

# August 2011 Paper-II

1. Spherical Bessel function of order  $n$  is a product of

$\sqrt{\frac{\pi}{2x}}$  with :

(a) Bessel function of order  $n + \frac{1}{2}$

(b) Bessel function of order  $n - \frac{1}{2}$

(c) Bessel function transformed in polar coordinates

(d) Spherical harmonic of order  $n$

2. The angle between vector  $\hat{i} + \hat{j}$  and  $\hat{j} + \hat{k}$  is (in radian) :

(a)  $\pi$

(b)  $\pi/2$

(c)  $\pi/6$

(d)  $\pi/4$

3. The cube roots of unity  $1, \omega, \omega^2$  form :

(a) a cyclic group of order 3

(b) a permutation group

(c)  $SU(3)$  group

(d)  $SU(2) \times U$  group

4. The value of

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

is :

(a)  $\sqrt{2}$

(b) 1.6

(c)  $\sqrt{3}$

(d) 0.8

5. The real matrix  $A = \begin{bmatrix} a & f & g \\ -f & a & -h \\ -g & h & a \end{bmatrix}$  is skew symmetric when :

(a)  $a = 0$

(b)  $f = 0$

(c)  $g = h$

(d)  $f = g$

6. The eigenvalues of the matrix  $\begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}$  are :

(a) 1

(b)  $\pm \omega$

(c)  $\pm \omega^2$

(d)  $\pm i$

7. A complex function  $f(z)$  is given by :

$$f(z) = \sqrt{z} + \frac{1}{z-a} + \exp(z)$$

The singularities of  $f(z)$  are :

(a) simple pole at  $z = a$

(b) branch point at  $z = 0$

(c) essential singularity at  $z \rightarrow \infty$

(d) all of the three above

8. The determinant of a  $3 \times 3$  real symmetric matrix is 36. If two of its eigenvalues are 2 and 3, then the sum of the eigenvalues is :

(a) 30

(b) 10

(c) 11

(d) 31

9. A harmonic oscillator in one dimension is perturbed by the potential  $\alpha x^3$ . The ground state energy of the oscillator to a first-order in perturbation is :

(a)  $\frac{\hbar\omega}{2} + \alpha$

(b)  $\frac{3}{2}\hbar\omega + \alpha$

(c)  $\frac{\hbar\omega}{2} + \alpha^3$

(d)  $\frac{\hbar\omega}{2}$

10. A particle moves in one-dimensional potential  $V(x)$ . At  $x = a$ , if  $V$  has a finite discontinuity (jump), then which of the following is true for its wave function  $\phi$  and its first derivative  $\phi'$  at  $x = a$ ?

(a)  $\phi$  is continuous and  $\phi'$  must be discontinuous

(b)  $\phi$  is discontinuous and  $\phi'$  must be continuous

(c) both  $\phi$  and  $\phi'$  are discontinuous

(d) both  $\phi$  and  $\phi'$  are continuous

11 In quantum mechanics, three dimensional wave function  $\psi(\vec{r})$  of a particle :

(a) has dimension of (energy  $\times$  time)

(b) has dimension of (length)<sup>-3/2</sup>

(c) has dimension of energy

(d) is dimensionless

12 A system is known to be in a state described by the wave function :  $\psi(\theta, \phi) = \frac{1}{\sqrt{30}} \{5Y_4^0 + Y_6^0 + 2Y_6^3\}$  where  $Y_l^m(\theta, \phi)$  are spherical harmonics. The probability of finding the system in a state with  $m = 0$  is :

(a) zero

(b)  $6/\sqrt{30}$

(c)  $6/30$

(d)  $13/15$

13 What is the degeneracy of the third excited state for a particle in 3-dimensional isotropic Harmonic oscillator potential ?

(Note : ground state is not an excited state)

(a) 10

(b) 6

(c) 4

(d) 3

14 The parity of wave function  $\psi$  is associated with which of the following transformation ?

(a) Space translation

(b) Space rotation

(c) Space inversion

(d) Space exchange of two particles

15 Which of the following processes involves tunnelling through a potential barrier ?

(a) Pair production (b)  $\alpha$ -decay

(c)  $\beta$ -decay (d)  $\gamma$ -decay

16 The variational method in perturbation theory, when applied to obtain the value of the ground state energy :

(a) gives energy value higher than or equal to the exact ground state energy

(b) always gives exact ground state energy

(c) gives energy value lower than the exact ground state energy

(d) gives energy value which is sometimes higher than or sometimes lower than the exact ground state energy

17 In a scintillation detector, the height of the output pulse is proportional to :

(a) Energy of the incident photon

(b) Intensity of the incident photon

(c) Energy and intensity of the photon

(d) Does not depend either on intensity or energy

18 If 'N' number of gadgets are connected to a power supply with a capacity of 'X' amperes without overloading then :

(a) Total current drawn by all the gadgets should be equal to  $\frac{X}{2}$  ampere

(b) Total current drawn by all the gadgets should be equal to N.X ampere

(c) Total current drawn by all the gadgets should be equal to X

(d) Total current drawn by all the gadgets should be equal to  $\frac{X}{4}$

19 If a square wave from a function generator is coupled to an oscilloscope in a.c. mode, what would be observed on the oscilloscope ?

(a) A perfect square wave

(b) Distorted square wave

(c) A sawtooth wave

(d) A perfect square wave with change in repetition rate

20 If a oscilloscope is operated in a d.c. mode, one can faithfully measure :

(a) only a.c. voltage

(b) only d.c. voltage

(c) both a.c. and d.c. voltage

(d) only low frequency a.c. voltage

21 In recording a powder X-ray diffraction pattern :

(a) the specimen and detector are both rotated

(b) the specimen alone is rotated

(c) the detector alone is rotated

(d) the specimen and the source are both rotated

22 In a Michelson Interferometer, the mirror  $M_2$  is moved such that 800 fringes are counted. The wavelength of the source used was  $6000\text{\AA}$ . Through what distance the mirror  $M_2$  must have been moved ?

(a) 0.24 mm

(b) 0.48 mm

(c) 0.36 mm

(d) 0.60 mm

23 For the measurement of pressure in a chamber evacuated by a diffusion pump and a rotary mechanical pump, one would require the following combination of gauges :

(a) Thermocouple/Pirani

(b) Penning-Pirani

(c) Thermocouple/Mercury Manometer

(d) Pirani/Mercury Manometer

24 The vapour diffusion pump works in the following region of air flow :

(a) Molecular flow

(b) Turbulent flow

(c) Lamellar flow

(d) Viscous flow

25 A cork is submerged in a pail of water by a spring attached to the bottom of the pail. The pail is held by a child in an elevator. During the initial acceleration as the elevator travels to the next lower floor, will the displacement of the spring :

(a) increase

(b) decrease

(c) remain the same

(d) indeterminate

26 A satellite is launched into a circular orbit of radius  $R$ . A second satellite is launched into an orbit of radius  $1.01R$ . Then, the period of the second satellite is :

(a) larger by 1.5%

(b) smaller by 1%

(c) larger by 2%

(d) smaller by 2%

27 A sphere of radius  $R$  is released in a liquid of viscosity  $\eta$ , so that by Stokes' law its drag is  $6\eta\pi Rv$ . Simultaneously, a second sphere of identical mass but with radius  $2R$  is released. Then the ratio of their terminal velocities is :

(a)  $\frac{V_R}{V_{2R}} = 1$

(b)  $\frac{V_R}{V_{2R}} = \frac{3}{2}$

(c)  $\frac{V_R}{V_{2R}} = 2$

(d)  $\frac{V_R}{V_{2R}} = \frac{2}{3}$

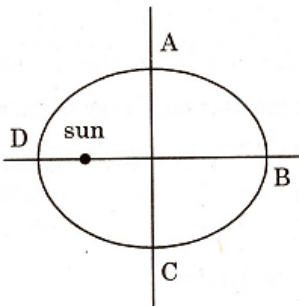
28 What would be the approximate length of a day if the earth spun so fast that bodies floated on the equator? Take the radius of the earth =  $6 \times 10^6$  m and  $g = 9.8 \text{ m/sec}^2$ .

- (a) 12hrs (b) 6hrs  
(c) 3hrs (d) 1.5hrs

29 An example of a scleronomic, holonomic, conservative and unilateral constraint is :

- (a) simple pendulum with rigid support  
(b) simple pendulum with variable length  
(c) a spherical container of fixed radius filled with gas  
(d) an expanding or contracting spherical container of gas

30 A planet has elliptical orbit with sun at the focus as shown in the figure. Which position of the orbit the planet has the highest speed ?



- (a) A (b) B  
(c) C (d) D

31 If the Lagrangian of the system is :

$$L(\rho, \theta, \dot{\rho}, \dot{\theta}) = \frac{m}{2} (\dot{\rho}^2 \dot{\theta}^2 + \dot{\rho}^2 \csc^2 \alpha) - mg \rho \cot \alpha$$

then conserved quantities are :

- (a)  $p_\rho$  (b)  $p_\rho$  and  $p_\theta$   
(c)  $p_\theta$  (d) none of the above

32 Example of a non-central force is :

- (a) Gravitational force  $-\frac{Gm_1m_2}{r^2} \hat{r}$   
(b) Coulomb force  $\frac{z_1z_2}{r^2} \hat{r}$   
(c) Hooke law  $k\vec{r}$   
(d) dipole-dipole interaction  $\frac{\vec{p} \cdot \vec{r}}{r^3}$  where  $\vec{p}$ , is the dipole moment

33 An infinitely long line-charge has a uniform linear charge density  $\lambda$ . If  $r$  denotes the distance of a point from the wire, then magnitude of the electric field at the point is :

- (a) proportional to  $\frac{1}{r}$  (b) proportional to  $\frac{1}{r^2}$   
(c) proportional to  $\frac{1}{r^3}$  (d) independent of  $r$

34 The dispersion relation for electromagnetic waves in a certain medium is  $\omega^2 = \alpha k$ , where  $\alpha$  is constant,  $\omega$  the angular frequency and  $k$  the magnitude of the wave vector. The velocity of the energy propagation by electromagnetic waves in this medium is :

- (a)  $\frac{\alpha}{\omega}$  (b)  $\frac{2\alpha}{\omega}$   
(c)  $\frac{\alpha}{2\omega}$  (d)  $\frac{\alpha}{4\omega}$

35 The dispersion relation for electromagnetic waves in a certain medium is  $\omega^2 = \alpha k^2$ , where  $\alpha$  is constant,  $\omega$  the angular frequency and  $k$  the magnitude of the wave vector. Which of the following statements is correct?

- (a) The phase velocity in the medium is  $\alpha$   
(b) The group velocity in the medium is  $\alpha$   
(c) The medium is dispersive  
(d) The medium is non-dispersive

36 The interaction energy of an electric dipole  $\vec{p}$  in an external electric field  $\vec{E}$  is :

- (a)  $\vec{p} \cdot \vec{E}$  (b)  $-\vec{p} \cdot \vec{E}$   
(c)  $|\vec{p} \times \vec{E}|$  (d)  $-\vec{p} \times \vec{E}$

37 In the Young's double slit experiment, the intensity of central maximum is  $I_2$ . If either of the slits is closed, the intensity at the same location is  $I_1$ . The relation between  $I_1$  and  $I_2$  is :

- (a)  $I_2 = 4I_1$  (b)  $I_2 = 2I_1$   
(c)  $I_2 = I_1$  (d)  $I_1 = 2I_2$

38 Maxwell introduced an additional term in :

- (a) Gauss's law (b) Faraday's law  
(c) Ampere's law (d) Coulomb's law

39 The skin depth  $\delta$  of a good metal, for the microwave frequency  $\omega$  follows the relation :

- (a)  $\delta \propto \omega$  (b)  $\delta \propto \frac{1}{\omega}$   
(c)  $\delta \propto \sqrt{\omega}$  (d)  $\delta \propto \frac{1}{\sqrt{\omega}}$

40 For a certain material  $\frac{g}{\epsilon W} \ll 1$ , where  $g$  and  $\epsilon$  are conductivity and permittivity of the medium. For a radiation of frequency  $\omega$ , this material is :

- (a) a good conductor (b) a good insulator  
(c) partially insulating (d) a semiconductor

41 The entropy of an ideal gas at absolute zero is :

- (a)  $\infty$   
(b) 0  
(c)  $Nk_B$   
(d) can not be calculated

42 For a system of  $N$  non-interacting fermions enclosed in a volume '  $V$  ' at constant temperature  $T$ , the average occupation number of the '  $r$  th ' energy level is given by :

- (a)  $\bar{n}_r = \frac{1}{e^{\beta(\epsilon_r - \mu)} + 1}$   
(b)  $\bar{n}_r = \frac{1}{(e^{\beta(\epsilon_r - \mu)} - 1)}$   
(c)  $\bar{n}_r = e^{-\beta(\epsilon_r - \mu)}$   
(d)  $\bar{n}_r = (e^{\beta(\epsilon_r - \mu)} + 1)$

43 A first order phase transition is characterized by :

- (a) a divergence of the specific heat at  $T_c$ , the critical temperature  
(b) A cusp in the average energy at  $T_c$   
(c) The constancy of entropy in the transition  
(d) A latent heat is involved in the transition process

44 A gas of molecules, each of mass '  $m$  ' is in thermal equilibrium at an absolute temperature '  $T$  '. If  $v_x, v_y, v_z$  are the components of the velocity  $\vec{v}$ , of each molecule, then the mean value of  $\vec{v}^2$  :

- (a) 0 (b)  $\frac{1}{2} k_B T$   
(c)  $\frac{3}{m} k_B T$  (d)  $N k_B T$

45 The Fermi energy of a free electron gas at absolute zero is of the order of :

- (a) electron-volts (b) MeV  
(c) keV (d) ergs

46 Consider an ideal gas of  $N$  molecules enclosed in a volume '  $V$  ' maintained at a temperature '  $T$  '. The

correct expression for the entropy of the system is :

$$(a) S = Nk_B \left[ \ln V + \frac{3}{2} \ln T + \sigma \right]$$

$$(b) S = Nk_B \left[ \ln \left( \frac{V}{N} \right) + \frac{3}{2} \ln T + \sigma \right]$$

$$(c) S = k_B \left[ \ln V + \frac{3}{2} \ln T + \sigma \right]$$

$$(d) S = k_B \left( \frac{N}{V} \right) \left[ \ln \left( \frac{V}{N} \right) + \frac{3}{2} \ln T + \sigma \right]$$

Answer Key				
1. a	2. d	3. a	4. b	5. a
6. d	7. d	8. c	9. d	10. d
11. b	12. d	13. a	14. c	15. b
16. d	17. a	18. c	19. c	20. c
21. a	22. a	23. d	24. d	25. b
26. a	27. d	28. d	29. a	30. d
31. c	32. d	33. a	34. c	35. d
36. b	37. d	38. c	39. d	40. b
41. b	42. a	43. a	44. c	45. a
46. d	47. d	48. c	49. a	50. d

47 If the temperature of a black body is increased by a factor of 2 , the amount of energy/ volume radiated increases by a factor of :

- (a) 2 (b) 4  
(c) 8 (d) 16

48 If the temperature of a free electron gas is increased by a factor of 2 , its specific heat increases by a factor of :

- (a) 2 (b) 4  
(c) 8 (d) 16

49 In Laue X-ray diffraction experiment in the study of single crystal structure, the following X-ray source is used :

- (a) Monochromatic  
(b) Non-monochromatic  
(c) Pulsed Monochromatic  
(d) By chromatic

50 5 boys and 3 girls are to stand in a straight line such that no two girls are adjacent. The number of ways in which this can be done is :

- (a) 5! (b) 3!  
(c) 5! × 3! (d) 5! × 5! .