September 2015 Paper-II

1. If $\vec{a}$ is a constant vector, then $\vec{\nabla} \cdot (\vec{a} \times \vec{r})$ is equal to :

   (a) a (b) 2a

   (c) 0 (d) a/2

2. The volume of the parallelepiped with sides :

   $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

   $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and

   $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$ is :

   (a) $2 + \sqrt{3}$ (b) $\sqrt{2}$

   (c) $\sqrt{3}$ (d) 3

3 The constant term (a_0) in the Fourier series expansion of the function $f(x) = x^2$ for $-\pi < x < \pi$ is :

   (a) π^2 (b) $\pi^2/3$

   (c) $\pi/2$ (d) $\pi^3/3$

4 Fourier transform of unit step function :

$$\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

   is :

   (a) $\frac{1}{2\pi k}$ (b) $\frac{1}{2\pi} k$

   (c) $\frac{1}{2\pi}\frac{1}{k^2}$ (d) $\frac{1}{2\pi k} e^{-k}$

5 The residue of $\frac{z^2}{z^2+a^2}$ at $z = ia$ is :

   (a) ia/2 (b) ia

   (c) ia^2 (d) $ia^2/2$

6 A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card ?

   (a) 1/5 (b) 4/25

   (c) 1/4 (d) 2/5

7 y(x) satisfies the differential equation :

$$\frac{d^2y}{dx^2} + y = \text{cosec}\,(x)$$

   The complementary function part y_c of y(x) is given by :

   (a) $y_c(x) = c_1 \sin x + c_2 \cos x$

   (b) $y_c(x) = xe^{ix}$

   (c) $y_c(x) = (c_1 \sin x + c_2 \cos x)e^x$

   (d) $y_c(x) = c_1 e^x + c_2 \cos x$

8 A, B, C, D are $(n \times n)$ matrices, each with non-zero determinant. If :

$$ABCD = I,$$

   then B^{-1} is :

   (a) $D^{-1}C^{-1}A^{-1}$ (b) CDA

   (c) ADC (d) does not exist

9 The degrees of freedom for a particle falling under gravity (without air resistance) are :

   (a) Zero (b) One

   (c) Two (d) Three

10 A particle of mass m is released from height h. Its velocity at the position when it has covered a distance equal to $\left(\frac{h}{2}\right)$ is :

   (a) $v = \sqrt{\frac{2g}{h}}$ (b) $v = \frac{2g}{h}$

   (c) $v = \sqrt{g \cdot h}$ (d) $v = g \cdot h$

11 Which of the following properties of the Lagrangian is due to "space is isotropic" characteristic of the inertial frame of reference?

   (a) Invariant to translation

(b) Invariant to time
(c) Invariant to rotation

(d) Explicit function of time

12 For a given system, Hamiltonian H(q, p) is defined as :
$H(q,p) = p + \sqrt{1-p^2} \cdot \sin q |p| \leq 1.$
The Hamilton's equations of motion are :

(a) $\dot{q} = 1 - \frac{p \sin q}{\sqrt{1-p^2}} - \dot{p} = \sqrt{1-p^2} \cos q$

(b) $\dot{q} = 1 - \frac{p \cos q}{\sqrt{1-p^2}}, \dot{p} = \sqrt{1-p^2} \cos q$

(c) $\dot{q} = 1 - \frac{2p \cos q}{\sqrt{1-p^2}} - \dot{p} = \sqrt{1-p^2} \cos q$

(d) $\dot{q} = 1 + \frac{p \sin q}{\sqrt{1-p^2}}, \dot{p} = \sqrt{1-p^2} \cos q$

13 For a particle under the action of central force, which of the following quantities is invariant?
(a) Linear velocity

(b) Angular velocity

(c) Linear momentum

(d) Angular momentum

14 Which of the following statements holds true for a rigid rotator (or rotor)?
(a) All the three principal moments of inertia are different

(b) All the three principal moments of inertia are equal

(c) Two of the three principal moments of inertia are equal and the third one is non-zero and different

(d) Two of the three principal moments of inertia are equal and the third one is zero

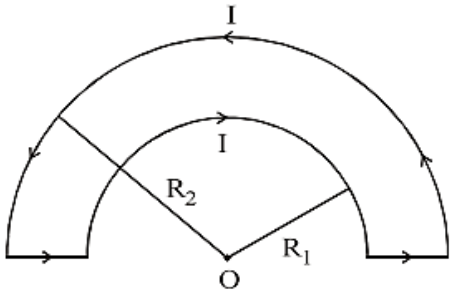
15 The plane of oscillation of Focoult pendulum rotates with infinite period, when it is located at (λ is the lattitude of location):
(a) $\lambda = 90°N$ (b) $\lambda = 60°N$

(c) $\lambda = 30°N$ (d) $\lambda = 0°$

16 An observer is moving with relativistic speed u. His length measurements along the direction of velocity are half of the measurements made when he was at rest. His time measurement of 1hr. when he was at rest, now becomes :
(a) 30 minutes (b) 01 hour

(c) 02 hours (d) Infinite hours

17 The magnetic induction $\vec{B}$ at point O in the figure is given as :



(a) $\frac{\mu_0 I}{4\pi}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ (b) $\frac{\mu_0 I}{4\pi} R_1 R_2$

(c) $\frac{\mu_0 I}{4\pi}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$ (d) $\frac{\mu_0 I}{4\pi}(R_1 + R_2)$

18 A cubical box of sides 5 cm encloses 100 electric dipoles of unit moments.
What would be the flux of electric field emanating from the cube ?
(a) $\frac{20}{\epsilon_0}$ V cm (b) $\frac{100}{30\epsilon_0}$ V cm

(c) Zero Vcm (d) $\frac{200}{\epsilon_0}$ V cm

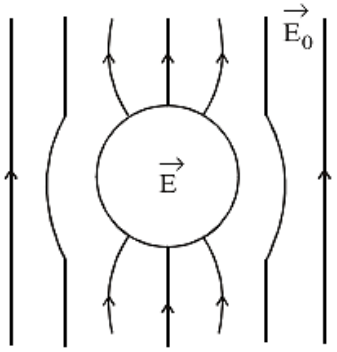
19 A uniform current 20 A flows through a copper pipe of inner radius 10 cm. Which of the following statements is valid?
(a) The magnetic field is non-zero except along the axial points of the pipe.

(b) The magnetic field everywhere inside the pipe is zero.

(c) The magnetic field everywhere inside the pipe is constant and non-zero.

(d) The magnetic field is nonuniform inside the pipe.

20 An uncharged sphere of homogeneous linear dielectric material is placed in an uniform electric field $\vec{E}_0$. The resultant electric field $\vec{E}$ inside the dielectric sphere is :



(a) Zero

(b) Uniform non-zero and parallel to $\vec{E_0}$

(c) Uniform non-zero and antiparallel to $\vec{E}_0$

(d) Non-uniform but along $\vec{E}_0$

21 In electrodynamics, the curling magnetic field is more appropriately expressed as :
(a) a bound magnetization current density $\vec{J}_M$

(b) a free current density $\vec{J}$

(c) stationary electric charges

(d) a free current density $\vec{J}$ and a time-varying electric field $\vec{D}$

22 The continuity equation for charges and currents is not explicitly written in the Maxwell's equations. This is because :
(a) It is not a universal law and holds good in certain cases only

(b) It is difficult to prove the continuity equation rigorously

(c) It is not required for time varying electromagnetic fields

(d) It is already included implicitly in the Maxwell's equations

If $\vec{A}$ and V are the vector and scalar potentials, respectively, then the Lorentz gauge in timevarying electromagnetic case is given by :
(a) $\vec{\nabla} \times \vec{A} = \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$ (b) $\vec{\nabla} \cdot \vec{A} = \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

(c) $\vec{\nabla} \cdot V = \mu_0 \epsilon_0 \frac{\partial \vec{A}}{\partial t}$ (d) $\vec{\nabla} \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

23 A plane polarized EM wave is propagating in certain medium such that the ratio of the electric energy density to the magnetic energy density is 0.01 . Which of the following statements is valid for this medium?
(a) Conduction current is about 100 times larger than the displacement current

(b) Conduction and displacement currents are of the same magnitude

(c) Displacement current is about 100 times larger than the conduction current

(d) The medium through which the EM wave is propagating is a dielectric

24 A proton of mass m and charge Q is moving in counter-clockwise direction in a crossed uniform magnetic field of induction $\vec{B}$ pointed into the plane of the paper. If v and R are the velocity and radius of the orbit, then angular momentum p acquired by the particle is given by :

(a) $p = QBR$ (b) $p = \frac{QB}{R}$

(c) $p = \frac{QB}{mR}$ (d) $p = \frac{QBR}{m}$

25 If two operators $\hat{A}$ and $\hat{B}$ have a common eigenfunction ψ. What is the value of the commutator $[\hat{A}, \hat{B}]$?

(a) 1 (b) $i\hbar$

(c) $\hat{A} - \hat{B}$ (d) Zero

26 Standing wave with wave function $\psi(x) =$ Asin kx, (corresponding to particles of mass m) gives rise to current density equal to :

(a) $|A|^2\hbar k/m$

(b) $|A|^2\hbar k$

(c) $|A|^2(\sin kx \cos kx)/m$

(d) Zero

28 A particle moves in one-dimensional potential V(x). At $x = a$, if V has a finite discontinuity (jump), then which of the following is true for its wave function ϕ and its first derivative ϕ' at $x = a$?

(a) Both ϕ and ϕ' are discontinuous

(b) Both ϕ and ϕ' are continuous

(c) ϕ is continuous and ϕ' must be discontinuous

(d) ϕ is discontinuous and ϕ' must be continuous

29 The wave function of a particle moving in one-dimensional time independent potential V(x) is represented by $\psi(x) = e^{-iax+b}$ where a and b are real constants. This means that the potential V(x) is of the form :

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$

(c) $V(x) =$ constant (d) $V(x) = e^{-ax} + b$

30 In a quantum mechanical one-dimensional system, the Hamiltonian is :

$$H = \frac{p_x^2}{2m} + 1/2m\omega_x^2x^2 + \alpha x = H_0 + \alpha x$$

where α is very small. The eigenvalues of H_0 are $\hbar\omega(n + 1/2)$ with $\psi_n^{(0)}(x)$ as eigenfunctions. Which of the following is correct?

(a) The energy levels are not changed since all eigenfunctions $\psi_n^{(0)}(x)$ have definite parity

(b) All energy levels are shifted by a constant amount

(c) Only $n =$ odd energy eigenvalues are shifted

(d) Only $n =$ even energy eigenvalues are shifted

31 Which of the following is an eigenfunction of linear momentum operator $\frac{\hbar}{i}\frac{\partial}{\partial x}$, such that the eigenfunction describes a particle moving in free space in the direction of positive x-axis, with zero uncertainty in the linear momentum ?

(a) e^{-kx} (b) e^{ikx}

(c) cos kx (d) sin kx

32 A system in one-dimension is in a state described by :

$$\psi(x, t) = A\exp\{i(pX - Et)/\hbar\} + B\exp\{-i(px + Et)/\hbar\}$$

A and B are complex numbers, p and E are real. The probability current density is given by :

(a) p/m (b) $(|A| - |B|)p/m$

(c) $(|A|^2 + |B|^2)p/m$ (d) $(|A|^2 - |B|^2)p/m$

33 The expectation value of momentum for a bound state of finite square well potential (width a and depth V_0) is :

(a) Zero (b) Depends on V_0

(c) Depends on a (d) Depends on $V_0 a^2$

34 $\vec{L}$ is quantum mechanical operator of orbital angular momentum. $\vec{L} \times \vec{L}$ is equal to :

(a) 0 (b) $i\hbar L$

(c) $1(1+1)\hbar^2$ (d) L^2

35 A constant volume ideal gas thermometer in thermal equilibrium with system A at temperature T_A measures a pressure p_A. When the same thermometer is brought in thermal equilibrium with another system B at temperature $T_A/3$, the pressure in the thermometer will now be :

(a) $p_A/3$ (b) $p_A^2/3$

(c) $3p_A$ (d) $p_A/\sqrt{3}$

36 Consider a process in which a system is in thermal equilibrium with a heat reservoir at temperature T.
The pressure of the system remains constant at the pressure of the reservoir p. Such a process is called :

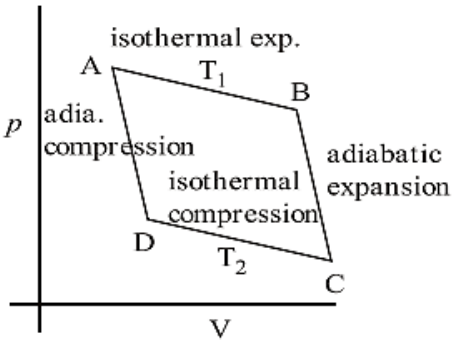
(a) Isothermal process

(b) Adiabatic and isochoric process

(c) Isochoric process

(d) Isobaric and isothermal process

37 A Carnot engine operates between two temperatures T_1 and T_2 as shown in the diagram.



The maximum efficiency of the engine η_{max} is given by :

(a) $\eta_{max} = 1 - \frac{T_1}{T_2}$ (b) $\eta_{max} = \frac{T_2}{T_1}$

(c) $\eta_{max} = 1 + \frac{T_2}{T_1}$ (d) $\eta_{max} = 1 - \frac{T_2}{T_1}$

38 A gas is contained in a box of volume V at pressure P and temperature T. One wall of the box is moved at a very high speed so that the volume is doubled. Which of the following is true?

(a) P, T both increase

(b) P, T both decrease

(c) P increases, T decreases

(d) P decreases, T increases

39 A thermodynamic system is characterized by independent variables entropy S and volume V and dependent variables pressure P and temperature T. Then which of the following is true?

(a) $\left(\frac{\partial P}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$ (b) $\left(\frac{\partial P}{\partial S}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_S$

(c) $\left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_S$ (d) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$

40 A system consists of 3 independent particles. Each particle can occupy any of the two energy states with energy 0 and $\in$. The system is in thermal equilibrium with a heat reservoir at temperature T. Its partition function will be :

(a) $Z = 1 + 3e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$

(b) $Z = 1 + e^{-\beta\epsilon}$

(c) $Z = 1 + e^{-3\beta\epsilon}$

(d) $Z = 1 + e^{-\beta\epsilon} + e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$

41 Considering the Maxwell-Boltzmann distribution of speeds of molecules, the average speed of the molecule of mass m at temperature T is :

(a) $\sqrt{\frac{3k\,T}{m}}$ (b) $\sqrt{\frac{8k\,T}{\pi m}}$

(c)0 (d) $\sqrt{\frac{2k\,T}{m}}$

42 In the Debye model of specific heat of solids, the specific heat at low temperature has the following temperature dependence :

(a) $T^{3/2}$ (b) T^2

(c) T^3 (d) $T^{1/2}$

43 In Signal generators :

(a) Energy is created

(b) Energy is generated

(c) Energy is converted from a simple d.c. source into a.c. energy at some specific frequency

(d) Energy is maintained

44 In an experiment, the voltage across a 10kΩ resistor is applied to CRO. The screen shows a sinusoidal signal of total vertical occupancy 3 cm and total horizontal occupancy of 2 cm. The front-panel controls of V/div and time/div are on 2 V/ div and 2 ms/ div respectively. Find out the frequency of the waveform :

(a) 250kHz (b) 250 Hz

(c) 2.5kHz (d) 25kHz

45 When two rotary pumps are connected in series to a vacuum chamber, the ultimate pressure can be achieved is closest to :

(a) 1×10^{-6} Torr (b) 1×10^{-5} Torr

(c) 1×10^{-3} Torr (d) 1×10^{-9} Torr

46 Which of the following noise can be reduced by integration of the signal ?

(a) White noise

(b) Shot noise

(c) Flicker noise

(d) Environmental noise

47 In which of the following detector P − N junction diode is used ?

(a) G.M. counter

(b) Scintillation counter

(c) Lithium drifted detector

(d) Ionisation chamber

48 An oil drop has a weight of 3.2×10^{-13} N. With an electric field of 5×10^5 V/m between the plates of Millikan's oil-drop apparatus, this drop is observed to be essentially balanced. The charge on the drop in electron charge units is :

(a) 2 electrons (b) 4 electrons

(c) 3 electrons (d) 6 electrons

49 A McLead gauge has a volume of 100ml and a capillary of 0.5 mm diameter. The pressure

indicated by a reading of 25 cm of mercury corresponds to :

(a) 24.54×10^{-9} (b) 12.27×10^{-10}

(c) 1.227×10^{-3} (d) 2.454×10^{-8}

50 A transparent film of glass of refractive index 1.5 is introduced normally in the path of one of the interfering beams of Michelson's interferometer which is illuminated with light of wavelength 4800Å. This causes 500 dark fringes to sweep across the field of view. The thickness of the film is :

(a) 0.012 cm (b) 0.024 cm

(c) 0.005 cm (d) 0.015 cm .

Answer Key				
1. c	2. d	3. b	4.	5. a
6. a	7. a	8. b	9. b	10. c
11. c	12. a	13. b	14. d	15. d
16. c	17. a	18. c	19. b	20. b
21. d	22. d	23. d	24. a	25. a
26. d	27. d	28. b	29. c	30. b
31. b	32. d	33. a	34. b	35. a
36. d	37. d	38. d	39. b	40. a
41. b	42. c	43. c	44. d	45. c
46. a	47. c	48. b	49.	50. b