❖ NET FEB- 2022

➢ Part- B

1. A particle in one dimension executes oscillatory motion in a potential $V(x) = A|x|$, where $A > 0$ is a constant of appropriate dimension. If the time period T of its oscillation depends on the total energy E as E^{α}, then the value of α is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

2. The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a dissipative force is described by $\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 16x = 6te^{-8t} + 4t^2e^{-2t}$. The general form of the particular solution, in terms of constants A, B etc, is

(a) $t(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}$
(b) $(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}$
(c) $t(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$
(d) $(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$

3. The vector potential for an almost point like magnetic dipole located at the origin is $A = \frac{\mu \sin\theta}{4\pi r^2}\hat{\phi}$, where (r, θ, ϕ) denote the spherical polar coordinates and $\hat{\phi}$ is the unit vector along ϕ. A particle of mass m and charge q, moving in the equatorial plane of the dipole, starts at time $= t = 0$ with an initial speed v_0 and an impact parameter b. Its instantaneous speed at the point of closest approach is

(a) v_0 (b) 0/0
(c) $v_0 + \frac{\mu q}{4\pi m b^2}$ (d) $\sqrt{v_0^2 + \left(\frac{\mu q}{4\pi m b^2}\right)^2}$

4. A particle, thrown with a speed v from the earth's surface, attains a maximum height h (measured from the surface of the earth). If v is half the escape velocity and R denotes the radius of earth, then $\frac{h}{R}$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

5. A particle of mass $\frac{1GeV}{c^2}$ and its antiparticle, both moving with the same speed v, produce a new particle X of mass $\frac{10\text{GeV}}{c^2}$ in a head-on collision. The minimum value of v required for this process is closest to

(a) $0.83c$ (b) $0.93c$
(c) $0.98c$ (d) 0.88c

6. The volume of the region common to the interiors of two infinitely long cylinders defined by $x^2 + y^2 = 25$ and $x^2 + 4z^2 = 25$ is best approximated by

(a) 225 (b) 333
(c) 423 (d) 625

7. The volume integral

$$I = \iiint_V A \cdot (\nabla \times A) d^3x$$

is over a region V bounded by a surface $\sum$ (an infinitesimal area element being $\hat{\mathbf{n}}ds$, where $\hat{\mathbf{n}}$ is the outward unit normal). If it changes to $I + \Delta I$, when the vector $\mathbf{A}$ is changed to $\mathbf{A} + \nabla\Lambda$, then ΔI can be expressed as

(a) $\iiint_V \nabla \cdot (\nabla\Lambda \times A) d^3x$

(b) $-\iiint_V \nabla^2 \Lambda d^3x$

(c) $-\oiint_{\Sigma} (\nabla\Lambda \times \mathbf{A}) \cdot \hat{\mathbf{n}} ds$

(d) $\oiint_{z} \nabla\Lambda . \hat{\mathbf{n}} ds$

8. A generic 3×3 real matrix A has eigenvalues 0,1 and 6 and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the

(a) eigenvalues $(I + A)^{-1}$

(b) eigenvalues of $(I + A^T A)$

(c) determinant of $A^T A$

(d) rank of A

9. A discrete random variable X takes a value from the set $\{-1,0,1,2\}$ with the corresponding probabilities $p(X) = \frac{3}{10}, \frac{2}{10}, \frac{2}{10}$ and $\frac{3}{10}$, respectively. The probability distribution $q(Y) = (q(0), q(1), q(4))$ of the random variable $Y = X^2$ is

(a) $\left(\frac{1}{5}, \frac{3}{5}, \frac{1}{5}\right)$ (b) $\left(\frac{1}{5}, \frac{1}{2}, \frac{3}{10}\right)$

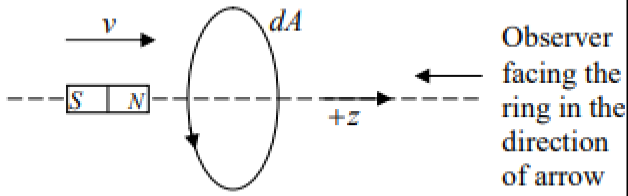
(c) $\left(\frac{2}{5}, \frac{2}{5}, \frac{1}{5}\right)$ (d) $\left(\frac{3}{10}, \frac{3}{10}, \frac{2}{5}\right)$

10. The components of the electric field, in a region of space devoid of any charge or current sources, are given to be $E_i = a_i + \sum_{j=1,2,3} b_{ij} x_j$, where a_i and b_{ij} are constants independent of the coordinates. The number of independent components of the matrix b_{ij} is

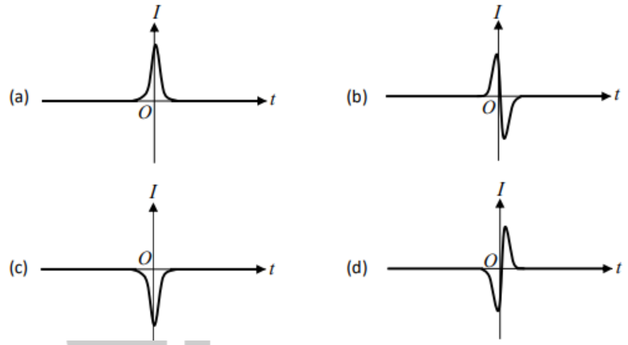
(a) 5 (b) 6

(c) 3 (d) 4

11. A conducting wire in the shape of a circle lies on the (x, y)-plane with its centre at the origin. A bar magnet moves with a constant velocity towards the wire along the z-axis (as shown in the figure below)



We take $t = 0$ to be the instant at which the midpoint of the magnet is at the centre of the wire loop and the induced current to be positive when it is counter-clockwise as viewed by the observer facing the loop and the incoming magnet. In these conventions, the best schematic representation of the induced current $I(t)$ as a function of t, is



12. In an experiment to measure the charge to mass ratio $\frac{e}{m}$ of the electron by Thomson's method, the values of the deflecting electric field and the accelerating potential are 6×10^6 N/C (newton per coulomb) and 150 V, respectively. The magnitude of the magnetic field that leads to zero deflection of the electron beam is closest to

(a) $0.6T$ (b) $1.2T$

(c) $0.4T$ (d) $0.8T$

13. A monochromatic source emitting radiation with a certain frequency moves with a velocity v away from a stationary observer A. It is moving towards another observer B (also at rest) along a line joining the two. The frequencies of the radiation recorded by A and B are ν_A and ν_B, respectively. If the ratio $\frac{\nu_B}{\nu_A} = 7$, then the value of $\frac{v}{c}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{\sqrt{3}}{2}$

14. The Hamiltonian of a particle of mass m in one-dimension is $H = \frac{1}{2m}p^2 + \lambda|x|^3$, where $\lambda > 0$ is constant. If E_1 and E_2, respectively, denote the ground state energies of the particle for $\lambda = 1$ and $\lambda = 2$ (in appropriate units) the ratio $\frac{E_2}{E_1}$ is best approximated by

(a) 1.260 (b) 1.414
(c) 1.516 (d) 1.320

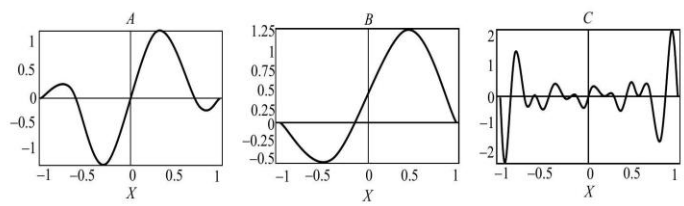
15. A particle of mass m is in a one dimensional infinite potential well of length L, extending from $x = 0$ to $x = L$. When it is in the energy eigenstate labelled by n, ($n = 1,2,3, \dots$) the probability of finding it in the interval $0 \leq x \leq \frac{L}{8}$ is $\frac{1}{8}$. The minimum value of n for which this is possible is

(a) 4 (b) 2
(c) 6 (d) 8

16. A two-state system evolves under the action of the Hamiltonian $H = E_0|A\rangle\langle A| + (E_0 + \Delta)|B\rangle\langle B|$, where $|A\rangle$ and $|B\rangle$ are its two orthonormal states. These states transform to one another under parity i.e., $P|A\rangle = |B\rangle$ and $P|B\rangle = |A\rangle$. If at time $t = 0$ the system is in a state of definite parity $P = 1$, the earliest time t at which the probability of finding the system in a state of parity $P = -1$ is one, is

(a) $\frac{\pi\hbar}{2\Delta}$ (b) $\frac{\pi\hbar}{\Delta}$
(c) $\frac{2\pi\hbar}{2\Delta}$ (d) $\frac{2\pi\hbar}{\Delta}$

17. The figures below depict three different wavefunctions of a particle confined to a one-dimensional box $-1 \leq x \leq 1$



The wavefunctions that correspond to the maximum expectation values $|\langle x\rangle|$ (absolute value of the mean position) and $\langle x^2\rangle$, respectively, are

(a) B and C (b) B and A
(c) C and B (d) A and B

18. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

(a) square of the radial position and z-component of angular momentum (r^2 and L_z)
(b) x-components of linear and angular momenta (p_x and L_x)
(c) y-component of position and z-component of angular momentum (y and L_z)
(d) squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

19. The ratio $\frac{C_P}{C_V}$ of the specific heats at constant pressure and volume of a monoatomic ideal gas in two dimensions is

(a) $\frac{3}{2}$ (b) 2
(c) $\frac{5}{3}$ (d) $\frac{5}{2}$

20. The volume and temperature of a spherical cavity filled with black body radiation are V and 300 K respectively. If it expands adiabatically to a volume 2 V, its temperature will be closest to

(a) 150 K (b) 300 K
(c) $250K$ (d) $240K$

21. The total number of phonon modes in a solid of volume V is $\int_0^{\omega_D} g(\omega)d\omega = 3N$, where N is the number of primitive cells, ω_3 is the Debye frequency and density of photon modes is

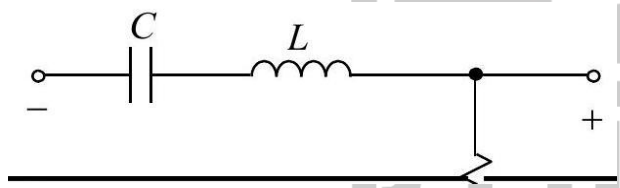
$g(\omega) = AV\omega^2$ (with $A > 0$ a constant). If the density of the solid doubles in a phase transition, the Debye temperature θ_D will
(a) increase by a factor of $2^{2/3}$
(b) increase by a factor of $2^{1/3}$
(c) decrease by a factor of $2^{2/3}$
(d) decrease by a factor of $2^{1/3}$

22. The position of a particle in one dimension changes in discrete steps. With each step it moves to the right, however, the length of the step is drawn from a uniform distribution from the interval $\left[\lambda - \frac{1}{2}w, \lambda + \frac{1}{2}w\right]$, where λ and w are positive constants. If X denotes the distance from the starting point after N steps, the standard deviation $\sqrt{\langle X^2\rangle - \langle X\rangle^2}$ for large values of N is

(a) $\frac{\lambda}{2} \times \sqrt{N}$ (b) $\frac{\lambda}{2} \times \sqrt{\frac{N}{3}}$

(c) $\frac{w}{2} \times \sqrt{N}$ (d) $\frac{w}{2} \times \sqrt{\frac{N}{3}}$

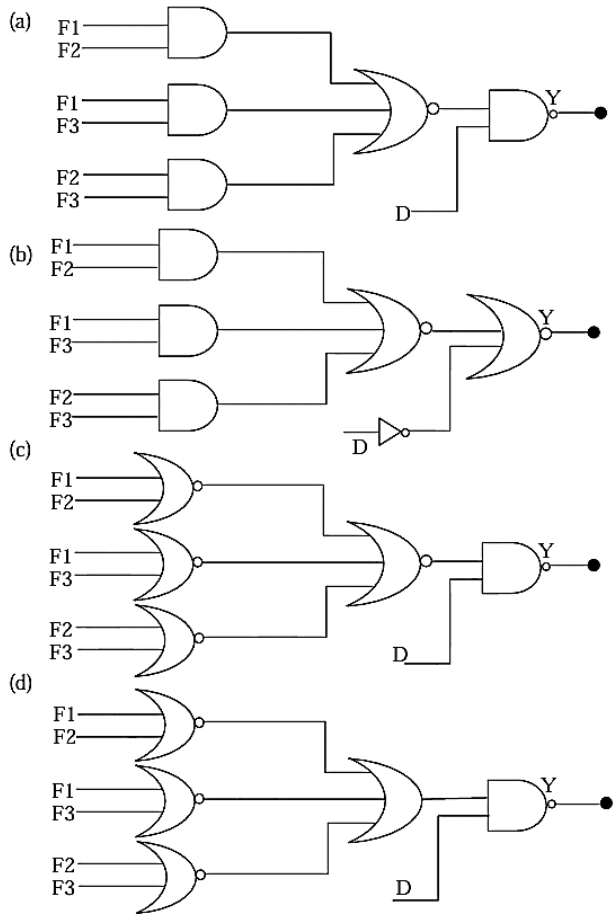
23. In the LCR circuit shown below, the resistance $R = 0.05\Omega$, the inductance $L = 1H$ and the capacitance $C = 0.04$ F.



If the input v_{in} is a square wave of angular frequency 1rad/s the output v_{out} is best approximated by a
(a) square wave of angular frequency 1rad/s
(b) sine wave of angular frequency 1rad/s
(c) square wave of angular frequency 5rad/s
(d) sine wave of angular frequency 5rad/s

24. In an experiment, the velocity of a non-relativistic neutron is determined by measuring the time (~ 50 ns) it takes to travel from the source to the detector kept at a distance L. Assume that the error in the measurement of L is negligibly small. If we want to estimate the kinetic energy T of the neutron to within 5% accuracy i.e., $\left|\frac{\delta T}{T}\right| \leq 0.05$, the maximum permissible error $|\delta T|$ in measuring the time of flight is nearest to
(a) 1.75 ns (b) 0.75 ns
(c) 2.25 ns (d) 1.25 ns

25. The door of an X - ray machine room is fitted with a sensor D (0 is open and 1 is closed). It is also equipped with three fire sensors F_1, F_2 and F_3 (each is 0 when disable and 1 when enabled).
The X-ray machine can operate only if the door is closed and at least 2 fire sensors are enabled.
The logic circuit to ensure that the machine can be operated is



➢ PART C

26. If we use the Fourier transform $\phi(x, y) = \int e^{ikx}\phi_k(y)dk$ to solve the partial differential equation $-\frac{\partial^2\phi(x,y)}{\partial y^2} - \frac{1}{y^2}\frac{\partial^2\phi(x,y)}{\partial x^2} + \frac{m^2}{y^2}\phi(x, y) = 0$ in the half-plane $\{(x, y): -\infty < x < \infty, 0 < y < \infty\}$ the Fourier modes $\phi_k(y)$ depend on y as y^α and y^β. The values of α and β are

(a) $\frac{1}{2} + \sqrt{1 + 4(k^2 + m^2)}$ and $\frac{1}{2} - \sqrt{1 + 4(k^2 + m^2)}$

(b) $1 + \sqrt{1 + 4(k^2 + m^2)}$ and $1 - \sqrt{1 + 4(k^2 + m^2)}$

(c) $\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4(k^2 + m^2)}$ and $\frac{1}{2} - \frac{1}{2}\sqrt{1 + 4(k^2 + m^2)}$

(d) $1 + \frac{1}{2}\sqrt{1 + 4(k^2 + m^2)}$ and $1 - \frac{1}{2}\sqrt{1 + 4(k^2 + m^2)}$

27. The Newton-Raphson method is to be used to determine the reciprocal of the number $x = 4$. If we start with the initial guess 0.20 then after the first iteration the reciprocal is

(a) 0.23 (b) 0.24

(c) 0.25 (d) 0.26

28. The Legendre polynomials $P_n(x), n = 0,1,2, \ldots$, satisfying the orthogonality condition $\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{nm}$ on the interval $[-1, +1]$ may be defined by the Rodrigues formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$. The value of the definite integral $\int_{-1}^{1}(4 + 2x - 3x^2 + 4x^3)P_3(x)dx$ is

(a) $\frac{3}{5}$ (b) $\frac{11}{15}$

(c) $\frac{23}{32}$ (d) $\frac{16}{35}$

29. A particle of mass m moves in a potential that is $V = \frac{1}{2}m(\omega_1^2x^2 + \omega_2^2y^2 + \omega_3^2z^2)$ in the coordinates of a non-inertial frame F. The frame F is rotating with respect to an inertial frame with an angular velocity $\hat{k}\Omega$, where $\hat{k}$ is the unit vector along their common z-axis. The motion of the particle is unstable for all angular frequencies satisfying

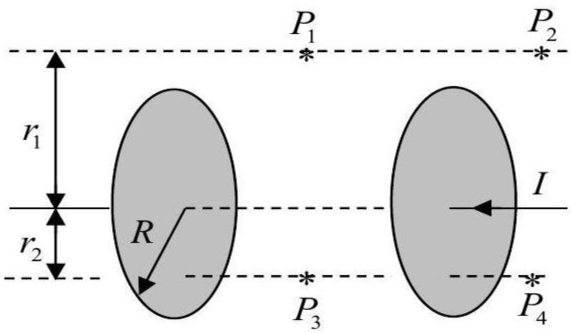
(a) $(\Omega^2 - \omega_1^2)(\Omega^2 - \omega_2^2) > 0$

(b) $(\Omega^2 - \omega_1^2)(\Omega^2 - \omega_2^2) < 0$

(c) $(\Omega^2 - (\omega_1 + \omega_2)^2)(\Omega^2 - |\omega_1 - \omega_2|^2) > 0$

(d) $(\Omega^2 - (\omega_1 + \omega_2)^2)(\Omega^2 - |\omega_1 - \omega_2|^2) < 0$

30. The figure below shows an ideal capacitor consisting of two parallel circular plates of radius R. Points P_1 and P_2 are at a transverse distance $r_1 > R$ from the line joining the centres of the plates, while points P_3 and P_4 are at a transverse distance $r_2 < R$.



If $B(x)$ denotes the magnitude of the magnetic fields at these points, which of the following holds while the capacitor is charging?

(a) $B(P_1) < B(P_2)$ and $B(P_3) < B(P_4)$

(b) $B(P_1) > B(P_2)$ and $B(P_3) > B(P_4)$

(c) $B(P_1) = B(P_2)$ and $B(P_3) < B(P_4)$

(d) $B(P_1) = B(P_2)$ and $B(P_3) > B(P_4)$

31. A perfectly conducting fluid, of permittivity ε and permeability μ, flows with a uniform velocity $\mathcal{V}$ in the presence of time dependent electric and magnetic fields E and B, respectively. If there is a finite current density in the fluid, then

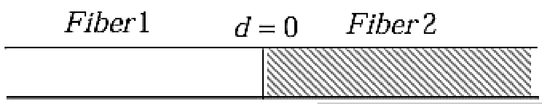
(a) $\nabla \times (v \times B) = \frac{\partial B}{\partial t}$

(b) $\nabla \times (v \times B) = -\frac{\partial B}{\partial t}$

(c) $\nabla \times (v \times B) = \sqrt{\varepsilon\mu}\frac{\partial E}{\partial t}$

(d) $\nabla \times (v \times B) = -\sqrt{\varepsilon\mu}\frac{\partial E}{\partial t}$

32. A laser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as shown in the figure). The loss factor of fiber 2 is 10 dB/km.
Fiber1 $d = 0$ Fiber 2



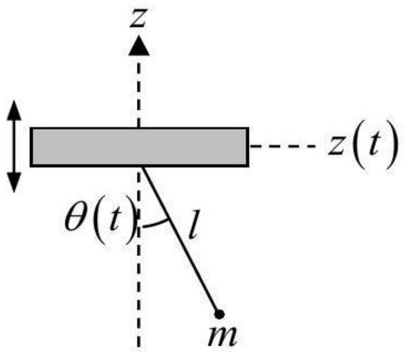
If $E_2(d)$ denotes the magnitude of the electric field in fiber 2 at a distance d from the interface, the ratio $\frac{E_2(0)}{E_2(d)}$ for $d = 10$ km, is

(a) 10^2 (b) 10^3
(c) 10^5 (d) 10^7

33. A particle in two dimensions is found to trace an orbit $r(\theta) = r_0\theta^2$. If it is moving under the influence of a central potential $V(r) = c_1 r^{-a} + c_2 r^{-b}$, where r_0, c_1 and c_2 are constants of appropriate dimensions, the values of a and b, respectively, are

(a) 2 and 4 (b) 2 and 3
(c) 3 and 4 (d) 1 and 3

34. The fulcrum of a simple pendulum (consisting of a particle of mass m attached to the support by a massless string of length l) oscillates vertically as sin $z(t) = a\sin\ \omega t$, where ω is a constant. The pendulum moves in a vertical plane and $\theta(t)$ denotes its angular position with



respect to the z-axis.

If $\ell\frac{d^2\theta}{dt^2} + \sin\ \theta(g - f(t)) = 0$ (where g is the acceleration due to gravity) describes the equation of motion of the mass, then $f(t)$ is

(a) $a\omega^2\cos\ \omega t$ (b) $a\omega^2\sin\ \omega t$
(c) $-a\omega^2\cos\ \omega t$ (d) $-a\omega^2\sin\ \omega t$

35. A satellite of mass m orbits around earth in an elliptic trajectory of semi-major axis α. At a radial distance $r = r_0$, measured from the centre of the earth, the kinetic energy is equal to half
the magnitude of the total energy. If M denotes the mass of the earth and the total energy is $-\frac{GMm}{2a}$, the value of $\frac{r_0}{a}$ is nearest to

(a) 1.33 (b) 1.48
(c) 1.25 (d) 1.67

36. A particle of mass m in one dimension is in the ground state of a simple harmonic oscillator described by a Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$ in the standard notation. An impulsive force at time $t = 0$ suddenly imparts a momentum $p_0 = \sqrt{\hbar m\omega}$ to it. The probability that the particle remains in the original ground state is

(a) e^{-2} (b) $e^{-3/2}$
(c) e^{-1} (d) $e^{-1/2}$

37. . The energies of a two-state quantum system are E_0 and $E_0 + \alpha\hbar$, (where $\alpha > 0$ is a constant) and the corresponding normalized state vectors are $|0\rangle$ and $|1\rangle$, respectively. At time $t=0$, when the system is in the state $|0\rangle$, the potential is altered by a time independent term V such that $\langle 1|V|0\rangle = \frac{\hbar\alpha}{10}$. The transition probability to the state $|1\rangle$ at times $t << \frac{1}{\alpha}$, is

(a) $\frac{\alpha^2 t^2}{25}$ (b) $\frac{\alpha^2 t^2}{50}$

(c) $\frac{\alpha^2 t^2}{100}$ (d) $\frac{\alpha^2 t^2}{200}$

38. In an elastic scattering process at an energy E, the phase shift $\delta_0 \approx 30^\circ, \delta_1 \approx 10^\circ$, while the other phase shifts are zero. The polar angle at which the differential cross section peaks is closest to

(a) 20° (b) 10°

(c) 0^0 (d) 30°

39. The unnormalized wavefunction of a particle in one dimension in an infinite square well with walls at $x=0$ and $x=a$, is $\psi(x) = x(a-x)$. If $\psi(x)$ is expanded as a linear combination of the energy eigenfunctions, $\int_0^a |\psi(x)|^2 dx$ is proportional to the infinite series (you may use $\int_0^a t\sin\ t dt = -a\cos\ a + \sin\ a$ and $\int_0^a t^2 \sin\ t dt = -2-(a^2-2)(\cos\ a + 2a\sin\ a)$)

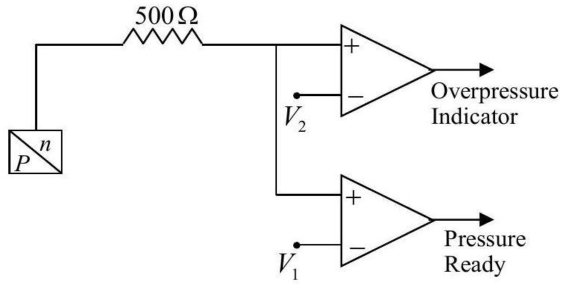
(a) $\sum_{n=1}^{\infty}(2n-1)^{-6}$

(b) $\sum_{n=1}^{\infty}(2n-1)^{-4}$

(c) $\sum_{n=1}^{\infty}(2n-1)^{-2}$

(d) $\sum_{n=1}^{\infty}(2n-1)^{-8}$

40. The pressure of a gas in a vessel needs be maintained between 1.5 bar to 2.5 bar in an experiment. The vessel is fitted with a pressure transducer that generates 4 mA to 20 mA current for pressure in the range 1 bar to 5 bar. The current output of the transducer has a linear dependence on the pressure.



The reference voltages V_1 and V_2 in the comparators in the circuit (shown in figure above) suitable for the desired operating conditions, are, respectively

(a) 2 V and 10 V (b) 2 V and 5 V

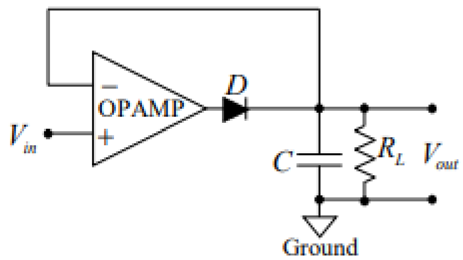
(c) 3 V and 10 V (d) 3 V and 5 V

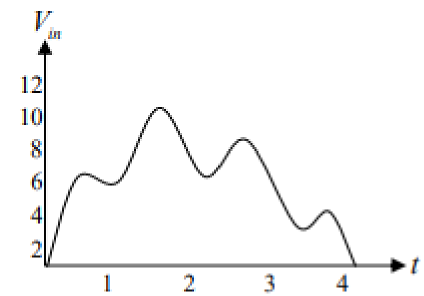
41. The nuclei of ^{137}Cs decay by the emission of β-particles with a half of 30.08 years. The activity (in units of disintegrations per second or Bq) of a 1mg source of ^{137}Cs, prepared on January 1 , 1980 as measured on January 1, 2021 is closest to

(a) 1.79×10^{16} (b) 1.79×10^{9}

(c) 1.24×10^{16} (d) 1.24×10^{9}

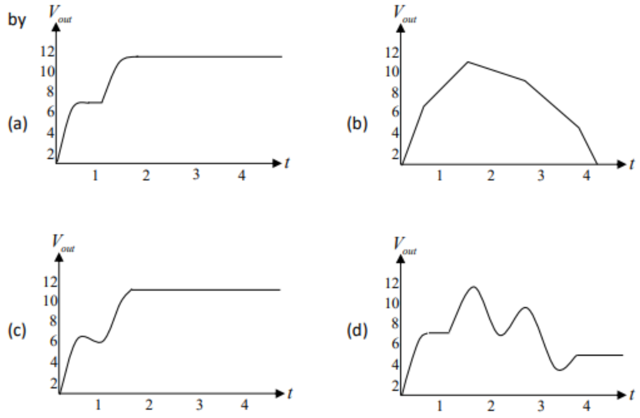
42. In the following circuit the input voltage V_{in} is such that $|V_{\text{in}}| < |V_{\text{out}}|$, where V_{sat} is the saturation voltage of the op-amp. (Assume that the diode is an ideal one and $R_L C$ is much large than the duration of the measurement.)



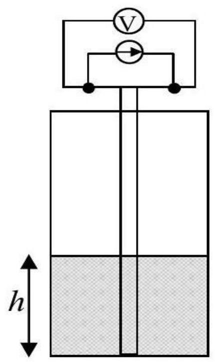


for the input voltage as shown in the figure above, the output voltage V_{out} is best

represented
by



43. To measure the height h of a column of liquid helium in a container, a constant current I is sent through an $NbTi$ wire of length l, as shown in the figure. The normal state resistance of the $NbTi$ wire is R. If the superconducting transition temperature of $NbTi$ is $\approx 10K$ then the measured voltage $V(h)$ is best described by the expression



(a) $IR\left(\frac{1}{2}-\frac{2h}{l}\right)$ (b) $IR\left(1-\frac{h}{l}\right)$

(c) $IR\left(\frac{1}{2}-\frac{h}{l}\right)$ (d) $IR\left(1-\frac{2h}{l}\right)$

44. The energy levels of a non-degenerate quantum system are $\epsilon_n = nE_0$, where E_0 is a constant and $n = 1,2,3, \ldots$. At a temperature T, the free energy F can be expressed in terms of the average energy E by

(a) $E_0 + k_B T \ln\frac{E}{E_0}$ (b) $E_0 + 2k_B T \ln\frac{E}{E_0}$

(c) $E_0 - k_B T \ln\frac{E}{E_0}$ (d) $E_0 - 2k_B T \ln\frac{E}{E_0}$

45. A polymer made up of N monomers, is in thermal equilibrium at temperature T. Each monomer could be of length a or $2a$. The first contributes zero energy, while the second one contributes $\in$. The average length (in units of Na) of the polymer at temperature $T = \frac{\epsilon}{k_B}$ is

(a) $\frac{5+e}{4+e}$ (b) $\frac{4+e}{3+e}$

(c) $\frac{3+e}{2+e}$ (d) $\frac{2+e}{1+e}$

46. Balls of ten different colours labeled by $a = 1,2, \ldots, 10$ are to be distributed among different coloured boxes. A ball can only go in a box of the same colour, and each box can contain at most one ball. Let n_a and N_a denote, respectively the numbers of balls and boxes of colour a. Assuming that $N_a \gg n_a \ggg 1$, the total entropy (in units of the Boltzmann constant) can be best approximated by

(a) $\sum_a \left(N_a \ln N_a + n_a \ln n_a - (N_a - n_a)\ln(N_a - n_a)\right)$

(b) $\sum_a \left(N_a \ln N_a - n_a \ln n_a - (N_a - n_a)\ln(N_a - n_a)\right)$

(c) $\sum_a \left(N_a \ln N_a - n_a \ln n_a + (N_a - n_a)\ln(N_a - n_a)\right)$

(d) $\sum_a \left(N_a \ln N_a + n_a \ln n_a + (N_a - n_a)\ln(N_a - n_a)\right)$

47. The dispersion relation of a gas of non-interacting bosons in d dimensions is $E(k) = ak^s$, where a and S are positive constants. Bose-Einstein condensation will occur for all values of

(a) $d > s$

(b) $d + 2 > s > d - 2$

(c) $s > 2$ independent of d

(d) $d > 2$ independent of S

48. Lead is superconducting below 7 K and has a critical magnetic field 800×10^{-4} tesla close to $0K$. At $2K$ the critical current that flows through a long lead wire of radius

5 mm is closest to

(a) 1760 A (b) 1670 A

(c) 1950 A (d) 1840 A

49. Potassium chloride forms an FCC lattice, in which K and Cl occupy alternating sites. The density of KCl is $1.98\ \mathrm{g/cm^3}$ and the atomic weights of K and Cl are 39.1 and 35.5 , respectively. The angles of incidence (in degrees) for which Bragg peaks will appear when X-ray of wavelength 0.4 nm is shone on a KCl crystal are

(a) 18.5,39.4 and 72.2

(b) 19.5 and 41.9

(c) 12.5,25.7,40.5 and 60.0

(d) 13.5,27.8,44.5 and 69.0

50. In the reaction $p + n \rightarrow p + K^+ + X$, mediated by strong interaction, the baryon number B, strangeness S and third component of isospin I_3 of the particle X are, respectively

(a) $-1, -1$ and -1 (b) $+1, -1$ and -1

(c) $+1, -2$ and $-\frac{1}{2}$ (d) $-1, -1$ and 0

51. A ^{60}Co nucleus β-decays from its ground state with $J^P = 5^+$to a state of ^{60}Ni with $J^P = 4^+$. From the angular momentum selection rules, the allowed values of the orbital angular momentum L and the total spin S of the election-antineutrino pair are

(a) $L = 0$ and $S = 1$

(b) $L = 1$ and $S = 0$

(c) $L = 0$ and $S = 0$

(d) $L = 1$ and $S = 1$

52. The Q-value of the α-decay of ^{232}Th to the ground state of ^{228}Ra is 4082keV. The maximum possible kinetic energy of the α-particle is closest to

(a) 4082keV (b) 4050keV

(c) 4035keV (d) 4012keV

53. The $|3,0,0\rangle$ state (in the standard notation $|n, l, m\rangle$) of the H-atom in the non-relativistic theory decays to the state $|1,0,0\rangle$ via two dipole transitions. The transition route and the corresponding probability are

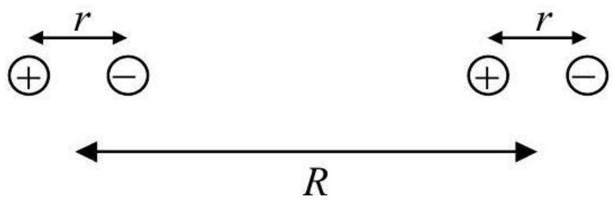
(a) $|3,0,0\rangle \rightarrow |2,1,-1\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{4}$

(b) $|3,0,0\rangle \rightarrow |2,1,1\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{4}$

(c) $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{3}$

(d) $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$ and $\frac{2}{3}$

54. A linear diatomic molecule consists of two identical small electric dipoles with an equilibrium separation R, which is assumed to be a constant. Each dipole has charges $\pm q$ of mass m separated by r when the molecule is at equilibrium. Each dipole can execute simple harmonic motion of angular frequency ω.



Recall that the interaction potential between two dipoles of moments p_1 and p_2, separated by $R_{12} = R_{12}\hat{n}$ is $\left(p_1 \cdot p_2 - 3(p_1 \cdot \hat{n})(p_2 \cdot \hat{n})\right)/(4\pi\epsilon_0 R_{12}^3)$.

Assume that $R \gg r$ and let $\Omega^2 = \frac{q^2}{4\pi\epsilon_0 m R^3}$.

The angular frequencies of small oscillations of the diatomic molecule are

(a) $\sqrt{\omega^2 + \Omega^2}$ and $\sqrt{\omega^2 - \Omega^2}$

(b) $\sqrt{\omega^2 + 3\Omega^2}$ and $\sqrt{\omega^2 - 3\Omega^2}$

(c) $\sqrt{\omega^2 + 4\Omega^2}$ and $\sqrt{\omega^2 - 4\Omega^2}$

(d) $\sqrt{\omega^2 + 2\Omega^2}$ and $\sqrt{\omega^2 - 2\Omega^2}$

55. Diffuse hydrogen gas within a galaxy may be assumed to follow a Maxwell distribution at temperature 10^6 K, while the temperature appropriate for the H gas in the inter-galactic space, following the same distribution, may be taken to be 10^4 K. The ratio of thermal broadening $\Delta v_G / \Delta v_{1G}$ of the Lyman- α line from the H-atoms within the galaxy to that from the intergalactic space is closest to

(a) 100 (b) $\frac{1}{100}$

(c) 10 (d) $\frac{1}{10}$

❖ ANSWER KEY

1. b	2. c	3. a	4. b	5. c
6. b	7. c	8. b	9. b	10. a
11. d	12. d	13. c	14. d	15. a
16. b	17. a	18. c	19. b	20. d
21. b	22. d	23. d	24. d	25. b
26. c	27. b	28. d	29. b	30. c
31. a	32. c	33. b	34. d	35. a
36. d	37. c	38. c	39. a	40. d
41. d	42. a	43. d	44. c	45. d
46. b	47. a	48. d	49. a	50. b
51. a	52. d	53. c	54. c	55. c