

# January 2018 Paper –II

1. Value of  $\oint_c \frac{dz}{(z+2)}, c: |z| = 1$  is :  
 (a)  $\pi i$  (b) zero  
 (c)  $2\pi i$  (d)  $-2\pi i$
2. The inverse Fourier transform of  $e^{-|k|}$  is :  
 (a)  $\frac{1}{\pi} \frac{1}{1+x^2}$  (b)  $e^{-x^2}$   
 (c)  $e^{-|x|}$  (d)  $\frac{1}{1+x}$
3. Two unbiased dice are thrown. The probability that both dice show the same number is :  
 (a)  $\frac{1}{3}$  (b)  $\frac{11}{36}$   
 (c)  $\frac{1}{6}$  (d)  $\frac{1}{36}$
4. Eigen values of a nilpotent matrix A, i.e., a matrix satisfying  $A^K = 0$  for some integer K are  
 (a) Real and distinct (b) Zero  
 (c) Purely imaginary (d) Real and negative
5. Residue of  $\cot(z)$  at  $z = 0$  is :  
 (a)  $\frac{1}{2}$  (b) 0  
 (c) 2 (d) 1
6. Trace of a  $3 \times 3$  matrix is 6. Two of its eigen values are 1 and 2. The third eigen value is :  
 (a) 1 (b) 2  
 (c) 3 (d) 6
7.  $\frac{d^2f}{dx^2} + 2\text{sech}^2 x f - f = 0$   
 One of the solution is :  
 (a)  $\text{sech } x$  (b)  $\text{sech}^2 x$   
 (c)  $\tanh x$  (d)  $\text{sech}^3 x$
8. The average of function  $f(x) = \sin x$  in the interval 0 to  $\pi$  is :  
 (a)  $1/2$  (b)  $2/\pi$   
 (c)  $1/\pi$  (d)  $4/\pi$
9. If  $r^2 = x^2 + y^2 + z^2$ ,  $\text{grad } r^n$  is :  
 (a) 0 (b)  $r^{n-1}\vec{r}$   
 (c)  $nr^{n-2}\vec{r}$  (d)  $n(n-2)r^n\vec{r}$
10. A simple pendulum of mass m and length 1 oscillates with a frequency  $\omega_0$ . A piece breaks off from the pendulum and the mass is reduced to m/2. The frequency of oscillation  $\omega$  then becomes  
 (a)  $\omega = 2\omega_0$  (b)  $\omega = \omega_0/2$   
 (c)  $\omega = \omega_0$  (d)  $\omega_0 = \sqrt{2}\omega_0$
11. A propagating wave is reflected at a barrier. The phase of the wave changes in the act of reflection by :  
 (a)  $\pi/2$  (b) zero  
 (c)  $\pi$  (d)  $-\pi/2$
12. Two particles of mass  $m_1$  and  $m_2$  and charge  $q_1$  and  $q_2$  respectively are constrained to move in a straight line along x-axis. The Lagrangian of the system (in esu) is :  
 (a)  $\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + q_1q_2/|x_1 - x_2|$   
 (b)  $\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - q_1q_2/|x_1 - x_2|$   
 (c)  $-\frac{1}{2}m_1\dot{x}_1^2 - \frac{1}{2}m_2\dot{x}_2^2 - q_1q_2/|x_1 - x_2|$   
 (d)  $-\frac{1}{2}m_1\dot{x}_1^2 - \frac{1}{2}m_2\dot{x}_2^2 + q_1q_2/|x_1 - x_2|$
13. Time-order of two events can be changed by making a Lorentz transformation only if they occur in :  
 (a) Time-like region

(b) Space-like region

(c) Light-like region

(d) Speed of light can be exceeded

- 14 A satellite is moving around the earth in a circular orbit at a height of  $2R$ , from the earth surface.  $R$  is the radius of the earth. The speed of satellite is :

(a)  $gR/3$  (b)  $(gR/2)^{1/2}$   
(c)  $(gR/3)^{1/2}$  (d)  $gR/2$

- 15 Virial theorem is expressed as

$$\left\langle \sum_{\alpha} p_{\alpha} \cdot \dot{\gamma}_{\alpha} \right\rangle = - \left\langle \sum_{\alpha} p_{\alpha} \cdot \gamma_{\alpha} \right\rangle$$

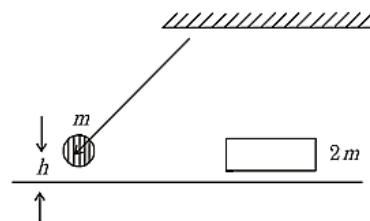
The theorem is valid, if :

- (a)  $\gamma_{\alpha}$  is bounded (remain finite over all values of time)  
(b)  $p_{\alpha}$  is bounded  
(c) both  $\gamma_{\alpha}$  and  $p_{\alpha}$  are bounded  
(d)  $\gamma_{\alpha}$  is unbounded

- 16 Electron and positron annihilate and produce a single photon only when :

- (a) Electron and positron are at rest  
(b) Electron is at rest  
(c) Positron is at rest  
(d) At least one particle is in a bound state

- 17 As shown in the figure a ball of mass  $m$  suspended by an inextensible massless string is released from height  $h$  and collides elastically when it is at its lowest point with a block of mass  $2m$  at rest on a frictionless surface. After the collision the ball will rise to a total height equal to

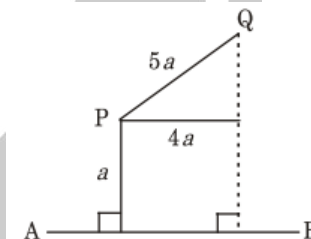


- (a)  $h/9$  (b)  $h/8$   
(c)  $h/3$  (d)  $2h/3$

- 18 Uniformly charged solid sphere of radius  $R$  has a volume charge density  $\rho$ . The strength of electric field, at a distance  $r$ , ( $r > R$ ), from its centre is :

- (a)  $\frac{\rho r^3}{3 \epsilon_0 R^2}$  (b)  $\frac{\rho R^3}{3 \epsilon_0 r^2}$   
(c)  $\frac{\rho R^3}{3 \epsilon_0 r}$  (d)  $\frac{\rho r^3}{3 \epsilon_0 R}$

- 19 Consider the figure shown.  $AB$  is an infinite, uniform line charge distribution ' $\lambda$ '.  $P$  and  $Q$  are two points as shown in the figure. The ratio of strength of electric fields at  $P$  and  $Q$ ; i.e.,  $E_P/E_Q$  is



(fig. not to the scale)

- (a) 3:1 (b) 1:3  
(c) 4:1 (d) 1:4

- 20 Points  $P$  and  $Q$  are at a distance of 10 cm and 20 cm respectively from the ideal magnetic dipole. The ratio of magnitude of vector potential at point  $P$  to that at point  $Q$  is :

- (a) 4 (b) 8  
(c)  $1/4$  (d)  $1/8$

21 A circular loop of radius  $r$  is placed in a magnetic field  $\vec{B}(t) = B_0 e^{-\alpha t} \hat{z}$ ,  $B_0$  is constant. If the loop is in  $xy$  plane, then the induced emf in the loop is :

- (a) Zero (b)  $\pi r^2 \alpha B_0 e^{-\alpha t}$   
(c)  $2\pi r B_0 e^{-\alpha t}$  (d)  $2\pi r \alpha B_0 e^{-\alpha t}$

22 Plane monochromatic electromagnetic wave is propagating through a conducting material of refractive index  $n + ik$ . The phase difference between the fields  $\vec{E}$  and  $\vec{B}$  associated with the wave is :

- (a)  $\tan^{-1}(n)$  (b)  $\tan^{-1}(k)$   
(c)  $\tan^{-1}(nk)$  (d)  $\tan^{-1}(n/k)$

23 Under the Lorentz gauge condition, the electric potential  $V$ , is related to magnetic vector potential  $\vec{A}$  as :

- (a)  $\vec{\nabla} \cdot \vec{A} + (1/\mu_0 \epsilon_0) \frac{\partial V}{\partial t} = 0$   
(b)  $\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$   
(c)  $-\vec{\nabla} \cdot \vec{A} - (1/\mu_0 \epsilon_0) \frac{\partial^2 V}{\partial t^2} = 0$   
(d)  $\vec{\nabla} \cdot \vec{A} - \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$

24 A point charge ' $q$ ' having mass ' $m$ ' follows a circular trajectory of radius ' $R$ ' under the action of magnetic field  $\vec{B}$ . The magnitude of the angular momentum of the particle is :

- (a) Zero (b)  $Bq^2 R^2$   
(c)  $Bmq^2 R^2$  (d)  $BqR^2$

25 If ' $A_1$ ' and ' $A_2$ ' are the amplitudes of two electromagnetic waves (same frequency) coming from two slits in Young's double slit experiment, then the maximum intensity of interference fringe is :

- (a)  $A_1 + A_2$  (b)  $A_1^2 + A_2^2$

- (c)  $(A_1 + A_2)^2$  (d)  $\sqrt{A_1^2 + A_2^2}$

26 Given  $[x_i, p_j] = i\hbar \delta_{ij}$ ,  $i, j = 1, 2, 3$ ,  $[x_1, p_1^3]$  is :

- (a)  $(i\hbar)^3 p_1$  (b)  $i^2 p_1$   
(c)  $3 i p_1^2$  (d) Zero

27 Which of the following is not an eigenstate of linear momentum operator  $p_x$  ?

- (a)  $Ae^{ikx}$  (b)  $Ae^{-ikx}$   
(c)  $A^{-x}$  (d)  $A \sin kx$

28 The electron in a hydrogen atom is in a superposition state described by the wave function

$$\psi(\vec{r}) = \frac{1}{6} [4\psi_{100}(\vec{r}) - 2\psi_{211}(\vec{r}) + \sqrt{6}\psi_{210}(\vec{r}) - \sqrt{10}\psi_{21-1}(\vec{r})],$$

where  $\psi_{n\ell m}(\vec{r})$  is a normalized eigenstate. The expectation value of  $L_z$  is :

- (a)  $-\hbar/6$  (b)  $\hbar/6$   
(c)  $6\hbar$  (d)  $-6\hbar$

29 Two coherent light sources of intensities  $I$  and  $9I$  are used in an interference experiment. The resultant intensity at points where the waves from the two sources superpose with a phase difference zero is :

- (a)  $16I$  (b)  $9I$   
(c)  $I$  (d) Zero

30 The number of distinct  $(n, l, m_l)$  states of hydrogen atom with  $n = 3$  is :

- (a) 5 (b) 9  
(c) 3 (d) 2

31 The condition for two distinct quantum mechanical states, represented by wave functions  $\psi_1(x)$  and  $\psi_2(x)$  respectively, to be orthogonal is:

- (a)  $\int_{-\infty}^{\infty} \psi_1^*(x) \psi_2(x) dx = 1$

(b)  $\int_{-\infty}^{\infty} \psi_1^*(x) \psi_2(x) dx = 0$

(c)  $\int_{-\infty}^{\infty} \psi_1^*(x) \psi_2(x) dx = \infty$

(d)  $\int_{-\infty}^{\infty} [\psi_1^*(x) \psi_2(x) + \psi_2^*(x) \psi_1(x)] dx = 1$

32 Addition of angular momentum states  $\vec{J}_1 = 1/2$  and  $\vec{J}_2 = 1/2$  will result in 4 states, of which number of linearly dependent states with magnetic quantum number  $m = 0$  is :

(a) Zero

(b) 1

(c) 2

(d) 4

33 In the Born approximation, the effective cross-section of scattering depends :

(a) Linearly on scattering angle  $\theta$

(b) on  $p \sin^2 \theta$

(c) on  $p \sin \theta/2$

(d) only on the momentum  $p$

34 A system consists of two indistinguishable fermions. Each particle can occupy only two energy levels  $E_1 = \epsilon$  and  $E_2 = 2\epsilon$ . The canonical partition function is :

(a)  $Z = e^{-\beta\epsilon} + e^{-2\beta\epsilon}$

(b)  $Z = e^{-3\beta\epsilon}$

(c)  $Z = 3e^{-\beta\epsilon}$

(d)  $Z = e^{-\beta\epsilon} + 2e^{-2\beta\epsilon}$

35 Consider 5 independent magnetic moments. Each moment has only two directions  $+m$  and  $-m$ .

Hamiltonian of such system is given by  $x = \sum_{i=1}^5 m_i$ . If the total magnetization of the system is equal to  $m$ , the entropy is equal to :

(a)  $k \ln(10)$

(b) Zero

(c)  $k \ln(20)$

(d)  $k \ln(5)$

36 Let  $f(T, l)$  be the tension in a rubber band of length  $l$ ,  $T$  being the absolute temperature. Let  $F$

denote Helmholtz free energy and  $S$  the entropy.

Then :

(a)  $\left. \frac{\partial F}{\partial l} \right|_T = S, \left. \frac{\partial F}{\partial T} \right|_l = f$

(b)  $\left. \frac{\partial F}{\partial l} \right|_T = f, \left. \frac{\partial F}{\partial T} \right|_l = -S$

(c)  $\left. \frac{\partial F}{\partial l} \right|_T = S, \left. \frac{\partial F}{\partial T} \right|_l = fl$

(d)  $\left. \frac{\partial F}{\partial l} \right|_T = 0, \left. \frac{\partial F}{\partial T} \right|_l = f$

37 An insulated chamber is divided into two halves of volume. The left half contains an ideal gas at temperature  $T_0$  and the right half is evacuated. A small hole is opened between the two halves, allowing the gas to flow through and the system comes to equilibrium. No heat is exchanged with walls. What is the final temperature of the system:

(a)  $2 T_0$

(b)  $\frac{3}{2} T_0$

(c)  $T_0$

(d)  $\frac{1}{2} T_0$

38 Consider a mixture of equal number of non-interacting single atomic gas and diatomic gas. The internal energy per particle is given by :

(a)  $2kT$

(b)  $4k T$

(c)  $\frac{3}{2} k T$

(d)  $\frac{5}{2} k T$

39 Consider an ensemble of systems having only two energy levels having energies 0 and  $\epsilon$  respectively. The average energy of the system at temperature  $T$  is :

(a)  $\frac{\epsilon}{e^{-\epsilon/kT} + 1}$

(b)  $\frac{\epsilon}{e^{\epsilon/kT} + 1}$

(c)  $\frac{\epsilon}{e^{-\epsilon/kT} - 1}$

(d)  $\frac{\epsilon}{e^{\epsilon/kT} - 1}$

40 What play the role of thermodynamic potential in microcanonical ensemble?

- (a) Entropy (b) Enthalpy  
(c) Gibbs potential (d) Internal energy

41 For a metal  $C_V = \alpha T + \beta T^3$  at low temperatures. The entropy is :

- (a)  $\alpha + 3\beta T^2$  (b)  $\alpha T + 3\beta T^2$   
(c)  $\alpha T + \frac{\beta T^3}{3}$  (d)  $\alpha + \frac{\beta T^3}{3}$

42 In which of the following detectors P – N junction diode is used ?

- (a) G.M. Counter  
(b) Scintillation detector  
(c) Surface Barrier detector  
(d) Proportional counter

43 The X-rays emitted by a target consist of continuous range of wavelengths superimposed by characteristic X-rays named  $K_\alpha$ ,  $K_\beta$  etc. It is found that :

- (a)  $K_\alpha$  has high intensity and lower wavelength compared to that of  $K_\beta$ .  
(b)  $K_\alpha$  has low intensity and lower wavelength as compared to that of  $K_\beta$ .  
(c)  $K_\alpha$  has high intensity and higher wavelength compared to that of  $K_\beta$ .  
(d)  $K_\alpha$  has lower intensity and higher wavelength compared to that of  $K_\beta$ .

44 Air Ballast valve in a rotary pump is used for :

- (a) Reducing the noise  
(b) Removing the condensable vapors

(c) Reducing the back streaming

(d) Increasing the ultimate vacuum to  $10^{-5}$  Torr

45 A signal of one  $\mu V$  is buried under 1V RMS noise. Which of the following devices is suitable to detect it ?

- (a) High gain linear amplifier  
(b) Band pass filter  
(c) Notch filter  
(d) Phase sensitive amplifier

46 Steady state response of an electric circuit is generally tested by giving following waveform at the input :

- (a) Sinusoidal (b) Square  
(c) Triangular (d) Sawtooth

47 An inductance 'L', resistance 'R' and capacitance 'C' are connected in series to resonate to frequency " $f_s$ ". When they are connected in parallel, the resonance frequency is " $f_p$ ".

- (a)  $f_p < f_s$   
(b)  $f_p > f_s$   
(c)  $f_p = f_s$   
(d) There is no relation between the two

48 Frequency Bandwidth of CRO is decided by :

- (a) Horizontal amplifier (b) Vertical amplifier  
(c) Delay line (d) Triggering circuit

49 Resolving power of a  $2 \text{ cm}^2$  site grating having a rulings of 600 J/ mm in order one is

approximately :

(a) 10

(b)  $10^2$

(c)  $10^3$

(d)  $10^4$

50 A dual slope digital voltmeter has a gate time of 400 ms reflecting the measurement cycle. The frequency bandwidth of this instrument is :

(a)  $2 \cdot 5$  Hz

(b) 1.25 Hz

(c) 0.8 Hz

(d) 0.6 Hz

#### Answer Key

1. b	2. a	3. c	4. b	5. d
6. c	7.	8. b	9. c	10. c
11. c	12. b	13. b	14. c	15. c
16. d	17.	18. b	19. c	20. a
21. b	22. d	23. b	24. d	25. c
26. c	27. d	28. a	29. a	30. b
31. b	32. c	33. c	34. d	35. a
36. b	37. c	38. a	39. b	40.
41. c	42. b	43. c	44. b	45. a
46. a	47. a	48. a	49.	50.