

## August 2010 Paper-II

- 1 If A & B are matrices such that  $AB = B$  and  $BA = A$  then  $A^2 + B^2 =$ 
  - (a)  $2AB$
  - (b)  $2BA$
  - (c)  $A + B$
  - (d)  $AB$
- 2 The number of values of  $k$  for which the system of equations  $(k + 1)x + 8y = 4k$  and  $kx + (k + 3)y = (3k - 1)$  has infinitely many solutions is
  - (a) 0
  - (b) 1
  - (c) 2
  - (d)  $\infty$
- 3 The eigen values of the matrix  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$  are
  - (a)  $\cos \theta, \sin \theta$
  - (b)  $\cos \theta, -\sin \theta$
  - (c)  $\cos \theta \pm \sin \theta$
  - (d)  $\cos \theta \pm i \sin \theta$
- 4 If  $\phi = yz + zx + xy$  then  $\nabla \phi$  at point  $(1, 1, 1)$  is
  - (a)  $\vec{0}$
  - (b)  $2\hat{x} + 2\hat{y} + 2\hat{z}$
  - (c) 3
  - (d) 6
- 5 Which of the following is not true the curve  $y^2 = x^2(x - 5)$ 
  - (a) The curve is symmetric about x axis
  - (b) The curve is symmetric about y-axis
  - (c) The curve lies in the region  $x \leq 5$
  - (d) The curve forms a loop
- 6 3 persons work independently on a same problem. If the respective probabilities that they will solve it are  $\frac{1}{3}, \frac{1}{4}$  and  $\frac{1}{5}$ , then the probability that none can solve is .....
  - (a)  $\frac{2}{5}$
  - (b)  $\frac{3}{5}$
  - (c)  $\frac{13}{60}$
  - (d)  $\frac{59}{60}$
7. Domain of the function  $\sqrt{\log \left( \frac{5x - x^2}{6} \right)}$  is
  - (a) (2,3)
  - (b) [2,3]
  - (c) [1,2]
  - (d) (1,3)
8. Let  $P_n(x)$  denote the Legendre polynomial of degree  $n$ . Then  $P_1\left(\frac{1}{2}\right) =$ 
  - (a) 0
  - (b)  $\frac{1}{2}$
  - (c) 1
  - (d) 2
9. The phase velocity of the wave described by  $\psi = A \sin(kx + \sigma t + \pi/2)$  is
  - (a)  $x/t$
  - (b)  $-\sigma/k$
  - (c)  $A/\sigma$
  - (d)  $\pi/2k$
- 10 A simple harmonic oscillator of natural frequency  $\omega$  put at the center of a 1 - d box of length  $L$  with non-penetrable walls. At what energy  $E$  of the particle it starts feeling the presence of the walls.
  - (a)  $E = \frac{1}{2} mL^2 \omega^2$
  - (b)  $E = \frac{1}{8} mL^2 \omega^2$
  - (c)  $E = \frac{1}{4} mL^2 \omega^2$
  - (d)  $E = mL^2 \omega^2$
- 11 A particle of mass  $m$  and charge  $q$  is moving in the combined field of gravity and a constant electric field  $E$  in the horizontal plane. Taking x-axis parallel to the electric field and z-axis vertically upward, the potential energy of the particle is given by
  - (a)  $mgz - q|E|x$
  - (b)  $-mgz - q|E|x$
  - (c)  $mgz + q|E|x$
  - (d)  $-mgz + q|E|x$
- 12 A particle of mass  $m$  is constrained to move on the circumference of a circle of radius  $R$ . using the angle  $\theta$  made to the polar axis passing through the center of the circle. what is the Hamiltonian of the system?
  - (a)  $\frac{1}{2} mR^2 \dot{\theta}^2$

(b)  $p_\theta^2/2mR^2 + \frac{1}{2}mR^2\dot{\theta}^2$

(c)  $p_\theta^2/2mR^2$

(d)  $\frac{1}{2}mR^2\dot{\theta}^2 - p_\theta^2/2mR^2$

- 13 A particle of charge  $q$  and mass  $m$  is moving along  $x$ -axis with velocity  $V$ . An oscillating electric field of frequency  $\omega$  and magnitude  $E$  is applied along the  $y$ -axis. What is the wavelength of the resulting wave motion?

(a)  $qE/m\omega^2$

(b)  $|V|/\omega$

(c)  $2\pi|V|/\omega$

(d)  $m|V|^2/qE$

- 14 A simple harmonic oscillator is moving in a potential  $V(x) = \frac{1}{2}kx^2$ . What happens to the frequency of the oscillator if a perfectly reflecting wall is kept at the origin?

(a) no change

(b) it is halved

(c) it is doubled

(d) the motion becomes aperiodic.

- 15 Let two unequal masses  $m_1$  and  $m_2$  ( $m_1 < m_2$ ) be connected by a string of length  $l$  which passes over a frictionless and massless pulley such that the distance of  $m_2$  from the pulley be  $x$ , then the Lagrangian of the system is

(a)  $\frac{1}{2}(m_1 - m_2)\dot{x}^2 + (m_1 + m_2)x$

(b)  $\frac{1}{2}(m_1 + m_2)\dot{x}^2 + (m_2 - m_1)x$

(c)  $\frac{1}{2}(m_1 + m_2)\dot{x}^2 - (m_2 - m_1)x$

(d)  $\frac{1}{2}(m_2 - m_1)\dot{x}^2 + -(m_2 - m_1)x$

- 16 A particle of mass  $m$  is attached to the ceiling with a spring of spring constant  $k$  and unstretched length  $L$ . The number of degrees of freedom for this system is

(a) 2

(b) 3

(c) 4

(d) 5

- 17 The electric field at a point just outside the conducting surface is always

(a) Zero

(b) Tangential to the surface

(c) Normal to the surface

(d) Infinite

- 18 Consider two spherical shells of radii  $R$  and  $2R$  respectively. Same amount of charge  $q$  is placed on each of them. The spheres are placed far apart. If  $E$  is the electric field just outside the shell of radius  $R$ , then the electric field just outside the shell of radius  $2R$  is

(a)  $\frac{E}{2}$

(b)  $2E$

(c)  $\frac{E}{4}$

(d)  $4E$

- 19 Let  $E(r)$  denote the electric field intensity due to an infinite uniformly charged plane surface at a point at a distance  $r$  from the surface. Then which of the following is correct?

(a)  $E(r)$  varies as  $r$

(b)  $E(r)$  varies as  $\frac{1}{r}$

(c)  $E(r)$  varies as  $\frac{1}{r^2}$

(d)  $E(r)$  is independent of  $r$

- 20 A cube of side  $l$  is placed in a uniform field. The total electric flux through the surface of a cube is  
 (a) 0 (b)  $6t^2$   
 (c)  $4l^2\epsilon$  (d)  $2l^2E$
- 21 A plane electromagnetic wave propagates in negative  $y$  direction. Which of the following pair of space and time varying fields can be fields associated with this wave  
 (a)  $\vec{E} = -E_0\hat{y}$  (b)  $\vec{E} = E_0\hat{y}$   
 $\vec{B} = -B_0\hat{y}$   $\vec{B} = B_0\hat{y}$   
 (c)  $\vec{E} = E_0\hat{z}$  (d)  $\vec{E} = E_0\hat{x}$   
 $\vec{B} = B_0\hat{x}$   $\vec{B} = B_0\hat{z}$
- 22 For good conductors, skin depth varies inversely with..... power of frequency.  
 (a)  $\frac{1}{2}$  (b) 1  
 (c) 2 (d) 0
- 23 The electric potential due to a linear quadrupole varies inversely with.....  
 (a)  $r$  (b)  $r^2$   
 (c)  $r^3$  (d)  $r^4$
- 24 A charged particle of mass  $m$  and charge  $q$  travels on a circular path of radius  $r$  that is perpendicular to a magnetic field  $B$ . The time taken by the particle to complete the circle.  
 (a)  $\frac{2\pi eB}{m}$  (b)  $\frac{2\pi m}{qB}$   
 (c)  $\frac{2\pi}{qBm}$  (d)  $\frac{2\pi rqB}{m}$
- 25 Two interfering rays are of intensities  $I$  and  $4I$ . The ratio of intensity of dark band to that of bright band is.....  
 (a) 1:3 (b) 1:4  
 (c) 1:9 (d) 0: $\infty$
- 26 Given the operators  $P_x = P_x$  and  $x = i\hbar \frac{\partial}{\partial P_x}$ , the value of  $[x, D_x]$  is  
 (a)  $-i\hbar$  (b)  $xP_x$   
 (c)  $-P_x x$  (d)  $i\hbar$
- 27 The probability current density for the unnormalized wave function  $\psi = e^{ikx}$  is  
 (a)  $\hbar^2 k^2 / 2m$  (b)  $\hbar k / m$   
 (c)  $\hbar^3 k^3 / m$  (d) 0
- 28 Which of the following Hamiltonian conserve angular momentum?  
 (a)  $\frac{p^2}{2m} + \frac{1}{2}k(x^2 + y^2 + z^2)$   
 (b)  $\frac{p^2}{2m} + \frac{1}{2}m\omega^2(x^2 + yz)$   
 (c)  $p^2/2m + qE \cdot r$   
 (d)  $p^2/2m + \frac{1}{2}k(r^2 - z^2)$
- 29 A quantum mechanical particle of mass  $m$  confined in a one-dimensional box of length  $L$  is in its ground state. What will happen to the particle energy when the size of the box is slowly increased to double its original size, i.e. made  $2L$   
 (a) Energy will not change  
 (b) Energy will increase to double its original value  
 (c) Energy will decrease to quarter its original value  
 (d) Energy will decrease to half its original value.
- 30 Consider the Hamiltonian  
 $H = (p_x^2 + p_y^2)/2m + \frac{1}{2}m\omega^2(x^2 + y^2 + 2xy)$ .  
 The ground state energy of this system is

(a) 0  
(c)  $\frac{1}{2}\hbar\omega$

(b)  $\hbar\omega$   
(d)  $\frac{3}{2}\hbar\omega$

31 A and B are two linear operators in some linear vector space. Let  $\lambda$  be a non-zero eigenvalue of AB corresponding to eigenvector  $|\lambda\rangle$  i.e.  $AB|\lambda\rangle = \lambda|\lambda\rangle$ . Define  $|W\rangle = B|\lambda\rangle$ .

Which of the following is definitely true? It is an

(a) eigenvector of  $A^2$  (b) eigenvector of  $B^2$

(c) eigenvector of AB (d) eigenvector of BA

32 Which of the following Hamiltonians conserves parity?

(a)  $\frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + bxyz$

(b)  $\frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + b(x + y + z)$

(c)  $\frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + b(x^2 - y^2 + z^2)$

(d)  $\frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + b/(xyz)$

33 A free particle is in a state

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \left[ \frac{1}{2} e^{ikx} + \frac{1}{2} e^{-2ikx} + \frac{1}{\sqrt{2}} e^{3ikx} \right]$$

what is the energy of the particle in units of  $\hbar^2 k^2 / 2m$

(a) 1

(b) 5.75

(c) 4

(d) zero

34 A simple harmonic oscillator has an energy spectrum  $E = (n + 1/2)\hbar\omega$ . What happens to the spectrum if a perfect reflector is kept at the origin so that the oscillator can move in the domain  $(0, \infty)$  instead of  $(-\infty, \infty)$ ?

(a) No change in the energy eigenvalues

(b) Only odd n eigen values are allowed

(c) Only even n eigen values are allowed

(d) Only ground state energy double

35 Consider a gas of 3 particles with four available single particle quantum states. The probability that there are 3 particles in a single state in case the particles obey Bose Einstein statistics is

(a) 36/64

(b) 4/64

(c) 4/20

(d)  $\frac{0}{4}$

36 Consider a system of conduction electrons in a metal at a temperature T. The specific heat of conduction electron is

(a) Proportional to T

(b) Proportional to  $T^3$

(c) equal to  $\frac{3}{2}R$ , R being gas constant

(d) independent of temperature.

37 Consider a system of two distinguishable particles, each of which can be found in one of the three possible quantum states. The number of distinct ways one can put the two particles into 3 single particle states is

(a) 6

(b) 9

(c) 3

(d) 12

38 Consider a Grand Canonical Ensemble for a system. In this case, the following condition apply in the primary consideration:

(a) system in contact with a heat reservoir, Energy can vary, Temperature is constant, number of particles is constant.

(b) System isolated, Energy is constant, Number of Particles is constant

(c) System in contact with a heat reservoir,

Energy, Temperature, Number of particles are constant

(d) System in contact with a heat reservoir.

Energy can vary, Temperature is constant, number of particles can vary.

39 The magnetic susceptibility of N non interacting spins follow Curie law at

(a) All temperatures

(b) High temperatures

(c) Very low temperatures

(d) Only at absolute zero

40 The de Broglie wavelength of helium atom at 300 K is  $\sim 0.6 \times 10^{-8}$  cm. The de Broglie wavelength of neon atom (5 times heavier than helium atom) at 600 K will be

(a)  $\frac{0.6}{\sqrt{10}} \times 10^{-8}$  cm

(b)  $6 \times 10^{-8}$  cm

(c)  $0.06 \times 10^{-8}$  cm

(d)  $0.6 \times \sqrt{10} \times 10^{-8}$  cm

41 Consider N independent particles with spin angular momentum S each. Each spin has  $2S + 1$  projections along the axis of quantization. The total number of microstates the system will be

(a)  $N(2S + 1)$

(b)  $N^{2S+1}$

(c)  $(2S + 1)^N$

(d)  $N(2S + 1)!$

42 A system has been taken from an initial state described by  $p_1 V_1$  to the final state described by  $p_2 V_2$  by two different paths. Let  $\Delta Q$  and  $\Delta W$  represent the heat given to the system and the work done by the system. The following quantity will be the same in both the ways.

(a)  $\Delta Q$

(b)  $\Delta W$

(c)  $\Delta Q + \Delta W$

(d)  $\Delta Q - \Delta W$

43 The plates (1 m<sup>2</sup> square size) of parallel plate condenser are separated by 1 cm. The potential difference across the plate is 1000 V, the force in Newton between the plates is

(a) 5.0

(b) 0.5

(c)  $5 \times 10^{-2}$

(d)  $5 \times 10^{-3}$

44 Floating voltage signals should be amplified by

(a) OP. AMP in inverting configuration

(b) OP-AMP in non-inverting configuration

(c) Instrumentation amplifier

(d) Buffer amplifier

45 An inductance L and capacitance C are connected in series to resonate at frequency ' $f_s$ '. When they are connected in parallel, the resonance frequency is ' $f_p$ '.

(a)  $f_p < f_s$

(b)  $f_p > f_s$

(c)  $f_p = f_s$

(d)  $f_p$  is not related to  $f_s$

46 The peak current in series resonance circuit depends on

(a) L

(b) R

(c) C

(d) LC

47 The resonance frequency of parallel resonance circuit is given by

(a)  $\frac{1}{2\pi\sqrt{LC}}$

(b)  $\frac{1}{2\pi\sqrt{RC}}$

$$(c) \frac{1}{2\pi\sqrt{RL}}$$

$$(d) \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

48 In a thermocouple, junctions of two dissimilar metal wires are used for measuring temperature. The temperature of a body is estimated by measuring the

- (a) Difference between the current flowing to the junctions
- (b) Difference of the voltages developed between the junctions
- (c) Difference of the frequencies of the electrical signals between the junctions
- (d) Changes in the length of the metal wires used for making the junctions

49 Sensitivity of a strain gauge is given by

$$(a) K = \frac{\Delta R/l}{\Delta l/R}$$

$$(b) K = \frac{\Delta R/R}{\Delta l/l}$$

$$(c) K = \frac{\Delta R \Delta l}{Rl}$$

$$(d) K = \frac{\Delta l/l}{\Delta R/R}$$

50 Which one of the following optical sources is more monochromatic?

- (a) Output of Nd: YAG laser
- (b) Output of Ruby laser
- (c) Laser pointer
- (d) He-Ne Laser beam.

#### Answer Key

1. c	2.	3. d	4. b	5. a
6. a	7. c	8. b	9. b	10. a
11. a	12.	13. d	14. d	15. c
16. b	17. c	18. c	19. d	20. a
21.	22. a	23. c	24. b	25. b
26. d	27. b	28. a	29. c	30. b
31. d	32.	33.	34. b	35.
36. b	37. b	38. d	39. b	40. a
41. c	42. a	43. b	44. a	45. b
46. b	47. d	48. b	49. b	50. d