April 2017 Paper-II

1 The Fourier transform of

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

is :

(a) $e^{-|k|}$ (b) $e^{-|k|^2}$

(c) $\frac{1}{\pi}\cdot\frac{1}{1+k^2}$ (d) $\ln(1+k^2)$

2 A card is drawn from a pack of 52 cards. The probability that one of the cards drawn is a 'ten' or a 'jack' or a 'queen' is :

(a) 1/13 (b) 2/13

(c) 6/13 (d) 3/13

3 The matrix $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is :

(a) diagonalizable by an orthogonal transformation

(b) diagonalizable by a unitary transformation

(c) not diagonalizable

(d) diagonalizable by a Hermitian transformation

4 Residue of the function

$$f(z) = \frac{z}{(z-a)(z-b)}$$

at infinity is :

(a) a/b (b) $-b/a$

(c) 1 (d) -1

5 Determinant of a 3×3 real symmetric matrix is 36 . Two of its eigenvalues are 2 and 3 . The third eigenvalue is :

(a) 1 (b) 6

(c) 4 (d) 9

6 One of the solutions of the differential equation $\frac{d^2f}{dx^2} + 2\text{sech}^2\ xf = 0$ is :

(a) sech x (b) tanh x

(c) $\text{sech}^2\ x$ (d) $\text{sech}^3\ x$

7 Average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is :

(a) 15 (b) 20

(c) 80 (d) 40

8 If $r = \sqrt{(x^2+y^2+z^2)}$, grad $r(\vec{\nabla}r)$ is :

(a) $\vec{r}/r$ (b) 0

(c) r (d) $\vec{r}$

9 Residue of $\int_C \frac{dz}{(z-z_0)^2}$, where C is any simple closed contour enclosing z_0, is :

(a) $2\pi i$ (b) $2\pi i z_0$

(c) πi (d) zero

10 A rigid body of N particles has the following number of degrees of freedom :

(a) 3 N (b) N

(c) 6 (d) 3

11 A particle of mass m moves in a central force field defined by $\vec{F} = (-K/r^3)\hat{r}$. If E is the total energy of the particle, then its speed is :

(a) $\frac{k}{mr^2} + \frac{2E}{m}$ (b) $\frac{k}{mr^2} - \frac{2E}{m}$

(c) $\sqrt{\frac{k}{2mr^2} + \frac{2E}{m}}$ (d) $\sqrt{\frac{k}{2mr^2} - \frac{2E}{m}}$

12 A block of mass m is sliding down an inclined plane at constant speed. The coefficient of kinetic

friction between the mass and the inclined plane is 2. The angle of inclination is :

(a) $\sin^{-1}(1/2)$ (b) $\cos^{-1}(1/2)$

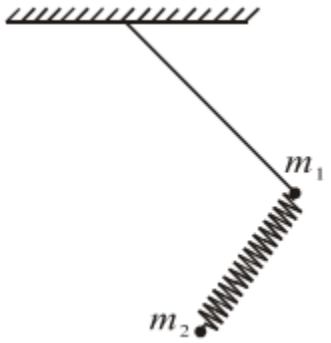
(c) $\tan^{-1}(1/2)$ (d) $\tan^{-1}(2)$

13 A particle of mass m under the influence of central force $\vec{F} = \frac{-k}{r^3}\hat{r}$, where k is constant. What is the potential energy of the particle if potential is zero at $r = \infty$.

(a) $\frac{k}{2r^2}$ (b) $\frac{-k}{2r^2}$

(c) $\frac{3k}{r^4}$ (d) $\frac{-3k}{r^4}$

14 A mass is hanged from ceiling with an inextensible massless string.
Another mass m_2 is hanged from m by a spring.
The number of degrees of freedom of the system is :



(a) 3 (b) 5

(c) 4 (d) 6

15 A particle of mass m falls vertically under gravity and the frictional force is obtainable from dissipation function $G(v) = \frac{1}{2}kv^2$. The Lagrangian equation of motion is :

(a) $m\ddot{y} + k\dot{y} + mg = 0$

(b) $m\ddot{y} + mg = 0$

(c) $m\ddot{y} + \frac{1}{2}k\dot{y} + mg = 0$

(d) $m\ddot{y} + \frac{1}{2}k\dot{y}^2 + mg = 0$

16 A particle of mass m and charge Q is moving in a constant magnetic field of strength H along Z-axis. The Hamiltonian of the system is (in esu):

(a) $p^2/2m - \frac{QH}{mc}xp_y + \frac{Q^2H^2x^2}{2mc^2}$

(b) $\frac{p^2}{2m} + \frac{QH}{mc}xp_y + \frac{Q^2H^2x^2}{2mc^2}$

(c) $\frac{p^2}{2m} + \frac{2HQ}{mc}(xp_y + yp_x)$

(d) $\frac{p^2}{2m} - \frac{2HQ}{mc}(xp_y + yp_x)$

17 A one-dimensional simple harmonic oscillator with generalized coordinate q is subject to an additional time-dependent potential energy of the form

$$V(t) = q^2t + q\dot{q}t^2$$

The Lagrangian equations of motion contain an additional term :

(a) Containing only t (b) Containing only t^2

(c) Containing t and t^2 (d) No additional term

18 The electric potential at a distance r from the center of the sphere of radius R having uniform surface charge density σ is equal to

(a) $\frac{\sigma R}{\epsilon_0}$ (b) $\frac{\sigma R^2}{\epsilon_0}$

(c) $\frac{\sigma R^2}{\epsilon_0 r}$ (d) $\frac{\sigma R^2}{\epsilon_0 r^2}$

19 A charged solid sphere of radius 3 units is made up of perfectly conducting material and is placed with center at the origin. If the total charge on the sphere is Q, the volume charge density at a point $(1,1,-1)$ is

(a) $\frac{Q}{36\pi}$ (b) $\frac{Q}{4\sqrt{3}\pi}$

(c) $\frac{3Q}{4\pi}$ (d) Zero

20 The electric potential at a large distance r from the ideal linear quadrupole varies with r as

(a) r^3 (b) r^4

(c) r^{-3} (d) r^{-4}

21 Which of the following equations signifies the conservative nature of the electric field $\overline{E}$?

(a) $\nabla \cdot \overline{E}(\bar{r}) = \frac{\rho(\bar{r})}{\epsilon_0}$

(b) $\nabla \times \overline{E}(\bar{r}) = \bar{0}$

(c) $\nabla \times \overline{E}(\bar{r}, t) = \frac{-\partial \overline{B}(\bar{r}, t)}{\partial t}$

(d) $\epsilon_0 \mu_0 \frac{\partial \overline{E}(\bar{r}, t)}{\partial t} = \nabla \times \overline{B}(\bar{r}, t) - \mu_0 \overline{J}(\bar{r}, t)$

22 Plane monochromatic electromagnetic wave is propagating through a perfect dielectric material of refractive index $\frac{3}{2}$. The phase difference between the fields $\overline{E}$ and $\overline{B}$ associated with the wave passing through the material is

(a) Zero

(b) π

(c) $\frac{3}{2}\pi$

(d) any non-zero value between $-\pi$ and π

23 An electromagnetic wave is propagating in a dielectric medium of permittivity $\in$ and permeability μ having an electric field vector $\overline{E}$ associated with the wave. The associated magnetic field $\overline{H}$ is

(a) Parallel to $\overline{E}$ with magnitude $E\sqrt{\mu/\epsilon}$

(b) Parallel to $\overline{E}$ with magnitude $E\sqrt{\epsilon/\mu}$

(c) Perpendicular to $\overline{E}$ with magnitude $E\sqrt{\mu/\epsilon}$

(d) Perpendicular to $\overline{E}$ with magnitude $E\sqrt{\epsilon/\mu}$

24 Power radiated by a point charge moving with constant acceleration of magnitude a is proportional to

(a) a (b) a^2

(c) a^{-1} (d) a^{-2}

25 The output of a laser has a bandwidth of 1.2×10^{14} Hz. The coherence length l_c of the output radiation is

(a) 3.6 mm (b) 50μm

(c) 2.5μm (d) 1.5 cm

26 Given $[x_i, P_j] = i\hbar\delta_{ij}, i, j = 1,2,3.$ (x_1, P_2^2) is :

(a) 0 (b) $i\hbar P_2$

(c) $2x_1$ (d) $2P_2$

27 Which of the following is an eigen state of square of linear momentum operator P_x^2 ?

(a) Ax^2

(b) $A(\sin\ kx + \cos\ kx)$

(c) $Ae^{-\alpha x^2}$

(d) $A\sin^2 kx$

28 The electron in a hydrogen atom is in a superposition state described by the wavefunction $\psi(\vec{r}) = \mathrm{A}[4\psi_{100}(\vec{r}) - 2\psi_{211}(\vec{r}) + \sqrt{6}\psi_{210}(\vec{r}) - \sqrt{10}\psi_{21-1}(\vec{r})]$
$\psi_{nlm}(\vec{r})$ normalized wave function.
The value of normalization constant, A, is :
(a) $\frac{1}{3}$ (b) $\frac{1}{6}$
(c)6 (d) 36

29 Two coherent light sources of I and 9I are used in an interference experiment. The resultant intensity at points where the waves from the two sources superpose with phase difference π is :
(a) 16I (b) 9I
(c) 4I (d) Zero

30 Non-relativistic hydrogen atom spectrum is proportional to $-1/n^2$. The degeneracy of nth level is :
(a) n (b) $2n + 1$
(c) n^2 (d) $1/n^2$

31 Uncertainty relation holds between :
(a) Time and space
(b) Life time and energy
(c) Position and energy
(d) Momentum and energy

32 Addition of angular momentum $\vec{j}_1 = 1$ and $\vec{j}_2 = \frac{1}{2}$ will result in 6 states, of which the number of linearly independent states with magnetic number $m = -\frac{1}{2}$ is :
(a) Zero (b) 6
(c) 3 (d) 2

33 In a scattering event by a spherically symmetric potential, only P-wave scattering occurs. The angular distribution of differential cross-section is proportional to :
(a) Constant (b) $\cos\ \theta$
(c) $\cos^2\ \theta$ (d) $a + \sin\ \theta$

34 If energy of a two-dimensional simple harmonic oscillator
$$x = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2)$$
is fixed to be $3\hbar\omega$, the entropy is given by (k_{B} is Boltzmann constant) :
(a) $k_{\mathrm{B}}\ln\ 3$ (b) $2k_{\mathrm{B}}\ln\ 3$
(c) $k_{\mathrm{B}}\ln\ 2$ (d) Zero

35 The equation of state for photon gas is :
(a) $p\ \mathrm{V} = \frac{5}{3}\mathrm{E}$
(b) $p\ \mathrm{V} = \frac{2}{3}\mathrm{E}$
(c) $p\ \mathrm{V} = \frac{1}{3}\mathrm{E}$
(d) $p\ \mathrm{V} = \hbar\omega$ for some fixed frequency ω

36 The energy density for photons in a cavity is proportional to :
(a) T^3 (b) T
(c) T^4 (d) $\mathrm{T}^{4/3}$

37 Let ρ be the density matrix for a system. Then :
(a) $\mathrm{Tr}\ (\rho) = 0$ (b) $\mathrm{Tr}(\rho) < 0$
(c) $0 \leq \mathrm{T}r(\rho) < 1$ (d) $\mathrm{Tr}(\rho) = 1$

38 A system has only two energy levels E_1 and E_2. In equilibrium at temperature T, the number of particles occupying level E_1 is double of those occupying level E_2. The value of $E_2 - E_1$ must be

[k = Boltzmann constant] :

(a) k Tln (2) (b) k Tln (3)

(c) $3k$ T (d) $2k$ T

39 The quantities (i) isothermal compressibility (ii) volume coefficient of expansion are :

(a) Extensive and intensive respectively

(b) Intensive and extensive respectively

(c) Both extensive

(d) Both intensive

40 The chemical potential in the classical limit is :

(a) Zero (b) Negative

(c) Positive (d) Complex quantity

41 van der Waals equation for one mole is $\left(p + \frac{a}{\mathrm{V}^2}\right)(\mathrm{V} - b) = \mathrm{RT}$. The equation for n moles would be :

(a) $\left(p + \frac{an^2}{\mathrm{V}^2}\right)(\mathrm{V} - nb) = \mathrm{RT}$

(b) $\left(p + \frac{a^2}{\mathrm{V}^2}\right)(\mathrm{V} - b) = n\mathrm{RT}$

(c) $\left(p + \frac{an^2}{\mathrm{V}^2}\right)(\mathrm{V} - nb) = n\mathrm{RT}$

(d) $\left(p + \frac{a}{n^2\,\mathrm{V}^2}\right)(n\,\mathrm{V} - b) = n\mathrm{RT}$

42 A Wheatstone's bridge is used to measure the pressure in the following vacuum gauge :

(a) McLeod gauge (b) Pirani gauge

(c) Penning gauge (d) Ionization gauge

43 The continuous X-rays are produced when :

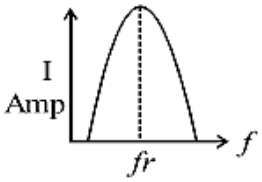
(a) Electrons of the target atom jump from a higher to lower orbital

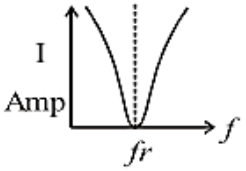
(b) Electrons from the valence electrons are de-excited to the hole in the inner orbitals

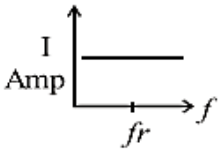
(c) Electrons are accelerated to fixed energy

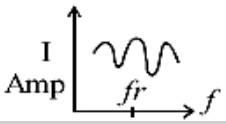
(d) Incident electrons are deaccelerated near heavy nuclei of the target atoms.

44 Which of the following curves refers to the series resonance circuit of inductance and capacitor ?

(A) 

(B) 

(C) 

(D) 

45 For detecting photons which of the following detectors is used ?

(a) Faraday Collector (b) Channel Tron

(c) Photo-multiplier (d) Micro-channel plate

46 For obtaining a Laue pattern of a single crystal the sample is held stationary in a beam of :

(a) Monochromatic K_αX-rays from Cu target

(b) Monochromatic K_αX-rays from Mo target

(c) Monochromatic K_β X-rays from Cu target

(d) Continuous X-rays from any target

47 The transient response of an electronic circuit is tested by giving the following wave form at the input :

(a) Sinusoidal (b) Square

(c) Triangular (d) Saw-tooth

48 Which of the following effects manifests particle nature of light?

(a) Photoelectric effect

(b) Black body radiation

(c) Interference

(d) Diffraction

49 A ruled grating having 1000 grooves per mm is used for diffraction. If light of wavelength 300 nm is incident at right angle and if ' θ ' is the diffraction angle, sin θ for first order diffraction will be equal to :

(a) 0.1 (b) 0.3

(c) 0.2 (d) 0.5

50 A gas laser has mirrors at its ends and produces a spectrum of lines which are separated according to various orders of interference between the two mirrors. If the separation of mirrors and the wavelength of laser are 30 cm and 6000 A respectively, the separation between two neighbouring lines is approximately :

(a) 6Å (b) 0.6Å

(c) 0.06Å (d) 0.006Å

Answer Key				
1.	2. d	3. c	4. d	5. b
6.	7. d	8. a	9. a	10.
11.	12. d	13. b	14. b	15. a
16.	17.	18. c	19. d	20. c
21. b	22. a	23.	24. b	25. c
26. a	27.	28.	29.	30.
31. b	32.	33.	34.	35.
36.	37.	38.	39.	40.
41. c	42.	43.	44.	45.
46.	47.	48.	49.	50.