $\frac{3\mu}{4}\left(\frac{\mu B}{k_B T}\right)$

56. A simple harmonic oscillator with an angular frequency ω is in thermal equilibrium with a reservoir at absolute temperature T, with $\omega = \frac{2k_B T}{\hbar}$. Which one of the following is the partition function of the system?

[GATE: 2023]

(a) $\frac{e}{e^2-1}$ (b) $\frac{e}{e^2+1}$

(c) $\frac{e}{e-1}$ (d) $\frac{e}{e+1}$

57. The canonical partition function of an ideal gas is

$$Q(T,V,N) = \frac{1}{N!}\left[\frac{V}{(\lambda(T))^3}\right]^N$$

where T,V,N and $\lambda(T)$ denote temperature, volume, number of particles, and thermal deBroglie wavelength, respectively. Let k_B be the Boltzmann constant and μ be the chemical potential. Take $\ln(N!) = N\ln(N) - N$.
If the number density $\left(\frac{N}{V}\right)$ is 2.5×10^{25} m^{-3} at a temperature T, then $\frac{e^{\mu/(k_B T)}}{(\lambda(T))^3} \times 10^{-25}$ is m^{-3} (rounded off to one decimal place).

[GATE: 2024]

58. The Hamiltonian of a system of N particles in volume V at temperature T is

$$H = \sum_{i=1}^{2N} a_i q_i^2 + \sum_{i=1}^{2N} b_i p_i^2$$

where a_i and b_i are positive constants. The ensemble average of the Hamiltonian is $\alpha N k_B T$, where k_B is the Boltzmann constant. The value of α is (in integer).

[GATE: 2024]

59. The canonical partition function of an ideal gas is

$$Q(T,V,N) = \frac{1}{N!}\left[\frac{V}{(\lambda(T))^3}\right]^N$$

where T,V,N and $\lambda(T)$ denote temperature, volume, number of particles, and thermal deBroglie wavelength, respectively. Let k_B be the Boltzmann constant and μ be the chemical potential. Take $\ln(N!) = N\ln(N) - N$.
If the number density $\left(\frac{N}{V}\right)$ is 2.5×10^{25} m^{-3} at a temperature T, then

$$\frac{e^{\mu/(k_B T)}}{(\lambda(T))^3} \times 10^{-25}$$

is m^{-3} (rounded off to one decimal place).

[GATE 2024]

60. A paramagnetic material containing paramagnetic ions with total angular momentum $J = \frac{1}{2}$ is kept at absolute temperature T. The ratio of the magnetic field required for 80% of the ions to be in the lowest energy state to that required for having 60% of the ions to be in the lowest energy state at the same temperature is

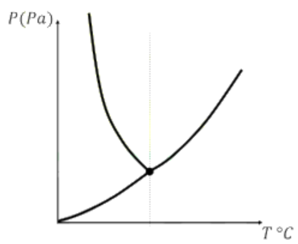
[GATE: 2025]

(a) $\frac{2\ln 2}{\ln\left(\frac{3}{2}\right)}$ (b) $\frac{\ln 2}{\ln\left(\frac{3}{2}\right)}$

(c) $\frac{3\ln 2}{\ln\left(\frac{3}{2}\right)}$ (d) $\frac{\ln 3}{\ln\left(\frac{3}{2}\right)}$

61. A schematic Pressure-Temperature diagram of water is shown in the figure. Which of the following option(s) is/are correct?

[GATE: 2025]



(a)Clausius-Clapeyron equation is valid across the melting curve and the vaporization Curve

(b)Melting curve has the highest slope

(c)The critical point exists only for the vaporization curve

(d)Clausius-Clapeyron equation is not valid across the melting curve and the vaporization curve

62. Consider a two-level system with energy states $+\varepsilon$ and $-\varepsilon$. The number of particles at $+\varepsilon$ level is $N_+$and the number of particles at $-\varepsilon$ level is N_-. The total energy of the system is E and the total number of particles is $N = N_+ + N_-$. In the thermodynamic limit, the inverse of the absolute temperature of the system is (Given: $\ln N! \simeq N\ln N - N$)

[GATE: 2025]

(a) $\frac{k_B}{2\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$ (b) $\frac{k_B}{\varepsilon} l\,n\,N$

(c) $\frac{k_B}{2\varepsilon} l\,n\,N$ (d) $\frac{k_B}{\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$

❖ JEST PYQ

1. Consider a system of particles in three dimension with momentum $\vec{p}$ and energy $E = c|\vec{p}|, c$ being a constant. The system is maintained at inverse temperature β, volume V and chemical potential μ. What is the grand partition function of the system?

[JEST 2012]

(a) $\exp\left[e^{\beta\mu}8\pi V/(\beta ch)^3\right]$

(b) $\exp\left[e^{\beta\mu}6\pi V/(\beta ch)^3\right]$

(c) $e^{\beta\mu}6\pi V/(\beta ch)^3$

(d) $e^{\beta\mu}8\pi V/(\beta ch)^2$

2. A collection of N two-level systems with energies 0 and $E > 0$ is in thermal equilibrium at temperatures T. For $T \to \infty$, the specific heat approaches

[JEST-2012]

(a) 0 (b) Nk_B

(c) $3Nk_B/2$ (d) ∞

3. Consider a system maintained at temperature T, with two available energy states E_1 and E_2 each with degeneracies g_1 and g_2. If p_1 and p_2 are probabilities of occupancy of the two energy states, what is the entropy of the system?

[JEST-2012]

(a) $S = -K_B[p_1 \text{In}\,(p_1/g_1) + p_2 \text{In}\,(p_2/g_2)]$

(b) $S = -K_B[p_1 \text{In}\,(p_1 g_1) + p_2 \text{In}\,(p_2 g_2)]$

(c) $S = -K_B\left[p_1 \text{In}\,(p_1^{g_1}) + p_2 \text{In}\,(p_2^{g_2})\right]$

(d) $S = -K_B[(1/p_1)\text{In}\,(p_1/g_1) + (1 + p_2)\text{In}\,(p_2/g_2)]$

4. Consider a particle with three possible spin states s = 0 and ±1. There is a magnetic field h present and the energy for a spin state s is −hs. The system is at a temperature T. Which of the following statements is true about the entropy S(T) ?

[JEST-2013]

(a) S(T) = In 3 at T = 0, and 3 at high T

(b) S(T) = In 3 at T0, and zero at high T

(c) S(T) = 0 at T = 0, and 3 at high T

(d) $S(T) = 0$ for $T = 0$, and In 3 at high T

5. Consider a system of two particles A and B. Each particle can occupy one of three possible quantum states $|1\rangle, |2\rangle$ and $|3\rangle$. The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwll-Boltzman) particles. They are respectively

[JEST-2013]

(a) 1,0 (b) 1/2,1

(c) 1,1/2 (d) 0,1/2

6. Consider three situations of 4 particles in a one dimensional box of width L with hard walls. In case (i), the particles are fermions, in case (ii) they are bosons, and in case (iii) they are classical. If the total ground state energy of the four particles in these three cases are E_F, E_B and E_{cl} respectively, which of the following is true?

[JEST-2013]

(a) $E_F = E_B = E_d$ (b) $E_F > E_B = E_d$

(c) $E_F < E_B < E_d$ (d) $E_F > E_B > E_d$

7. A monoatomic gas consists of atoms with two internal energy levels, ground state $E_0 = 0$ and an excited state $E_1 = E$. The specific heat of the gas is given by

[JEST-2014]

(a) $\frac{3}{2}k$

(b) $\frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

(c) $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

(d) $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2(1+e^{E/kT})^2}$

8. Consider a system of 2 N non-interacting spin 1/2 particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of spin up particle for which the entropy of the system will be maximum is

[JEST-2014]

(a) 0 (b) N

(c) 2 N (d) N/2

9. A particle in thermal equilibrium has only 3 possible states with energies $-\epsilon, 0, \epsilon$. If the system is maintained at a temperature $T \gg \frac{\epsilon}{k_B}$, then the average energy of the particle can be approximated to,

[JEST-2015]

(a) $\frac{2\in}{2k_BT}$ (b) $\frac{-2\epsilon^2}{3k_BT}$

(c) $\frac{-\epsilon^2}{k_BT}$ (d) 0

10. For a system in thermal equilibrium with a heat bath at temperature T, which one of the following equalities is correct? $\left(\beta = \frac{1}{k_BT}\right)$

[JEST-2015]

(a) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E\rangle^2 - \langle E^2\rangle$

(b) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E^2\rangle - \langle E^2\rangle^2$

(c) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E\rangle^2 + \langle E^2\rangle$

(d) $\frac{\partial}{\partial\beta}\langle E\rangle = -\langle E^2\rangle + \langle E^2\rangle^2$

11. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T. The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential is $U = mgz$ where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is :

[JEST-2016]

(a) $\frac{\exp\left(-\frac{mg\left(z-\frac{L}{2}\right)}{k_BT}\right)}{\sinh\left(\frac{mgL}{k_BT}\right)}$

(b) $P(z) = \frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg\left(z-\frac{L}{2}\right)}{k_BT}\right)}{\cosh\left(\frac{mgL}{2k_BT}\right)}$

(c) $P(z) = \frac{k_BTN}{V}$

(d) $P(z) = \frac{N}{V}mgz$

12. A two dimensional box in a uniform magnetic field B contains $N/2$ localized spin-1/2 particles with magnetic moment μ, and $N/2$ free spinless particles which do not interact with each other. The average energy of the system at a temperature T is :

[JEST-2016]

(a) $3NkT - \frac{1}{2}N\mu B\sinh\left(\frac{\mu B}{k_BT}\right)$

(b) $NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$

(c) $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$

(d) $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh\left(\frac{\mu B}{k_B T}\right)$

13. For a quantum mechanical harmonic oscillator with energies, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0,1,2 \ldots$, the partition function is: **[JEST-2016]**

(a) $\frac{e^{\frac{\hbar\omega}{e^{B}T}}}{e^{\frac{\hbar\omega}{2kgT-1}}}$

(b) $e^{\frac{\hbar ie}{2x^5y^5}} - 1$

(c) $e^{\frac{\hbar\omega}{2x_B 7}} + 1$

(d) $\frac{e^{\frac{\hbar\omega}{2k_H T}}}{e^{\frac{\hbar\omega}{k_G T}} - 1}$

14. A system of particles of N lattice sites is in equilibrium at temperature T and chemical potential μ. Multiple occupancy of the sites is forbidden. The binding energy of a particle at each site is $-\varepsilon$.The probability of onsite being occupied is

[JEST-2017]

(a) $\frac{1 - e^{\beta(\mu+\varepsilon)}}{1 - e^{(N+1)\beta(\mu+\varepsilon)}}$

(b) $\frac{1}{[1 + e^{\beta(\mu+\varepsilon)}]^N}$

(c) $\frac{1}{[1 + e^{-\beta(\mu+\varepsilon)}]^N}$

(d) $\frac{1 - e^{-\beta(\mu+\varepsilon)}}{1 - e^{-(N+1)\beta(\mu+\varepsilon)}}$

15. Two classical particles are distributed among $N(> 2)$ sites on a ring. Each site can accommodate only one particle. If two particles occupy two nearest neighbor sites, then the energy of the system is increased by ε The average energy of the system at temperature T is

[JEST-2017]

(a) $\frac{2\varepsilon e^{-\beta c}}{(N-3) + 2e^{-\beta c}}$

(b) $\frac{2N\varepsilon e^{-\beta\varepsilon}}{(N-3) + 2e^{-\beta\varepsilon}}$

(c) $\frac{\varepsilon}{N}$

(d) $\frac{2\varepsilon e^{-\beta\varepsilon}}{(N-2) + 2e^{-\beta\varepsilon}}$

16. Suppose that the number of microstates available to a system of N particles depends on N and the combined variable UV^2, where U is the internal energy and V is the volume of the system. The system initially has volume $2\ \text{m}^3$ and energy 200 J. It undergoes an isentropic expansion to volume $4\ \text{m}^3$. What is the final pressure of the system in SI units?

[JEST-2017]

17. A collection of N interacting magnetic moments, each of magnitude μ, is subjected to a magnetic field H along the z direction. Each magnetic moment has a doubly degenerate level of energy zero, and two nondegenerate levels of energies $-\mu H$ and μH respectively. The collection is in thermal equilibrium at temperature T. the total energy E(T, H) of the collection is

[JEST-2018]

(a) $-\frac{\mu H N \sinh(\mu H/k_B T)}{1 + \cosh(\mu H/k_b T)}$

(b) $-\frac{\mu H N}{2(1 + \cosh(\mu H/k_B T)}$

(c) $-\frac{\mu H N \cosh(\mu H/k_B T)}{1 + \cosh(\mu H/k_b T)}$

(d) $-\mu H N \frac{\sinh(\mu H/k_B T)}{\cosh(\mu H/k_B T)}$

18. When a collection of two-level systems is in equilibrium at temperature T_0, the ratio of the population in the lower and upper levels is 2: 1. When the temperature is changed to T, the ratio is 8: 1. Then

[JEST-2018]

(a) $\text{T} = 2\ \text{T}_0$

(b) $\text{T}_0 = 2\ \text{T}$

(c) $\text{T}_0 = 3\ \text{T}$

(d) $\text{T}_0 = 4\ \text{T}$

19. Consider a grand ensemble of a system of one dimensional non-interacting classical harmonic oscillators (each of frequency ω). Which one of the following equations is correct? Here the angular bracket $\langle \cdot \rangle$ indicate the ensemble average. N, E and T represent the number of particles, energy and temperature, respectively. k_B is the Boltzmann constant.

[JEST 2019]

(a) $\langle E \rangle = N\frac{k_B T}{2}$

(b) $\langle E \rangle = \langle N \rangle \frac{k_B T}{2}$

(c) $\langle E \rangle = N k_B T$

(d) $\langle E \rangle = \langle N \rangle k_B T$

20. Consider a system of N distinguishable particles with two energy levels for each particle, a ground state with energy zero and an excited state with energy $\varepsilon > 0$. What is the average energy per particle as the system temperature $T \rightarrow \infty$?

[JEST-2019]

(a) 0 (b) $\frac{\varepsilon}{2}$

(c) ε (d) ∞

21. Consider a diatomic molecule with an infinite number of equally spaced non-degenerate energy levels. The spacing between any two adjacent levels is ε and the ground state energy is zero. What is the single particle partition function Z ?

[JEST-2019]

(a) $Z = \dfrac{1}{1-\frac{\varepsilon}{k_B T}}$ (b) $Z = \dfrac{1}{1-e^{\frac{\varepsilon}{k_B T}}}$

(c) $Z = \dfrac{1}{1-e^{\frac{2\varepsilon}{k_B T}}}$ (d) $Z = \dfrac{1-\frac{\varepsilon}{k_B T}}{1+\frac{\varepsilon}{k_B T}}$

22. The energy spectrum of a particle consists of four states with energies $0, \in, 2\in, 3\in$. Let $Z_B(T), Z_F(T)$ and $Z_C(T)$ denote the canonical partition functions for four non-interacting particles at temperature T. The subscripts B, F and C corresponds to bosons, fermions and distinguishable classical particles, respectively. Let $y = \exp\left(-\frac{\epsilon}{k_B T}\right)$. Which one of the following statements is true about $Z_B(T), Z_F(T)$ and $Z_C(T)$?

[JEST-2019]

(a) They are polynomials in y of degree 12,6 and 12 , respectively.

(b) They are polynomials in y of degree 16,10 and 16 , respectively

(c) They are polynomials in y of degree 9,6 and 12 , respectively.

(d) They are polynomials in y of degree 12,10 and 16 , respectively.

23. Consider a system of two particles at temperature $T \rightarrow \infty$. Each of them can occupy three different quantum energy levels having energies $0, \in$ and $2\in$, and both of them cannot occupy the same energy level. What is the average energy of the system?

[JEST-2020]

(a) $\in$ (b) $\frac{3\in}{2}$

(c) $2\in$ (d) $4\in$

24. A classical gas of N particles is kept at a temperature T and is confined to move on a two dimensional surface (xy-plane). If an external linear force field is applied along the x-axis, then the partition function of the system will be proportional to

[JEST-2020]

(a) T^N (b) T^{2N}

(c) $T^{N/2}$ (d) $T^{3N/2}$

25. Consider an ideal gas whose entropy is given by

$$S = \frac{n}{2}\left[\sigma + 5R\ln\frac{U}{n} + 2R\ln\frac{V}{n}\right],$$

where n is the number of moles, σ is a constant, R is the universal gas constant, U is the internal energy and V is the volume of the gas. The specific heat at constant pressure is then given by

[JEST-2020]

(a) $\frac{5}{2}nR$ (b) $\frac{7}{2}nR$

(c) $\frac{3}{2}nR$ (d) nR

26. The free energy density of a gas at a constant temperature is given by $f(\rho) = C_\rho \ln(\rho/\rho_0)$, where ρ represents the density of the gas, while C and ρ_0 are positive constants. The pressure of the system is

[JEST-2021]

(a) C_ρ (b) $C\rho^2/\rho_0$

(c) $C\rho_0\ln(\rho/\rho_0)$ (d) $C\rho\ln(\rho/\rho_0)$

27. Consider a system consisting of three non-degenerate energy levels, with energies $0, \in$ and $2\in$. In the limit of infinite temperature $T \rightarrow \infty$, the probability of finding a particle in the ground state is

[JEST-2021]

(a) 0 (b) ½

(c) 1/3 (d) 1

28. A particle can access only three energy levels $E_1 = 1\text{eV}, E_2 = 2\text{eV}$, and $E_3 = 6$ eV. The average energy $\langle E \rangle$ of the particle changes as temperature T changes. What is the ratio of the minimum to the maximum average energy of the particle ?

[JEST-2022]

29. A system with two energy levels is in thermal equilibrium with a heat reservoir at temperature 600 K. The energy gap between the levels is 0.1eV. Let p be the the probability that the system is in the higher energy level. Which of the following statement is correct? [Note: $1\text{eV} \simeq 11600$ K]

[JEST-2022]

(a) $0.1 < p \leq 0.2$ (b) $0 < p \leq 0.1$

(c) $0.2 < p \leq 0.3$ (d) $p \geq 0.3$

30. A system of N classical non-identical particles moving in one dimensional space is governed by the Hamiltonian

$$H = \sum_{i=1}^{N} \left(A_i p_i^2 + B_i |q_i|^\alpha\right),$$

where p_i and q_i are momentum and position of the i-th particle, respectively, and the constant parameters A_i and B_i characterize the individual particles. When the system is in equilibrium at temperature T, then the internal energy is found to be

$$E = \langle H \rangle = \frac{2}{3} N k_B T,$$

where k_B is the Boltzmann constant. What is the value of α ? **[JEST-2022]**

31. There are three states of energy $E, 0, -E$ available for the population of two identical noninteracting spinless fermions. If they are in equilbrium at temperature T, what is the average energy of the system? The Boltzmann constant is k_B and consider β to be $\frac{1}{k_B T}$.

[JEST-2023]

(a)0

(b) $\frac{E\left(e^{\beta E} - e^{-\beta E}\right)}{1 + e^{-\beta E} + e^{\beta E}}$

(c) $\frac{E\left(e^{-\beta E} - e^{\beta E}\right)}{1 + e^{-\beta E} + e^{\beta E}}$

(d) $\frac{E\left(2e^{-\beta E} + e^{-\beta E} - e^{\beta E} - 2e^{2\beta E}\right)}{2e^{-\beta E} + e^{-\beta E} + 2 + e^{\beta E} + e^{2\beta E}}$

32. A cylinder of height L and cross-section A placed vertically along the central axis is filled with noninteracting particles each of mass m which are acted upon by a gravitational force of magnitude mg in the downward direction. The system is maintained at a temperature T. What is the ratio

$$\frac{C_v(T \to 0)}{C_v(T \to \infty)},$$

where C_v is the specific heat at constant volume.

[JEST-2023]

(a) $\frac{3}{5}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{3}$

33. Which of the following functions is not a valid thermodynamic function of internal energy U in terms of entropy S, volume V, and number of particles N ? Here U_0, α, β, A, B and C are constants. **[JEST-2024]**

(a) $\frac{BS^2V^2}{N^3}$

(b) $\left(\frac{AV^2}{N}\right) \exp\left(\frac{\beta VN}{S^2}\right)$

(c) $U_0 \exp\left(\frac{\alpha V^2 N}{S^2}\right)$

(d) $\frac{CN^2}{\sqrt{SV}}$

34. The density of states of a system of N particles at energy E is

$$g(E,N) = \begin{cases} \frac{1}{(\hbar\omega)^N} \frac{E^{N-1}}{(N-1)!} & \text{for } E \geq 0 \\ 0 & \text{for } E < 0 \end{cases}$$

where $\hbar$ is the Planck's constant and ω is a natural frequency of the system. Taking k_B to be the Boltzmann constant, compute the temperature of the system at energy E.

[JEST-2024]

(a) $\frac{1}{k_B}\left(\frac{E}{N}+\frac{1}{2}\hbar\omega\right)$

(b) $\frac{E}{Nk_B}$

(c) $\frac{1}{k_B}\left(\frac{E}{N}+\hbar\omega\right)$

(d) $\frac{1}{k_B}\sqrt{\left(\frac{E}{N}\right)^2+(\hbar\omega)^2}$

35. consider a system of N noninteracting spin- $\frac{1}{2}$ atoms subjected to a magnetic field with the Hamiltonian given by

$$H=-g\mu_B B\sum_{i=1}^{N} S_i^z,$$

where g is the dimensionless Landé factor, μ_B is the Bohr magneton, B is the strength of the magentic field, and S_i^z is the z-component of the spin of the i th atom (S_i^z takes values $\pm\frac{1}{2}$). The system is in equilibrium at temperature T. What is the probability that the z-component of the spins corresponding to two given atoms have the same value? Take $\beta=\frac{1}{k_BT}$, where k_B is the Boltzmann constant. **[JEST-2024]**

(a) $\frac{\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$

(b) $\frac{\exp(-\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$

(c) $\frac{\exp(\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$

(d) $\frac{1}{4}$

36. The energy spectrum for a system of spinless noninteracting fermions consists of $(N+1)$ nondegenerate energy levels $0,\varepsilon,2\varepsilon,\ldots,N\varepsilon(\varepsilon>0)$. Let $x=\exp\left(-\frac{\varepsilon}{k_BT}\right)$, where k_B is the Boltzmann constant and T is the temperature. For N identical fermions in thermal equilibrium at temperature T, what is the average occupancy of the highest energy level? **[JEST-2024]**

(a) $\frac{x}{1-x^N}$

(b) $\frac{x-x^{N+1}}{1+x^{N+1}}$

(c) $\frac{x-x^{N+1}}{1-x^{N+1}}$

(d) $\frac{x^N}{1+x^N}$

37. A system of two noninteracting identical bosons is in thermal equilibrium at temperature T. The particles can be in one of three states with nondegenerate energy eigenvalues $-\varepsilon,0$ and ε. The temperature T is such that

$$\exp\left(-\frac{\varepsilon}{k_BT}\right)=\frac{1}{2}$$

where k_B is the Boltzmann's constant. The average energy of the system is found to be $\langle E\rangle=-\frac{n}{35}\varepsilon$, where n is an integer. What is the value of n ?

[JEST-2024]

38. A two-level quantum system has the Hamiltonian$H=\hbar\omega_0\begin{pmatrix}0&1\\1&0\end{pmatrix}$

At $t=0$, the system is in the state$|\psi(0)\rangle=\begin{pmatrix}1\\0\end{pmatrix}$ What is the earliest time $t>0$ at which a measurement of $\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ will yield the value -1 with probability one?

[JEST-2024]

(a) Never

(b) $\frac{2\pi}{\omega_0}$

(c) $\frac{\pi}{\omega_0}$

(d) $\frac{\pi}{2\omega_0}$

39. Consider a system of N noninteracting spin- $\frac{1}{2}$ atoms subjected to a magnetic field with the Hamiltonian given by

$$H=-g\mu_B B\sum_{i=1}^{N} S_i^z,$$

where g is the dimensionless Landé factor, μ_B is the Bohr magneton, B is the strength of the magentic field, and S_i^z is the z-component of the spin of the i th atom (S_i^z takes values $\pm\frac{1}{2}$). The system is in equilibrium at temperature T. What is the probability that the z-component of the spins corresponding to two given atoms have the same value? Take $\beta = \frac{1}{k_B T}$, where k_B is the Boltzmann constant.

[JEST 2024]

(a) $\dfrac{\exp(-\beta g\mu_B B) + \exp(\beta g\mu_B B)}{2 + \exp(-\beta g\mu_B B) + \exp(\beta g\mu_B B)}$

(b) $\dfrac{\exp(-\beta g\mu_B B)}{2 + \exp(-\beta g\mu_B B) + \exp(\beta g\mu_B B)}$

(c) $\dfrac{\exp(\beta g\mu_B B)}{2 + \exp(-\beta g\mu_B B) + \exp(\beta g\mu_B B)}$

(d) $\frac{1}{4}$

40. Calculate the partition function for two indistinguishable bosonic particles at a temperature T, which can be distributed in two single-particle energy levels ϵ_1 and ϵ_2. Consider $\beta = \frac{1}{k_B T}$.

[JEST 2025]

(a) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$

(b) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-2\beta(\epsilon_1+\epsilon_2)}$

(c) $\left(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}\right)^2$

(d) $\dfrac{1}{2!}\left(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}\right)^2$

❖ **TIFR PYQ**

1. A quantum system has three energy levels $-0.12\text{eV}, -0.2\text{eV}$ and -0.44eV respectively. Three electrons are distributed among these levels. At a temperature of 1727°C the system has total energy -0.68eV. The free energy of the system is approximately

[TIFR: 2009]

(a) $+1.5\text{eV}$ (b) $+0.3\text{Ev}$

(c) -0.1eV (d) -0.3eV

(e) -1.0eV (f) -1.5eV

2. A system having N non-degenerate energy eigenstates is populated by N identical spinzero particles and 2 N identical spin-half particles. There are no interactions between any of these particles. If $N = 1000$, the entropy of the system is closest to

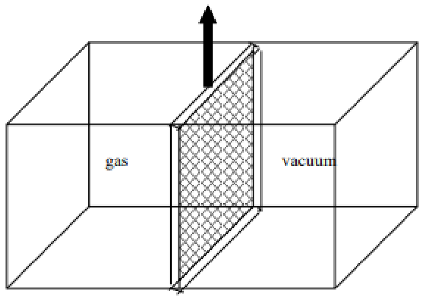
[TIFR-2011]

(a) $13.82k_B$ (b) zero

(c) $693.1k_B$ (d) $1000k_B$

(e) $5909.693k_B$ (f) $6909k_B$

3. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure).



If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

[TIFR-2012]

(a) $2k_B$ (b) $k_B \ln(1/2)$

(c) $k_B \ln 2$ (d) $(k_B \ln 2)/2$

4. A classical ideal gas, consisting of N particles $(N \to \infty)$ is confined in a box of volume V at temperature T and pressure p. The probability that, at any instant of time, a small sub-volume v_0 becomes totally void (i.e. no particles inside), due to a spontaneous statistical fluctuation, is

[TIFR-2013]

(a) $\exp(-v_0/V)$ (c) $\dfrac{v_0}{V}\exp(-pV/NT)$

(b) $\exp(-Nv_0/V)$ (d) pv_0/NT

5. An system at temperature T has three energy states $0, \pm\varepsilon$. The entropy of the system in the low temperature $(T \to 0)$ and high temperature $(T \to \infty)$ limits are, respectively,

[TIFR-2013]

(a) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \exp(-3)$

(b) $S_{T\to 0} = S_{T\to\infty} = k_B \ln 3$

(c) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \ln 3$

(d) $S_{T\to 0} = 0$ and $S_{T\to\infty} = 3k_B/2$

6. The entropy S of a black hole is known to be of the form $S = \alpha k_B A$
where A is the surface area of the black hole and α is a constant, which can be written in terms of c (velocity of light in vacuum), $\hbar$ (reduced Planck's constant) and G_N (Newton's constant of gravitation). Taking the radius of the black hole as $R = \frac{2G_N M}{c^2}$ it follows that the entropy S is [λ is a numerical constant]

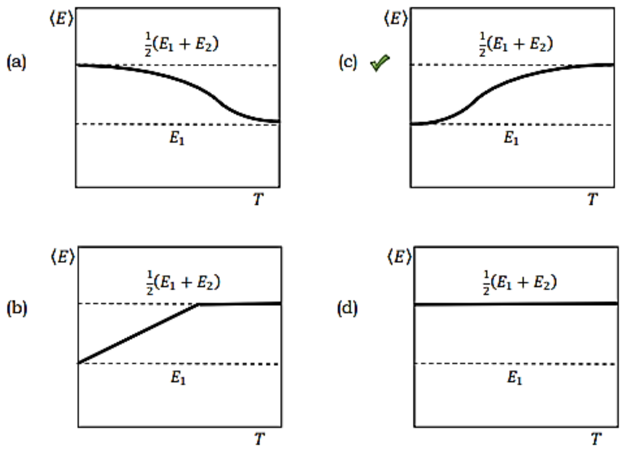
[TIFR-2013]

(a) $\dfrac{G_N^2 M^2 k_B}{\lambda(\hbar c)^4}$ (b) $\dfrac{\hbar c k_B}{\lambda G_N M}$

(c) $\dfrac{G_N^2 M^2 k_B}{\lambda \hbar c^4}$ (d) $\dfrac{G_N M^2 k_B}{\lambda \hbar c}$

7. Consider an ensemble of microscopic quantum mechanical systems with two energy levels E_1 and E_2, where $E_1 < E_2$. Which of the following graphs best describes the temperature dependence of the average energy $\langle E\rangle$ of the system?

[TIFR-2013]



8. In a monatomic gas, the first excited state is only 1.5eV above the ground state, while the other excited states are much higher up. The ground state is doubly-degenerate, while the first excited state has a four-fold degeneracy. If now, the gas is heated to a temperature of 7000 K, the fraction of atoms in the excited state will be approximately

[TIFR-2015]

(a) 0.14 (b) 0.07

(c) 0.42 (d) 0.3

9. The energy per oscillator of an isolated system of a large number of identical non-interacting fermions in a one-dimensional harmonic oscillator potential is $5\hbar\omega/4$, where ω is the angular frequency of the harmonic oscillator. The entropy of the system per oscillator is given by

[TIFR-2016]

(a) 0.25 (b) 0.56

(c) 0.63 (d) 0.75

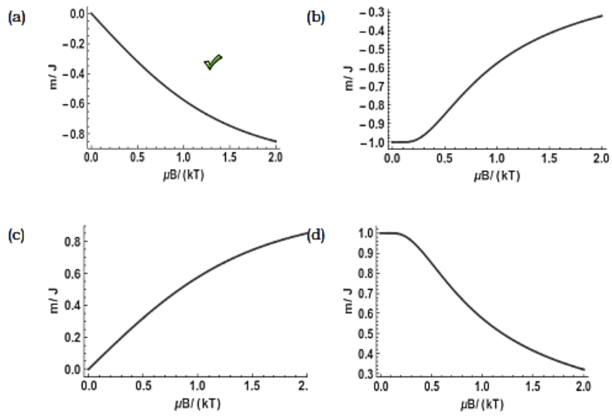
10. N distinguishable particles are distributed among three states having energies $E = 0, k_B T$ and $2k_B T$ respectively. If the total equilibrium energy of the system is $138.06 k_B T$, find the number N of particles.

[TIFR-2016]

11. Consider a system of non-interacting particles with integer angular momentum J at a temperature T. This system is placed in a magnetic field B in the z direction. The energy of a state with $J_z = m\hbar$ is

$$E_m = m\mu_B B$$

with $\mu_B > 0$. The fractional magnetization of the particles as a function of $\mu_B B/k_B T$ can be represented as **[TIFR-2017]**



12. Two identical bosons may occupy any of two energy levels $0, \varepsilon$, where $\varepsilon > 0$. The lowest energy state is doubly-degenerate and the excited state is non-degenerate. Assume that the two-particle system is in thermal equilibrium at a temperature T. Calculate the average energy $\langle E\rangle$. What will be the leading term of

$$\frac{\langle E\rangle}{\exp\left(-\frac{\varepsilon}{k_B T}\right)}$$

at low temperature? **[TIFR-2017]**

13. A one-dimensional quantum harmonic oscillator of natural frequency ω is in thermal equilibrium with a heat bath at temperature T. The mean value $\langle E\rangle$ of the energy of the oscillator can be written as

[TIFR-2017]

(a) $\frac{\hbar\omega}{2}\operatorname{sech}\left(\frac{\hbar\omega}{2k_B T}\right)$ (b) $\frac{\hbar\omega}{2}\operatorname{csch}\left(\frac{\hbar\omega}{2k_B T}\right)$

(c) $\frac{\hbar\omega}{2}\coth\left(\frac{\hbar w}{2k_B T}\right)$ (d) $\frac{\hbar\omega}{2}\tanh\left(\frac{\hbar\omega}{2k_B T}\right)$

14. Hydrogen atoms in the atmosphere of a star are in thermal equilibrium, with an average kinetic energy of 1eV. The ratio of the number of hydrogen atoms in the 2nd excited state ($n = 3$) to the number in the ground state ($n = 1$) is

[TIFR-2017]

(a) 3.16×10^{-11} (b) 1.33×10^{-8}

(c) 3.16×10^{-8} (d) 5.62×10^{-6}

15. A system of particles occupying single-particle levels and obeying Maxwell Boltzmann statistics is in thermal equilibrium with a heat reservoir at temperature T. If the population distribution in the non-degenerate energy levels is as shown in the table below, what would be the temperature of the system in degree Kelvin?

[TIFR-2017]

16. N particles are distributed among three energy levels having energies: 0, $k_B T$ and $2k_B T$ respectively. If the total equilibrium energy of the system is approximately $42.5k_B T$ then find the value of N (to the closest integer).

[TIFR-2018]

17. A statistical system, kept at a temperature T, has n discrete energy levels with equal levelspacing ε, starting from energy 0 . If, now, a single particle is placed in the system what will be the mean energy of the system in the limit as $n \to \infty$? [The answer should not be left as a summation]

[TIFR-2018]

18. An ideal monatomic gas at chemical potential $\mu = -1\text{eV}$ and a temperature given by $k_B T = 0.1\text{eV}$ is in equilibrium with an adsorbing metal surface, i.e. there are isolated sites distributed randomly on the metal surface where the gas atom can get bound. Each such binding site can adsorb 0,1 , or 2 atoms with the released energy being $0, -1\text{eV}$ and -1.9eV respectively. The average number of adsorbed molecules at each site would be

[TIFR-2019]

(a) $\frac{1+e}{1-e}$ (b) $\frac{1+e}{1+2e}$

(c) $\frac{2+e}{1+2e}$ (d) $\frac{1+2e}{1+e}$

19. Consider N non-interacting distinguishable particles in equilibrium at an absolute temperature T. Each particle can only occupy one of two possible states of energy 0 and ϵ respectively ($\epsilon > 0$). The entropy of the system, in terms of $\beta = \epsilon/k_B T$ is **[TIFR-2019]**

(a) $Nk_B\left\{ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1+e^{-\beta}}\right\}$

(b) $Nk_B\left\{ln\left(1+e^{-\beta}\right) + \frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$

(c) $Nk_B\left\{\ln\left(1-e^{-\beta}\right) + \frac{\beta e^{-\beta}}{1-e^{-\beta}}\right\}$

(d) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1-e^{-\beta}}\right\}$

20. Consider a thermal ensemble at temperature T which is composed of identical quantum harmonic oscillators of frequency ω_0 with non-overlapping wavefunctions. The probability that there will be an even number of energy quanta in the system is

[TIFR-2019]

(a) $\frac{1}{\exp(-\hbar\omega_0/k_B T)+1}$ ↓

(c) $\frac{1}{\exp(-\hbar\omega_0/k_B T)-1}$

(b) $\frac{1}{\exp(\hbar\omega_0/k_B T)+1}$

(d)tanh $(\hbar\omega_0/2k_BT)$

21. A

Energy (eV)	Population %
30.30	3.16
21.60	8.69
13.01	23.54
4.31	64.61

system is composed of a large number of non-interacting classical particles moving in two dimensions, which individually obey the Hamiltonian

$$\frac{p_x^2+p_y^2}{2m}+\frac{1}{2}m\omega^2(x^2+y^2)$$

and the system is connected to a heat bath at a temperature T.
The probability of finding a particle within a radius R from the origin is given by

[TIFR-2020]

(a) $1-\exp\left(-\frac{m\omega^2R^2}{2T}\right)$ (b) $\exp\left(-\frac{m\omega^2R^2}{2T}\right)$

(c) $\mathrm{erf}\left(\sqrt{\frac{m}{2T}}\omega R\right)$ (d) $1-\frac{m\omega^2R^2}{2T}$

22. A gas has the following equation of state

$$U=\frac{aS^5}{N^2V^2}$$

where U is the internal energy, V is the volume and N is the number of particles. Here a is a constant of the appropriate dimension. It follows that the equation of state of this gas relating its pressure P to its temperature T and its density $\rho=N/V$ is given by **[TIFR 2020]**

(a) $\frac{P^4}{T^5\rho^2}=$ constant (b) $\frac{P^5}{T^4\rho^3}=$ constant

(c) $\frac{P}{T\rho}=$ constant (d) $\frac{P^3}{T^2\rho^3}=$ constant

23. In a certain atom, the ground state and first excited state of the valence electron are -7.8eV and -3.9eV, while all the higher excited states have energies very close to zero. The ground state has a degeneracy of 2 , while the first excited state has a degeneracy of 6 . It follows that if these atoms reside in the outer layers of a blue giant star at a temperature around 2.32×10^4 K, the average energy per atom will be approximately

[TIFR-2020]

(a) -5.1eV (b) -5.9eV

(c) -6.8eV (d) -4.4eV

24. A square lattice consists of $2N$ sites, of which alternate sites are labeled A and B. An example with $N=6$ is shown on the right. Now, N identical classical particles are distributed over these sites, such that each site can accommodate at most one particle.
The fraction of the total number N of particles occupying A sites is denoted α and the fraction occupying B sites is
denoted β, so that $\alpha+\beta=1$.
If α,β are fixed and $N\gg1$, the entropy S of the system can be written **[TIFR-2020]**

(a) $S=-2Nk_BT(\alpha\ln\,\alpha+\beta\ln\,\beta)$

(b) $S=2Nk_BT(\alpha\ln\,\alpha+\beta\ln\,\beta)$

(c) $S=-2Nk_BT(\alpha\ln\,\alpha-\beta\ln\,\beta)$

(d) $S=2Nk_BT(\alpha\ln\,\alpha-\beta\ln\,\beta)$

25. A certain system has one state with energy E, two states with energy $2E$, three states with energy $3E$ and so on, where $E>0$. The partition function Z of the system at temperature T is given by

[TIFR-2021]

(a) $\frac{1}{Z}=4\sinh^2\frac{E}{2T}$ (b) $\frac{1}{Z}=2\cosh^2\frac{E}{4T}$

(c) $\frac{1}{Z}=4\coth^2\frac{E}{2T}$ (d) $\frac{1}{Z}=2\tanh^2\frac{E}{4T}$

26. A vertical cylinder of height H is filled with an ideal gas of classical point particles each of mass m and is allowed to come to equilibrium under gravity at a temperature T. The mean height of these particles is

[TIFR-2022]

(a) $\frac{k_BT}{mg}\left(1-\frac{mgH/k_BT}{e^{mgH/k_BT}-1}\right)$

(b) $\frac{H}{3}\frac{mgH/k_BT}{e^{mgH/k_BT}+1}$

(c) $\frac{k_BT}{mg}\left(1-\frac{2mgH/k_BT}{e^{mgH/k_BT}+1}\right)$

(d) $\frac{H}{3}\frac{mgH/k_BT}{e^{mgH/k_BT}-1}$

27. A quantum dot is constructed such that it has just three energy levels, with energies E, $2E$ and $3E$ respectively. The chemical potential in the system has the value $\mu = 2E$ and the temperature is given by

$$T = \frac{E}{2k_B}$$

The expected number of electrons populating the quantum dot will be

[TIFR-2022]

(a) 3.0 (b) 2.5

(c) 1.5 (d) 4.0

28. Consider a fermionic system with a Hamiltonian

$$\hat{H} = \begin{bmatrix} 0 & E_0 & 0 \\ E_0 & 0 & 2E_0 \\ 0 & 2E_0 & 0 \end{bmatrix}$$

Consider the grand canonical ensemble of this system at temperature T and zero chemical potential, where k_B is the Boltzmann constant. The grand canonical partition function of the system is given by

[TIFR 2023]

(a) $4+\cosh\left(\sqrt{5}\frac{E_0}{k_BT}\right)$ (b) $\cosh\left(\sqrt{5}\frac{E_0}{k_BT}\right)$

(c) $\frac{1}{4+\cosh\left(\sqrt{5}\frac{E_0}{k_BT}\right)}$ (d) $\operatorname{sech}\left(\sqrt{5}\frac{E_0}{k_BT}\right)$

29. A two-level system with zero ground state energy is in equilibrium at a nonzero finite temperature. If α is defined as the ratio

$$\alpha = \frac{\langle E^2\rangle}{\langle E\rangle^2}$$

where $\langle E\rangle$ denotes the mean energy and $\langle E^2\rangle$ denotes the mean squared energy, then

[TIFR-2023]

(a) $2 < \alpha < \infty$ (b) $1 < \alpha \le 2$

(c) $\frac{1}{2} < \alpha < 1$ (d) $0 < \alpha < \frac{1}{2}$

30. A lattice in the three-dimensional space has N sites, each occupied by an atom whose magnetic moment is μ. The lattice is in contact with a heat reservoir at a fixed temperature T. The atoms interact with an applied magnetic field

$$\vec{H} = H(\vec{x})\hat{\boldsymbol{z}}$$

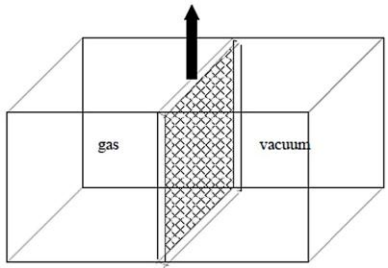
Ignoring the interactions between the atoms, the average magnetic susceptibility per lattice site is given by

[TIFR 2023]

(a) $\frac{\mu^2}{3k_BT}$ (b) $\frac{\mu^2}{9k_BT}$

(c) $\frac{\mu}{3k_BT}$ (d) $\frac{\mu H}{3k_BT}$

31. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin partition, which is thermally conducting but impermeable to particles. One of the chambers contains an ideal gas, while the other is a vacuum.



If the partition is removed suddenly and the ideal gas is allowed to expand and fill the entire container, then, once equilibrium has been reached, the entropy per molecule will increase by an amount **[TIFR-2023]**

(a) $\frac{1}{2}k_B\ln 2$ (b) $-k_B\ln 2$

(c) $2k_B$ (d) $+k_B\ln 2$

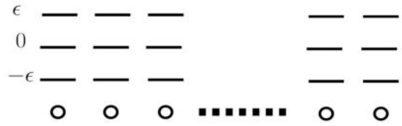
32. A string has 8 beads in a row, with n identical red beads and $(8-n)$ identical blue beads. When one of the red beads is replaced by a blue one, the entropy of the given system changes from S to $S + k_B\ln 2$. All configurations of the beads are equally probable. What is the value of n ?

[TIFR-2024]

(a) 8 (b) 2

(c) 4 (d) 6

33. Each site of a linear chain of N sites has a spin which can be in three different states with energies $0, \pm\epsilon$, as shown in the figure below.



The system has a constraint that the neighboring spins cannot be in the same state. At infinite temperature, the entropy of the system is given by:

[TIFR-2024]

(a) $N\ln\ 2 + \ln\frac{3}{2}$ (b) $N\ln\ 3$

(c) $(N-1)\ln\ 2$ (d) $N\ln\ 2$

❖ Answer Key

CSIR-NET				
1. b	2. a	3. d	4. a	5. c
6. b	7. d	8. c	9. d	10. c
11. d	12. d	13. a	14. d	15. c
16. b	17. c	18. a	19. a	20. c
21. c	22. a	23. a	24. d	25. d
26. d	27. d	28. a	29. a	30. a
31. a,b	32. b	33. b	34. d	35. d
36. b	37. b	38. b	39. c	40. d
41. b	42.	43. d	44. b	45. b
46. d	47. a	48. b	49. b	50. b
51. d	52. b	53. b	54. d	55. d
GATE				
1.	2.	3. b	4.	5. a
6. a	7. a	8. c	9. d	10. d
11. d	12. c	13. b	14. a	15. a
16. b	17. c	18. d	19. b	20. a
21. a	22. a	23. b	24. b	25. b
26. d	27. a	28. d	29. a	30. d
31. b	32. d	33. b	34. c	35. a
36.	37. d	38.	39. b	40. a
41. a	42. a	43. d	44. 0.	45. 2
46. a	47. 0.42	48. 6.20	49. 12	50. 14
51. a	52. 14	53. b	54. c	55. a
56. a	57. 2.5	58. 2	59. 2.	60. a
61. abc	62. a			
JEST				
1. d	2. a	3. a	4. d	5. c
6. b	7. c	8. b	9. d	10. a
11. a	12. c	13. d	14. b	15.
16.	17. a	18. c	19. d	20. b
21.	22. a	23. c	24. d	25. b
26. a	27. c	28. 0.33	29. a	30. 6
31. c	32. d	33. c	34. b	35. a
36. c	37. 36.1	38. d	39. a	40. a
TIFR				
1.	2.	3. c	4. b	5. c
6. d	7. c	8. a	9. b	10. 32
11. a	12.	13. c	14. b	15.
16. 10	17.	18. c	19. b	20. a
21. a	22. a	23. c	24. a	25. a
26. a	27. a	28. a	29. a	30. a
31. d	32. d	33. a		

❖ CSIR-NET PYQ

1. The number of ways in which N identical bosons can be distributed in two energy levels, is

[CSIR: JUNE-2012]

(a) $N+1$ (b) $N(N-1)/2$

(c) $N(N+1)/2$ (d) N

2. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ and $5\varepsilon_0, (\varepsilon_0 > 0)$. Onc system is populated by spin$\frac{1}{2}$ fermions and the other by bosons; What is the value of $\mathrm{E_F} - \mathrm{E}_B$ where E_p and $\mathrm{E_B}$ are the ground state energies of the fermionic and bosonic systems respectively?

[CSIR JUNE 2013]

(a) $6\varepsilon_0$ (b) $2\varepsilon_0$

(c) $4\varepsilon_0$ (d) ε_0

3. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

[CSIR DEC 2013]

(a) 8 (b) 4

(c) 3 (d) 2

4. The number of ways of distributing 11 indistinguishable bosons in 3 different energy levels is

[CSIR: JUNE 2018]

(a) 3^{11} (b) 11^3

(c) $\frac{(13)!}{2!\,(11)!}$ (d) $\frac{(11)!}{3!\,8!}$

5. In a system of N distinguishable particles, each particle can be in one of two states with energies 0 and $-E$, respectively. The mean energy of the system at temperature T, is

[CSIR JUNE 2018]

(a) $-\frac{1}{2}N\left(1+e^{E/k_gT}\right)$ (b) $-\frac{NE}{\left(1+e^{E/k_BT}\right)}$

(c) $-\frac{1}{2}NE$ (d) $-\frac{NE}{\left(1+e^{-E/\kappa_BT}\right)}$

6. The energy levels available to each electron in a system of N non-interacting electrons are $E_n = nE_0 n = 0,1,2,\cdots$. A magnetic field, which does not affect the energy spectrum, but completely polarizes the electron spins, is applied to the system. The change in the ground state energy of the system is

[CSIR JUNE 2023]

(a) $\frac{n^2E_0}{2}$ (b) n^2E_0

(c) $\frac{n^2E_0}{8}$ (d) $\frac{n^2E_0}{4}$

❖ GATE PYQ

1. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

[GATE: 2002]

(a) $n/T^{L/2} < 1$

(b) $n/T^{3/2} \ll 1$

(c) $n/T^{3/2} \gg 1$

(d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value

2. At temperature T Kelvin (K), the value of the Fermi function at an energy 0.5eV above the Fermi energy is 0.01. Then T, to the nearest integer, is ($k_\mathrm{B} = 8.62 \times 10^{-5}$eV/K) 1262

[GATE: 2019

3. Choose the correct statement related to the Fermi energy (E_F) and the chemical potential (μ) of a metal **[GATE: 2020]**

(a) $\mu = E_F$ only at $0K$

(b) $\mu = E_F$ at finite temperature

(c) $\mu < E_F$ at $0K$

(d) $\mu > E_F$ at finite temperature

❖ JEST PYQ

1. A gas contains particle of type A with fraction 0.8 , and particles of type B with fraction 0.2 . the

probability that among 3 randomly chosen particles at least one is of type A is:

[JEST 2016]

(a) 0.8 (b) 0.25

(c) 0.33 (d) 0.992

2. There are three states of energy $E, 0, -E$ available for the population of two identical noninteracting spinless fermions. If they are in equilbrium at temperature T, what is the average energy of the system? The Boltzmann constant is k_B and consider β to be $\frac{1}{k_B T}$.

[JEST 2023]

(a) 0

(b) $\dfrac{E\left(e^{\beta E}-e^{-\beta E}\right)}{1+e^{-\beta E}+e^{\beta E}}$

(c) $\dfrac{E\left(e^{-\beta E}-e^{\beta E}\right)}{1+e^{-\beta E}+e^{\beta E}}$

(d) $\dfrac{E\left(2e^{-\beta E}+e^{-\beta E}-e^{\beta E}-2e^{2\beta E}\right)}{2e^{-\beta E}+e^{-\beta E}+2+e^{\beta E}+e^{2\beta E}}$

3. The energy spectrum for a system of spinless noninteracting fermions consists of $(N+1)$ nondegenerate energy levels $0, \varepsilon, 2\varepsilon, \ldots, N\varepsilon (\varepsilon > 0)$. Let

$$x = \exp\left(-\frac{\varepsilon}{k_B T}\right)$$

, where k_B is the Boltzmann constant and T is the temperature. For N identical fermions in thermal equilibrium at temperature T, what is the average occupancy of the highest energy level?

[JEST 2024]

(a) $\dfrac{x}{1-x^N}$ (b) $\dfrac{x-x^{N+1}}{1+x^{N+1}}$

(c) $\dfrac{x-x^{N+1}}{1-x^{N+1}}$ (d) $\dfrac{x^N}{1+x^N}$

4. A system of two noninteracting identical bosons is in thermal equilibrium at temperature T. The particles can be in one of three states with nondegenerate energy eigenvalues $-\varepsilon, 0$ and ε. The temperature T is such that $\exp\left(-\frac{\varepsilon}{k_B T}\right) = \frac{1}{2}$, where k_B is the Boltzmann's constant. The average energy of the system is found to be $\langle E \rangle = -\frac{n}{35}\varepsilon$, where n is an integer. What is the value of n ?

[JEST 2024]

Answer Key				
CSIR-NET				
1. a	2. b	3. d	4. c	5. d
6. d				
GATE				
1. c	2. 1262	3. a		
JEST				
1. d	2. c	3. c	4.	

Ideal Fermi Gas

❖ **CSIR-NET PYQ**

1. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temperature will

[CSIR: DEC-2011]

(a) increase by a factor of 4

(b) decrease by a factor of 4

(c) increase by a factor of 8

(d) decrease by a factor of 8

2. The pressure of a non-relativistic free Fermi gas in three-dimensions depends, at $T = 0$, on the density of fermions n as

[CSIR: JUNE-2014]

(a) $n^{5/3}$ (b) $n^{1/3}$

(c) $n^{2/3}$ (d) $n^{4/3}$

3. An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single-particle quantum states. If the probability that a particular quantum state is unoccupied is 0.1 , then the average number of bosons in that state is

[CSIR: DEC-2014]

(a) 8 (b) 9

(c) 10 (d) 11

4. Consider a quantum system of non-interacting bosons in contact with a particle bath. The probability of finding no particle in a given single particle quantum state is 10^{-6}. The average number of particles in that state is of the order of

[CSIR: DEC-2017]

(a) 10^9 (b) 10^6

(c) 10^9 (d) 10^{12}

5. The single particle energies of a system of N non-interacting fermions of spin s(at $T = 0$) are $E_n = n^2 E_0, n = 1,2,3, \dots ....$ The ratio of

$$\frac{\varepsilon_F\left(\frac{3}{2}\right)}{\varepsilon_F\left(\frac{1}{2}\right)}$$

the Fermi energy of Fermions of spin $\frac{3}{2}$ and $\frac{1}{2}$ is

[CSIR: JUNE-2023]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) 1

6. The dispersion relation of electrons in three dimensions is $\varepsilon(k) = \hbar v_F k$, where v_F is the Fermi. If at low temperature $T << T_F$ the Fermi energy ε_F depends on the number density n as $\varepsilon_F(n) \sim n^\alpha$, the value of α is

[CSIR: JUNE-2023]

(a) 1/3 (b) 2/3

(c) 1 (d) 3/5

7. The electron cloud (of the outermost electrons) of an ensemble of atoms of atomic number Z is described by a continuous charge density $\rho(r)$ that adjusts itself so that the electrons at the Fermi level have zero energy. If $V(r)$ is the local electrostatic potential, then $\rho(r)$ is

[CSIR: JUNE-2023]

(a) $\frac{e}{3\pi^2\hbar^3}[2m_e eV(r)]^{3/2}$

(b) $\frac{Ze}{3\pi^2\hbar^3}[2m_e eV(r)]^{3/2}$

(c) $\frac{Ze}{3\pi^2\hbar^3}[Zm_e eV(r)]^{3/2}$

(d) $\frac{e}{3\pi^2\hbar^3}[m_e eV(r)]^{3/2}$

8. For an ideal Bose gas, the density of states is given by $\rho(E) = CE^2$, where C is a positive constant. Assume that the number of bosons is not conserved. The variation of the specific heat of the gas with temperature T is closest to

[CSIR MARCH 2025]

(a)T^2 (b)T^3

(c)T (d)T^4

9. Bose condensation experiments are carried out on two samples A and B of an ideal Bose gas. The same gas species is used in both. The condensate densities achieved at a given temperature below the critical temperature are $n_A = 1.80 \times$

10^{14} cm^{-3} and $n_B = 1.44 \times 10^{15}$ cm^{-3}, respectively. If P_A and P_B are the pressures of the two gas samples, the ratio $\frac{P_A}{P_B}$ is

[CSIR MARCH 2025]

(a) 1 (b) $\left(\frac{1}{8}\right)^{\frac{3}{2}}$

(c) $\left(\frac{1}{8}\right)^{\frac{2}{3}}$ (d) 8

10. Consider a free fermion gas in a hypercubic infinite potential well in hypothetical 4-dimensional space. What will be the expression for ground state energy per particle in term of the Fermi energy E_F ? (Ignore spin degeneracy of the fermions)

[CSIR MARCH 2025]

(a) $\frac{4}{5}E_F$ (b) $\frac{2}{3}E_F$

(c) $\frac{1}{3}E_F$ (d) $\frac{2}{5}E_F$

❖ **GATE PYQ**

1. The distribution function $f(\varepsilon)$ for a photon gas is given by

[GATE 1995]

(a) $\exp(-\varepsilon/kT)$ (b) $\frac{1}{\exp(\varepsilon/kT)+1}$

(c) $\frac{1}{\exp(\varepsilon/kT)-1}$ (d) $\frac{1}{\exp(-\varepsilon/kT)-1}$

2. Choose the correct alternatives
At the same temperature

[GATE 1999]

(a) a fermion gas will exert the greatest pressure

(b) a boson gas will exert the greatest pressure

(c) a fermion gas will exert the least pressure

(d) a boson gas will exert the least pressure

3. The Fermi energy of a free electron gas depends on the electron density ρ as,

[GATE 2000]

(a) $p^{1/3}$ (b) $\rho^{2/3}$

(c) $\rho^{-1/3}$ (d) $\rho^{-2/3}$

4. For an energy state E of a photon 'gas', the density of states is proportional to

[GATE 2001]

(a) $\sqrt{3}$ (b) E

(c) $E^{1/2}$ (d) E^2

5. The specific heat of an ideal Fermi gas in 3-dimension at very low temperatures (T) varies as

[GATE: 2004]

(a) T (b) $T^{3/2}$

(c) T^2 (d) T^3

6. The equation of state of a dilute gas at very high temperature is described by

$$\frac{pv}{k_B T} \approx 1 + \frac{B(T)}{v}$$

, where v is the volume per particle and $B(T)$ is a negative quantity. One can conclude that this is a property of

[GATE: 2004]

(a) a van der Waals gas (b) an ideal Fermi gas

(c) an ideal Bose gas (d) an ideal inert gas

7. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is

[GATE: 2005]

(a) $p = \frac{3}{2}\frac{U}{V}$ (b) $p = \frac{2}{3}\frac{U}{V}$

(c) $p = \frac{3}{5}\frac{U}{V}$ (d) $p = \frac{1}{2}\frac{U}{V}$

8. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T = 0$ is

[GATE: 2005]

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$

(c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

9. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by

[GATE: 2008]

(a) $\frac{1}{3}NE_F$ (b) $\frac{1}{2}NE_F$

(c) $\frac{2}{3}NE_F$ (d) $\frac{3}{5}NE_F$

10. A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is

[GATE: 2008]

(a) $\exp\left(\frac{\hbar\omega}{k_BT}\right) + 1$

(b) $\exp\left(\frac{\hbar\omega}{k_BT}\right) - 1$

(c) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right) + 1\right)^{-1}$

(d) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right) - 1\right)^{-1}$

11. For a Fermi gas of N particles in three dimensions at $T = 0K$, The Fermi energy, E_F is proportional to

[GATE: 2009]

(a) $N^{2/3}$ (b) $N^{3/2}$

(c) N^3 (d) N^2

12. For a two-dimensional free electron gas, the electronic density n, the Fermi energy E_F are related by **[GATE: 2010]**

(a) $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ (b) $n = \frac{mE_F}{\pi\hbar^2}$

(c) $n = \frac{mE_F}{2\pi\hbar^2}$ (d) $n = \frac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

13. For an ideal Fermi gas in three dimensions the electron velocity v_F at the Fermi surface is related to electron concentration n as

[GATE: 2012]

(a) $v_F \propto n^{2/3}$ (b) $v_F \propto n$

(c) $v_F \propto n^{1/2}$ (d) $v_F \propto n^{1/3}$

14. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ Where N is the number of particles and V is the volume of the gas. Identify the correct equation of state (P being the pressure),

[GATE: 2012]

(a) $PV = \frac{1}{3}E$ (b) $PV = \frac{2}{3}E$

(c) $PV = E$ (d) $PV = \frac{5}{3}E$

15. Given that the Fermi energy of gold is 5.54eV, the number density of electrons is (upto one decimal place)
(Mass of electron $= 9.11 \times 10^{-31}$ kg; $h = 6.626 \times 10^{-34}$ J = s; leV $= 1.6 \times 10^{-19}$ J) 5.9

[GATE: 2015]

16. Consider a 2-dimensional electron gas with a density of 10^{19} m^{-2}. The Fermi energy of the system is____ eV (up to two decimal places).
($m_e = 9.31 \times 10^{-31}$ kg. $h = 6.626 \times 10^{-34}$Js, $e = 1.602 \times 10^{-19}$C)

[GATE: 2017]

❖ **JEST PYQ**

1. An ideal gas of non-relativistic fermions in 3-dimensions is at 0 K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

[JEST 2014]

(a) $2^{1/2}$ (b) 1

(c) $2^{1/3}$ (d) $2^{-1/3}$

2. Consider N non-interacting electrons ($N \sim N_A$) in a box of sides L_x, L_y, L_z Assume that the dispersion relation is $\epsilon(k) = Ck^4$ where C is a constant, the ratio of the ground state energy per particle to the Fermi energy is :

[JEST 2016]

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$

(c) $\frac{3}{5}$ (d) $\frac{5}{7}$

3. A cylinder at temperature $T = 0$ is separated into two compartments A and B by a free sliding piston. Compartments A and B are filled by Fermi gases made of spin 1/2 and 3/2 particles respectively. If particle in both the compartments have same mass, the ratio of equilibrium density of the gas in compartment A to that of gas in compartment B is

[JEST-2017]

(a) 1 (b) $\frac{1}{3^{2/5}}$

(c) $\frac{1}{2^{2/5}}$ (d) $\frac{1}{2^{2/3}}$

4. Consider a non-relativistic two-dimensional gas of N electrons with the Fermi energy E_F. What is the average energy per particle at temperature $T=0$?

[JEST-2019]

(a) $\frac{3}{5}E_F$ (b) $\frac{2}{5}E_F$

(c) $\frac{1}{2}E_F$ (d) E_F

❖ TIFR PYQ

1. Consider two energies of a free electron gas in a metal at an absolute temperature T, viz.,

$E_{\pm}=E_F\pm\Delta$

where E_F is the Fermi level. If the corresponding electron populations $n(E_{\pm})$ satisfy the relation $n(E_-)/n(E_+)=2$, then $\Delta=$

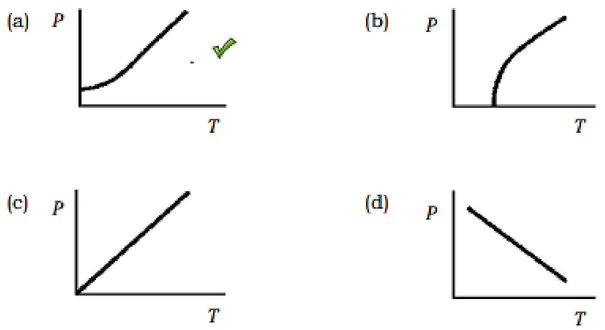
[TIFR-2013]

(a) $k_BT\ln\ 2$ (b) $2k_BT$

(c) $k_BT/2$ (d) k_BT

2. Which of the following graphs qualitatively describes the pressure P of a gas of non-interacting fermions in thermal equilibrium at a constant volume as a function of temperature?

[TIFR-2015]



3. In two dimensions, two metals A and B, have the number density of free electrons in the ratio

$$n_A:n_B=1:2$$

The ratio of their Fermi energies is

[TIFR-2017]

(a) $2:3$ (b) $1:2$

(c) $1:4$ (d) $1:8$

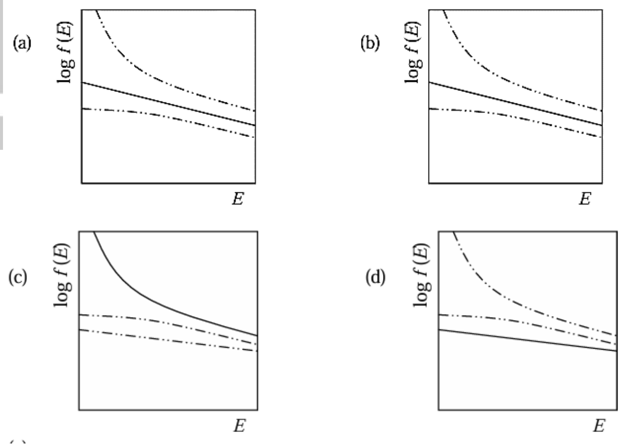
4. Assume that the crystal structure of metallic copper (Cu) results in a density of atoms $\rho_{\mathrm{Cu}}=8.46\times10^{28}\ \mathrm{m}^{-3}$. Each Cu atom in the crystal donates one electron to the conduction band, which leads, for the 3-D Fermi gas, to a density of states $g(\varepsilon)=\frac{1}{2\pi^2}\left(\frac{2m^x}{\hbar^2}\right)^{3/2}\varepsilon^{1/2}$

where m^* is the effective mass of the conduction electrons. In the low temperature limit (i.e. $T=0$ K), find the Fermi energy E_F, in units of eV. You may assume m* to be equal to the free electron mass m_e.

[TIFR-2017]

5. Four students are asked to draw on the same semi-logarithmic plot the energy distributions $f(E)$ of a classical gas (with a solid line), a boson gas (with a dashed line) and a fermion gas (with a dash-dot line) respectively, each as a function of energy E. Only one student's answer was correct. The graphs submitted by the four students are given below. The correct one is

[TIFR-2020]



6. A pseudo-potential V_{12} between every pair of particles in an ideal gas is to be constructed which will reproduce the effects of quantum statistics if the gas particles are bosonic in nature. A correct formula for this, in terms of the inter-particle distance r_{12} and a mean distance λ, will be of the form

[TIFR-2022]

(a) $V_{12} = -k_B T \ln\left(1 + e^{-2\pi r_{12}^2/\lambda^2}\right)$

(b) $V_{12} = -k_B T \ln\left(1 - e^{-2\pi r_{12}^2/\lambda^2}\right)$

(c) $V_{12} = +k_B T \ln\left(1 + e^{-2\pi r_{12}^2/\lambda^2}\right)$

(d) $V_{12} = +k_B T \ln\left(1 - e^{-2\pi r_{12}^2/\lambda^2}\right)$

7. A particle of mass m in a three-dimensional potential well has a Hamiltonian of the form

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 y^2 + 2m\omega^2 z^2$$

where ω is a constant. If there are two identical spin-1/2 particles in this potential having a total energy

the entropy of the system will be

$E = 6\hbar\omega$

[TIFR-2022]

(a) $k_B \ln\ 14$ (b) $k_B \ln\ 16$

(c) $k_B \ln\ 12$ (d) $k_B \ln\ 10$

8. Two types of particles A and B have the same mass, but are distinguished by an internal degree of freedom. A classical ideal gas in a volume V at temperature T contains (X) $2N$ particles of A-type and $(Y)N$ particles of A-type and N particles of B-type. Which of the following is true?

[TIFR 2025]

(a) Pressure of (X) and (Y) are same; (Y) has more entropy than (X)

(b) Pressure of (X) and (Y) are same; (X) has more entropy than (Y)

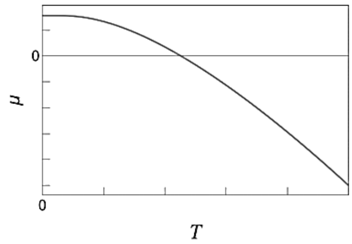
(c) Pressure of (X) is greater than pressure of (Y); (X) has more entropy than (Y)

(d) Pressure of (X) is greater than pressure of (Y); (Y) has more entropy than (X)

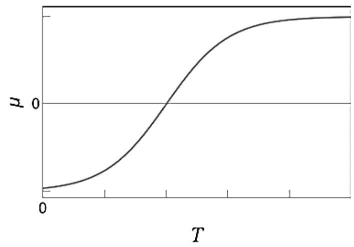
9. Consider a (non-relativistic) gas of fermions in a container with a fixed density n. Which plot best describes how the chemical potential μ changes with T ?

[TIFR 2025]

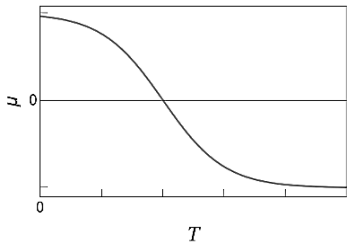
(a)



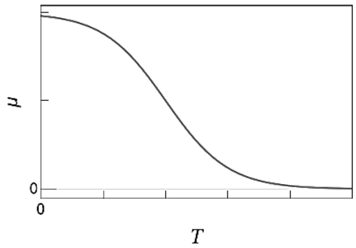
(b)



(c)



(d)



❖ Answer Key				
CSIR-NET				
1. a	2.	3. b	4. b	5. b
6. a	7. a	8. b	9.	10. b
GATE				
1.	2.	3.	4. d	5.
6. a	7. b	8. d	9. b	10. d
11. a	12. b	13. d	14. b	15. 5.9
16. 2.34				
JEST PYQ				
1. d	2. a	3. c	4. c	
TIFR PYQ				
1. a	2. a	3. b	4. 6.87	5. a
6. a	7. a	8. a	9. a	

Bose Einstein Condensation

❖ CSIR-NET PYQ

1. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a nonzero temperature is:

[CSIR: JUNE-2011]

(a) $1 < s < 3$ (b) $0 < s < 2$

(c) $0 < s < 3$ (d) $0 < s < \infty$

2. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4 ? (Assume that it is a perfect Bose gas)

[CSIR: JUNE-2012]

(a) 2.17mK (b) 21.7mK

(c) 21.7μK (d) 2.17μK

3. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

[CSIR: DEC-2012]

(a) $\hbar\omega/2$ (b) $\hbar\omega$

(c) $3\hbar\omega/2$ (d) 0

4. An ideal Bose gas in d-dimensions obeys the dispersion relation $s(\hat{k}) = Ak^s$, where A and s are constants. For Bose-Einstein condensation to occur, the occupancy of excited states

$$N_e = c\int_0^\infty \frac{\varepsilon^{(d-s)/s}}{e^{\beta(\varepsilon-\mu)}-1}d\varepsilon$$

where c is a constant, should remain finite even for $\mu = 0$. This can happen if

[CSIR: JUNE-2015]

(a) $\frac{d}{s} < \frac{1}{4}$ (b) $\frac{1}{4} < \frac{d}{s} < \frac{1}{2}$

(c) $\frac{d}{s} > 1$ (d) $\frac{1}{2} < \frac{d}{s} < 1$

5. The dispersion relation of a gas of non-interacting bosons in d dimensions is $E(k) = ak^s$, where a and S are positive constants. Bose-Einstein condensation will occur for all values of

[CSIR: JUNE -2021]

(a) $d > s$

(b) $d + 2 > s > d - 2$

(c) $s > 2$ independent of d

(d) $d > 2$ independent of S

6. A system of non-relativistic and non-interacting bosons of mass m in two dimensions has a density n. The Bose-Einstein condensation temperature T_c is

[CSIR DEC -2023]

(a) $\frac{12nh^2}{\pi m k_B}$ (b) $\frac{3n\hbar^2}{\pi m k_B}$

(c) $\frac{6nh^2}{\pi m k_B}$ (d)0

❖ GATE PYQ

1. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

[GATE: 2002]

(a)$T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\zeta\left(\frac{3}{2}\right)}\right\}^{3/2}$

(b)$T < \frac{h^2}{2\pi m k_B}\left\{\frac{V}{N_{\zeta}\left(\frac{3}{2}\right)}\right\}^{3/2}$

(c)$T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

(d)$T < \frac{h^2}{2\pi m k_B}\left\{\frac{V}{N\zeta\left(\frac{3}{2}\right)}\right\}^{1/2}$

where m is the mass of each particle of the system, k_B is the Boltzmann constant, h is the Planck's constant and ζ is the well known Zeta function.

2. Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle

[GATE: 2010]

(a) $^{1}H_1$ (b) $^{4}He_2$

(c) $^{23}Na_{11}$ (d) $^{30}K_{19}$

3. In Bose-Einstein condensates, the particles

[GATE: 2015]

(a) have strong interparticle attraction

(b) condense in real space

(c) have overlapping wave functions

(d) have large and positive chemical potential

4. A large number N of ideal bosons, each of mass m, are trapped in a three-dimensional potential

$$V(r) = \frac{m\omega^2 r^2}{2}$$

The bosonic system is kept at temperature T which is much lower than the Bose-Einstein condensation temperature T_c. The chemical potential (μ) satisfies **[GATE: 2019]**

(a) $\mu \leq \frac{3}{2}\hbar\omega$ (b) $2\hbar\omega > \mu > \frac{3}{2}\hbar\omega$

(c) $3\hbar\omega > \mu > 2\hbar\omega$ (d) $\mu = 3\hbar\omega$

❖ Answer Key				
CSIR-NET				
1. c	2. b	3. c	4. c	5. c
6. d				
GATE				
1. a	2. d	3. c	4. a	

❖ CSIR-NET PYQ

1. Consider a system of three spins S_1, S_2 and S_3 each of which can take values +1 and -1. The energy of the system is given by $E = -J[S_1 S_2 + S_2 S_3 + S_3 S_1]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,

[CSIR: DEC-2012]

(a) J and 1 (b) -3 J and 1

(c) -3 J and 2 (d) -6 J and 2

2. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2 J. In a configuration with r spin flipss, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_r$; r varies from 0 to N. The partition function is

[CSIR: DEC-2012]

(a) $\left(\frac{J}{k_B T}\right)^N$ (b) $e^{-NJ/k^T T}$

(c) $\left(\sinh \frac{J}{k_B T}\right)^N$ (d) $\left(\cosh \frac{J}{k_B T}\right)^N$

3. Consider a system of two Ising spins S_1 and S_2 taking value ± 1 with interaction energy given by $\varepsilon = -JS_{[}S_2$, when it is in thermal equilibrium at temperature T. For large T, the average energy of the system varies as $C/k_B T$, with C given by

[CSIR: JUNE-2013]

(a) $-2\,J^2$ (b) $-J^2$

(c) J^2 (d) 4 J

4. A collection N of non-interacting spins $S_i, i = 1,2,\ldots,N, (S_i = \pm 1)$ is kept in an external magnetic field B at a temperature T. The Hamiltonian of the system is $H = -\mu B \sum_i S_i$. What should be the minimum value of $\frac{\mu B}{k_B T}$ for which the mean value $\langle S_i \rangle \geq \frac{1}{3}$?

[CSIR: DEC-2014]

(a) $\frac{1}{2} N \ln 2$ (b) $2 \ln 2$

(c) $\frac{1}{2} \ln 2$ (d) $N \ln 2$

5. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength J. The partition function of the system at temperature T is given by $\left(\beta = \frac{1}{k_g T}\right)$:

[CSIR: JUNE-2015]

(a) $2e^{3,\beta/} + 6e^{-\beta\rho}$

(b) $2e^{-3\beta J} + 6e^{\beta J}$

(c) $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$

(d) $(2\cosh \beta J)^3$

6. The partition function of a system of N Ising spins is $Z = \lambda_1^N + \lambda_2^N$, where λ_1 and λ_2 are functions of temperature, but are independent of N. If $\lambda_1 > \lambda_2$, the free energy per spin in the limit $N \to \infty$ is

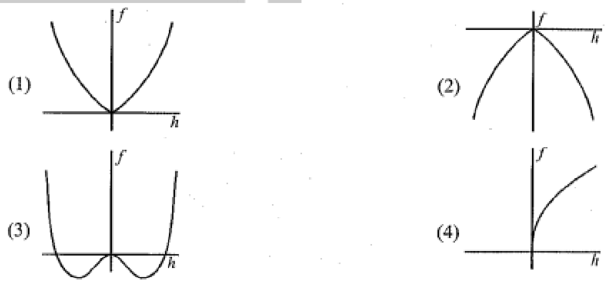
[CSIR DEC-2015]

(a) $-k_B T \ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (b) $-k_B T \ln \lambda_2$

(c) $-k_\beta T \ln(\lambda_1 \lambda_2)$ (d) $-k_B T \ln \lambda_1$

7. Which of the following graphs shows the qualitative dependence of the free energy $f(h,T)$ of a ferromagnet in an external magnetic field h, and at a fixed temperature $T < T_v$, where T_c is the critical temperature?

[CSIR DEC-2015]



8. An ensemble of non-interacting spin-$\frac{1}{2}$ particles is in contact with a heat bath at temperature T and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm\epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is

[CSIR DEC-2015]

(a) $-2\epsilon_0 \frac{\ln(4/\sqrt{3})}{\ln 3}$ (b) $-\epsilon_0 \ln(3/2)$

(c) $-2\epsilon_0 \ln 2$ (d) $-\epsilon_0 \frac{\ln 2}{\ln 3}$

9. The Hamiltonian of a system of N non-interacting spin- $\frac{1}{2}$ particles is $H = -\mu_0 B \sum_i S_i^z$, where $S_i^2 = \pm 1$ are the components of i^{th} spin along an external magnetic field B. At a temperature T such that $e^{\mu_0//k_a T} = 2$, the specific heat per particle is

[CSIR: DEC-2015]

(a) $\frac{16}{25} k_B$ (b) $\frac{8}{25} k_B \ln 2$

(c) $k_B (\ln 2)^2$ (d) $\frac{16}{25} k_B (\ln 2)^2$

10. The Hamiltonian for three Ising spins S_0, S_1 and S_2, taking values ± 1, is $H = -jS_0(S_1 + S_2)$. If the system is in equilibrium at temperature T, the average energy of the system, in terms of $\beta = (k_B T)^{-1}$ is

[CSIR: JUNE-2017]

(a) $-\frac{1 + \cosh(2\beta j)}{2\beta \sinh(2\beta j)}$

(b) $-2j[1 + \cosh(2\beta j)]$

(c) $-2/\beta$

(d) $-2j \frac{\sinh(2\beta j)}{1 + \cosh(2\beta j)}$

11. The Hamiltonian of a one-dimensional Ising model of N spins (N large) is

$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1}$$

where the spin $\sigma_i = \pm 1$ and J is a positive constant. At inverse temperature $\beta = \frac{1}{k_B T}$, the correlation function between the nearest neighbour spins $\langle \sigma_i \sigma_{i+1} \rangle$ is

[CSIR: DEC-2018]

(a) $\frac{e^{-\beta J}}{(e^{\beta J} + e^{-\beta J})}$ (b) $e^{-2\beta J}$

(c) $\tanh(\beta J)$ (d) $\coth(\beta J)$

12. The free energy of a magnetic system, as a function of its magnetization m, is F $\frac{1}{2} am^2 - \frac{1}{4} bm^4 + \frac{1}{6} m^6$, where a and b are positive constants. At a fixed value of a, the critical value of b, above which the minimum of F will be at a non-zero value of magnetisation, is

[CSIR JUNE 2019]

(a) $\sqrt{10a/3}$ (b) $\sqrt{16a/3}$

(c) $\frac{10}{3}\sqrt{a}$ (d) $\frac{16}{3}\sqrt{a}$

13. The Hamiltonian of three Ising spins S_1, S_2 and S_3, each taking values ± 1, is $H = -J(S_1 S_2 + S_2 S_3) - hS_1$, where J and h are positive constants. The mean value of S_3 in equilibrium at a temperatrue $T = 1/(k_B \beta)$, is

[CSIR JUNE 2019]

(a) $\tanh^3(\beta J)$

(b) $\tanh(\beta h) \tanh^2(\beta J)$

(c) $\sinh(\beta h) \sinh^2(\beta J)$

(d) 0

14. The Hamiltonian of a system of 3 spins is $H = J(S_1 S_2 + S_2 S_3)$, where $S_i = \pm 1$ for $i = 1,2,3$. Its canonical partition function, at temperature T, is

[CSIR: JUNE-2020]

(a) $2\left(2\sin h \frac{J}{k_B T}\right)^2$ (b) $2\left(2cos\, h \frac{J}{k_B T}\right)^2$

(c) $2\left(2cos\, h \frac{J}{k_B T}\right)$ (d) $2\left(2\cosh \frac{J}{k_B T}\right)^3$

15. In a one-dimensional system of N spins the allowed values of each spin are $\sigma_i = \{1,2, \ldots.. q\}$ where $q \geq 2$ is an integer. The energy of the system is $-J \sum \delta_{\sigma i, \sigma i+1}$

Where $j > 0$ is a constant. If periodic boundary conditions are imposed, the number of ground states of the

[CSIR: JUNE-2023]

(a) q (b) Nq

(c) q^N (d) 1

16. Two electrons in thermal equilibrium at temperature $T = \frac{k_B}{\beta}$ can occupy two sites. The energy of the configuration in which they occupy the different sites is $JS_1 \cdot S_2$ (where J>0 is a constant and S denotes the spin of an electron), while it is U if they are at the same site. If $U = 10J$, the probability for the system to be in the first excited state is

[CSIR: JUNE-2023]

(a) $e^{-3\beta J/4}/(3e^{\beta J/4} + e^{-3\beta J/4} + 2e^{-10\beta})$

(b) $3e^{-\beta J/4}/(3e^{-\beta/4} + e^{3\beta J/4} + 2e^{-10\beta J})$

(c) $e^{-\beta J/4}/(2e^{-\beta J/4} + 3e^{3\beta/4} + 2e^{-10\beta})$

(d) $3e^{-3\beta J/4}/(2e^{\beta J/4} + 3e^{-3\beta J/4} + 2e^{-10\beta J})$

17. Five classical spins are placed at the vertices of a regular pentagon. The Hamiltonian of the system is $H = J\sum S_i S_j$, where $J > 0, S_i = \pm 1$ and the sum is over all possible nearest neighbour pairs. The degeneracy of the ground state is

[CSIR JUNE-2024]

(a) 8 (b) 5

(c) 4 (d) 10

18. Energy of two Ising spins $\left(s = \pm\frac{1}{2}\right)$ is given by

$$E = s_1 s_2 + s_1 + s_2$$

At temperature T, the probability that both spins take the value $-\frac{1}{2}$ is 16 times the probability that both take the value $+\frac{1}{2}$. At the same temperature, what is the probability that the spins take opposite values?

[CSIR MARCH 2025]

(a) $\frac{16}{25}$ (b) $\frac{8}{25}$

(c) $\frac{8}{33}$ (d) $\frac{16}{33}$

19. Consider $2N$ Ising spins, $s_i(s_i = \pm 1)$ in a one-dimensional lattice with periodic boundary conditions. The Hamiltonian is given by

$$H = -J\sum_{i=1}^{2N} s_i s_{i+1}$$

where J denotes the strength of the nearest-neighbour interactions with $J > 0$. Let F be the fully ferromagnetic state and let A be the lowest energy state with zero magnetization. The energy difference between these two states is

(a) $\frac{3J}{2}$ (b) 4 J

(c) $\frac{J}{2}$ (d) $2J$

❖ JEST PYQ

1. The energy of two Ising spins ($s_1 = \pm 1, s_2 = \pm 1$) is given by

$$E = -s_1 s_2 - \frac{1}{2}(s_1 + s_2)$$

. At certain temperature T probability that both spins take $+1$ values is 4 times than they both take -1 values. What is the probability that they have opposite spins? $[\beta = 1/k_B T]$

[JEST 2022]

(a) $\frac{e^\beta}{1+e^{2\beta}}$ (b) $e^\beta \tanh \beta$

(c) $\frac{1}{6}$ (d) $\frac{1}{2}$

❖ Answer Key				
CSIR-NET				
1. c	2. d	3. b	4. c	5. a
6. d	7. b	8. a	9. d	10. d
11. c	12. b	13. b	14. b	15. a
16. b	17. d	18. d	19. b	
JEST PYQ				
1. c				

❖ **CSIR-NET PYQ**

1. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = \gamma T^3$ where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is:

[CSIR JUNE 2011]

(a) $4\gamma T^4$ (b) $2\gamma T^4$

(c) $\frac{\gamma T^4}{4}$ (d) γT^4

2. Consider black body radiation contained in a cavity whose walls are at temperature T. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to $2T$ and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

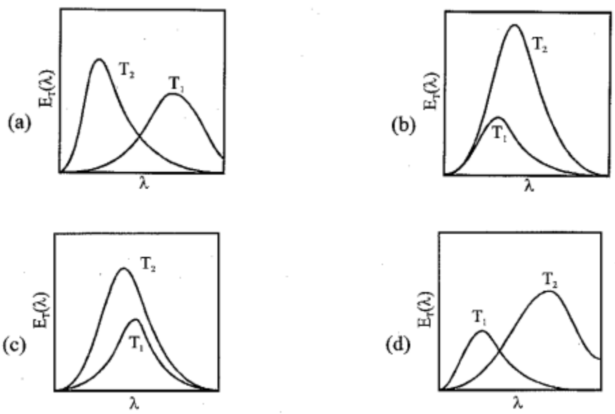
[CSIR JUNE 2012]

(a) 2 (b) 4

(c) 8 (d) 16

3. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and $T_2 (T_1 < T_2)$?

[CSIR: JUNE-2014]



4. A gas of photons inside a cavity of volume V is in equilibrium at temperature T. If the temperature of the cavity is changed to $2T$, the radiation pressure will change by a factor of

[CSIR: JUNE-2017]

(a) 2 (b) 16

(c) 8 (d) 4

5. Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1, respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\to1}$ and $W_{1\to0}^t$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$. Which of the fol lowing equations hold in equilibrium ?

[CSIR: DEC 2017]

(a) $P_0 n W_{0\to1} = P_1 W_{1\to0}$

(c) $P_0 n W_{0\to1} = P_1 W_{1\to0} - P_1 n W_{1\to0}$

(b) $P_0 W_{0\to1} = P_1 n W_{1\to0}$

(d) $P_0 n W_{0\to1} = P_1 W_{1\to0} + P_1 n W_{1\to0}$

6. The maximum intensity of solar radiation is at the wavelength of $\lambda_{\text{sun}} \sim 5000\text{Å}$ and corresponds to its surface temperature $T_{\text{sun}} \sim 10^4$ K. If the wavelength of the maximum intensity of an X-ray star is 5Å, its surface temperature is of the order of

[CSIR JUNE 2018]

(a) 10^{16} K (b) 10^{14} K

(c) 10^{10} K (d) 10^7 K

7. The range of the inter-atomic potential in gaseous hydrogen is approximately 5 A. In thermal equilibrium, the maximum temperature for which the atom-atom scattering is dominantly s-wave, is

[CSIR JUNE 2019]

(a) 500 K (b) 100 K

(c) 1 K (d) I mk

8. Consider black body radiation in thermal equilibrium contained in a two-dimensional box. The dependence of the energy density on the temperature T is

[CSIR DEC 2019]

(a) T^3 (b) T

(c) T^2 (d) T^4

9. The temperatures of two perfect black bodies A and B are $400K$ and $200K$, respectively. If the surface area of A is twice that of B, the ratio of total power emitted by A to that by B is

[CSIR: JUNE-2020]

(a) 4 (b) 2

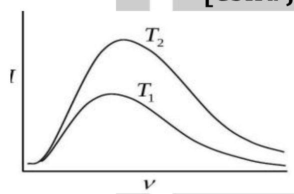
(c) 32 (d) 16

10. The energy density I of a black body radiation at temperature T is given by the Planck's distribution function

$$I(v,T) = \frac{8\pi v^2}{c^3}\frac{hv}{\left(e^{\frac{hv}{k_BT}}-1\right)}$$

, where v is the frequency. The function $I(v,T)$ for two different temperatures T_1 and T_2 are shown in figure.

[CSIR: JUNE-2020]



If the two curves coincide when $I(v,T)v^a$ is plotted against v^b/T, then the values of a and b are, respectively,

(a) 2 and 1 (b) -2 and 2

(c) 3 and -1 (d) -3 and 1

11. The volume and temperature of a spherical cavity filled with black body radiation are V and 300 K respectively. If it expands adiabatically to a volume 2 V, its temperature will be closest to

[CSIR JUNE -2021]

(a) 150 K (b) 300 K

(c) $250K$ (d) $240K$

12. A spherical cavity of volume V is filled with thermal radiation at temperature T. The cavity expands adiabatically to 8 times its initial volume. If σ is Stefan's constant and c is the speed of light in vacuum, what is the closest value of the work done in the process?

[CSIR MARCH 2025]

(a) $8\left(\frac{\sigma T^4 V}{c}\right)$ (b) $4\left(\frac{\sigma T^4 V}{c}\right)$

(c) $\frac{1}{2}\left(\frac{\sigma T^4 V}{c}\right)$ (d) $2\left(\frac{\sigma T^4 V}{c}\right)$

❖ GATE PYQ

1. Consider the Fermi-Dirac distribution function f(E) at room temperature (300 K) where E refers to energy. If E_F is the Fermi energy, which of the following is true?

[GATE: 2003]

(a) f(E) is a step function

(b) $f(E_F)$ has a value of ½

(c) States with $E < E_F$ are filled completely

(d) f(E) is large and tends to infinity as E decreases much below E_F

2. Suppose temperature of the sun goes down by a factor of two, then the total power emitted by the sun will go down by a factor of

[GATE 1997]

(a) 2 (b) 4

(c) 8 (d) 16

3. A cavity of volume V contains blackbody radiation at an absolute temperature T . Which of the following option is incorrect?

[GATE 1991]

(a) The internal energy of the radiation is proportional to T^4.

(b) The specific heat of radiation is proportional to T^3.

(c) The pressure of the radiation is proportional to T^4

(d) Although the radiation is in thermal equilibrium, the number, of photons in the cavity is not constant in time.

4. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?

[GATE: 2003]

(a) its energy and the number of photons both increase 8 times

(b) its energy increases 8 times and the number of photons increases 16 times

(c) its energy increases 16 times and the number of photons increases 8 times

(d) its energy and the number of photons both increase 16 times

5. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to

[GATE: 2004]

(a) 800 K (b) 700 K

(c) 600 K (d) 500 K

Statement for Linked Answer Question 6 and 7 :
Consider a radiation cavity of volume V at temperature T.
Consider a radiation cavity of volume V at temperature T.

6. The density of states at energy E of the quantized radiation (photons) is

[GATE: 2006]

(a) $\frac{8\pi V}{h^3c^3}E^2$ (b) $\frac{8\pi V}{h^3c^3}E^{3/2}$

(c) $\frac{8\pi V}{h^3c^3}E$ (d) $\frac{8\pi V}{h^3c^3}E^{1/2}$

7. The average number of photons in equilibrium inside the cavity is proportional to

[GATE: 2006]

(a) T (b) T^2

(c) T^3 (d) T^4

8. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the

[GATE: 2010]

(a) visible range of frequencies

(b) infrared range of frequencies

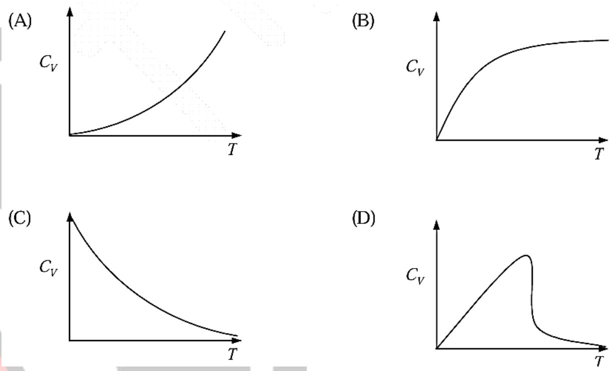
(c) ultraviolet range of frequencies

(d) microwave range of frequencies

Common Data for Question 9 and 10:
Partition function for a gas of photons is given as

$$\ln Z = \frac{\pi^2 V (k_B T)^3}{45\hbar^3 C^3}$$

9. The specific heat of the photon gas varies with temperature as **[GATE: 2010]**



10. The pressure of the photon gas is

(a) $\frac{\pi^2 (k_B T)^3}{15\hbar^3 C^3}$ (b) $\frac{\pi^2 (k_B T)^4}{8\hbar^3 C^3}$

(c) $\frac{\pi^2 (k_B T)^4}{45\hbar^3 C^3}$ (d) $\frac{\pi^2 (k_B T)^{3/2}}{45\hbar^3 C^3}$

11. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

[GATE: 2015]

(a) the chemical potential of the photons is zero

(b) photons obey Pauli exclusion principle

(c) photons are spin-1 particles

(d) the entropy of the photons is very large

12. The total power emitted by a spherical black body of radius R at a temperature T is P_1. Let P_2 be the total power emitted by another spherical black body of radius $R/2$ kept at temperature $2T$. The ratio, P_1/P_2 is ________ (Give your answer upto two decimal places)

[GATE: 2016]

13. The energy density and pressure of a photon gas are given by $u = aT^4$ and $P = u/3$, where T is the temperature and a is the radiation constant. The entropy per unit volume is given by $\alpha a T^3$. The value of α is (up to two decimal places). [GATE: 2017]

14. Two solid spheres A and B have same emissivity. The radius of A is four times the radius of B and temperature of A is twice the temperature of B. The ratio of the rate of heat radiated from A to that from B is -. [GATE: 2018]

15. The Planck's energy density distribution is given by

$$u(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/k_B T} - 1)}$$

At long wavelengths, the energy density of photons in thermal equilibrium with a cavity at temperature T varies as T^α, where α is

[GATE: 2020]

16. A light source having its intensity peak at the wavelength 289.8 nm which is calibrated as 10,000 K which is the temperature of an equivalent black body radiation. Considering the same calibration, the temperature of light source (in K) having its intensity peak at the wavelength 579.6 nm (rounded off to the nearest integer) is

[GATE: 2021]

❖ **JEST PYQ**

1. The blackbody at a temperature of 6000 K emits a radiation whose intensity spectrum peaks at 600 nm. If the temperature is reduced to 300 K, the spectrum will peak at,

[JEST-2015]

(a) 120μm (b) 12μm

(c) 12 mm (d) 120 mm

❖ **TIFR PYQ**

1. A binary star is observed to consist of a blue star B (peak wavelength 400 nm) and a red star R (peak wavelength 800 nm) orbiting each other. As observed from the Earth, B and R appear equally bright. Assuming that the stars radiate as perfect blackbodies, it follows that the ratio of volumes V_B/V_R of the two stars is

[TIFR 2013]

(a) 1/64 (b) 64

(c) 16 (d) 1/16

2. A gas of photons is enclosed in a container of fixed volume at an absolute temperature T. Noting that the photon is a massless particle (i.e., its energy and momentum are related by $E = pc$), the number of photons in the container will vary as

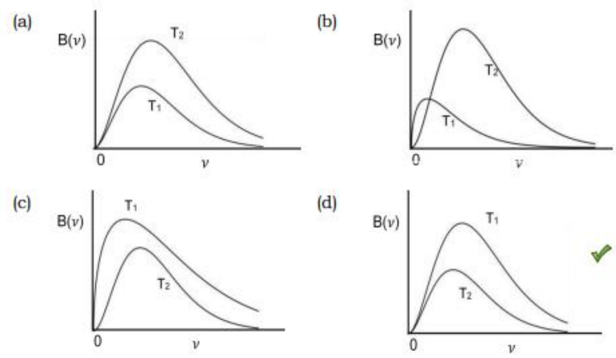
[TIFR-2014]

(a) T (b) T^2

(c) T^3 (d) T^4

3. Two blackbodies radiate energy at temperatures T_1 and $T_2 (T_1 > T_2)$. The energy emitted per unit time per unit solid angle per unit surface area of a blackbody in the frequency range v to $v + dv$ is given by $B(v)dv$. Which one of the following graphs has the correct form?

[TIFR-2015]



4. The cosmic microwave background radiation in the Universe has a blackbody distribution corresponding to a temperature 2.735 K. In a certain cosmological model, it was assumed that the universe consists purely of radiation and is undergoing adiabatic expansion. In this model it was predicted that the volume of the Universe will be tripled in the next 10^{10}yrs. The corresponding blackbody radiation temperature would be

[TIFR-2017]

(a) 0.9116 K (b) 2.078 K

(c) 1.896 K (d) 1.526 K

5. In a Universe with only two spatial dimensions, the total energy radiated by a perfect blackbody across all wavelengths per unit surface area per unit time is proportional to

[TIFR-2021]

(a) T^2 (d) T^4

(c) T^3 (d) $T^{3/2}$

6. The equilibrium temperature (T_0) on the surface of a planet results from the balance between the energy received from their host star and the energy they emit back into space. In the case of the Earth, $T_0 = 300$ K and the orbit is almost circular. We may safely assume that planets absorb and emit radiation like perfect blackbodies

Now consider an exoplanet of the same size as the Earth, which orbits a fainter star having a power output only 25% of that of the Sun, in an almost-circular orbit of radius 25% that of the Earth-Sun distance.

The equilibrium temperature T_0' on the surface of this exoplanet will be about

[TIFR-2023]

(a) 212 K (b) 424 K

(c) 300 K (d) 600 K

7. A closed, thermally-insulated box contains one mole of an ideal monatomic gas G in thermodynamic equilibrium with blackbody radiation B. The total internal energy of the system is $U = U_G + U_B$ where U_G and $U_B(\propto T^4)$ are the energies of the ideal gas and the radiation respectively. If $U_G = U_B$ at a certain temperature T_0 K, then the energy required to raise the temperature from T_0 K to $(T_0 + 1)$K, in terms of the gas constant R, is

[TIFR-2011]

(a) $7.5R$ (b) $6R$

(c) 1.5R (d) $0.33R$

❖ Answer Key				
CSIR-NET				
1. c	2. c	3. c	4. b	5. d
6. c	7. c	8. a	9. c	10. d
11. d	12. d			
GATE				
1. b	2.	3.	4. c	5. d
6. a	7. c	8. b	9. b	10. c
11. a	12. 0.25	13. 1.33	14. 256	15. 0.5
16. 5000				
JEST				
1. b				
TIFR				
1. a	2. c	3. d	4. c	5. c
6. b	7.			

❖ **CSIR-NET PYQ**

1. A random walker takes a step of unit length in the positive direction with probability 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is

[CSIR: DEC-2014]

(a) $n/3$ (b) $n/8$

(c) $2n/3$ (d) 0

2. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than once is

[CSIR: DEC-2015]

(a) 12/27 (b) 27/64

(c) 3/8 (d) 9/16

3. A box of volume V containing N molecules of an ideal gas, is divided by a wall with a hole into two compartments. If the volume of the smaller compartment is $V/3$, the variance of the number of particles in it, is

[CSIR: JUNE-2016]

(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$

(c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$

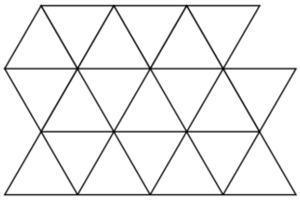
4. Consider a continuous time random walk. If a step has taken place at time $t = 0$, the probability that the next step takes place between t and $t + dt$ is given by $btdt$, where b is a constant. What is the average time between successive steps ?

[CSIR DEC 2016]

(a) $\sqrt{\frac{2\pi}{b}}$ (b) $\sqrt{\frac{\pi}{b}}$

(c) $\frac{1}{2}\sqrt{\frac{\pi}{b}}$ (d) $\sqrt{\frac{\pi}{2b}}$

5. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities of moving to any of the nearest neighbor sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps ?

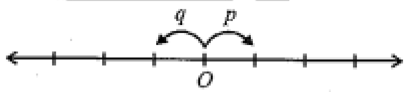
[CSIR: DEC-2016]

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$

(c) $\frac{1}{18}$ (d) $\frac{1}{12}$

6. A particle hops on a one-dimensional lattice with lattice spacing a. The probability of the particle to hop to the neighbouring site to its right is p, while the corresponding probability to hop to the left is $q = 1 - p$. The root-mean-squared deviation $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ in displacement after N steps, is

[CSIR DEC 2018]



(a) $a\sqrt{Npq}$ (b) $aN\sqrt{pq}$

(c) $2a\sqrt{Npq}$ (d) $a\sqrt{N}$

7. At each time step, a random walker in one-dimension either remains at the same point with probability $\frac{1}{4}$, or moves by a distance Δ to the right or left with probabilities 3/8 each. After N time steps, its root mean squared displacement is

[CSIR JUNE 2019]

(a) $\Delta\sqrt{N}$ (b) $\Delta\sqrt{\frac{9N}{16}}$

(c) $\Delta\sqrt{\frac{3N}{4}}$ (d) $\Delta\sqrt{\frac{3N}{8}}$

8. A particle hops randomly from a site to its nearest neighbour in each step on a square lattice of unit lattice constant. The probability of hopping to the positive x-direction is 0.3 , to the negative x direction is 0.2 , to the positive y-direction is 0.2 and to the negative y-direction is 0.3 . If a particle starts from the origin, its mean position after N steps is

[CSIR DEC 2019]

(a) $\frac{1}{10}N(-\hat{\imath}+\hat{\jmath})$ (b) $\frac{1}{10}N(\hat{\imath}-\hat{\jmath})$

(c) $N(0.3\hat{\imath}-0.2\hat{\jmath})$ (d) $N(0.2\hat{\imath}-0.3\hat{\imath})$

9. A photon inside the sun executes a random walk process. Given the radius of the sun $\approx 7\times 10^8$ km and mean free path of a photon $\approx 10^{-3}$ m, the time taken by the photon to travel from the centre to the surface of the sun is closest to

[CSIR: DEC-2023]

(a) 10^6sec (b) 10^{24}sec

(c) 10^{12}sec (d) 10^{18}sec

❖ Answer Key

CSIR-NET				
1. b	2. d	3. b	4. d	5. c
6. c	7. c	8. b	9. c	

❖ **CSIR-NET PYQ**

1. A heater and a thermocouple are used to measure and control temperature T of a sample at $T_0 = 250^\circ C$. A feedback circuit supplies power ' P ' to the heater according to the equation $P = P_0 + G(T_0 - T) - D\frac{dT}{dt}$ with appropriately tuned values of the coefficients G and D. In order to maintain temperature stability in the presence of an external heat perturbation which causes small but rapid fluctuations of temperature, it is necessary to

[CSIR: DEC-2011]

(a) decrease D (b) increase D

(c) decrease G (d) increase G

2. One gram of salt is dissolved in water that is filled to a height of 5 cm in a beaker of diameter 10 cm. The accuracy of length measurement is 0.01 cm while that of mass measurement is 0.01mg. When measuring the concentration C, the fractional error $\Delta C/C$ is

[CSIR: JUNE-2014]

(a) 0.8% (b) 0.14%

(c) 0.5% (d) 0.28%

❖ **Answer Key**

CSIR-NET				
1. a	2. d			