D PHYSICS

CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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PHYSICAL SCIENCE

Quantum Mechanics

Previous Year Questions [Topic-Wise)

With Answer Key

CSIR-NET/JRF,GATE,JEST,TIFR

❖ **CSIR NET PYQ**

1. The wavefunction of a particle is given by

$$\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1\right)$$

where ϕ_0 and ϕ_1 are the normalized eigenfunctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is

[CSIR JUNE 2011]

(a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$

(c) $\frac{E_0 - 2E_1}{3}$ (d) $\frac{E_0 + 2E_1}{3}$

2. Consider a particle in a one-dimensional potential that satisfies $V(x) = V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$ be a normalized state with α_0 and α_1 being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $|\psi\rangle$ is given by

[CSIR DEC 2011]

(a) $\alpha_0^2\langle\psi_0|x|\psi_0\rangle + \alpha_1^2\langle\psi_1|x|\psi_1\rangle$

(b) $\alpha_0\alpha_1[\langle\psi_0|x|\psi_1\rangle + \langle\psi_1|x|\psi_0\rangle]$

(c) $\alpha_0^2 + \alpha_1^2$

(d) $2\alpha_0\alpha_1$

3. The wave function of a particle at time $t = 0$ is given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$$

, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues E_1 and E_2 respectively, $(E_2 > E_1)$. The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is

[CSIR DEC 2011]

(a) $\frac{-\hbar\pi}{2(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{E_2 - E_1}$

(c) $\frac{\sqrt{2}\hbar\pi}{E_2 - E_1}$ (d) $\frac{2\hbar\pi}{E_2 - E_1}$

4. A particle in one-dimension is in the potential

$$V(x) = \begin{cases} \infty, & \text{if } x < 0 \\ -V_0 & \text{, if } 0 \le x \le l \\ 0 & \text{, if } x > l \end{cases}$$

If there is at least one bound state, the minimum depth of potential is

[CSIR JUNE 2012]

(a) $\frac{\hbar^2\pi^2}{8ml^2}$ (b) $\frac{\hbar^2\pi^2}{2ml^2}$

(c) $\frac{2\hbar^2\pi^2}{ml^2}$ (d) $\frac{\hbar^2\pi^2}{ml^2}$

5. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}(a^2 - x^2)}{4a^{5/2}}, & \text{for } -a < x < a \\ 0 & \text{, otherwise} \end{cases}$$

the uncertainty Δp in its momentum is

[CSIR DEC 2012]

(a) $2\hbar/5a$ (b) $5\hbar/2a$

(c) $\sqrt{10}\hbar/a$ (d) $\sqrt{5}\hbar/\sqrt{2}a$

6. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential $V(x)$ are -4 and -1 , respectively, (in units in which $\hbar = 1$). If the corresponding wavefunctions are related by $\psi_1(x) = \psi_0(x)\sin h\, x$, then the ground state eigenfunction is

[CSIR DEC 2012]

(a) $\psi_0(x) = \sqrt{\mathrm{se\,c}\,h\, x}$ (b) $\psi_0(x) = \mathrm{se\,c}\,h\, x$

(c) $\psi_0(x) = \mathrm{se\,c}\,h^2\, x$ (d) $\psi_0(x)\mathrm{se\,c}\,h^3\, x$

7. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one-dimensional Hamiltonian with eigen value $E = 0$, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is **[CSIR DEC 2013]**

(a) $12x^2$ (b) $16x^6$

(c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

8. A particle is in the ground state of an infinite square well potential given by,

$$V(x) = \begin{cases} 0 & \text{for } -a \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is **[CSIR DEC 2013]**

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$

(c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

9. A particle of mass m in three dimensions is in the potential

$$V(r) = \begin{cases} 0, & r < a \\ \infty, & r > a \end{cases}$$

Its ground state energy is **[CSIR JUNE 2014]**

(a) $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$

(c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

10. An electron is in the ground state of a hydrogen atom. The probability that it is within th Bohr radius is approximately equal to

[CSIR JUNE 2014]

(a) 0.60 (b) 0.90

(c) 0.16 (d) 0.32

11. A particle in the infinite square well potential

$$V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{otherwise} \end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x) = \begin{cases} A\sin^3\left(\frac{\pi x}{a}\right), & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$

The expectation value of the energy of the particle is `

[CSIR JUNE 2014]

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$

(c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

12. Let ψ_1 and ψ_2 denote the normalized eigenstates of a particle with energy eigenvalues E_1 and E_2 respectively, with $E_2 > E_1$. At time $t = 0$ the particle is prepared in a state

$$\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

The shortest time T at which $\Psi(t = T)$ will be orthogonal to $\Psi(t = 0)$ is

[CSIR DEC 2014]

(a) $\frac{2\hbar\pi}{(E_2-E_1)}$ (b) $\frac{\hbar\pi}{(E_2-E_1)}$

(c) $\frac{\hbar\pi}{2(E_2-E_1)}$ (d) $\frac{\hbar\pi}{4(E_2-E_1)}$

13. A Hermitian operator $\hat{O}$ has two normalized eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2 , respectively. The two states $|u\rangle = \cos\,\theta|1\rangle + \sin\,\theta|2\rangle$ and $|v\rangle = \cos\,\phi|1\rangle + \sin\,\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u \mid v\rangle = 0$. Which of the following are possible values of θ and ϕ?

[CSIR DEC 2015]

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$ (d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

14. The ratio of the energy of the first excited state E_1, to that of the ground state E_0 of a particle in a three-dimensional rectangular box of sides L, L and $L/2$ is **[CSIR JUNE 2015]**

(a) 3: 2 (b) 2: 1

(c) 4: 1 (d) 4: 3

15. The state of a particle of mass m in a one dimensional rigid box in the interval 0 to L is given by the normalized wavefunction

$$\psi(x) = \sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right) + \frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right)$$

If its energy is measured the possible outcomes and the average value of energy are, respectively

[CSIR JUNE 2016]

(a) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{50}\frac{h^2}{mL^2}$

(b) $\frac{h^2}{8mL^2}, \frac{h^2}{2mL^2}$ and $\frac{19}{40}\frac{h^2}{mL^2}$

(c) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{19}{10}\frac{h^2}{mL^2}$

(d) $\frac{h^2}{8mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{200}\frac{h^2}{mL^2}$

16. A particle of mass m moves in one dimension under the influence of the potential $V(x) = -\alpha\delta(x)$, where α is a positive constant. The uncertainty in the product $(\Delta x)(\Delta p)$ in its ground state is

[CSIR JUNE 2016]

(a) $2\hbar$ (b) $\frac{\hbar}{2}$

(c) $\frac{\hbar}{\sqrt{2}}$ (d) $\sqrt{2}\hbar$

17. Consider the two lowest normalized energy eigenfunctions $\psi_0(x)$ and $\psi_1(x)$ of a one-dimensional system. They satisfy $\psi_0(x) = \psi_0^*(x)$ and

$$\psi_1(x) = \alpha \frac{d\psi_0}{dx}$$

where α is a real constant. The expectation value of the momentum operator in the state ψ_1 is

[CSIR DEC 2016]

(a) $-\frac{\hbar}{\alpha^2}$ (b) 0

(c) $\frac{\hbar}{\alpha^2}$ (d) $\frac{2\hbar}{\alpha^2}$

18. A particle in one dimension is in a potential $V(x) = A\delta(x-a)$. Its wavefunction $\psi(x)$ is continuous everywhere. The discontinuity in $\frac{d\psi}{dx}$ at $x = a$ is **[CSIR DEC 2016]**

(a) $\frac{2m}{\hbar^2}A\psi(a)$ (b) $A(\psi(a) - \psi(-a))$

(c) $\frac{\hbar^2}{2m}A$ (d) 0

19. The eigenstates corresponding to eigenvalues E_1 and E_2 of a time-independent Hamiltonian are $|1\rangle$ and $|2\rangle$ respectively. If at $t = 0$, the system is in a state $|\psi(t=0)\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle$ the value of $\langle\psi(t) \mid \psi(t)\rangle$ at time t will be

[CSIR JUNE 2016]

(a) 1

(b) $\frac{(E_1\sin^2\theta + E_2\cos^2\theta)}{\sqrt{E_1^2 + E_2^2}}$

(c) $e^{iE_1t/\hbar}\sin\theta + e^{iE_2/\hbar}\cos\theta$

(d) $e^{-iE_1//\hbar}\sin^2\theta + e^{-iE_2t/\hbar}\cos^2\theta$

20. Consider a potential barrier A of height V_0 and width b, and another potential barrier B of height $2V_0$ and the same width b. The ratio T_A/T_B of tunnelling probabilities T_A and T_B, through barriers A and B respectively, for a particle of energy $V_0/100$ is best approximated by

[CSIR JUNE 2017]

(a) $\exp\left[(\sqrt{1.99} - \sqrt{0.99})\sqrt{8mV_0b^2/\hbar^2}\right]$

(b) $\exp\left[(\sqrt{1.98} - \sqrt{0.98})\sqrt{8mV_0b^2/\hbar^2}\right]$

(c) $\exp\left[(\sqrt{2.99} - \sqrt{0.99})\sqrt{8mV_0b^2/\hbar^2}\right]$

(d) $\exp\left[(\sqrt{2.98} - \sqrt{0.98})\sqrt{8mV_0b^2/\hbar^2}\right]$

21. The two vectors $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} b \\ c \end{pmatrix}$ are orthonormal if

[CSIR JUNE 2017]

(a) $a = \pm 1, b = \pm 1/\sqrt{2}, c = \pm 1/\sqrt{2}$

(b) $a = \pm 1, b = \pm 1, c = 0$

(c) $a = \pm 1, b = 0, c = \pm 1$

(d) $a = \pm 1, b = \pm 1/2, c = 1/2$

22. The normalized wavefunction in the momentum space of a particle in one dimension is

$$\phi(p) = \frac{\alpha}{p^2 + \beta^2}$$

where α and β are real constants. The uncertainty Δx in measuring its position is

[CSIR DEC 2017]

(a) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^2}$ (b) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^3}$

(c) $\frac{\hbar}{\sqrt{2}\beta}$ (d) $\sqrt{\frac{\pi}{\beta}}\frac{\hbar\alpha}{\beta}$

23. At $t = 0$, the wavefunction of an otherwise free particle confined between two infinite walls at $x = 0$ and $x = L$ is

$$\psi(x, t=0) = \sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)$$

Its wave function at a later time $t = \frac{mL^2}{4\pi h}$ is

[CSIR JUNE 2018]

(a) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)e^{i\pi/6}$

(b) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}+\sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

(c) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}+\sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$

(d) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}+\sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

24. A particle of mass m is confined in a three-dimensional box by the potential

$$V(x,y,z)=\begin{cases}0, & 0\le x,y,z\le a\\ \infty, & \text{otherwise}\end{cases}$$

The number of eigenstates of Hamiltonian with energy $\frac{9\hbar^2\pi^2}{2ma^2}$ is **[CSIR JUNE 2018]**

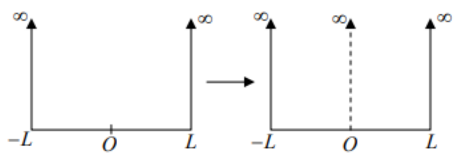
(a) 1
(b) 6
(c) 3
(d) 4

25. The energy eigenvalues of a particle of mass m, confined to a rigid one-dimensional box of width L, are $E_n(n=1,2,\ldots)$. If the walls of the box are moved very slowly toward each other, the rate of change of time-dependent energy $\frac{dE_2}{dt}$ of the first excited state is

[CSIR DEC 2019]

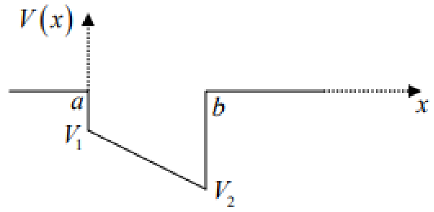
(a) $\frac{E_2}{L}\frac{dL}{dt}$
(b) $\frac{2E_2}{L}\frac{dL}{dt}$
(c) $-\frac{2E_2}{L}\frac{dL}{dt}$
(d) $-\frac{E_1}{L}\frac{dL}{dt}$

26. A quantum particle of mass m in one dimension, confined to a rigid box as shown in the figure, is in its ground state. An infinitesimally thin wall is very slowly raised to infinity at the centre of the box, in such a way that the system remains in its ground state at all times. Assuming that no energy is lost in raising the wall, the work done on the system when the wall is fully raised, eventually separating the original box into two compartments, I **[CSIR JUNE 2019]**



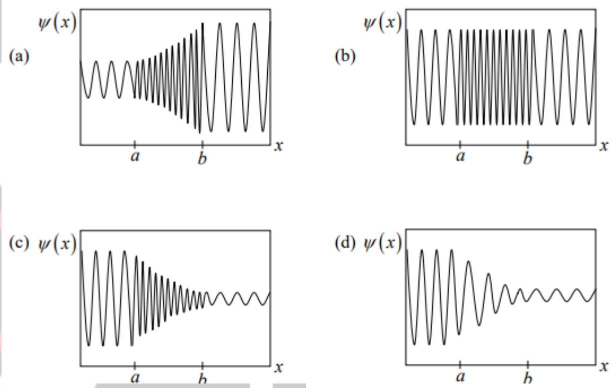
(a) $\frac{3\pi^2\hbar^2}{8mL^2}$
(b) $\frac{\pi^2\hbar^2}{8mL^2}$
(c) $\frac{\pi^2\hbar^2}{2mL^2}$
(d) 0

27. A particle of mass in and energy $E>0$. in one dimension is scattered by the potential



If the particle was moving from $x=-\infty$ to $x=\infty$, which of the following graphs gives the best qualitative representation of the wavefunction of this particle?

[CSIR JUNE 2019]



28. Let the normalized eigenstates of the Hamiltonian,

$$H=\begin{pmatrix}2&1&0\\1&2&0\\0&0&2\end{pmatrix}$$

be $|\psi_1\rangle,|\psi_2\rangle$ and $|\psi_3\rangle$. The expectation value $\langle H\rangle$ and the variance of H in the state

$$|\psi\rangle=\frac{1}{\sqrt{3}}(|\psi_1\rangle+|\psi_2\rangle-i|\psi_3\rangle)\text{are}$$

[CSIR DEC 2019]

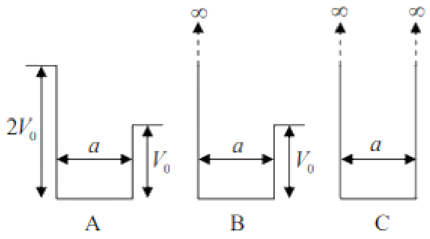
(a) $\frac{4}{3}$ and $\frac{1}{3}$
(b) $\frac{4}{3}$ and $\frac{2}{3}$
(c) 2 and $\frac{2}{3}$
(d) 2 and 1

29. The wavefunction of a free particle of mass m, constrained to move in the interval $-L\le x\le L$, is $\psi(x)=A(L+x)(L-x)$, where A is the normalization constant. The probability that the particle will be found to have the energy $\frac{\pi^2\hbar^2}{2mL^2}$ is

[CSIR JUNE 2019]

(a) 0
(b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{2\sqrt{3}}$
(d) $\frac{1}{\pi}$

30. For the one dimensional potential wells A, B and C, as shown in the figure, let E_A, E_B and E_C denote the ground sate energies of a particle, respectively.



The correct ordering of the energies is

[CSIR NOV 2020]

(a) $E_C > E_B > E_A$ (b) $E_A > E_B > E_C$

(c) $E_B > E_C > E_A$ (d) $E_B > E_A > E_C$

31. The state of an electron in a hydrogen atom is

$$|\psi\rangle = \frac{1}{\sqrt{6}}|1,0,0\rangle + \frac{1}{\sqrt{3}}|2,1,0\rangle + \frac{1}{\sqrt{2}}|3,1,-1\rangle$$

where $|n, l, m\rangle$ denotes common eigenstates of $\hat{H}, \hat{L}^2$ and $\hat{L}_z$ operators in the standard notation.In a measurement of $\hat{L}_z$ for the electron in this state, the result is recorded to be 0 . Subsequently a measurement of energy is performed. The probability that the result is E_2 (the energy of the $n = 2$ state) is

[CSIR NOV 2020]

(a) 1 (b) 1/2

(c) 2/3 (d) 1/3

32. Let the normalized eigenstates of the Hamiltonian $H = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ be $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$. The expectation value $\langle H\rangle$ and the variance of H in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|\psi_1\rangle + |\psi_2\rangle - i|\psi_3\rangle)$$

are

[CSIR DEC2021]

(a) $\frac{4}{3}$ and $\frac{1}{3}$ (b) $\frac{4}{3}$ and $\frac{2}{3}$

(c) 2 and $\frac{2}{3}$ (d) 2 and 1

33. The momentum space representation of the Schrodinger equation of a particle in a potential $V(\vec{r})$ is

$$\left(|\vec{p}|^2 + \beta(\nabla_p^2)^2\right)\psi(\vec{p}, t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{p}, t)$$

where $(\nabla_p)_i = \dfrac{\partial}{\partial p_i}$

, and β is a constant. The potential is (in the following V_0 and a are constants)

[CSIR SEP 2022]

(a) $V_0 e^{-r^2/a^2}$ (b) $V_0 e^{-r^4/a^4}$

(c) $V_0\left(\frac{r}{a}\right)^2$ (d) $V_0\left(\frac{r}{a}\right)^4$

34. If the expectation value of the momentum of a particle in one dimension is zero, then its (box-formalizable) wave function may be of the form

[CSIR SEPT 2022]

(a) $\sin\, kx$ (b) $e^{ikx}\sin\, kx$

(c) $e^{ikx}\cos\, kx$ (d) $\sin\, kx + e^{ikx}\cos\, kx$

35. The energy/energies E of the bound state(s) of a particle of mass m in one dimension in the potential $V(x) = \begin{cases} \infty, & x \leq 0 \\ -V_0, & 0 < x < a \\ 0, & x \geq a \end{cases}$ (where $V_0 >$ 0) is/are determined by

[CSIR SEPT 2022]

(a) $\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = \frac{E - V_0}{E}$

(b) $\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = -\frac{E}{E + V_0}$

(c) $\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = -\frac{E}{E + V_0}$

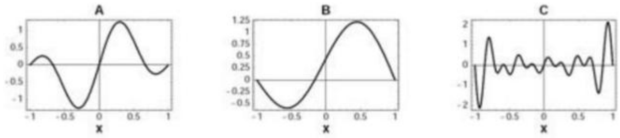
(d) $\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = \frac{E - V_0}{E}$

36. A particle of mass m is in a one dimensional infinite potential well of length L, extending from $x = 0$ to $x = L$. When it is in the energy Eigen-state labelled by n, $(n = 1,2,3,..)$*the* probability of finding in the interval $0 \leq x \leq L/8$ is 1/8. The minimum value of n for which this is possible is

[CSIR FEB 2022]

(a) 4 (b) 2 (c) 6 (d) 8

37. The figures below depict three different wave functions of a particle confined to a one dimensional box $-1 \le x \le 1$



The wave functions that correspond to the maximum expectation values $|\langle x\rangle|$ (absolute value of the mean position) and $\langle x^2\rangle$, respectively, are

[CSIR FEB 2022]

(a) B and C (b) B and A

(c) C and B (d) A and B

38. The unnormalized wave function of a particle in one dimension in an infinite square well with walls at $x = 0$ and $x = a$, is $\psi(x) = x(a - x)$. If $\psi(x)$ is expanded as a linear combination of the energy eigenfunctions, $\int_0^a |\psi(x)|^2 dx$ is proportional to the infinite series (You may use $\int_0^a t\sin\, tdt = -a\cos\, a + \sin\, a$ and $\int_0^a t^2 \sin\, tdt = -2 - (a^2 - 2)\cos\, a + 2a\sin\, a$

[CSIR FEB 2022]

(a) $\sum_{n=1}^{\infty}(2n-1)^{-6}$ (b) $\sum_{n=1}^{\infty}(2n-1)^{-4}$

(c) $\sum_{n=1}^{\infty}(2n-1)^{-2}$ (d) $\sum_{n=1}^{\infty}(2n-1)^{-8}$

39. A quantum particle of mass m is moving in a one dimensional potential

$$V(x) = V_0\theta(x) - \lambda\delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E \gg V_0 > \lambda$ and $\sqrt{2mE} \gg \frac{V_0 h}{\lambda}$ is

[CSIR DEC 2023]

(a) $\frac{V_0^2}{4E^2}$ (b) $\frac{V_0^2}{8E^2}$ (c) $\frac{m\lambda^2}{2E\hbar^2}$ (d) $\frac{m\lambda^2}{4E\hbar^2}$

40. A particle of mass m is in the third energy eigenstate of an infinite potential well of width a. The time interval in which the phase of this wave function changes by 2π is

[CSIR JUNE 2025]

(a) $\frac{4ma^2}{3\pi\hbar}$ (b) $\frac{4ma^2}{9\pi\hbar}$

(c) $\frac{8ma^2}{3\pi\hbar}$ (d) $\frac{8ma^2}{9\pi\hbar}$

41. The probability density of a free particle of mass m at time $t = 0$, is given by

$$A\exp\left(-\frac{x^2}{2\sigma^2(0)}\right)$$

At $t > 0$, its probability density is proportional to $\exp\left(-\frac{x^2}{2\sigma^2(t)}\right)$, where $\sigma^2(t)$ is

[CSIR JUNE 2025]

(a) $\sigma^2(0) + \frac{\hbar^2 t^2}{\sigma^2(0)m^2}$

(b) $\sigma^2(0) + \frac{\hbar^2 t^2}{4\sigma^2(0)m^2}$

(c) $\sigma^2(0) + \frac{4\hbar^2 t^2}{\sigma^2(0)m^2}$

(d) $\sigma^2(0) + \frac{2\hbar^2 t^2}{\sigma^2(0)m^2}$

42. A particle of mass m is in a cubic box of side a. The potential inside the box $(0 \le x \le a, 0 \le y \le a, 0, \le z \le a\ )$ is zero and infinite outside. If the particle is in an energy eigenstate with $E = \frac{7\pi^2\hbar^2}{ma^2}$, a possible wavefunction is

[CSIR DEC 2024]

(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)\sin\left(\frac{2\pi z}{a}\right)$

(b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{3\pi y}{a}\right)\sin\left(\frac{\pi z}{a}\right)$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)\sin\left(\frac{3\pi z}{a}\right)$

(d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)\sin\left(\frac{2\pi z}{a}\right)$

Answer Key				
1. d	2. b	3. a	4. a	5. d
6. c	7. d	8. b	9. a	10. d
11. c	12. b	13. a	14. a	15. a
16. c	17. b	18. b	19. a	20. a
21. c	22. c	23. c	24. c	25. c
26. a	27. c	28. c	29. a	30. a
31. c	32. c	33. d	34. a	35. c
36. a	37. a	38. b	39. c	40. b
41.	42. c	43.	44.	45.

❖ **GATE PYQ**

1. Which of the following functions represents an acceptable wavefunction of the particle in the range $-\infty \leq x \leq \infty$?

[GATE 2001]

(a) $\psi(x) = A\tan x,\ A > 0$

(b) $\psi(x) = B\cos x; B$ is real

(c) $\psi(x) = C\exp\left(-\frac{D}{x^2}\right), C > 0, D < 0$

(d) $\psi(x) = Ex\exp(-Fx^2); E, F > 0$

2. A quantum particle of mass m is confined to a square region in xoy-plane whose vertices are given by (0,0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for l, m, n positive integers)?

[GATE 2001]

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$

(b) $\frac{2}{L}\cos\left(\frac{l\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$

(d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{l\pi y}{L}\right)$

3. If the wave function of a particle trapped in space between $x = 0$ and $x = L$ is given by

$$\psi(x) = A\sin\left(\frac{2\pi x}{L}\right)$$

, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?

[GATE 2002]

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$

(c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

4. The normalized wave functions ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$.The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is

[GATE 2003]

(a) -0.32 (b) 0.0

(c) 0.75 (d) 0.96

5. The normalized wave functions ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$.Which of the following are eigenfunctions of $\hat{A}^2$?

[GATE 2003]

(a) ψ_1 and ψ_2

(b) ψ_2 and not ψ_1

(c) ψ_1 and not ψ_2

(d) neither ψ_1 nor ψ_2

6. A particle is located in a three dimensional cubic well of width L with impenetrable walls.

The sum of the energies of the third and the fourth levels is

[GATE 2003]

(a) $10\pi^2\hbar^2/mL^2$ (b) $10\pi^2\hbar^2/3mL^2$

(c) $11\pi^2\hbar^2/2mL^2$ (d) $15\pi^2\hbar^2/2mL^2$

7. A particle is located in a three dimensional cubic well of width L with impenetrable walls.The degeneracy of the fourth level is given by

[GATE 2003]

(a) 1 (b) 2

(c) 3 (d) 4

8. The wave function of a spin-less particle of mass m in a one-dimensional potential $V(x)$ is $\psi(x) = A\exp(-\alpha^2x^2)$ corresponding to an eigenvalue $E_0 = \hbar^2\alpha^2/m$. The potential $V(x)$ is

[GATE 2004]

(a) $2E_0(1 - \alpha^2x^2)$ (b) $2E_0(1 + \alpha^2x^2)$

(c) $2E_0\alpha^2x^2$ (d) $2E_0(1 + 2\alpha^2x^2$

9. A particle is confined to the region $0 < x < L$ in one dimension

If the particle is in the first excited state, then the probability of finding the particle is maximum at

[GATE 2004]

(a) $x = \frac{L}{6}$ (b) $x = \frac{L}{2}$

(c) $x = \frac{L}{3}$ (d) $x = \frac{L}{4}$ and $\frac{3L}{4}$

10. A particle is confined to the region $0 < x < L$ in one dimension

If the particle is in the lowest energy state, then the probability of finding the particle in the region $0 < x < \frac{L}{4}$ is

[GATE 2004]

(a) $\frac{1}{4} - \frac{1}{(2\pi)}$ (b) $\frac{1}{4}$

(c) $\frac{1}{4} + \frac{1}{(2\pi)}$ (d) $\frac{1}{2}$

11. A free particle is moving in $+x$-direction with a linear momentum p. The wavefunction of the particle normalized in a length L is

[GATE 2006]

(a) $\frac{1}{\sqrt{L}} \sin \frac{p}{\hbar} x$ (b) $\frac{1}{\sqrt{L}} \cos \frac{p}{\hbar} x$

(c) $\frac{1}{\sqrt{L}} e^{-i\frac{p}{\hbar}x}$ (d) $\frac{1}{\sqrt{L}} e^{i\frac{p}{\hbar}x}$

12. The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t = 0) = \frac{1}{\sqrt{5}}[2\psi_0(x) - \psi_1(x)],$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time t is

[GATE 2006]

(a) $\frac{1}{\sqrt{5}} e^{\frac{-i(E_0 E_1)t}{2\hbar}}[2\psi_0(x) - \psi_1(x)]$

(b) $\frac{1}{\sqrt{5}} e^{\frac{-iE_0 t}{\hbar}}[2\psi_0(x) - \psi_1(x)]$

(c) $\frac{1}{\sqrt{5}} e^{\frac{-iE_1 t}{\hbar}}[2\psi_0(x) - \psi_1(x)]$

(d) $\frac{1}{\sqrt{5}}\left[2\psi_0(x) e^{\frac{-E_0 t}{\hbar}} - \psi_1(x) e^{\frac{-E_1 t}{\hbar}}\right]$

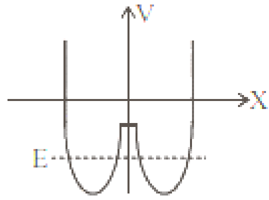
13. The wavefunction of a particle, moving in a one-dimensional time-independent potential $V(x)$, is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form.

[GATE 2007]

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$

(c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

14. A particle with energy E is in a time-independent double well potential as shown in the figure.



Which of the following statements about the particle is NOT correct?

[GATE 2007]

(a) The particle will always be in a bound state

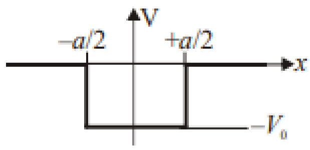
(b) The probability of finding the particle in one well will be time-dependent

(c) The particle will be confined to any one of the wells

(d) The particle can tunnel from one well to the other, and back

15. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies

[GATE 2007]



(a) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$

(b) $\frac{\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$

(d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

16. A particle is placed in a one dimensional box size L along the x-axis $(0 < x < \mathrm{L})$. Which of the following is true?

[GATE 2008]

(a) In the ground state, the probability of finding the particle in the interval (L/4, L/4) is half.

(b) In the first excited state, the probability of finding the particle in the interval (L/4,3 L/4) is half. This also holds for states with $\mathrm{n} = 4,6,8, \ldots \ldots \ldots$

(c) For an arbitrary state $|\Psi\rangle$, the probability of finding the particle in the left half of the well is half.

(d) None

17. A particle is placed in a one-dimensional box size L along the x-axis $(0 < x < \mathrm{L})$. Which of the following is true? Throughout $0 < x < L$, the wave function

[GATE 2008]

(b) is exponential decaying

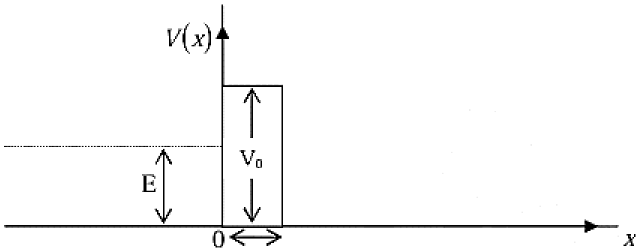
(a) can be chosen to be real

(c) is generally complex

(d) is zero

Common Data Questions

Common Data for Questions 18, 19 and 20: A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure $(E < V_0)$.



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_R and A_T denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

18. Throughout $0 < x < L$, the wave function

[GATE 2008]

(a) can be chosen to be real

(b) is exponential decaying

(c) is generally complex

(d) is zero

19. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient then

[GATE 2008]

(a) the probability current vanishes in the classically forbidden region

(b) the probability current is $\mathrm{T}S_0$ for $x > L$

(c) for, $x < 0$ the probability current is $S_0(1 + R)$

(d) for $x > L$, the probability current is complex

20. The ratio of the reflected to the incident amplitude $\mathrm{A_R}/\mathrm{A_0}$ is

[GATE 2008]

(a) $I - A_T/A_0$

(b) $\sqrt{(1 - T)}$ in magnitude

(c) a real negative number

(d) $\sqrt{\left(1 - \left|\frac{A_y}{A_0}\right|^2\right)\frac{E}{V_0 - E}}$

21. For a physical system, two observables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below:

[GATE 2008]

(a) every eigen state of O_1 must necessarily be an eigen state of O_2

(b) every non-degenerate eigen state of O_1 must necessarily be an eigen state of O_2

(c) when an observation of O_1 is carried out on an arbitrary state $|\Psi\rangle$ of the physical system a subsequent observation of O_2 leads to an unambiguous result

(d) observation of O_1 and O_2 carried out on an arbitrary state $|\Psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observation are made

22. The De-Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimensions is given by q

[GATE 2009]

(a) $\lambda = \dfrac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \dfrac{h}{\sqrt{3mk_BT}}$

(c) $\lambda = \dfrac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \dfrac{h}{\sqrt{3m}}$

23. A particle is in normalized state $|\psi\rangle$ which is a superposition of the energy eigenstates $|E_0 = 10\text{eV}\rangle$ and $|E_1 = 30\text{eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20eV. The state $|\psi\rangle$ is given by

[GATE 2009]

(a) $\dfrac{1}{2}|E_0 = 10\text{eV}\rangle + \dfrac{\sqrt{3}}{4}|E_1 = 30\text{eV}\rangle$

(b) $\dfrac{1}{\sqrt{3}}|E_0 = 10\text{eV}\rangle + \sqrt{\dfrac{2}{3}}|E_1 = 30\text{eV}\rangle$

(c) $\dfrac{1}{2}|E_0 = 10\text{eV}\rangle - \dfrac{\sqrt{3}}{4}|E_1 = 30\text{eV}\rangle$

(d) $\dfrac{1}{\sqrt{2}}|E_0 = 10\text{eV}\rangle - \dfrac{1}{\sqrt{2}}|E_1 = 30\text{eV}\rangle$

24. The monochromatic waves having frequencies ω and $\omega + \Delta\omega (\Delta\omega \ll \omega)$ and corresponding wave lengths λ and $\lambda - \Delta\lambda (\Delta\lambda << \lambda)$ of same polarization traveling along x axis are superimposed on each other. The phase velocity and group velocity of the resultant of the resultant wave are respectively given by

[GATE 2009]

(a) $\dfrac{\omega\lambda}{2\pi}, \dfrac{\Delta\omega\lambda^2}{2\pi\Delta\lambda}$ (b) $\omega\lambda, \dfrac{\Delta\omega\lambda^2}{\Delta\lambda}$

(c) $\dfrac{\omega\Delta\lambda}{2\pi}, \dfrac{\Delta\omega\Delta\lambda}{2\pi}$ (d) $\omega\Delta\lambda, \omega\Delta\lambda$

25. Which one of the function given below represents the bound state eigen function of the operator $-\dfrac{d^2}{dx^2}$ in the region, $0 \le x \le \infty$, with the eigen value -4 ?

[GATE 2009]

(a) $A_0 e^{2x}$ (b) $A_0 \cosh\ 2x$

(c) $A_0 e^{-2x}$ (d) $A_0 \sinh\ 2x$

26. Which of the following is an allowed wave function for a particle in a bound state? N is a constant and $\alpha, \beta > 0$

[GATE 2010]

(a) $\Psi = N\dfrac{e^{-ar}}{r^3}$

(b) $\Psi = N(1 - e^{-\alpha r})$

(c) $\Psi = Ne^{-\alpha x}e^{-\beta(x^2+y^2+z^2)}$

(d) $\Psi = \begin{cases} \text{non-zero constant}, & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

27. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by **[GATE 2011]**

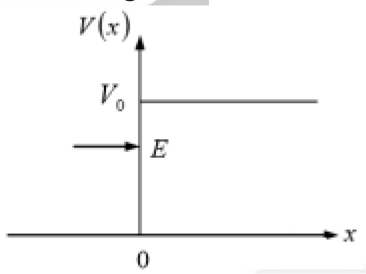
(a) $i\hbar\dfrac{d}{dx}$ (b) $-i\hbar\dfrac{d}{dx}$

(c) $i\hbar\dfrac{\partial}{\partial t}$ (d) $-\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}$

28. An electron with energy E is incident from left on a potential barrier given by

$$V(x) = \begin{cases} 0, & \text{for } x < 0 \\ V_0, & \text{for } x > 0 \end{cases}$$

as shown in the figure



For $E < V_0$, the space part of the wave function for $x > 0$ is of the form

[GATE 2011]

(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$

(c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

29. In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by

$$\psi_1 = \varphi_0 - 2\varphi_1 + 3\varphi_2$$
$$\psi_2 = \varphi_0 - \varphi_1 + \alpha\varphi_2$$

where α is a constant? **[GATE 2011]**

The value of α for which ψ_2 is orthogonal to ψ_1 is

(a) 2 (b) 1

(c) -1 (d) -2

30. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by$V(x,y) = 0$ for $-a < x < a$ and $-a < y < a = \infty$ elsewhere The energy of the first excited state for this particle is given by

[GATE 2012]

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{5\pi^2\hbar^2}{8ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

31. A proton is confined to a cubic box, whose sides have length 10^{-12} m. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27} kg and Planck's constant is 6.63×10^{-34}Js

[GATE 2012]

(a) 1.1×10^{-17} J (b) 3.3×10^{-17} J

(c) 9.9×10^{-17} J (d) 6.6×10^{-17} J

32. Consider the wavefunction $A\left(\frac{r_0}{r}\right)e^{ikr}$ where A is normalization canst. for $r = 2r_0$. The magnitude of probability current density up to two decimal places in unit of $A^2\frac{\hbar k}{m}$ is, ...

[GATE 2013]

33. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

[GATE 2014]

(a) 1: 1 (b) $1:\sqrt{3}$

(c) 1: 3 (d) 3: 2

34. The dispersion relation for phonons in a one dimensional monatomic Bravias lattice with lattice spacing a and consisting of ions of masses M is given by,

$$\omega(k) = \sqrt{\frac{2C}{M}[1-\cos(ka)}]$$

, where ω is the frequency of oscillation, k is the wave vector and C is the spring constant. For the long wavelength modes ($\lambda >> a$), the ratio of the phase velocity to the group velocity is

[GATE 2015]

(a)1: 1 (b) 2;2

(c) 3:5 (d) 7:6

35. A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0K$. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is___________.

[GATE 2016]

36. The state of a system is given by

$$|\Psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$$

Where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is____ (Give your answer upto two decimal places)

[GATE 2016]

37. A particle of mass m and energy E, moving in the positive x direction, is incident on a step , where $x_0 > 0$, the probability of finding the electron is $\frac{1}{e}$ times the probability of finding it at. $x = 0$. If

$$\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

, the value of x_0 is

[GATE 2016]

(a) $\frac{2}{\alpha}$ (b) $\frac{1}{\alpha}$

(c) $\frac{1}{2\alpha}$ (d) $\frac{1}{4\alpha}$

38. A free electron of energy 1eV is incident upon a one dimensional finite potential step of height 0.75eV. The probability of its reflection from the barrier is (up to two decimal places).

[GATE 2017]

39. Consider a one-dimensional potential well of width 3 nm. Using the uncertainty principle ($\Delta x \cdot \Delta p \geq \hbar/2$) an estimate of the minimum depth of the well such that it has at least one bound state for an electron is ($m_e = 9.31 \times 10^{-31}$ kg, $h = 6.626 \times 10^{-34} Js, e = 1.602 \times 10^{-19}$C)

[GATE 2017]

(a) 1μeV (b)1 MeV

(c) leV (d) 2 MeV

40. A two-state quantum system has energy eigenvalues $\pm\in$ corresponding to the normalized states $|\psi_\pm\rangle$. At time $t = 0$, the system is in quantum state

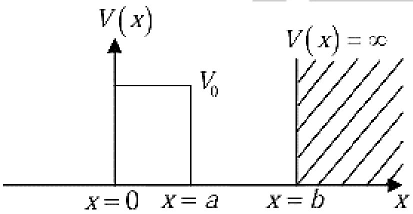
$$\frac{1}{\sqrt{2}}[|\psi_+\rangle + |\psi_-\rangle]$$

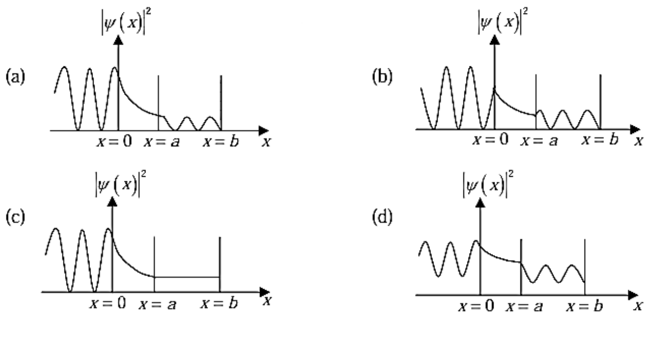
The probability that the system will be in the same state at $t = h/(6\in)$ is (up to two decimal places).

[GATE 2018]

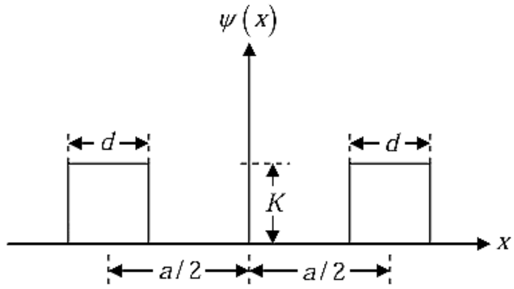
41. Consider a potential barrier $v(x)$ as in fig. where v_0 is a const for particles of energy $E < V_0$ incident on this barrier from the left. Which of the following schematic diagrams best represents the probability density $|\psi(x)|^2$ as a function of x ?

[GATE 2019]





42. The wave function $\psi(x)$ of a particle is as shown below



Here K is a constant, and $a > d$. The position uncertainty (Δx) of the particle is

[GATE 2019]

(a) $\sqrt{\dfrac{a^2 + 3d^2}{12}}$ (b) $\sqrt{\dfrac{3a^2 + d^2}{12}}$

(c) $\sqrt{\dfrac{d^2}{6}}$ (d) $\sqrt{\dfrac{d^2}{24}}$

43. A quantum particle is subjected to the potential

$$V(x) = \begin{cases} \infty, & x \leq -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \geq \frac{a}{2} \end{cases}$$

The ground state wave function of the particle is proportional to **[GATE 2020]**

(a)sin $\left(\frac{\pi x}{2a}\right)$ (b) sin $\left(\frac{\pi x}{a}\right)$

(c)cos $\left(\frac{\pi x}{2a}\right)$ (d) cos $\left(\frac{\pi x}{a}\right)$

44. A free particle of mass M is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth excited state of the particle is

[GATE 2020]

45. The wave function of a particle in a one-dimensional infinite well of size $2a$ at a certain time is

$$\psi(x) = \frac{1}{\sqrt{6a}}\left[\sqrt{2}\sin\left(\frac{\pi x}{a}\right) + \sqrt{3}\cos\left(\frac{\pi x}{2a}\right) + \cos\left(\frac{3\pi x}{2a}\right)\right]$$

. Probability of finding the particle in $n = 2$ state at that time is % (Round off to the nearest integer)

[GATE 2022]

46. A particle of mass m is moving inside a hollow spherical shell of radius a so that the potential is

$$V(r) = \begin{cases} 0 \text{ for } r < a \\ \infty \text{ for } r \geq a \end{cases}$$

The ground state energy and wave function of the particle are E_0 and $R(r)$, respectively. Then which of the following options are correct? **[GATE 2022]**

(a) $E_0 = \dfrac{\hbar^2 \pi}{2ma^2}$

(b) $-\dfrac{\hbar^2}{2m}\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 \dfrac{dR}{dr}\right) = E_0 R(r < a)$

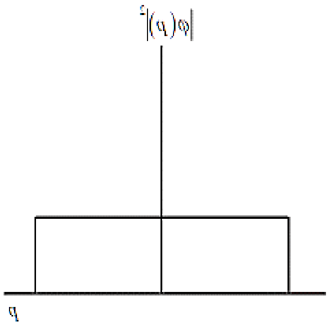
(c) $-\dfrac{\hbar^2}{2m}\dfrac{1}{r^2}\dfrac{d^2R}{dr^2} = E_0 R\ (r < a)$

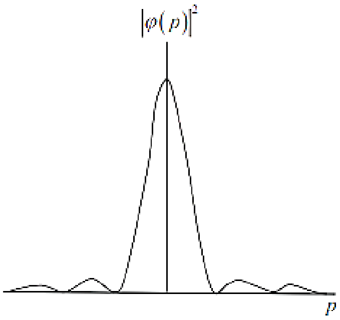
(d) $R(r) = \dfrac{1}{r}\sin\left(\dfrac{\pi r}{a}\right)\ (r < a)$

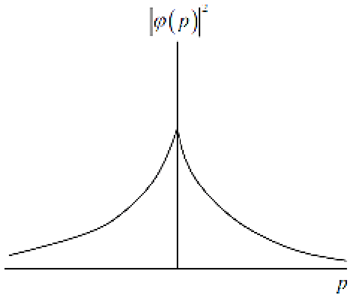
47. The wavefunction of a particle in one dimension is given by

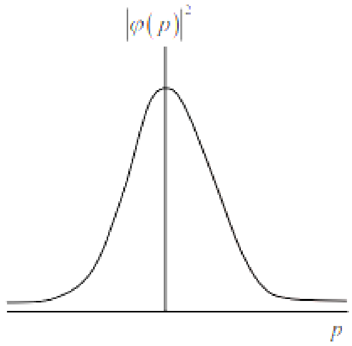
$$\psi(x) = \begin{cases} M, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

Here M and a are positive constants. If $\varphi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\varphi(p)|^2$? **[GATE 2023]**









48. Consider a particle in a two-dimensional infinite square well potential of side L, with $0 \leq x \leq L$ and $0 \leq y \leq L$. The wavefunction of the particle is zero only along the line $y = \frac{L}{2}$, apart from the boundaries of the well. If the energy of the particle in this state is E, what is the energy of the ground state?

[GATE 2023]

(a) $\dfrac{1}{4}E$ (b) $\dfrac{2}{5}E$

(c) $\dfrac{3}{8}E$ (d) $\dfrac{1}{2}E$

49. The wavefunction of a particle in an infinite one-dimensional potential well at time t is

$$\Psi(x,t) = \sqrt{\frac{2}{3}}e^{-iE_1t/\hbar}\psi_1(x) + \frac{1}{\sqrt{6}}e^{i\pi/6}e^{-iE_2t/\hbar}\psi_2(x) + \frac{1}{\sqrt{6}}e^{i\pi/4}e^{-iE_3t/\hbar}\psi_3(x)$$

where ψ_1, ψ_2 and ψ_3 are the normalized ground state, the normalized first excited state and the normalized second excited state, respectively. E_1, E_2 and E_3 are the eigen-energies corresponding to ψ_1, ψ_2 and ψ_3, respectively. The expectation value of energy of the particle in state $\Psi(x,t)$ is

[GATE 2024]

(a) $\dfrac{17}{6}E_1$ (b) $\dfrac{2}{3}E_1$

(c) $\frac{3}{2}E_1$ (d) $14E_1$

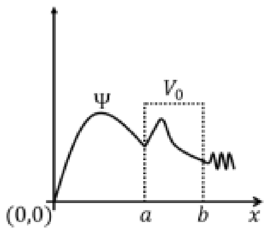
50. A particle is subjected to a potential

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ V_0, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

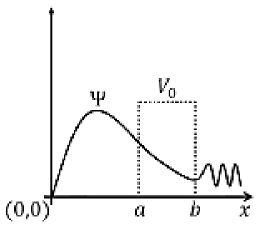
Here, $a > 0$ and $b > a$. If the energy of the particle $E < V_0$, which one of the following schematics is a valid quantum mechanical wavefunction (Ψ) for the system?

[GATE 2024]

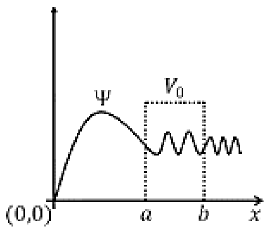
(A)



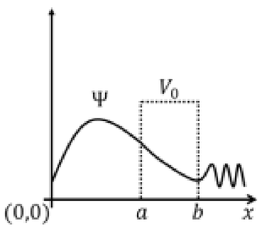
(B)



(C)



(D)



51. The wavefunction for a particle is given by the form $e^{-(i\alpha x+\beta)}$, where α and β are real constants. In which one of the following potentials $V(x)$, the particle is moving?

[GATE 2024]

(a) $V(x) \propto \alpha^2 x^2$ (b) $V(x) \propto e^{-\alpha x}$

(c) $V(x) = 0$ (d) $V(x) \propto \sin(\alpha x)$

52. An electron in the Coulomb field of a proton is in the following state of coherent superposition of orthonormal states ψ_{nlm}

$$\Psi = \frac{1}{3}\psi_{100} + \frac{1}{\sqrt{3}}\psi_{210} - \frac{\sqrt{5}}{3}\psi_{320}$$

Let E_1, E_2, and E_3 represent the first three energy levels of the system. A sequence of measurements is done on the same system at different times. Energy is measured first at time t_1 and the outcome is E_2. Then total angular momentum is measured at time $t_2 > t_1$ and finally energy is measured again at $t_3 > t_2$. The probability of finding the system in a state with energy E_2 after the final measurement is $P/9$. The value of P is (in integer).

[GATE 2024]

53. The wavefunction of the particle is given by. $\psi = Ae^{ikx} + Be^{-ikx}$. Find current density.

[GATE]

(a) zero (b) $\frac{\hbar k}{m}(|A|^2 + |B|^2)$

(c) $\frac{\hbar k}{m}|A||B|$ (d) $\frac{\hbar k}{m}(|A|^2 - |B|^2)$

54. A particle of mass m is represented by the wavefunction $\psi = Ae^{ikx}$ where k is wave vector and A is constant. The magnitude of probability of current density of a particles

[GATE]

(a)$|A|^2\frac{\hbar k}{m}$ (b) $|A|^2\frac{\hbar k}{2m}$

(c)$|A|^2\frac{(\hbar k)^2}{m}$ (d) $|A|^2\frac{(\hbar k)^2}{2m}$

55. The wavefunction of a particle in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$. The energy of the particle is,

[GATE]

(a) $\frac{5\hbar^2 k^2}{2m}$ (b) $\frac{3\hbar^2 k^2}{4m}$

(c) $\frac{\hbar^2 k^2}{2m}$ (d) $\frac{\hbar^2 k^2}{m}$

56. The prob. current density for the real part of the wavefunction is,

[GATE]

(a) 1

(b) $\frac{\hbar k}{m}$

(c) $\frac{\hbar k}{2m}$

(d) *zero*

57. Q. A beam of monoenergetic particle having speed v is described by the wavefunction $\psi(x) = u(x)e^{ikx}$ where $u(x)$ is a real function. This corresponds to a current density,

[GATE]

(a) $u^2(x) \cdot v$

(b) v

(c) zero

(d) $u^2(x)$

58. A particle of mass m in one D is in a state described by

[GATE]

$$\psi(x,t) = Ae^{\frac{ipx-iEt}{\hbar}} + Be^{-\frac{ipx-iEt}{\hbar}}$$

(a) $\frac{p}{m}$

(b) $(|A| - |B|)\frac{p}{m}$

(c) $(|A|^2 - |B|^2)\frac{p}{m}$

(d) $(|A|^2 + |B|^2)\frac{p}{m}$

59. the

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

and corresponding eigen states is $\phi_n(x)$ for a particle trapped in ID infinite potential box of width ' a ' centered at $x = 0$, the state $\psi(t = 0)$ is defined $\psi = 3\phi_1 + 4\phi_3(x)$. Find $\langle E\rangle$ on ψ.

[GATE]

(a) $\frac{75}{25}\frac{\hbar^2\hbar^2}{ma^2}$

(b) $\frac{151}{50}\frac{\hbar^2\hbar^2}{ma^2}$

(c) $\frac{149}{50}\frac{\hbar^2\hbar^2}{ma^2}$

(d) $\frac{153}{50}\frac{\pi^2\hbar^2}{ma^2}$

60. Consider a system whose wave function at time $t = 0$ is given by

$$(x,0) = \frac{5}{\sqrt{50}}\Phi_0(x) + \frac{4}{\sqrt{50}}\Phi_1(x) + \frac{3}{\sqrt{50}}\Phi_2(x)$$

where $\Phi_n(x)$ is the wavefunction of the n^{th} excited state for a harmonic oscillator of energy $E_n = (n + y_2)\hbar\omega$. find average energy of the system.

[GATE]

(a)3.5 $\hbar\omega$

(b) 1.18$\hbar\omega$

(c) 5.23$\hbar\omega$

(d) 6.33$\hbar\omega$

61. Which one of the following is correct for the phase velocity v_p and group velocity v_g? (c is the speed of light in vacuum)

[GATE 2025]

(a)For matter waves in the relativistic case, $v_p v_g > c^2$

(b)For electromagnetic waves in a medium, v_p represents the speed with which energy Propagates

(c)For electromagnetic waves in a medium, both v_p and v_g can be more than c

(d)For matter waves in free space, $v_p \neq v_g$

Answer Key

1. d	2. c	3. d	4. d	5. a
6. a	7. c	8. c	9. d	10. a
11. d	12. -	13. c	14. b	15. a
16. c	17. b	18. a	19. b	20. a
21. d	22. c	23. d	24. a	25. c
26. c	27. b	28. b	29. c	30. c
31. c	32. 0.25	33. c	34. a	35. 24
36. 0.28	37. c	38. 0.11	39. b	40. 0.25
41. a	42. b	43. d	44. 6	45. 33.3
46. abc	47. b	48. b	49. a	50. b
51. c	52. 9	53. d	54. a	55. c
56. d	57. a	58. c	59. a	60. b

❖ **JEST PYQ**

1. The ground state (apart from normalization) of a particle of unit mass moving in a one-dimensional potential V(x) is $\exp(-x^2/2)\cosh(\sqrt{2}x)$. The potential V(x), in suitable units so that h = 1, is (up to an addiative constant.) **[JEST 2012]**

(a) $\pi^2/2$

(b) $\pi^2/2 - \sqrt{2}x\tanh(\sqrt{2}x)$

(c) $\pi^2/2 - \sqrt{2}x\tan(\sqrt{2}x)$

(d) $\pi^2/2 - \sqrt{2}x\coth(\sqrt{2}x)$

2. The wave function of a free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of **[JEST 2012]**

(a) the position operator

(b) the Hamiltonian

(c) the momentum operator

(d) the parity operator

3. Consider a particle of mass m moving inside a two dimensional square box whose sides are described by the equations $x = 0, x = L, y = 0, y = L$. What is the lowest eigen value which changes sign under the exchange of x and y ? **[JEST 2012]**

(a) $\hbar^2/(mL^2)$ (b) $3\hbar^2/(2mL^2)$

(c) $5\hbar^2/(2mL^2)$ (d) $7\hbar^2/(2mL^2)$

4. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow|\downarrow\rangle = 0$ and x, ϕ are real is orthogonal to: **[JEST 2012]**

(a) $\sin x|\uparrow\rangle$

(b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

(c) $-\cos x|\uparrow\rangle - \exp(i\phi)\sin x|\downarrow\rangle$

(d) $-\exp(-i\phi\cos x)|\uparrow\rangle + \sin x|\downarrow\rangle$

5. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, the mean of x is **[JEST 2013]**

(a) λ (b) 2λ

(c) $\lambda/2$ (d) 0

6. A particle of mass m is contained in a one-dimensional infinite well extending from $x = -\frac{L}{2}$ to $x = \frac{L}{2}$. The particle is in its ground state given by $\varphi_0(x) = \sqrt{2/L}\cos(\pi x/L)$. The walls of the box are moved suddenly to form a box extending from $x = -L$ to $x = L$. what is the probability that the particle will be in the ground state after this sudden expansion? **[JEST 2013]**

(a) $(8/3\pi)^2$ (b) 0

(c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

7. A quantum mechanical particle in a harmonic oscillator potential has the initial wave function $\psi_0(x) + \psi_1(x)$, where ψ_0 and ψ_1 are the real wavefunctions in the ground and first excited state of the harmonic oscillator Hamiltonian. For convenience we take $m = \hbar = \omega = 1$ for the oscillator. What is the probability density of finding the particle at x at time $t = \pi$? **[JEST 2013]**

(a) $(\psi_1(x) - \psi_0(x))^2$

(b) $(\psi_1(x))^2 - (\psi_0(x))^2$

(c) $(\psi_1(x) + \psi_0(x))^2$

(d) $(\psi_1(x))^2 + (\psi_0(x))^2$

8. If the expectation value of the momentum is $\langle p\rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/h}\psi(x)$ is **[JEST 2013]**

(a) k (b) $\langle p\rangle - k$

(c) $\langle p\rangle + k$ (d) $\langle p\rangle$

9. The Hamiltonian operator for a two-state system is given by

$$H = \alpha(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|),$$

where α is a positive number with the dimension of energy. The energy eigenstates corresponding to the larger and smaller eigenvalues respectively are:

[JEST 2014]

(a) $|1\rangle-(\sqrt{2}+1)|2\rangle, |1\rangle+(\sqrt{2}-1)|2\rangle$

(b) $|1\rangle+(\sqrt{2}-1)|2\rangle, |1\rangle-(\sqrt{2}+1)|2\rangle$

(c) $|1\rangle+(\sqrt{2}-1)|2\rangle, (\sqrt{2}+1)|1\rangle-|2\rangle$

(d) $|1\rangle-(\sqrt{2}+1)|2\rangle, (\sqrt{2}-1)|1\rangle+|2\rangle$

10. A ball bounces off earth. You are asked to solve this quantum mechanically assuming the earth is an infinitely hard sphere. Consider surface of earth as the origin implying $V(0)=\propto$ and a linear potential elsewhere (i.e. $V(x)=-mgx$ for $x>0$). Which of the following wave functions is physically admissible for this problem (with $k>0$):

[JEST 2014]

(a) $\psi=e^{-kx}/x$ (b) $\psi=xe^{-kx^2}$

(c) $\psi=-Axe^{kx}$ (d) $\psi=Ae^{-kx^2}$

11. Consider a square well of depth $-V_0$ and width a with V_0 as fixed. Let $V_0\rightarrow\infty$ and $a\rightarrow 0$. This potential well has **[JEST 2014]**

(a) No bound states

(b) 1 bound state

(c) 2 bound states

(d) Infinitely many bound states

12. A particle of mass m moves in 1 -dimensional potential $V(x)$, which vanishes at infinity. The exact ground state eigenfunction is $\psi(x)=A\sec h(\lambda x)$, where A and λ are constants. The ground state energy eigenvalue of this system is,

[JEST 2015]

(a) $E=\frac{\hbar^2\lambda^2}{m}$ (b) $E=-\frac{\hbar^2\lambda^2}{m}$

(c) $E=-\frac{\hbar^2\lambda^2}{2m}$ (d) $E=\frac{\hbar^2\lambda^2}{2m}$

13. Given that ψ_1 and ψ_2 are eigenstates of a Hamiltonian with eigenvalues E_1 and E_2 respectively, what is the energy uncertainty in the state $(\psi_1+\psi_2)$? **[JEST 2015]**

(a) $-\sqrt{E_1E_2}$ (b) $\frac{1}{2}|E_1-E_2|$

(c) $\frac{1}{2}(E_1+E_2)$ (d) $\frac{1}{\sqrt{2}}|E_2-E_1|$

14. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen functions $\psi_{mlm}(\vec{r})(n,l,m$ are the usual quantum numbers):

$$\psi(\vec{r})=\frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r})-\frac{3}{\sqrt{14}}\psi_{210}(\vec{r})+\frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

[JEST 2016]

(a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$

(c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

15. An e- confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep 1D potential well. the difference in energies between first and second energy is

$$\delta t=\frac{3\pi^2\hbar^2}{2ma^2}$$

find the width of the well

[JEST 2016]

(a) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$

(c) $\sqrt{\frac{\pi^2\hbar^2}{2m\delta E}}$ (d) $\sqrt{\frac{\pi\hbar^2}{m\delta E}}$

16. A spin-$\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $\frac{(|1\rangle+|2\rangle)}{\sqrt{2}}$ at time $t=0$. It evolves to the state described by the ket $\frac{(|1\rangle-|2\rangle)}{\sqrt{2}}$ in time T. The minimum energy difference between two levels is:

[JEST 2016]

(a) $\frac{h}{6T}$ (b) $\frac{h}{4T}$ (c) $\frac{h}{2T}$ (d) $\frac{h}{T}$

17. The energy of a particle is given by $E = |p| + |q|$ where p and q are the generalized momentum and coordinate, respectively. All the states with $E \le E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate q, with $q > 0$ is:

[JEST 2016]

(a) $\frac{(E_0 + q)}{E_0^2}$ (b) $\frac{q}{E_0^2}$

(c) $\frac{(E_0 - q)}{E_0^2}$ (d) $\frac{1}{E_0}$

18. If the ground state wavefunction of a particle moving in a one dimensional potential is proportional to $\exp(-x^2/2)\cosh(\sqrt{2}x)$, then the potential in suitable units such that $\hbar = 1$, is proportional to

[JEST 2017]

(a) x^2

(b) $x^2 - 2\sqrt{2}x\tanh(\sqrt{2}x)$

(c) $x^2 - 2\sqrt{2}x\tan(\sqrt{2}x)$

(d) $x^2 - 2\sqrt{2}x\coth(\sqrt{2}x)$

19. The normalized eigenfunctions and eigenvalues of the Hamiltonian of a Particle confined to move between $0 \le x \le a$ in one dimension are

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \text{ and } E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

respectively. Here 1,2,3 ... Suppose the state of the particle is

$$\psi(x) = A\sin\left(\frac{\pi x}{a}\right)\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]$$

where A is the normalization constant. If the energy of the particle is measured, the probability to get the result as $\frac{\pi^2\hbar^2}{2ma^2}$ is $\frac{x}{100}$. What is the value of x ?

[JEST 2018]

20. Consider a wave packet defined by

$$\psi(x) = \int_{-\infty}^{\infty} dk f(k)\exp[i(kx)]$$

Further, $f(k) = 0$ for $|k| > \frac{K}{2}$ and $f(k) = a$ for $|k| \le \frac{K}{2}$. Then, the form of normalized $\psi(x)$ is

[JEST 2018]

(a) $\frac{\sqrt{8\pi K}}{x}\sin\frac{Kx}{2}$ (b) $\sqrt{\frac{2}{\pi K}\frac{\sin\frac{Kx}{2}}{x}}$

(c) $\frac{\sqrt{8\pi K}}{x}\cos\frac{Kx}{2}$ (d) $\sqrt{\frac{2}{\pi K}}\frac{\sin\frac{Kx}{2}}{x}$

21. What is the binding energy of an electron in the ground state of a He^+ion? **[JEST 2019]**

(a) 6.8eV (b) 13.6Ev

(c) 27.2eV (d) 54.4eV

22. The wave function $\psi(x) = A\exp\left(-\frac{b^2x^2}{2}\right)$ (for real constants A and b) is a normalized eigen-function of the Schrodinger equation for a particle of mass m and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

[JEST 2019]

(a)$V = \frac{\hbar^2b^4x^2}{m}$ (b) $V = \frac{\hbar^2b^4x^2}{2m}$

(c)$E = \frac{\hbar^2b^2}{4m}$ (d) $E = \frac{\hbar^2b^2}{m}$

23. Consider a quantum particle in a one-dimensional box of length L. The coordinates of the leftmost wall of the box is at $x = 0$ and that of the rightmost wall is at $x = L$. The particle is in the ground state at $t = 0$. At $t = 0$, we suddenly change the length of the box to $3L$ by moving the right wall. What is the probability that the particle is in the ground state of the new system immediately after the change?

[JEST 2019]

(a)0.36 (b) $\frac{9}{8\pi}$

(c) $\frac{81}{64\pi^2}$ (d) $\frac{0.5}{\pi}L$

24. The wave function of an electron in one dimension is given by

$$\psi(x) = \begin{cases} 0, & \text{for } x < 0 \\ 2\sqrt{3}e^{-x}(1 - e^{-x}), & \text{for } x \ge 0 \end{cases}$$

The ratio between the expected position $\langle x \rangle$ and the most probable position x_m is

[JEST 2020]

(a) 0.856 (b) 1.563

(c) 2.784 (d) 3.567

25. A free particle of energy E, characterized by a plane wave of wavelength λ enters a region of constant potential $-V$ (where $E > V > 0$). Within the region of the potential, the wavelength of the particle is $\frac{\lambda}{2}$. The ratio $\frac{V}{E}$ is:

[JEST 2020]

(a) $\frac{-1}{3}$ (b) -3

(c) 3 (d) $\frac{1}{3}$

26. A two state quantum system has energy eigenvalues $\pm\epsilon$ corresponding to normalised states ψ_{+}. At
time $t = 0$ the system is in the quantum state

$$\frac{[\psi_+ + \psi_-]}{\sqrt{2}}$$

Find the $10000 \times$ probability that the system will be in the same state at time $t = \frac{h}{(6\epsilon)}$, where h is the Planck's constant.

[JEST 2020]

27. A particle with energy E is in a bound state of the one-dimensional Hamiltonian

$$H = \frac{h^2}{2m}\frac{d^2}{dx^2} + V(x)$$

The expectation value of the momentum $\langle p \rangle$

(a) is always zero

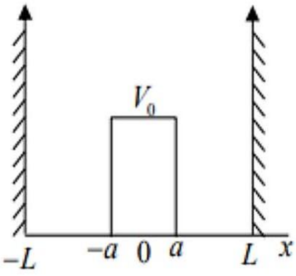
(b) depends on the degeneracy of the eigenstate

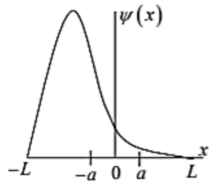
(c) is zero if and only if the potential symmetric $V(-x) = V(x)$

(d) depends on the energy E of the eigenstate

28. A quantum particle is moving in one dimension between rigid walls at $x = -L$ and $x = L$, under the influence of a potential (see figure). The potential has the uniform value V_0 between $-a < x < a$, and is 0 otherwise. Which one of the following graphs qualitatively represent the ground state wavefunction of this system? (You can assume that $a \ll LV_0 \gg \pi^2/8mL^2$).

[JEST 2021]



(a)


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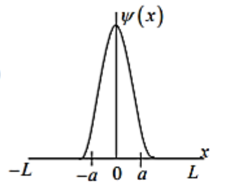
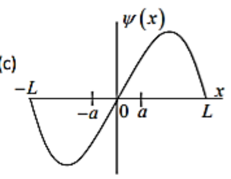
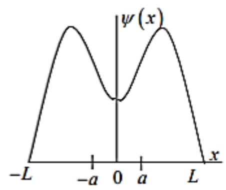


(c)


(d)


29. Consider a 4-dimensional vector space V that is a direct product of two 2-dimensional vector spaces
V_1 and V_2. A linear transformation H acting on V is specified by the direct product of linear transformations H_1 and H_2 acting on V_1 and V_2, respectively. In a particular basis,

$$H_1 = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}, H_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

what is the lowest eigenvalue of H ?

[JEST 2021]

(a) 1 (b) $\frac{3}{2}$

(c) $3 - \sqrt{5}$ (d) $\frac{1}{2}(3 - \sqrt{5})$

30. A particle is in the nth energy eigenstate of an infinite one-dimensional potential well between $x = 0$ and $x = L$. Let P be the probability of finding the particle between $x = 0$ and $x = 1/3$. In the limit $n \to \infty$, the value of P is

[JEST 2021]

(a) 1/9 (b) 2/3

(c) 1/3 (d) $1/\sqrt{3}$

31. A beam of high energy neutrons is scattered from a metal lattice, where the spacing between nuclei
is around 0.4 nm. In order to see quantum diffraction effects, the kinetic energy of the neutrons must be of the order [Mass of neutron $= 1.67 \times 10^{-27}$ kg, Planck's constant $= 6.62 \times 10^{-34}\ \mathrm{m^2\ kg\ s^{-1}}$]

[JEST 2022]

(a) meV (b) MeV

(c) eV (d) keV

32. If θ and ϕ are respectively the polar and azimuthal angles on the unit sphere, what is $\langle\cos^2(\theta)\rangle$ and $\langle\sin^2(\theta)\rangle$, where $\langle\mathcal{O}\rangle$ denotes the average of $\mathcal{O}$?

[JEST 2022]

(a) $\langle\cos^2(\theta)\rangle = 3/4$ and $\langle\sin^2(\theta)\rangle = 1/4$

(b) $\langle\cos^2(\theta)\rangle = 1/2$ and $\langle\sin^2(\theta)\rangle = 1/2$

(c) $\langle\cos^2(\theta)\rangle = 1/3$ and $\langle\sin^2(\theta)\rangle = 2/3$

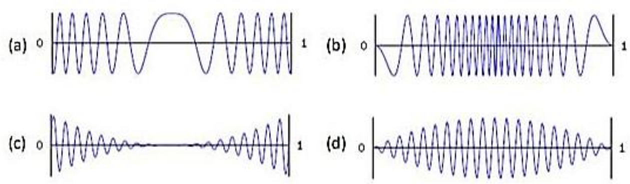
(d) $\langle\cos^2(\theta)\rangle = 2/3$ and $\langle\sin^2(\theta)\rangle = 1/3$

33. A-A particle is confined in an infinite potential well of the form given below.

$$V(x) = \begin{cases} 4V_0 x(1-x), \forall 0 \le x \le 1 \\ \infty, \text{otherwise} \end{cases}$$

If the particle has energy $E \ge V_0$, which of the following could be the form of its wavefunction?

[JEST 2022]



34. A particle of mass m moves in one dimension. The exact eigenfunction for the ground state of the
system is

$$\psi(x) = \frac{A}{\cosh(\lambda x)'}$$

where, λ is a constant and A is the normalization constant. If the potential $V(x)$ vanishes at infinity, the ground state energy of the system is

[JEST 2022]

(a) $-\frac{\hbar^2\lambda^2}{2m}$ (b) $\frac{\hbar^2\lambda^2}{2m}$

(c) $\frac{\hbar^2\lambda}{2m}$ (d) $-\frac{\hbar^2\lambda}{2m}$

35. The frequency dispersion relation of the surface waves of a fluid of density ρ and temperature T, is
given by $\omega^2 = gk + Tk^3/\rho$, where ω and k are the angular frequency and wavenumber, respectively, g is the acceleration due to gravity. The first term in r.h.s. describes the gravity waves and the second term describes the surface tension wave. What is the ratio of the first term to the second term, when the phase velocity is equal to the group velocity?

[JEST 2023]

36. Consider a spin- 1/2 particle in the quantum state

$$|\psi(\beta,\alpha)\rangle = \cos\left(\frac{\beta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}|\downarrow\rangle$$

where
$0 \le \beta \le \pi$ and $0 \le \alpha \le 2\pi$. For which values of (δ,γ) is the state $|\psi(\delta,\gamma)\rangle$ orthogonal to $|\psi(\beta,\alpha)\rangle$? **[JEST 2023]**

(a) $(\pi+\beta, \pi-\alpha)$ (b) $(\pi-\beta, \pi-\alpha)$

(c) $(\pi+\beta, \pi+\alpha)$ (d) $(\pi-\beta, \pi+\alpha)$

37. Consider a free particle in one dimension described by the wavefunction:

$$\psi(x,t=0) = A\exp\frac{ipx}{\hbar} + B\exp\frac{-ipx}{\hbar}$$

The probability current density corresponding to $\psi(x,t)$ at a later time t is:

[JEST 2023]

(a) $\frac{p(|A|^2-|B|^2)}{m}\cos\left(\frac{p^2}{2m\hbar}t\right)$

(b) $\frac{p(|A|^2+|B|^2)}{m}$

(c) $\frac{p(|A|^2-|B|^2)}{m}$

(d) $\frac{p(|A|^2+|B|^2)}{m}\cos\left(\frac{p^2}{2m\hbar}t\right)$

38. A quantum particle moving in one dimension is in a state having the wave function

$$\psi(x) = \sinh(\lambda x)\exp\left(\frac{-ax^4 + bx + ipx}{\hbar}\right)$$

where a, b, λ and p are all positive real numbers. What is the expectation value of momentum?

[JEST 2023]

(a) $\hbar\lambda$ (b) p

(c) b (d) $-p$

39. Consider a quantum particle incident from the left on a step potential given by $V_0\theta(x)$, with energy $E(> V_0)$; here $\theta(x)$ is the unit step function. The scattering state of the particle is given by

[JEST]

$$\psi(x) = \begin{cases} \exp\frac{ipx}{\hbar} + r\exp\frac{-ipx}{\hbar}, & x < 0 \\ t\exp\frac{ip'x}{\hbar} & x > 0 \end{cases}$$

where p and p' are the momenta of the particle corresponding to the energy E. Which of the following is true?

[JEST 2023]

(a) $|r|^2 + \frac{p'}{p}|t|^2 = 1$ (b) $|r|^2 + |t|^2 = 1$

(c) $|r|^2 + \frac{p}{p'}|t|^2 = 1$ (d) $r + t = 1$

40. Consider a particle subjected to the symmetric one-dimensional infinite square well potential:

$$V(x) = \begin{cases} 0, & |x| \leq \frac{L}{2} \\ \infty, & |x| > \frac{L}{2}. \end{cases}$$

Find the time evolution of the wavefunction $\psi(x,t)$, if at time $t = 0$ the particle is prepared in an equal superposition of the ground and the first excited states:

$$\psi(x,0) = \frac{1}{\sqrt{2}}(\phi_1(x) + \phi_2(x))$$

where $\phi_1(x)$ and $\phi_2(x)$ are normalized eigenfunctions of the ground state and the first excited state respectively. If τ is the smallest time at which the particle is equally likely to be in either half of the well, select the correct value of $\frac{\tau h}{mL^2}$ where h is the Planck's constant, m is the mass of the particle and L is the width of the well as defined above.

[JEST 2023]

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{4}{3}$

41. A and B are 2×2 Hermitian matrices. $|a_1\rangle$ and $|a_2\rangle$ are two linearly independent eigenvectors of A. Consider the following statements:

1.If $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B, then $[A, B] = 0$.

2.If $[A, B] = 0$, then $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B.
Mark the correct option.

[JEST 2024]

(a) Both statements 1 and 2 are true.

(b) Statement 2 is true but statement 1 is false.

(c) Statement 1 is true but statement 2 is false.

(d) Both statements 1 and 2 are false.

42. A particle moving in one dimension has the wave function

$$\psi(x) = \exp\left[-\alpha\left(x - \frac{ik_0}{\alpha}\right)^2\right]\sin^2(k_1 x),$$

where α is real positive and k_0, k_1 are real. The expectation value of momentum is

[JEST 2024]

(a) $\hbar k_0$ (b) 0

(c) $2\hbar k_0$ (d) $\hbar k_1$

43. A quantum particle is subjected to the potential $V(x) = ax + bx^2$, where a and b are constants. What is the mean position of the particle in the first excited state?

[JEST 2024]

(a) $\frac{a}{b}$ (b) $\frac{a}{2b}$

(c) $-\frac{a}{b}$ (d) $-\frac{a}{2b}$

44. A particle with energy $E > 0$ is incident from the right $(x > 0)$ on a one-dimensional potential composed of a delta-function barrier at $x = 0$ and a hard wall at $x = -a$:

$$V(x) = \begin{cases} \alpha\delta(x), & x > -a \\ \infty, & x \leq -a \end{cases}$$

where $\alpha > 0$ and $a > 0$. Let us define

$$\kappa^2 = \frac{2mE}{\hbar^2}$$

and the dimensionless quantities: $\xi = \kappa a$ and

$$\beta = \frac{\hbar^2}{2m\alpha a}$$

For some energy E the particle reflects from the barrier without any phase shift. Which of the following transcendental equations determines this energy? [Note that in the presence of the delta function barrier, the derivative of the wave function has a discontinuity at

$$x = 0: \psi'(0^+) - \psi'(0^-) = \frac{\psi(0)}{\beta a}.]$$

[JEST 2024]

(a) $\tan\ \xi = \beta\xi$ (b) $\tan\ \xi = -\beta\xi$

(c) $\tanh\ \xi = \beta\xi$ (d) $\tanh\ \xi = -\beta\xi$

45. A quantum mechanical particle of mass m is confined in a one dimensional infinite potential well whose walls are located at $x = 0$ and $x = 1$. The wave function of the particle inside the well is $\psi(x) = \mathcal{N}[x\ln\ x + (1-x)\ln\ (1-x)]$ for some normalization constant $\mathcal{N}$. An experimentalist measures the position of the particle on an ensemble of a large number of identical systems in the same state. The mean of the outcomes is found to be $\frac{1}{n}$, where n is an integer. What is n ?

[JEST 2024]

46. Consider a particle of mass m moving in a three-dimensional delta-function potential well $V(\vec{r}) = -\alpha\delta^3(\vec{r})$, where $\alpha > 0$. Which of the following is an allowed expression for the energy of a bound state for some dimensionless proportionality constant $\beta > 0$?

(a) $-\frac{\beta\hbar^6}{\alpha^2 m^3}$ (b) $\frac{\beta\hbar^6}{\alpha^2 m^3}$

(c) $-\frac{\beta\alpha^2 m}{\hbar^2}$ (d) $\frac{\beta\alpha^2 m}{\hbar^2}$

47. Suppose the wave function of a free particle in one dimension obeys $\frac{d^2\psi}{dx^2} = -4\psi$ in units where $\hbar = 1$. What is the magnitude of the momentum of the particle?

ANS: 2

48. For a particle in a one-dimensional box of width L, the uncertainty Δp in momentum in the n-th eigenstate of energy for large n is

[JEST 2025]

(a) $\frac{n\pi\hbar}{L}$ (b) $\frac{2n\pi\hbar}{L}$

(c) $\frac{2n\hbar}{L}$ (d) $\frac{\hbar}{n\pi L}$

49. Consider the time-independent Schrödinger equation with a real potential and suppose $\psi(x)$ is a solution of this equation. Which of the following is true?

(a) ψ^* is never a solution of the same equation.

(b) ψ^* is a solution of the same equation.

(c) ψ^* is a solution of the same equation only if the potential is symmetric about $x = 0$.

(d) ψ^* is a solution of the same equation only if the potential vanishes at infinity.

50. A quantum mechanical system is spanned by the eigenstates $|a_1\rangle$ and $|a_2\rangle$ of a Hermitian operator A with eigenvalues a_1 and a_2 respectively. If there is no degeneracy, what is the expectation value of the operator $(A - a_1)(A - a_2)$ in the state $\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}$?

(a) $\frac{(a_2 - a_1)(a_1 - a_2)}{2}$

(b) $(a_2 - a_1)(a_1 - a_2)$

(c) 1

(d) 0

Answer Key				
1. b	2. d	3. c	4. d	5. b
6. a	7. a	8. c	9. b	10. b
11. b	12. d	13. b	14. a	15. a
16. c	17. c	18. b	19. 80	20. b
21. d	22. b	23. c	24. b	25. c
26. 2500	27. c	28. d	29. c	30. c
31. a	32. c	33. a	34. a	35. 1
36. d	37. b	38. b	39. a	40. d
41. c	42. c	43. d	44. b	45.
46. a	47. 2	48. a	49. b	50. d

❖ **TIFR PYQ**

1. The wave function Ψ of a quantum mechanical system described by a Hamiltonian $\hat{H}$ can be written as a linear combination of Φ_1 and Φ_2 which are the eigenfunctions of $\hat{H}$ with eigenvalues E_1 and E_2 respectively. At $t = 0$, the system is prepared in the state

$$\Psi_0 = \frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$$

and then allowed to evolve with time. The wavefunction at time

$$T = \frac{1}{2}h/(E_1 - E_2)$$

will be (accurate to within a phase)

[TIFR 2010]

(a) $\frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ (b) Φ_1

(c) $\frac{4}{5}\Phi_1 - \frac{3}{5}\Phi_2$ (d) Φ_2

(e) $\frac{3}{5}\Phi_1 + \frac{4}{5}\Phi_2$ (f) $\frac{3}{5}\Phi_1 - \frac{4}{5}\Phi_2$

2. A particle P_1 is confined in a one-dimensional infinite potential well with walls at $x = \pm 1$. Another particle P_2 is confined in a one-dimensional infinite potential well with walls at $x = 0,1$. Comparing the two particles, one can conclude that

(a) the no. of nodes in the n^{th} excited state of P_1 is twice that of P_2

(b) the no. of nodes in the n^{th} excited state of P_1 is half that of P_2

(c) the energy of the n^{th} level of P_1 is the same as that of P_2

(d) the energy of the n^{th} level of P_1 is one quarter of that of P_2 **[TIFR 2010]**

3. A particle in a one-dimensional potential has the wavefunction

$$\psi(x) = \frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$$

where a is a constant. It follows that for a positive constant V_0, the potential $V(x) =$

[TIFR 2012]

(a) $V_0 x^2$ (b) $V_0|x|$

(c) $-V_0\delta(x)$ (d) $-V_0/|x|$

4. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

[TIFR 2012]

(a) free particles

(b) particles in a box

(c) harmonic oscillators

(d) rigid rotators

(e) hydrogenic atoms

5. In a quantum mechanical system, an observable A is represented by an operator $\hat{A}$. If $|\psi\rangle$ is a state of the system, but not an eigenstate of $\hat{A}$, then the quantity

$$r = \langle\psi|\hat{A}|\psi\rangle^2 - \langle\psi|\hat{A}^2|\psi\rangle$$

satisfies the relation

[TIFR 2013]

(a) $r < 0$ (b) $r = 0$

(c) $r > 0$ (d) $r \geq 0$

6. The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be $+1$ is

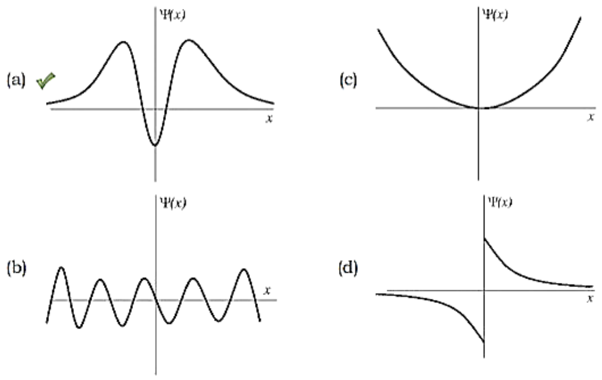
[TIFR 2013]

(a) $1/\sqrt{2}$ (b) 1

(c) ½ (d) ¼

7. A particle of energy E moves in one dimension under the influence of a potential $V(x)$. If $E > V(x)$ for some range of x, which of the following graphs can represent a bound state wave function of the particle?

[TIFR 2013]



8. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle/\hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is

[TIFR 2014]

(a)2 (b) $\frac{1+\lambda}{1-\lambda}$

(c) $\frac{1}{\lambda}$ (d) 1

9. A particle moving in one dimension has the un-normalized wave function

$$\psi(x) = x\exp\left(-\frac{x^2}{\lambda^2}\right)$$

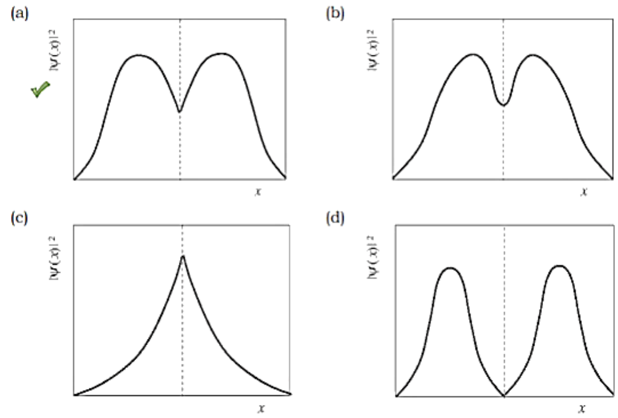
where λ is a real constant. The expectation value of its momentum is $\langle p\rangle$,

[TIFR 2014]

(a) $\frac{\hbar}{\lambda}\exp\left(-\frac{x^2}{\lambda^2}\right)$ (b) $\frac{\hbar^2}{\lambda^2} - 2\frac{\hbar}{\lambda}$

(c) $\frac{\hbar}{\lambda}\exp(-1)$ (d) zero

10. A particle is confined to a one-dimensional box of length L. If a vanishingly thin but strongly repulsive partition is introduced in the exact centre of the box, and the particle is allowed to come to its ground state, then the probability density for finding the particle will appear as **[TIFR 2014]**



11. In a quantum mechanical system, an observable A is represented by an operator $\hat{A}$. If $|\psi\rangle$ is a state of the system, but not an eigenstate of $\hat{A}$, then the quantity

$$r = \langle\psi|\hat{A}|\psi\rangle^2 - \langle\psi|\hat{A}^2|\psi\rangle$$

satisfies the relation **[TIFR 2014]**

(a) $r < 0$ (b) $r = 0$

(c) $r > 0$ (d) $r \geq 0$

12. A one-dimensional box contains a particle whose ground state energy is ϵ. It is observed that a small disturbance causes the particle to emit a photon of energy $h\nu = 8\epsilon$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, \ldots\}$ would be

[TIFR 2015]

(a) $\frac{\psi_1 - \psi_2}{\sqrt{2}}$

(b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$

(c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$

(d) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

13. A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics,

[TIFR 2015]

(a) the particle will shoot out of the well and move out towards infinity in one of the two possible directionas

(b) the particle will stop oscillating and as time increases it may be found farther and farther away from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

14. It is required to construct the quantum theory of a particle of mass m moving in one dimension x under the influence of a constant force F. The characteristic length-scale in this problem is

[TIFR 2015]

(a) $\frac{\hbar}{mF}$ (b) $\left(\frac{\hbar^2}{mF}\right)^{1/3}$

(c) $\left(\frac{\hbar}{m^2F}\right)^{1/3}$ (d) $\frac{mF}{\hbar^2}$

15. 1000 neutral spinless particles are confined in a one-dimensional box of length 100 nm. At a given instant of time, if 100 of these particle have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$, then the number of particles in the left half of the box will be approximately

[TIFR 2015]

(a) 625 (b) 500

(c) 441 (d) 100

16. If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \to \infty$ according to the distribution $f(x) = \exp(-4x)$
then the expectation value ofcos $4x$ is

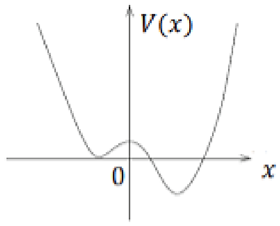
[TIFR 2016]

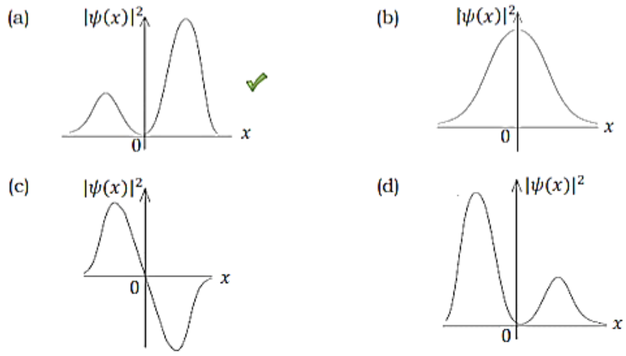
(a) zero (b) ½

(c) ¼ (d) 1/16

17. A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right.One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is

[TIFR 2016]





18. A particle moving in one dimension is confined inside a rigid box located between $x = -a/2$ and $x = a/2$. If the particle is in its ground state

$$\psi_0(x) = \sqrt{2/a}\cos\frac{\pi x}{a}$$

the quantum mechanical probability of its having a momentum p is given by

[TIFR 2016]

(a) $\frac{8\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$

(b) $\frac{\pi^2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\sin^2\frac{pa}{2\hbar}$

(c) $\frac{2\hbar^4}{(\pi^2\hbar^2 + p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$

(d) $\frac{16\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}$

19. The normalized wave function of a particle can be written as

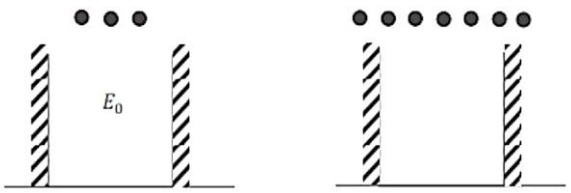
$$\Psi(x) = N\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{7}}\right)^n \varphi_n(x)$$

where $\varphi_n(x)$ are the normalized energy eigenfunctions of a given Hamiltonian. The value of N is **[TIFR 2017]**

(a) $\sqrt{1/7}$ (b) $\sqrt{6/7}$

(c) $\sqrt{3/7}$ (d) $\sqrt{(6-2\sqrt{7})/7}$

20. A quantum mechanical system consists of a one-dimensional infinite box, as indicated in the figures below.



3 (three) identical non-interacting spin-1/2 particles, are first placed in the box, and the ground state energy of the system is found to be $E_0 = 18$eV. If 7 (seven) such identical particles are placed in the box, what will be the ground state energy, in units of eV?

[TIFR 2017]

(a)132 (b)134

(c)136 (d)122

21. A particle of mass m, confined to one dimension x, is in the ground state of a harmonic oscillator potential with a normalized wave function

$$\Psi_0(x) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-ax^2}$$

where $a = m\omega/2\hbar$. Find the expectation value of x^8 in terms of the parameter a.

[TIFR 2017]

22. A particle is in the ground state of a cubical box of side ℓ. Suddenly one side of the box changes from ℓ to 4ℓ. If p is the probability of finding the particle in the ground state of the new box, what is $1000p$? **[TIFR 2018]**

23. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wavefunction changes to $\psi(\lambda x, t)$ then the expectation value of kinetic energy T and the potential energy V will change, respectively, to

[TIFR 2018]

(a) $\lambda^2 T$ and V/λ^2 (c) T/λ^2 and $\lambda^2 V$

(b) T/λ^2 and V/λ^2 (d) $\lambda^2 T$ and $\lambda^2 V$

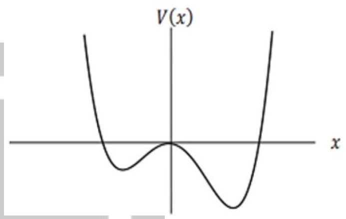
24. Given a particle confined in a one-dimensional box between $x = -a$ and $x = +a$, a student attempts to find the ground state by assuming a wave-function

$$\psi(x) = \begin{cases} A(a^2 - x^2)^{3/2} & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

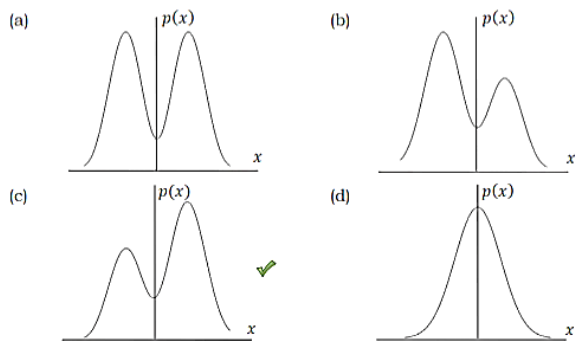
The ground state energy E_m is estimated by calculating the expectation value of energy with this trial wave-function. If E_0 is the true ground state energy, what is the ratio E_m/E_0 ?

[TIFR 2018]

25. Consider the 1-D asymmetric double-well potential $V(x)$ as sketched below.



The probability distribution $p(x)$ of a particle in the ground state of this potential is best represented by **[TIFR 2018]**



26. The wave-function Ψ of a particle in a one-dimensional harmonic oscillator potential is given by

$$\Psi = \left(\frac{1}{\pi\ell^2}\right)^{1/4} \left(1 + \frac{\sqrt{2}x}{\ell}\right) \exp\left(-\frac{x^2}{2\ell^2}\right)$$

where $\ell = 100\mu$m. Find the expectation value of the position x of this particle, in μm. 071

[TIFR 2018]

27. A particle of mass m, moving in one dimension, satisfies the modified Schrödinger Equation

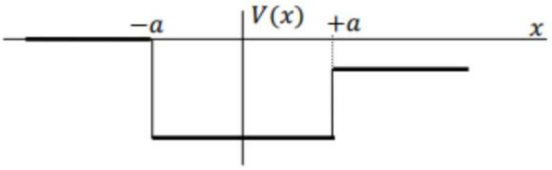
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + i\hbar u \frac{d\psi}{dx} = i\hbar \frac{d\psi}{dt}$$

where u is the velocity of the substrate. If, now, this particle is treated as a Gaussian wave packet peaked at wavenumber k, its group velocity will be $v_g =$ **[TIFR 2019]**

(a) $\dfrac{\hbar k}{2m} - u$ (c) $\dfrac{\hbar k}{m} - u$

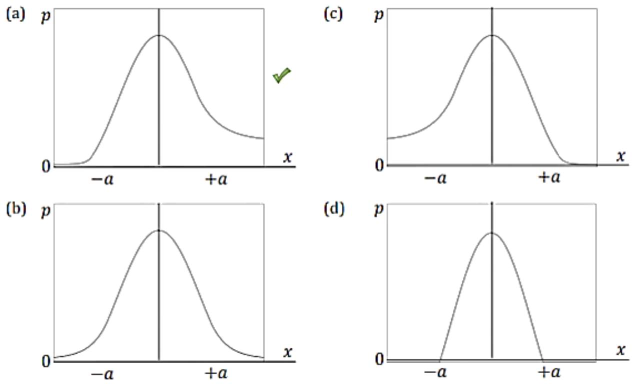
(b) $\dfrac{\hbar k}{m} + u$ (d) $-\dfrac{\hbar k}{2m} + u$

28. A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.



The probability density $p(x)$ in the ground state will most closely resemble

[TIFR 2019]



29. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\Psi(\vec{r}) = \frac{1}{\sqrt{10}}\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\psi_{21,-1}(\vec{x})$$

where $\psi_{n\ell m}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number ℓ and magnetic quantum number m. Neglecting the spin-orbit interaction, the expectation values of $\hat{L}_z$ and $\widehat{L^2}$ for this state are

[TIFR 2019]

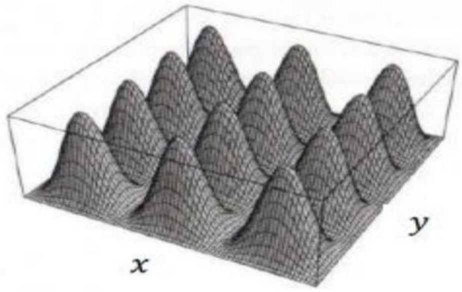
(a) $3\hbar/10, 9\hbar^2/5$ (c) $3\hbar/4, 9\hbar^2/25$

(b) $3\hbar/5, 9\hbar^2/10$ (d) $8\hbar/10, 3\hbar^2/5$

30. An electron is confined to a two-dimensional square box with the following potential

$$V = \begin{cases} 0 \text{ for } 0 < x < L \text{ and } < 0 < y < L, \\ \infty \text{ otherwise} \end{cases}$$

The probability distribution of the electron in one of its eigenstates is shown below



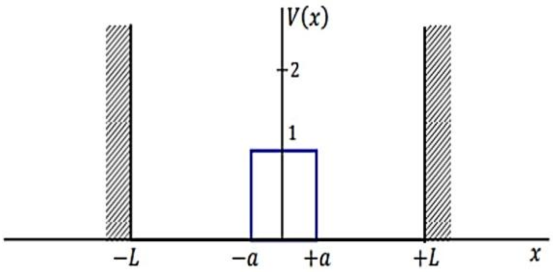
How many total different eigenstates of the electron have the same energy as this state?

[TIFR 2021]

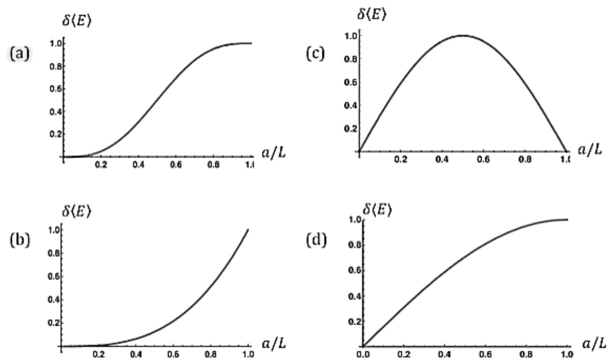
(a) 4 (b) 2

(c) 6 (d) 1

31. A particle of mass m, confined in a one-dimensional box between $x = -L$ and $x = L$, is in its first excited quantum state. Now, a rectangular potential barrier of height $V(x) = 1$ and extending from $x = -a$ to $x = a$ is suddenly switched on, as shown in the figure below.

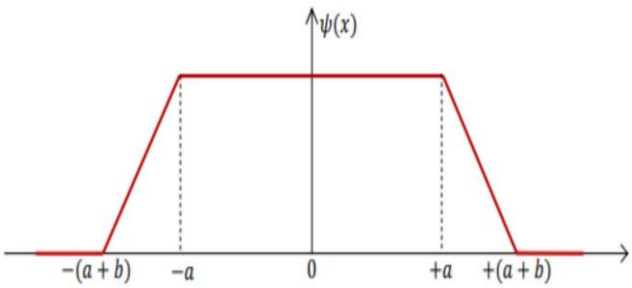


Which of the following curves most closely represents the resulting change in average energy $\delta\langle E\rangle$ of the system when plotted as a function of a/L, immediately after the barrier is created? **[TIFR 2021]**



32. The wave function of a one-dimensional particle of mass m is shown below. The average kinetic energy of the particle can be written as

[TIFR 2021]



(a) $\frac{3\hbar^2}{2mb(3a+b)}$ (b) 0

(c) $\frac{\hbar^2}{2mb^2}$ (d) $\frac{\hbar^2}{2mb(a+b)}$

33. In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever

[TIFR 2021]

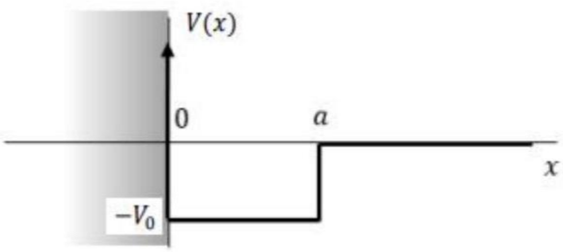
(a) the wavefunction $\psi(x)$ is itself continuous everywhere.

(b) there is a bound state and the potential is piecewise continuous.

(c) there is a bound state and the potential has no singularity anywhere.

(d) there are bound or scattering states with definite momentum.

34. A particle moves in one dimension x under the influence of a potential $V(x)$ as sketched in the figure below. The shaded region corresponds to infinite V, i.e., the particle is not allowed to penetrate there.



If there is an energy eigenvalue $E = 0$, then a and V_0 are related by **[TIFR 2022]**

(a) $a^2V_0 = \frac{\left(n+\frac{1}{2}\right)^2\pi^2}{2m}$ (b) $a^2V_0 = \frac{n^2\pi^2}{2m}$

(c) $a^2V_0 = \frac{\left(n+\frac{1}{2}\right)\pi^2}{2m}$ (d) $a^2V_0 = \frac{n\pi^2}{2m}$

35. A particle is in ground state of a one-dimensional box $-\frac{L}{2} \le x \le +\frac{L}{2}$. The uncertainty product $\Delta x \Delta p$ for this state satisfies **[TIFR 2023]**

(a) $\frac{3\hbar}{2} < \Delta x\Delta p \le 2\hbar$ (b) $\hbar < \Delta x\Delta p \le \frac{3\hbar}{2}$

(c) $\frac{\hbar}{2} < \Delta x\Delta p \le \hbar$ (d) $\Delta x\Delta p = \frac{\hbar}{2}$

36. A random positive variable x follows an exponential distribution

$$p(x) \propto e^{-\lambda x}$$

with $\lambda > 0$. The probability of observing an event $x > 3\langle x\rangle$, where $\langle x\rangle$ represents the average value of x, is **[TIFR 2023]**

(a) $1/e^3$ (b) $1/e$

(c) $1/e^4$ (d) $1/e^2$

37. Consider a particle of mass m in a quartic potential

$$H = \frac{p^2}{2m} + ax^4$$

If we take a variational wavefunction

$$\psi(x,\lambda) = e^{-\lambda x^2}$$

with $\lambda > 0$ and try to estimate the ground state energy, the value of λ should be chosen as [You may use the integral

$$\int_{-\infty}^{+\infty} dx(A + Bx^2 + Cx^4)e^{-\lambda x^2}$$

$$= A\sqrt{\frac{\pi}{\lambda}} + \frac{B}{2}\sqrt{\frac{\pi}{\lambda^3}} + \frac{3C}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

where A, B, C and $\lambda > 0$ are all constants.

[TIFR 2023]

(a) $\left(\frac{3ma}{4\hbar^2}\right)^{1/3}$ (b) $\left(\frac{5ma}{3\pi^2\hbar^2}\right)^{1/3}$

(c) $\left(\frac{15ma}{8\hbar^2}\right)^{1/3}$ (d) $\left(\frac{ma}{2\pi\hbar^2}\right)^{1/3}$

38. Consider a particle of mass m moving in a one-dimensional potential of the form

$$V(x) = \begin{cases} \frac{1}{2}kx^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

In a quantum mechanical treatment, what is the ground state energy of the particle?

[TIFR 2024]

(a) $\frac{3}{2}\hbar\sqrt{\frac{k}{m}}$

(b) $\frac{1}{2}\hbar\sqrt{\frac{k}{m}}$

(c) $\hbar\sqrt{\frac{k}{m}}$

(d) $\frac{5}{2}\hbar\sqrt{\frac{k}{m}}$

39. An electron confined in a two-dimensional square box, is in the ground state. The length of the side
of this square is unknown, but it is seen that the electron jumps to the first excited energy state by absorbing electromagnetic radiation of wavelength 4,040 nm. What is the length of one side of the square well?

[TIFR 2024]

(a) 1.91 nm

(b) 1.68 nm

(c) 2.55 nm

(d) 3.82 nm

40. Consider $\hat{x}$ and $\hat{p}_x$ as the quantum mechanical position and linear momentum operators with eigenstates $|x\rangle$ and $|p_x\rangle$ and eigenvalues x and p_x, respectively
The eigenvalue of $\hat{x}$ acting on the state $|\psi\rangle = e^{i\hat{p}_x a/2\hbar}|x\rangle$. is **[TIFR 2024]**

(a) $x + \frac{a}{2}$

(b) $x - \frac{a}{2}$

(c) $x + a$

(d) $x - a$

41. A particle of mass m moving in 1 dimension has the wavefunction

$$\psi(x) = \frac{1}{\pi^{1/4}\sqrt{a}} e^{ipx/h} e^{-x^2/2a^2}$$

Its average kinetic energy is given by
(You might find the following integral useful: $\int_{-\infty}^{+\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$
for $\alpha > 0$)

[TIFR 2024]

(a) $\frac{p^2}{2m} + \frac{\hbar^2}{4a^2 m}$

(b) $\frac{\left(p + \frac{\sqrt{\pi}\hbar}{a}\right)^2}{2m}$

(c) $\frac{\left(-p + \frac{\sqrt{\pi}\hbar}{a}\right)^2}{2m}$

(d) $\frac{p^2}{2m} + \frac{\hbar^2}{4a^2 m} + \frac{p\hbar}{2ma}$

42. The un-normalized energy eigenfunction of a one-dimensional simple quantum harmonic oscillator in dimensionless units ($m = \hbar = \omega = 1$) is

$$\psi_a(x) = (2x^3 - 3x)e^{-x^2/2}$$

Which of the following are two other (un-normalized) eigenfunctions which are closest in energy to ψ_a ? **[TIFR 2024]**

(a) $(2x^2 - 1)e^{-x^2/2}; (4x^4 - 12x^2 + 3)e^{-x^2/2}$

(b) $e^{-x^2/2}; (2x^2 - 1)e^{-x^2/2}$

(c) $xe^{-x^2/2}; (4x^5 - 20x^3 + 15x)e^{-x^2/2}$

(d) $(2x^2 - 1)e^{-x^2/2}; (4x^5 + 20x^3 + 15x)e^{-x^2/2}$

43. Consider a free-particle in 3 spatial dimensions described by the Hamiltonian

$$\hat{H} = \frac{\hat{\boldsymbol{p}}^2}{2m}$$

It is initially in a state described by a normalized wavefunction

$$\psi(\mathbf{r}, t = 0) = \left(\frac{\gamma}{\pi}\right)^{3/4} e^{-\gamma r^2/2}$$

What is the probability density of finding the particle with energy E at time t ?
(Hint: Express the wavefunction in momentum space.)
(The following integral might be useful:

[TIFR 2024]

$\int_{-\infty}^{+\infty} dx \frac{1}{\sqrt{2\pi}} e^{-ikx} e^{-\gamma x^2/2} = \frac{1}{\sqrt{\gamma}} e^{-k^2/(2\gamma)}$.)

(a) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{2mE}\, e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(b) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{\frac{2m\hbar}{t}}\, e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(c) $\frac{2m}{\hbar}\frac{1}{\sqrt{2mE}}(\gamma\pi)^{-1/2} e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(d) $\frac{2\pi m}{\hbar^2}(\gamma\pi)^{-1}e^{-2mE/(\gamma\hbar^{\wedge}2)}$

44. Consider a free-particle in 3 spatial dimensions described by the Hamiltonian

$$\hat{H} = \frac{\hat{\boldsymbol{p}}^2}{2m}$$

It is initially in a state described by a normalized wavefunction

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What is the probability density of finding the particle with energy E at time t ?
(Hint: Express the wavefunction in momentum space.)
(The following integral might be useful:
$\int_{-\infty}^{+\infty} dx \frac{1}{\sqrt{2\pi}} e^{-ikx} e^{-\gamma x^2/2} = \frac{1}{\sqrt{\gamma}} e^{-k^2/(2\gamma)}.$)

[TIFR 2025]

(a) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{2mE}\,e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(b) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{\frac{2m\hbar}{t}}\,e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(c) $\frac{2m}{\hbar}\frac{1}{\sqrt{2mE}}(\gamma\pi)^{-1/2}e^{-2mE/(\gamma\hbar^{\wedge}2)}$

(d) $\frac{2\pi m}{\hbar^2}(\gamma\pi)^{-1}e^{-2mE/(\gamma\hbar^{\wedge}2)}$

Answer Key				
1.	2.	3.	4. d	5. a
6. d	7. a	8. a	9. d	10. a
11. c	12. -	13. -	14. b	15. b
16. c	17. a	18. -	19. b	20. a
21.	22. -	23. a	24. 1.5	25. c
26. 0.71	27. a	28. a	29. d	30. a
31. a	32. a	33. a	34. a	35. c
36. a	37. a	38. a	39. a	40. b
41. a	42. a	43. a	44. a	

Q.21. Ans = 105/(256*a^4)

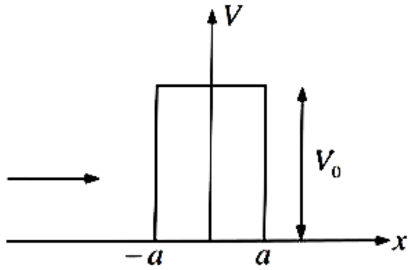
❖ JAM PYQ's

1. Electrons of energy E coming from $x = -\infty$ impinge upon a potential barrier of width $2a$ and height V_0 centered at the origin with $V_0 > E$, as shown in the figure below. Let

$$k = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

In the region $-a \le x \le a$, the electrons is a linear combination of

[JAM 2006]



(a) e^{kx} and e^{-kx} (b) e^{ikx} and e^{-kx}

(c) e^{ikx} and e^{-ikx} (d) e^{ikx} and e^{kx}

2. The wave function of a quantum mechanical particle is given by

$$\psi(x) = \frac{3}{5}\varphi_1(x) + \frac{4}{5}\varphi_2(x)$$

where $\phi_1(x)$ and $\phi_2(x)$ are eigenfunctions with corresponding energy eigenvalues -1 eV and -2 eV , respectively. The energy of the particle in the state ψ is

[JAM 2011]

(a) $\frac{-41}{25}$ eV (b) $\frac{-11}{5}$ eV

(c) $\frac{36}{25}$ eV (d) $\frac{-7}{5}$ eV

3. Four particles of mass m each are inside a two dimensional square box of side L. If each state obtained from the solution of the Schrodinger equation is occupied by only one particle, the minimum energy of the system in units of $\frac{h^2}{mL^2}$ is

[JAM 2011]

(a) 2 (b) $\frac{5}{2}$

(c) $\frac{11}{2}$ (d) $\frac{25}{4}$

4. A particle with energy E is incident on a potential given by

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x \ge 0 \end{cases}.$$

The wave function of the particle for $E < V_0$ in the region $x > 0$ (in terms of positive constants A, B and k) is

[JAM 2015]

(a) $Ae^{kx} + Be^{-kx}$ (b) Ae^{-kx}

(c) $Ae^{ikx} + Be^{-ikx}$ (d) Zero

5. A particle is moving in a two dimensional potential well

$$V(x,y)=\begin{cases}0, & 0\le x\le L, 0\le y\le 2L\\ \infty, & \text{elsewhere}\end{cases}$$

which of the following statements about the ground state energy E_1 and ground state eigenfunction φ_0 are true?

[JAM 2015]

(a) $E_1=\dfrac{\hbar^2\pi^2}{mL^2}$

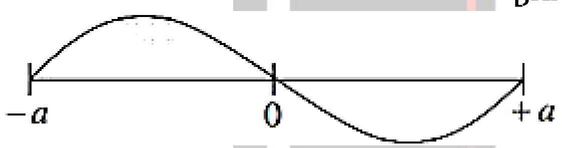
(b) $E_1=\dfrac{5\hbar^2\pi^2}{8mL^2}$

(c) $\varphi_0=\dfrac{\sqrt{2}}{L}\sin\dfrac{\pi x}{L}\sin\dfrac{\pi y}{2L}$

(d) $\varphi_0=\dfrac{\sqrt{2}}{L}\cos\dfrac{\pi x}{L}\cos\dfrac{\pi y}{2L}$

6. A particle is confined in a one dimensional box with impenetrable walls at $x=\pm a$. Its energy eigenvalue is 2 eV and the corresponding eigenfunction is as shown below. The lowest possible energy of the particle is

[JAM]



(a) 4 eV
(b) 2 eV
(c) 1 eV
(d) 0.5 eV

7. A free particle of energy E collides with a one-dimensional square potential barrier of height V and width W. Which one of the following statement(s) is/are correct?

[JAM]

(a) For $E>V$, the transmission coefficient for the particle across the barrier will always be unity

(b) For $E<V$, the transmission coefficient changes more rapidly with W than with V

(c) For $E<V$, if V is doubled, the transmission coefficient will also be doubled.

(d) Sum of the reflection and the transmission coefficients is always one

8. A particle of mass m is placed in a three-dimensional cubic box of side a. What is the degeneracy of its energy level with energy

$$14\left(\frac{\hbar^2\pi^2}{2ma^2}\right)?$$

(Express your answer as an integer)

[JAM]

9. A particle of mass m is in a one dimensional potential $V(x)=\begin{cases}0, & 0<x<L\\ \infty, & \text{otherwise}\end{cases}$.
At some instant its wave function is given by

$$\psi(x)=\frac{1}{\sqrt{3}}\psi_1(x)+i\sqrt{\frac{2}{3}}\psi_2(x)$$

, where $\psi_1(x)$ and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

[JAM]

(a) $\langle x\rangle=\dfrac{L}{2};\langle E\rangle=\dfrac{\hbar^2}{2m}\dfrac{3\pi^2}{L^2}$

(b) $\langle x\rangle=\dfrac{2L}{3};\langle E\rangle=\dfrac{\hbar^2}{2m}\dfrac{\pi^2}{L^2}$

(c) $\langle x\rangle=\dfrac{L}{2};\langle E\rangle=\dfrac{\hbar^2}{2m}\dfrac{8\pi^2}{L^2}$

(d) $\langle x\rangle=\dfrac{2L}{3};\langle E\rangle=\dfrac{\hbar^2}{2m}\dfrac{4\pi^2}{3L^2}$

10. Given the wave function of a particle

$$\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right)0<x<L$$

and 0 elsewhere the probability of finding the particle between $x=0$ and $x=\frac{L}{2}$ is .
(Round off to 1 decimal places)

[JAM]

11. If the ground state energy of a particle in an infinite potential well of width L_1 is equal to the energy of the second excited state in another infinite potential well of width L_2, then the ratio $\frac{L_1}{L_2}$ is equal to

[JAM]

(a) 1
(b) 1/3
(c) $1/\sqrt{3}$
(d) 1/9

12. Consider a particle trapped in a three-dimensional potential well such that $U(x,y,z)=0$ for $0\le x\le a, 0\le y\le a, 0\le z\le a$ and $U(x,y,z)=\infty$ everywhere else. The degeneracy of the 5$^{\text{th}}$ excited state is

[JAM]

(a) 1 (b) 3

(c) 6 (d) 9

13. Consider a quantum particle trapped in a one-dimensional potential well in the region $[-L/2<x<L/2]$, with infinitely high barriers at $x=-L/2$ and $x=L/2$. The stationary wave function for the ground state is

$$\psi(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$$

. The uncertainties in momentum and position satisfy

[JAM]

(a) $\Delta p=\frac{\pi\hbar}{L}$ and $\Delta x=0$

(b) $\Delta p=\frac{2\pi\hbar}{L}$ and $0<\Delta x<\frac{L}{2\sqrt{3}}$

(c) $\Delta p=\frac{\pi\hbar}{L}$ and $\Delta x>\frac{L}{2\sqrt{3}}$

(d) $\Delta p=0$ and $\Delta x=\frac{L}{2}$

14. A particle of mass m is in an infinite square well potential of length L. It is in a superposed state of the first two energy eigenstates, as given by

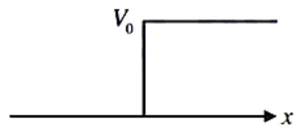
$$\psi(x)=\frac{1}{\sqrt{3}}\left|\psi_{n=1}(x)+\sqrt{\frac{2}{3}}\psi_{n=2}(x)\right.$$

. Identify the correct statement(s). h is Planck's constant. **[JAM]**

(a) $\langle p\rangle=0$ (b) $\Delta p=\frac{\sqrt{3}h}{2L}$

(c) $\langle E\rangle=\frac{3h^2}{8mL^2}$ (d) $\Delta x=0$

15. Consider the motion of a quantum particle of mass m and energy E under the influence of a step potential of height V_0. If R denotes the reflection coefficient, which one of the following statements is true? **[JAM]**



(a) If $E=\frac{4}{3}V_0, R=1$ (b) $E=\frac{4}{3}V_0, R=0$

(c) $E=\frac{1}{2}V_0, R=1$ (d) $E=\frac{1}{2}V_0, R=0.5$

16. A linear operator $\hat{O}$ acts on two orthonormal states of a system ψ_1 and ψ_2 as per following:

$$\hat{O}\psi_1=\psi_2, \hat{O}\psi_2=\frac{1}{\sqrt{2}}(\psi_1+\psi_2)$$

The system is in a superposed state defined by

$$\psi=\frac{1}{\sqrt{2}}\psi_1+\frac{i}{\sqrt{2}}\psi_2$$

The expectation value of $\hat{O}$ in the state ψ is

(a) $\frac{1}{2\sqrt{2}}(1+i(\sqrt{2}+1))$

(b) $\frac{1}{2\sqrt{2}}(1-i(\sqrt{2}+1))$

(c) $\frac{1}{2\sqrt{2}}(1+i(\sqrt{2}-1))$

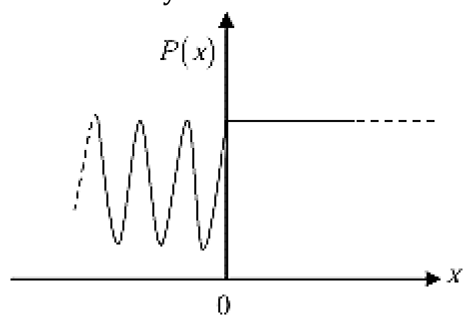
(d) $\frac{1}{2\sqrt{2}}(1-i(\sqrt{2}-1))$

17. For a quantum particle confined inside a cubic box of side L, the ground state energy is given by E_0. The energy of the first excited state is

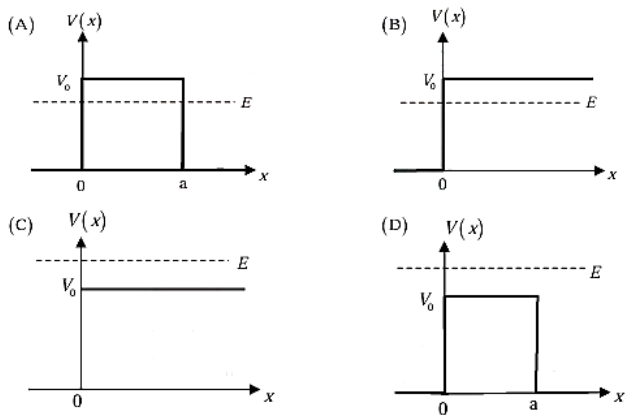
(a) $2E_0$ (b) $\sqrt{2}E_0$

(c) $3E_0$ (d) $6E_0$

18. A particle moving along the x-axis approaches $x=0$ from $x=-\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t\to\infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.



The correct option for the potential $V(x)$ is:

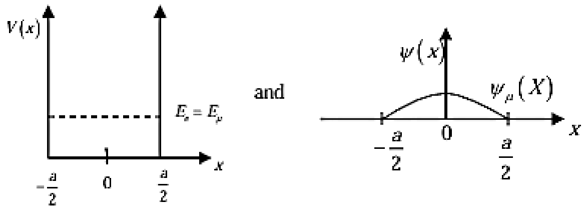


19. A one-dimensional infinite square-well potential is given by:

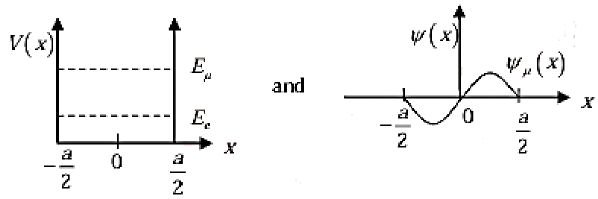
$$V(x) \quad = 0 \text{ for } -\frac{a}{2} < x < +\frac{a}{2}$$
$$\quad\quad = \infty \text{ elsewhere}$$

Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be the corresponding quantities, if a muon (μ) is trapped in the well. Choose the correct option:

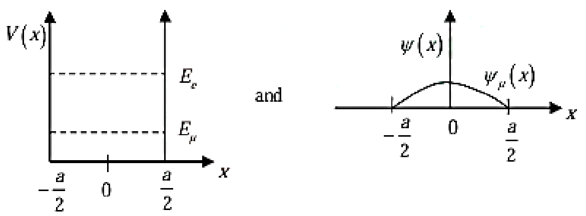
(A)



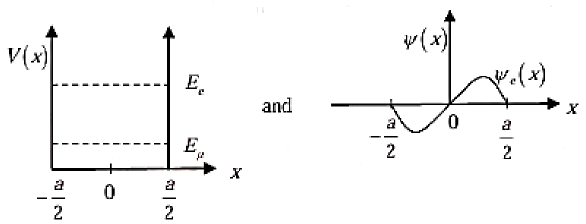
(B)



(C)



(D)



Answer Key				
1. a	2. a	3. b	4. b	5. bc
6. d	7. bd	8. 6	9. a	10. 0.5
11. b	12. c	13. b	14. abc	15. c
16. d	17. a	18. c	19. c	

❖ **CSIR-NET PYQ's**

1. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The wavefunction

$$\Psi=\frac{1}{4}[\Psi_{210}+\sqrt{5}\Psi_{21-1}+\sqrt{10}\Psi'_{213}]$$

is an eigenfunction only of

[CSIR JUNE 2012]

(a) H, L^2 and L_z (b) H and L_2

(c) H and L^2 (d) L^2 and L_z

2. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\bar{r})=zf(r)$ where f(r) is a function of the radial variable r. The eigenvalue of the operator $\hat{L}^2$ (namely the square of the orbital angular momentum) is

[CSIR JUNE 2013]

(a) $\hbar^2/4$ (b) $\hbar^2/2$

(c) $\hbar^2$ (d) $2\hbar^2$

3. If ψ_{mim} denotes the eigenfunction of the Hamiltonian with a potential $V=V(r)$ then the expectation value of the operator $L_x^2+L_y^2$ in the state

$$\psi=\frac{1}{5}[3\psi_{211}+\psi_{210}-\sqrt{15}\psi_{21-1}]$$

is

[CSIR JUNE 2013]

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$

(c) $2\hbar^2$ (d) $26\hbar^2/25$

4. Let ψ_{mim} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of L_z in the state

$$\psi=\frac{1}{6}[\psi_{200}+\sqrt{5}\psi_{210}+\sqrt{10}\psi_{21-1}+\sqrt{20}\psi_{211}]$$

is

[CSIR DEC 2013]

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$

(c) $\hbar$ (d) $\frac{5}{18}\hbar$

5. A particle moves in a potential

$$V=x^2+y^2+\frac{z^2}{2}$$

Which component (s) of the angular momentum is / are constant (s) of motion?

[CSIR DEC 2013]

(a) none (b) L_x, L_y and L_z

(c) only L_x and L_y (d) only L_z

6. The expectation value of the x-component of the orbital angular momentum L_x in the state

$$\psi=\frac{1}{5}[3\psi_{2,1,-1}+\sqrt{5}\psi_{2,1,0}-\sqrt{11}\psi_{2,1,+1}]$$

(where ψ_{ntm} are the eigenfunctions in usual notation), is

[CSIR DEC 2013]

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0

(c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

7. A particle is prepared in a simultaneous eigenstate of L^2 and L_{z^2}. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2\rangle$ of the particle in this state satisfies

[CSIR DEC 2013]

(a) $\langle L_x^2\rangle=0$

(b) $0\le\langle L_x^2\rangle\le\ell^2\hbar^2$

(c) $0\le\langle L_x^2\rangle\le\frac{\ell(\ell+1)\hbar^2}{3}$

(d) $\frac{\ell\hbar^2}{2}\le\langle L_x^2\rangle\le\frac{\ell(\ell+1)\hbar^2}{2}$

8. Consider the normalized wavefunction $\phi=a_1\psi_{11}+a_2\psi_{10}+a_3\psi_{1-1}$ where $\psi_{i\,\mathrm{ms}}$ is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hat{\hbar}$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then

[CSIR DEC 2014]

(a) $a_1=-a_3=\frac{1}{2}, a_2=\frac{1}{\sqrt{2}}$

(b) $a_1 = a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(c) $a_1 = a_3 = \frac{1}{2}, a_2 = -\frac{1}{\sqrt{2}}$

(d) $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

9. If L_i are the components of the angular momentum operator $\vec{L}$, then the operator $\sum i = 1,2,3 \left[[\vec{L}, L_i], L_i\right]$ equals

[CSIR JUNE 2015]

(a) $\vec{L}$ (b) $2\vec{L}$

(c) $3\vec{L}$ (d) $-\vec{L}$

10. Let ψ_{slm} denote the eigenstates of a hydrogen atom in the usual notation. The state $\frac{1}{5}\left[2\psi_{200} - 3\psi_{211} + \sqrt{7}\psi_{210} - \sqrt{5}\psi_{21-1}\right]$ is an eigenstate of

[CSIR DEC 2015]

(a) L^2, but not of the Hamiltonian or L_z

(c) the Hamiltonian, L^2 and L_z

(b) the Hamiltonian, but not of L^2 or L_z

(d) L^2 and L_z, but not of the Hamiltonian

11. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ (where $|l, m\rangle$ denotes an eigenstate of L^2 and L_z) will be a minimum for

[CSIR DEC 2015]

(a) $a = \pm ib$ (b) $a = 0$ and $b = 1$

(c) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (d) $a = \pm b$

12. If $\hat{L}_x, \hat{L}_y$ and $\hat{L}_x$ are the components of the angular momentum operator in three dimensions, the commutator $[\hat{L}_x, \hat{L}_x\hat{L}_y\hat{L}_z]$ may be simplified to

[CSIR JUNE 2016]

(a) $i\hbar L_x(\hat{L}_z^2 - \hat{L}_y^2)$ (b) $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$

(c) $i\hbar L_i(2\hat{L}_i^2 - \hat{L}_y^2)$ (d) 0

13. A particle moving in a central potential is described by a wavefunction $\psi(r) \cong zf(r)$, where $\boldsymbol{r} = (x, y, z)$ is the position vector of the particle and $f(r)$ is a function of $\boldsymbol{r}, \boldsymbol{r}$ |. If $\boldsymbol{L}$ is the total angular momentum of the particle, the value of L^2 must be

[CSIR JUNE 2019]

(a) $2\hbar^2$ (b) $\hbar^2$

(c) $4\hbar^2$ (d) $\frac{3}{4}n^2$

14. The normalized wavefunction of a particle in three-dimensions is given by

$$\psi(x, y, z) = Nz\exp\left[-a(x^2 + y^2 + z^2)\right]$$

where a is a positive constant and N is a normalization constant. If L is the angular momentum operator, the eigenvalues, of L^2 and L_z, respectively, are

[CSIR DEC 2019]

(a) $2\hbar^2$ and $\hbar$ (b) $\hbar^2$ and 0

(c) $2\hbar^2$ and 0 (d) $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$

15. The wavefunction of a particle of mass m, constrained to move on a circle of unit radius centered at the origin in the xy-plane, is described by $\psi(\phi) = A\cos^2\phi$, where ϕ is the azimuthal angle. All the possible outcomes of measurements of the z-component of the angular momentum L_z in this state, in units of h, are

[CSIR DEC 2019]

(a) ± 1 and 0 (b) ± 1

(c) ± 2 (d) ± 2 and 0

16. An angular momentum eigenstate $|j, 0\rangle$ is rotated by an infinitesimally small angle ε about the positive y-axis in the counter clockwise direction. The rotated state, to order ε (upto a normalization constant), is

[CSIR JUNE 2020]

(a) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j, 1\rangle + |j, -1\rangle)$

(b) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j, 1\rangle - |j, -1\rangle)$

(c) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}(|j,1\rangle - |j,-1\rangle)$

(d) $|j,0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}|j,1\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}|j,-1\rangle$

17. If we take the nuclear spin I into account, the total angular momentum is $\vec{F} = \vec{L} + \vec{S} + \vec{I}$, where $\vec{L}$ and $\vec{S}$ are the orbital and spin angular momenta of the electron. The Hamiltonian of the hydrogen atom is corrected by the additional interaction $\lambda\vec{I}\cdot(\vec{L}+\vec{S})$, where $\lambda > 0$ is a constant. The total angular momentum quantum number F of the p - orbital state with the lowest energy is

[CSIR JUNE 2020]

(a) 0 (b) 1

(c) 1/2 (d) 3/2

18. The value of $\langle L_x{}^2\rangle$ in the state $|\varphi\rangle$ for which $L_x{}^2|\varphi\rangle = 6\hbar^2|\varphi\rangle$ and $L_z|\varphi\rangle = 2\hbar|\varphi\rangle$ is

[CSIR JUNE 2023]

(a) 0 (b) $4\hbar^2$

(c) $2\hbar^2$ (d) $\hbar^2$

19. The Schrödinger wave function for a stationary state of an atom in spherical polar coordinates (r,θ,ϕ) is

$$\psi = Af(r)\sin\theta\cos\theta e^{i\phi}$$

where A is the normalization constant. The eigenvalue of $\widehat{L_z}$ for this state is

[CSIR DEC 2023]

(a) $2\hbar$ (b) h

(c) $-2\hbar$ (d) $-\hbar$

20. The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\varphi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\varphi)\chi_+\right].$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}\hbar$. The expectation value of the z component of the total angular momentum in the above state is

[CSIR DEC 2023]

(a) $-\frac{3}{4}\hbar$ (b) $\frac{3}{4}\hbar$

(c) $-\frac{9}{8}\hbar$ (d) $\frac{9}{8}\hbar$

21. The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$, is

$$\hat{H} = \frac{\epsilon}{\hbar^2}\left[(\hat{L}_1+\hat{L}_2)\cdot\hat{L}_2 - (\hat{L}_{1z}+\hat{L}_{2z})^2\right].$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

[CSIR DEC 2023]

(a) $3\epsilon, 2\epsilon, -\epsilon$ (b) $6\epsilon, 5\epsilon, 2\epsilon$

(c) $3\epsilon, 2\epsilon, \epsilon$ (d) $-3\epsilon, -2\epsilon, \epsilon$

22. If $\vec{L}$ is the orbital angular momentum operator and $\vec{\sigma}$ are the Pauli matrices, which of the following operators commutes with $\vec{\sigma}\cdot\vec{L}$?

[CSIR JUNE 2024]

(a) $\vec{L} - \frac{\hbar}{2}\vec{\sigma}$ (b) $\vec{L} + \frac{\hbar}{2}\vec{\sigma}$

(c) $\vec{L} + \hbar\vec{\sigma}$ (d) $\vec{L} - \hbar\vec{\sigma}$

23. A quantum mechanical system is in the angular momentum state $|l = 4, l_z = 4\rangle$. The uncertainty in L_x is **[CSIR JUNE 2024]**

(a) $\hbar\sqrt{2}$ (b) $2\hbar$

(c) 0 (d) $\hbar$

24. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

[CSIR JUNE 2021]

(a) square of the radial position and z-component of angular momentum (r^2 and L_z)

(b) x-components of linear and angular momenta (p_x and L_x)

(c) y-component of position and z-component of angular momentum (y and L_z)

(d) squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

25. The constant B which makes e^{-ax^2} an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Bx^2\right)$ is (a)$4a^2$ (b)0

(c)$2a$ (d)1

❖ **GATE PYQ's**

1. Let $\vec{L} = (L_x, L_y, L_z)$ denote the orbital angular momentum operators of a particle and let $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\hbar^2\ell(\ell+1)$ and $\hbar\ell$ respectively. The expectation value of L_+L_- in this state is

[GATE 2001]

(a) $\ell\hbar^2$ (b) $2\ell\hbar^2$

(c) 0 (d) $\ell\hbar$

2. The Hamiltonian of a particle is given by

$$H = \frac{p^2}{2m} + V(|\vec{r}|) + \phi(+|\vec{r}|)\vec{L}\cdot\vec{S}$$

where $\vec{S}$ is angular momentum. The Hamiltonian does NOT commute with

[GATE 2004]

(a) $\vec{L}+\vec{S}$ (b) $\vec{S}^2$

(c) L_z (d) $\vec{L}^2$

3. The commutator, $[L_z, Y_{lm}(\theta,\phi)]$, where L_z is the z-component of the orbital angular momentum and $Y_{lm}(\theta,\phi)$ is a spherical harmonic, is

[GATE 2005]

(a) $l(l+1)\hbar Y_{lm}(\theta,\phi)$ (b) $-m\hbar Y_{lm}(\theta,\phi)$

(c) $m\hbar Y_{lm}(\theta,\phi)$ (d) $+l\hbar Y_{lm}(\theta,\phi)$

4. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denote the quantum number associated with the total angular momentum and let $\langle\mathfrak{M}\rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B in Bohr magneton)?

[GATE 2008]

(a) $I = 0, \langle m\rangle = 0$

(b) $I = 1/2, \langle m\rangle = 1\mu_B$

(c) $I = 1, \langle m\rangle = 1\mu_B$

(d) $I = 0, \langle m\rangle = 2\mu_B$

5. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutator $[L_xL_y, L_z]$ is

[GATE 2011]

(a) $i\hbar(L_x^2 + L_y^2)$ (b) $2i\hbar L_z$

(c) $i\hbar(L_x^2 - L_y^2)$ (d) zero

6. An atom with one outer electron having orbital angular momentum l is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is

[GATE 2011]

(a) $2l+2$ (b) $2l+1$

(c) $2l$ (d) $2l-1$

7. Which one of the following commutation relations is not correct? Here symbols have their usual meanings.

[GATE 2013]

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$

(c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

8. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum then $\vec{L}\cdot\vec{S}$ does not commute with

[GATE 2014]

(a) S_z (b) L^2

(c) S^2 (d) $(\vec{L}+\vec{S})^2$

9. A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\Psi_{200} - \sqrt{\frac{3}{7}}\Psi_{310} + \sqrt{\frac{4}{21}}\Psi_{321},$$

Where n, l, m in Ψ_{nlm} denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is__________ $\hbar^2$.

[GATE 2014]

10. If $L_+$and $L_-$are angular momentum ladder operators, then the expectation value of $(L_+L_- +$

L_-L_+)in the state $|l=1,m=1\rangle$ of an atom is………. $\hbar^2$.

[GATE 2014]

11. Let $|l,m\rangle$ be the simultaneous eigenstates of L^2 and L_z. Here $\vec{L}$ is the angular momentum operator with Cartesian components (L_x, L_y, L_z), l is the angular momentum quantum number and m is the azimuthal quantum number. The value of $\langle 1,0|(L_x+iL_y)|1,-1\rangle$ is

[GATE 2016]

(a) 0 (b) $\hbar$

(c) $\sqrt{2}\hbar$ (d) $\sqrt{3}\hbar$

12. Particle A with angular momentum $j=\frac{3}{2}$ decays into two particles B and C with angular momenta j_1 and j_2, respectively. If

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle_A = \alpha|1,1\rangle_B \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle_C$$

the value of α is

[GATE 2020]

13. An electron in a hydrogen atom is in the state $n=3, l=2, m=-2$. Let $\hat{L}_y$ denote the y-component of the orbital angular momentum operator. If $(\Delta\hat{L}_y)^2=\alpha\hbar^2$, the value of α is

[GATE 2020]

14. From the pairs of operators given below, identify the ones which commute. Here l and j correspond to the orbital angular momentum and the total angular momentum, respectively.

[GATE 2022]

(a) l^2, j^2 (b) j^2, j_z

(c) j^2, l_z (d) l_z, j_z

15. H is the Hamiltonian, $\vec{L}$ the orbital angular momentum and L_z is the z-component of $\vec{L}$. The $1s$ state of the hydrogen atom in the non-relativistic formalism is an eigen function of which one of the following sets of operators?

[GATE 2023]

(a) H, L^2 and L_z (b) $H, \vec{L}, L^2$ and L_z

(c) L^2 and L_z only (d) H and L_z only

16. The commutator $[L_x, y]$, where L_x is the x-component of the angular momentum operator and y is the y component of the position operator, is equal to **[GATE 2006]**

(a) 0 (b) $i\hbar x$

(c) $i\hbar y$ (d) $i\hbar z$

17. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators; respectively for a particle. The commentator $[L_x, p_y]$ gives

[GATE 2015]

(a) $-i\hbar p_z$ (b) 0

(c) $i\hbar p_x$ (d) $i\hbar p_z$

❖ JEST PYQ's

1. If Jx, Jy, are angular momentum operators, the eigenvalues of the operator $(J_x+J_y)/\hbar$ are

[JEST 2013]

(a) real and discrete with rational spacing

(b) real and discrete with irrational spacing

(c) real and continuous

(d) not all real

2. Suppose a spin 1/2 particle is in the state,

$$|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix}1+i\\2\end{bmatrix}.$$ If S_x(x component of the spin angular momentum operator) is measured what is the probability of getting $+\frac{\hbar}{2}$?

[JEST 2014]

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

3. A spin 1/2 particles is in a state $\frac{(|\uparrow\rangle+|\downarrow\rangle)}{\sqrt{2}}$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstate of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

[JEST 2016]

(a) $\hbar$ (b) $-\hbar$

(c) 0 (d) $\frac{\hbar}{2}$

4. A quantum particle of mass m is moving in a horizontal circular path of radius a. The particle is prepared in a quantum state described by the wave function **[JEST 2018]**

$$\psi = \sqrt{\frac{4}{3\pi}}\cos^2\ \phi,$$

ϕ being the azimuthal angle. If a measurement of the z-component of orbital angular momentum of the particle is carried out, the possible outcomes and the corresponding probabilities are

[JEST 2018]

(a) $L_z = 0, \pm\hbar \pm 2\hbar$ with $P(0) = 1/5, P(\pm\hbar) = 1/5$ and $P(\pm 2h) = 1/5$

(b) $\mathrm{L}_2 = 0$, with $\mathrm{P}(0) = 1$

(c) $L_z = 0, \pm\hbar$ with $\mathrm{P}(0) = 1/3$ and $P(\pm\hbar) = 1/3$

(d) $L_z = 0, \pm 2\hbar$ with $\mathrm{P}(0) = 2/3$ and $P(\pm 2\hbar) = 1/6$

5. A particle in a spherically symmetric potential is known to be in an eigenstate of $\vec{L}^2$ and L_2 with eigenvalues $1(1+1)\hbar^2$ and $m\hbar$, respectively. What is the value of $(1, m|L_x^2|1, m)$?

[JEST 2020]

(a) $\frac{\hbar^2}{2}(1^2+1+m^2)$ (b) $\hbar^2(1^2+1-m^2)$

(c) $\frac{\hbar^2}{2}(1^2+1-m^2)$ (d) $\frac{\hbar^2}{2}(1^2+1-m^2)$

6. The wave function of a particle subjected to a spherically symmetric potential $V(r)$ is given by $\psi(\vec{r}) = (x - y + 2z)f(r)$. Which one of the following statements is true about $\psi(\vec{r})$?

[JEST 2020]

(a) It is an eigenfunction of $\vec{L}^2$ with $1 = 0$

(b) It is an eigenfunction of $\vec{L}^2$ with $l = 1$

(c) It is an eigenfunction of $\vec{L}^2$ with $1 = 2$

(d) It is not an eigenfunction of $\vec{L}^2$

7. If $\vec{L}$ is the angular momentum operator in quantum mechanics, the value of $\vec{L} \times \vec{L}$ will be

[JEST 2021]

(a) 0 (b) $i\hbar\vec{L}$

(c) $|\vec{L}|$ (d) $\hbar\vec{L}$

8. Two classical particles moving in three dimensions interact via the potential
$V = K[(x_1^2 + y_1^2) + (x_2^2 + y_2^2) + (z_1 - z_2)^2]$,
where K is a constant, and (x_1, y_1, z_1) and (x_2, y_2, z_2) are the Cartesian coordinates of the two particles. Let (p_1^x, p_1^y, p_1^z) and (p_2^x, p_2^y, p_2^z) be the components of the linear momenta of the two particles, and (L_1^x, L_1^y, L_1^z) and (L_2^x, L_2^y, L_2^z) the components of the corresponding angular moment(a) Which of the following statements is true?

[JEST 2024]

(a) L_1^z, L_2^z, and $(p_1^z + p_2^z)$ are conserve(d)

(b) L_1^z and L_2^z are not separately conserved but $L_1^z + L_2^z$ is conserve(d)

(c) $(L_1^x + L_2^x), (L_1^y + L_2^y), (L_1^z + L_2^z)$ are conserve(d)

(d) $(L_1^x + L_2^x)$ and $(L_1^y + L_2^y)$ are conserve(d)

9. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with $m = +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1 , what is the state after the measurement

[JEST 2013]

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

10. Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|1, m\rangle$. Let $A = \hat{n} \cdot \vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parametrized in terms of two angles as $(n_x, n_y n_z) = (\sin\ \theta\cos\ \phi, \sin\ \phi, \cos\ \theta)$. The width ΔA in $|l, m\rangle$ state is:

[JEST 2014]

(a) $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\cos\,\theta$

(b) $\sqrt{\frac{l(l+1)-m^2}{2}}\hbar\sin\,\theta$

(c) $\sqrt{l(l+1)-m^2}\hbar\sin\,\theta$

(d) $\sqrt{l(l+1)-m^2}\hbar\cos\,\theta$

11. If $Y_{xy}=\frac{1}{\sqrt{2}}\left(Y_{22}-Y_{2,-2}\right)$ where $Y_{l,m}$ are spherical harmonics, then which of the following is true?

[JEST 2016]

(a) Y_{xy} is an eigen function of both L^2 and L_z

(b) Y_{xy} is an eigen function of L^2 but not L_z

(c) Y_{xy} is an eigen function both of L_z but not L^2

(d) Y_{xy} is not an eigen function of either L^2 and L_z

❖ **TIFR PYQ's**

1. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta,\varphi)=\sqrt{\frac{3}{4\pi}}\sin\,\theta\sin\,\varphi$$

where θ and φ are the usual polar angles. If two successive measurements of L_z are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

[TIFR 2014]

(a) 0.25 (b) 0.33

(c) 0.5 (d) negligible

2. An rigid rotator has the wave function

$$\psi(\theta,\varphi)=N\left[2iY_{1,0}(\theta,\varphi)+(2+i)Y_{2,-1}(\theta,\varphi)+3iY_{1,1}(\theta,\varphi)\right]$$

where $Y_{l,m}(\theta,\varphi)$ are the spherical harmonics, and N is a normalization constant. If $\vec{L}$ is the orbital angular momentum operator, and $L_\pm=L_x\pm iL_y$ the expectation value of $L_+L_-$is

[TIFR 2015]

(a) $21\hbar^2/9$ (b) $23\hbar^2/9$

(c) $25\hbar^2/9$ (d) 0

3. A quantum mechanical plane rotator consists of two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the $x-y$ plane about their centre of mass. Suppose that the initial state of the rotor is given by$\psi(\varphi,t=0)=A\cos^2\varphi$,
where φ is the angle between one mass and the x axis, while A is a normalization constant. Find the expectation value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$.

[TIFR 2016]

4. The wave function of a particle subjected to a three-dimensional spherically symmetric potential $V(r)$ is given by

$$\psi(\vec{x})=(x+y+3z)f(r)$$

The expectation value for the operator $\vec{\boldsymbol{L}}^2$ for this state is **[TIFR 2020]**

(a) $\hbar^2$ (b) $2\hbar^2$

(c) $5\hbar^2$ (d) $11\hbar^2$

5. The momentum operator $i\hbar\frac{d}{dx}$ acts on a wavefunction $\psi(x)$. This operator is Hermitian **[TIFR 2020]**

(a) provided the wavefunction $\psi(x)$ is normalized

(b) provided the wavefunction $\psi(x)$ and derivate $\psi'(x)$ are continuous everywhere

(c) provided the wavefunction $\psi(x)$ vanishes as $x\to\pm\infty$

(d) by its very definition

6. A quantum-mechanical state of a particle, with Cartesian coordinates x,y and z, is described by the normalized wave function

$$\psi(x,y,z)=\frac{\alpha^{5/2}}{\sqrt{\pi}}ze^{-\alpha\sqrt{x^2+y^2+z^2}}$$

For this state what are the angular quantum number ℓ,L^2 and L_Z respectively?

[TIFR 2024]

(a) $2;6\hbar^2;0$ (b) $0;0;0$
(c) $1;2\hbar^2;\hbar$ (d) $1;2\hbar^2;0$

7. In a matrix mechanics formulation, a spin-1 particle has angular momentum components

$$L_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & -1 \\ 1 & \sqrt{2} & 0 \\ -1 & 0 & -\sqrt{2} \end{pmatrix} \quad L_z = \frac{\hbar}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

It follows that $L_y =$ **[TIFR 2022]**

(a) $\frac{\hbar}{2}\begin{pmatrix} 0 & -i & i \\ i & 0 & -i\sqrt{2} \\ -i & i\sqrt{2} & 0 \end{pmatrix}$

(b) $\frac{\hbar}{2}\begin{pmatrix} 0 & i & -i \\ -i & 0 & i\sqrt{2} \\ i & -i\sqrt{2} & 0 \end{pmatrix}$

(c) $\sqrt{2}\hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(d) $\sqrt{2}\hbar\begin{pmatrix} 0 & -\sqrt{2} & 0 \\ -\sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

❖ Answers key				
CSIR-NET				
1. c	2. d	3. d	4. d	5. d
6. a	7. d	8. b	9. b	10. b
11. d	12. a	13. a	14. c	15. d
16. b	17. d	18. d	19. b	20. b
21. a	22. b	23. a	24. c	25. a
GATE				
1. b	2. a	3. c	4. b	5. c
6. a	7. d	8. d	9. 2	10. 2
11. c	12. 1	13. 1	14. abd	15. a
16. d	17. d			
JEST				
1. a	2. c	3. d	4. d	5. d
6. b	7. b	8. a	9. b	10. b
11. b				
TIFR				
1. c	2. b	3. 004	4. b	5. c
6. d	7. b			

❖ **CSIR-NET PYQ's**

1. The energy of the first excited quantum state of a particle in the two-dimensional potential

$$V(x,y)=\frac{1}{2}m\omega^2(x^2+4y^2)$$

is:

[CSIR DEC 2011]

(a) $2\hbar\omega$ (b) $3\hbar\omega$

(c) $\frac{3}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

2. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and first excited state of a one-dimensional harmonic oscillator. The uncertainty Δp in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, is:

[CSIR DEC 2011]

(a) $\Delta p=\sqrt{\hbar m\omega}/2$ (b) $\Delta p=\sqrt{\hbar m\omega/2}$

(c) $\Delta p=\sqrt{\hbar m\omega}$ (d) $\Delta p=\sqrt{2\hbar m\omega}$

3. Two identical bosons of mass m are placed in a one-dimensional potential

$$V(x)=\frac{1}{2}m\omega^2x^2$$

The bosons interact via a weak potential,

$$V_{12}=V_0\exp\left[-m\Omega(x_1-x_2)^2/4\hbar\right]$$

where x_1 and x_2 denote coordinates of the particles. Given that the ground state wavefunction of the harmonic Oscillator is

$$\psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}e^{-\frac{m\omega x^2}{2\hbar}}$$

The ground state energy of the two-boson system, to the first order in V_0, is:

[CSIR JUNE 2013]

(a) $\hbar\omega+2V_0$

(b) $\hbar\omega+\frac{V_0\Omega}{\omega}$

(c) $\hbar\omega+V_0\left(1+\frac{\Omega}{2\omega}\right)^{-1/2}$

(d) $\hbar\omega+V_0\left(1+\frac{\omega}{\Omega}\right)$

4. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator: $|\psi\rangle=b_1|0\rangle+b_2|1\rangle$ where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when

[CSIR JUNE 2013]

(a) $b_2=0,(b_1=1)$ (b) $b_2=\frac{1}{\sqrt{2}}b_1$

(c) $b_2=\frac{1}{2}b_1$ (d) $b_2=-b_1$

5. A particle of mass m in the potential

$$V(x,y)=\frac{1}{2}m\omega^2(4x^2+y^2),$$

is in an eigenstate of energy $E=\frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is

[CSIR JUNE 2014]

(a) $y\exp\left[-\frac{m\omega}{2\hbar}(2x^2+y^2)\right]$

(b) $x\exp\left[-\frac{m\omega}{2\hbar}(2x^2+y^2)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}(x^2+y^2)\right]$

(d) $xy\exp\left[-\frac{m\omega}{2\hbar}(x^2+y^2)\right]$

6. A particle of mass m is in a potential

$$V=\frac{1}{2}m\omega^2x^2,$$

where ω is a constant. Let

$$\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+\frac{\hat{p}\hat{p}}{m\omega}\right)$$

In the Heisenberg picture $\frac{d\hat{a}}{dt}$ is given by

[CSIR JUNE 2015]

(a) $\omega\hat{a}$ (b) $-i\omega\hat{a}$

(c) $\omega\hat{a}^{\dagger\dagger}$ (d) $i\omega\hat{a}^{\dagger}$

7. Let $a=\frac{1}{\sqrt{2}}(x+ip)$ and $a^{\dagger}=\frac{1}{\sqrt{2}}(x-ip)$ be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and h have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle\psi|x|\psi\rangle$ in the state $|\psi\rangle=\exp\left(\lambda a^{\dagger}-\lambda^{*}a\right)|0\rangle$, is

[CSIR DEC 2016]

(a) $|\lambda|$ (b) $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$

(c) $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$ (d) $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

8. If the root-mean-squared momentum of a particle in the ground state of a one-dimensional simple harmonic potential is p_0, then its root-mean-squared momentum in the first excited state is

[CSIR JUNE 2017]

(a) $p_0\sqrt{2}$ (b) $p_0\sqrt{3}$

(c) $p_0\sqrt{2/3}$ (d) $p_0\sqrt{3/2}$

9. The state vector of a one-dimensional simple harmonic oscillator of angular frequency ω, at time $t = 0$, is given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |2\rangle]$$

where $|0\rangle$ and $|2\rangle$ are the normalized ground state and the second excited state, respectively. The minimum time t after which the state vector $|\psi(t)\rangle$ is orthogonal to $|\psi(0)\rangle$, is

[CSIR DEC 2017]

(a) $\frac{\pi}{2\omega}$ (b) $\frac{2\pi}{\omega}$

(c) $\frac{\pi}{\omega}$ (d) $\frac{4\pi}{\omega}$

10. The product $\Delta x \Delta p$ of uncertainties in the position and momentum of a simple larmonic oscillator of mass m and angular frequency ω in the ground state $|0\rangle$, is $\hbar/2$. The value of the product $\Delta x \Delta p$ in the state $e^{-i\vec{p}l/\hbar}|0\rangle$, where l is a constant and $\hat{p}$ is the momentum operator) is

[CSIR DEC 2018]

(a) $\frac{\hbar}{2}\sqrt{\frac{m\omega l^2}{\hbar}}$ (b) $\hbar$

(c) $\frac{\hbar}{2}$ (d) $\frac{\hbar^2}{m\omega l^2}$

11. The ground state energy of an anisotropic harmonic oscillator described by the potential $V(x,y,z) = \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2 + 8m\omega^2z^2$ (in units of $\hbar\omega$) is

[CSIR DEC 2018]

(a) 5/2 (b) 7/2

(c) 3/2 (d) 1/2

12. The operator $A = \sum_{n=0}^{\infty}|n+1\rangle\langle n|$ is defined in terms of the eigenstates $|n\rangle$ of the number operator of the simple
harmonic oscillator. Which of the following relations is obeyed by A and its hermitian conjugate A' ? (In the following 1 is the identity operator).

[CSIR DEC 2019]

(a) $A^\dagger A = 1$ and $AA^\dagger = 1$

(b) $A^\dagger A = 1$ but $AA^\dagger \neq 1$

(c) $A^\dagger A \neq 1$ and $AA^4 = 1$

(d) $A^\dagger A \neq 1$ and $AA^\dagger \neq 1$

13. Let $|n\rangle$ denote the energy eigenstates of a particle in a one-dimensional simple harmonic potential

$$V(x) = \frac{1}{2}m\omega^2x^2$$

If the particle is initially prepared in the state $|\psi(t=0)\rangle = \sqrt{\frac{1}{2}}(|0\rangle + |1\rangle)$, the minimum time after which the oscillator will be found in the same state is

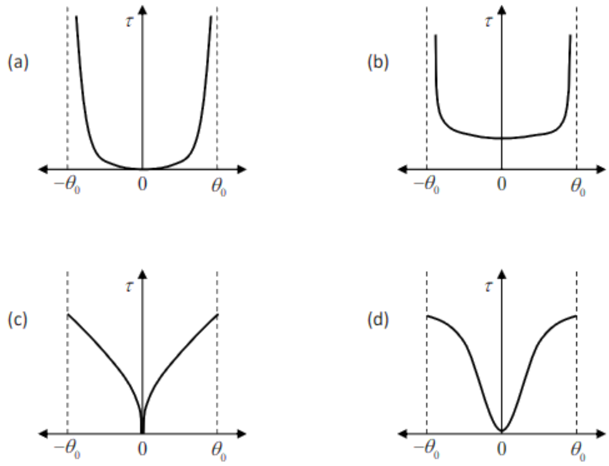
[CSIR JUNE 2020]

(a) $3\pi/(2\omega)$ (b) π/ω

(c) $\pi/(2\omega)$ (d) $2\pi/\omega$

14. A pendulum executes small oscillations between angles $+\theta_0$ and $-\theta_0$. If $\tau(\theta)d\theta$ is the time spent between θ and $\theta + d\theta$, then $\tau(\theta)$ is best represented by

[CSIR JUNE 2020]



15. A particle of mass m in one dimension is in the ground state of a simple harmonic oscillator described by a Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

in the standard notation. An impulsive force at time $t = 0$ suddenly imparts a momentum $p_0 = \sqrt{\hbar m\omega}$ to it. The probability that the particle remains in the original ground state is

[CSIR FEB 2022]

(a) e^{-2} (b) $e^{-3/2}$

(c) e^{-1} (d) $e^{-1/2}$

16. In terms of a complete set of orthonormal basis kets $|n\rangle$, $n = 0, \pm1, \pm2, \cdots$, the Hamiltonian is $H = \sum_n (E|n\rangle\langle n| + \epsilon|n+1\rangle\langle n| + \epsilon|n\rangle\langle n+1|)$ where E and ϵ are constants. The state $|\varphi\rangle = \sum_n e^{in\varphi}|n\rangle$ is an eigenstate with energy

[CSIR SEP 2022]

(a) $E + \epsilon\cos\varphi$ (b) $E - \epsilon\cos$

(c) $E + 2\epsilon\cos\varphi$ (d) $E - 2\epsilon\cos$

17. The Hamiltonian of a two-dimensional quantum harmonic oscillator is

$$H = \frac{p_x{}^2}{2m} + \frac{p_y{}^2}{2m} + \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2$$

where m and ω are positive constants. The degeneracy of the energy level $\frac{27}{2}\hbar\omega$ is

[CSIR JUNE 2023]

(a) 14 (b) 13

(c) 8 (d) 7

18. In a quantum harmonic oscillator problem, $\hat{a}$ and $\hat{N}$ are the annihilation operator and the number operator, respectively. The operator $e^{\hat{N}}\hat{a}e^{-\hat{N}}$ is

[CSIR DEC 2023]

(a) $\hat{a}$ (b) $e^{-1}\hat{a}$

(c) $e^{-(\hat{I}+\hat{a})}$ (d) $e^{\hat{a}}$

(where $\hat{I}$ is the identity operator)

19. The Hamiltonian for a one-dimensional simple harmonic oscillator is given by

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

The harmonic oscillator is in the state

$$|\psi\rangle = \frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle + \lambda e^{i\vartheta}|2\rangle\right)$$

where $|1\rangle$ and $|2\rangle$ are the normalised first and second excited states of the oscillator and λ, ϑ are positive real constants. If the expectation value

$$\langle\psi|x|\psi\rangle = \beta\sqrt{\frac{\hbar}{m\omega}}$$

the value of β is

[CSIR JUNE 2024]

(a) $\frac{1}{\sqrt{2}(1+\lambda^2)}$ (a) $\frac{\sqrt{2}\lambda\cos\vartheta}{1+\lambda^2}$

(c) $\frac{2\lambda\cos\vartheta}{1+\lambda^2}$ (d) $\frac{\lambda^2\cos\vartheta}{1+\lambda^2}$

20. Quantum particles of unit mass, in a potential

$$V(x) = \begin{cases} \frac{1}{2}\omega^2x^2 & x > 0 \\ \infty & x \leq 0 \end{cases}$$

are in equilibrium at a temperature T. Let n_2 and n_3 denote the numbers of the particles in the second and third excited states respectively. The ratio n_2/n_3 is given by

[CSIR JUNE 2024]

(a) $\exp\left(\frac{2\hbar\omega}{k_BT}\right)$ (b) $\exp\left(\frac{\hbar\omega}{k_BT}\right)$

(c) $\exp\left(\frac{3\hbar\omega}{k_BT}\right)$ (d) $\exp\left(\frac{4\hbar\omega}{k_BT}\right)$

21. Let $|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$ (where c_0 and c_1 are constants with $c_0^2 + c_1^2 = 1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of c_0 is the expectation value $\langle x\rangle$ a maximum?

[CSIR JUNE 2024]

(a) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, c_0 = \frac{1}{\sqrt{2}}$

(b) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, c_0 = \frac{1}{2}$

(c) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, c_0 = \frac{1}{\sqrt{2}}$

(d) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, c_0 = \frac{1}{2}$

22. The energy eigenstates of a one-dimensional harmonic oscillator are denoted by $|i\rangle$, where $i = 0,1,2,3$ .... If the momentum operator $\hat{p}$ satisfies $\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|\hat{p}|1\rangle} = \sqrt{2}$, then the value of n is

[CSIR JUNE 2025]

(a)0 (b)1

(c)2 (d)3

23. The Hamiltonian of the 1 -dimensional quantum harmonic oscillator is given by

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

. The expectation value of $[D, H]$ in the ground state, where

$$D = \frac{1}{2\hbar}(xp + px)$$

, is (in units of $\hbar\omega$) **[CSIR JUNE 2025]**

(a)i (b) $\frac{1}{2}$

(c) $\frac{-3i}{2}$ (d)0

24. $|n\rangle$ denotes the eigenvector of the number operator for a particle of mass m in a one-dimensional potential $V = \frac{1}{2}m\omega^2x^2 (n = 0,1,2,\cdots)$. For the state vector

$$|\varphi(x, t = 0)\rangle = \frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle, \langle\hat{x}(t)\rangle$$

is **[CSIR JUNE 2025]**

(a) $\frac{2\sqrt{2}}{3}\sqrt{\frac{\hbar}{2m\omega}}\cos \omega t$ (b) $\frac{4}{3}\sqrt{\frac{\hbar}{2m\omega}}\cos \omega t$

(c) $\frac{2\sqrt{2}}{3}\sqrt{\frac{\hbar}{2m\omega}}\cos 2\omega t$ (d) $\frac{4}{3}\sqrt{\frac{\hbar}{2m\omega}}\cos 2\omega t$

❖ GATE PYQ's

1. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda(a^t a)^2$ acts on the particle, where λ is a constant of suitable dimensions and a and $a^\dagger$ are lowering and raising operators respectively. Then the first order energy shift is given by

[GATE 2001]

(a) λn (b) $\lambda^2 n$

(c) λn^2 (d) $(\lambda n)^2$

2. Consider the harmonic oscillator in the form $H = (p^2 + x^2)/2$ (we have set $m = 1, \omega = 1$ and $\hbar = 1$). The harmonic oscillator is in its nth energy eigenstate and subjected to a time-independent perturbation $\lambda(xp + px)$, for λ real. Calculate the first-order energy shift and first-order correction to the wave function.

[GATE 2001]

3. The wave function of a one-dimensional harmonic oscillator is

$$\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$$

for the ground state $E_0 = \frac{\hbar\omega}{2}$, where $\alpha^2 = \frac{mw}{\hbar}$. In the presence of a perturbing potential of $E_0\left(\frac{\alpha x}{10}\right)^4$, the first order change in the ground state energy is

[GATE 2004]

[Given : $\Gamma(x + 1) = \int_0^\infty t^x \exp(-t)dt$]

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$

(c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)10^{-4}$

4. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants c_1 and c_2 must satisfy the condition

[GATE 2005]

(a) $|c_1| \cdot |c_2| = 1$ (b) $|c_1| + |c_2| = 1$

(c) $(|c_1| + |c_2|)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

5. A one-dimensional harmonic oscillator carrying a charge $-\bar{q}$ is placed in a uniform electric field $\vec{E}$ along the positive x-axis. The corresponding Hamiltonian operator is

[GATE 2005]

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$

(b) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$

(d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

6. The degeneracy of the $n = 2$ level for a three dimensional isotropic oscillator is

[GATE 2005]

(a) 4 (b) 6
(c) 8 (d) 10

7. A one-dimensional harmonic oscillator is in the state

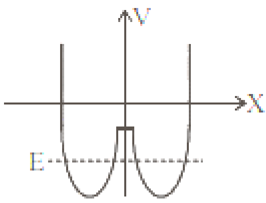
$$\psi(x) = \frac{1}{\sqrt{14}}[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)]$$

where $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is

[GATE 2006]

(a) 0 (b) $\frac{3}{\sqrt{14}}$
(c) $\frac{9}{14}$ (d) 1

8. A particle with energy E is in a time-independent double well potential as shown in the figure.



Which of the following statements about the particle is NOT correct?

[GATE 2007]

(a) The particle will always be in a bound state
(b) The probability of finding the particle in one well will be time-dependent
(c) The particle will be confined to any one of the wells
(d) The particle can tunnel from one well to the other, and back

9. The energy levels of a particle of mass m in a potential of the form

$$V(x) = \infty, x \leq 0$$
$$= \frac{1}{2}m\omega^2 x^2, x > 0$$

are given, in terms of quantum number $n = 0,1,2,3 \ldots$, by

[GATE 2007]

(a) $\left(n + \frac{1}{2}\right)\hbar\omega$ (b) $\left(2n + \frac{1}{2}\right)\hbar\omega$

(c) $\left(2n + \frac{3}{2}\right)\hbar\omega$ (d) $\left(n + \frac{3}{2}\right)\hbar\omega$

10. For a particle of mass m in a one-dimensional harmonic oscillator potential of the form

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

the first excited energy eigenstate is $\psi(x) = xe^{-ax^2}$. The value of a is

[GATE 2007]

(a) $m\omega/4\hbar$ (b) $m\omega/3\hbar$
(c) $m\omega/2\hbar$ (d) $2m\omega/3\hbar$

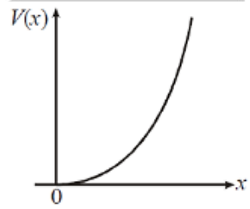
11. An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$. [The SHO has energy levels $E_n(n = 0,1,2,..) andassociated$ normalized wave-functions ψ_n]. Subsequently, an exact measurement of energy E is made. Using the general notation $\Pr(E = E)$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

[GATE 2008]

(a) $\Pr(E = E_0) = 0$
(b) $\Pr(E = E_n) = 1$ for some value of n
(c) $\Pr(E = E_n) \propto \psi_n(x)$
(d) $\Pr(E > E'') > 0$ for any E''

12. A particles of mass m is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & \text{for} \quad x > 0 \\ \infty & \text{for} \quad x \leq 0 \end{cases}$$



Let the wave-function of the particle be given by

$$\Psi(x) = -\frac{1}{\sqrt{5}}\Psi_0 + \frac{2}{\sqrt{5}}\Psi_1$$

where Ψ_0 and Ψ_1 are the eigen functions of the ground state and the first excited state respectively. The expectation value of the energy is

[GATE 2010]

(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$

(c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

Linked Answer Questions

Statement for Linked Answer Question 13 and 14:

In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by

$$\psi_1 = \varphi_0 - 2\varphi_1 + 3\varphi_2$$
$$\psi_2 = \varphi_0 - \varphi_1 + \alpha\varphi_2$$

where α is a constant?

13. The value of α for which ψ_2 is orthogonal to ψ_1 is

[GATE 2011]

(a) 2 (b) 1

(c) -1 (d) -2

14. For the value of α determined in $Q.52$, the expectation value of energy of the oscillator in the state ψ_2 is

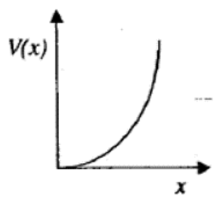
[GATE 2011]

(a) $\hbar\omega$ (b) $3\hbar\omega/2$

(c) $3\hbar\omega$ (d) $9\hbar\omega/2$

15. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is correct?

[GATE 2012]



(a) The parity of the first excited state is even

(b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$

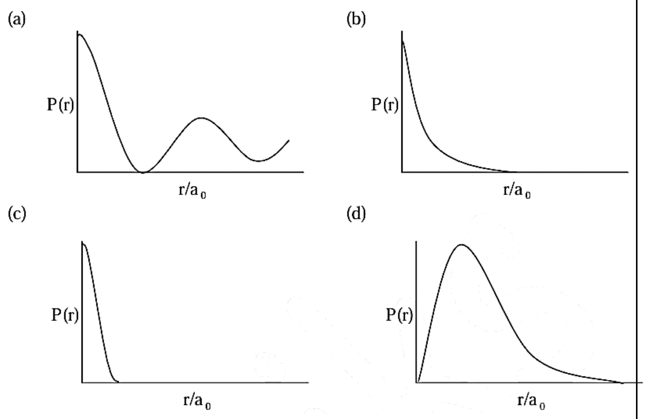
(d) The first excited state energy id $\frac{7}{2}\hbar\omega$

16. The ground state wave function for the hydrogen atom is given by

$$\Psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$$

where a_0 is the Bohr radius. The plot of the radial probability density, $P(r)$ for the hydrogen atom in the ground state is

[GATE 2012]



17. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\Psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is____________ $\hbar\omega$

[GATE 2014]

18. Suppose a linear harmonic oscillator of frequency ω and mass m is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$$

at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\Psi|x|\Psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is

[GATE 2015]

19. The wave function of which orbital is spherically symmetric:

[GATE 2017]

(a) p_x (b) p_y

(c) s (d) d_{xy}

20. The degeneracy of the third energy level of a 3-dimensinal isotropic quantum harmonic oscillator is

[GATE 2017]

(a) 6 (b) 12

(c) 8 (d) 10

21. A one dimensional simple harmonic oscillator with Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

is subjected to a small perturbation, $H_1 = \alpha x + \beta x^3 + \gamma x^4$. The first order correction to the ground state energy is dependent on

[GATE 2017]

(a) only β (b) α and γ
(c) α and β (d) only γ

22. Which one of the following operators is Hermitian?

[GATE 2017]

(a) $i\dfrac{(p_x x^2 - x^2 p_x)}{2}$ (b) $i\dfrac{(p_x x^2 + x^2 p_x)}{2}$

(c) $e^{ip_x a}$ (d) $e^{-ip_x a}$

23. The Hamiltonian for a quantum harmonic oscillator of mass m in three dimensions is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2$$

where ω is the angular frequency. The expectation value of r^2 in the first excited state of the oscillator in units of $\frac{\hbar}{m\omega}$ (rounded off to one decimal place) is

[GATE 2019]

24. Let $|\psi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |\psi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ represent two possible states of a two-level quantum system. The state obtained by the incoherent superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$ is given by a density matrix that is defined as $\rho \equiv c_1|\psi_1\rangle\langle\psi_1| + c_2|\psi_2\rangle\langle\psi_2|$. If $c_1 = 0.4$ and $c_2 = 0.6$, the matrix element ρ_{22} (rounded off to one decimal place) is

[GATE 2019]

25. Let $\hat{a}$ and $\hat{a}^\dagger$, respectively denote the lowering and raising operators of a one-dimensional simple harmonic oscillator. Let $|n\rangle$ be the energy eigenstate of the simple harmonic oscillator. Given that $|n\rangle$ is also an eigen state of $\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}$, the corresponding eigenvalue is

[GATE 2020]

(a) $n(n-1)$ (b) $n(n+1)$
(c) $(n+1)^2$ (d) n^2

26. Consider a state described by $\psi(x,t) = \psi_2(x,t) + \psi_4(x,t)$, where $\psi_2(x,t)$ and $\psi_4(x,t)$ are respectively the second and fourth normalized harmonic oscillator wave functions and ω is the angular frequency of the harmonic oscillator. The wave function $\psi(x, t=0)$ will be orthogonal to $\psi(x,t)$ at time t equal to

[GATE 2021]

(a) $\dfrac{\pi}{2\omega}$ (b) $\dfrac{\pi}{\omega}$

(c) $\dfrac{\pi}{4\omega}$ (d) $\dfrac{\pi}{6\omega}$

27. For a one-dimensional harmonic oscillator, the creation operator $(a^\dagger)$ acting on the n^{th} state $|\psi_n\rangle$ where $n = 0,1,2,\ldots,$ gives

$$a^\dagger|\psi_n\rangle = \sqrt{n+1}|\psi_{n+1}\rangle$$

The matrix representation of the position operator

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$$

for the first three rows and columns is

[GATE 2022]

(a) $\sqrt{\dfrac{\hbar}{2m\omega}}\begin{pmatrix}1 & 0 & 0\\ 0 & \sqrt{2} & 0\\ 0 & 0 & \sqrt{3}\end{pmatrix}$

(b) $\sqrt{\dfrac{\hbar}{2m\omega}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$

(c) $\sqrt{\dfrac{\hbar}{2m\omega}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & \sqrt{2}\\ 0 & \sqrt{2} & 0\end{pmatrix}$

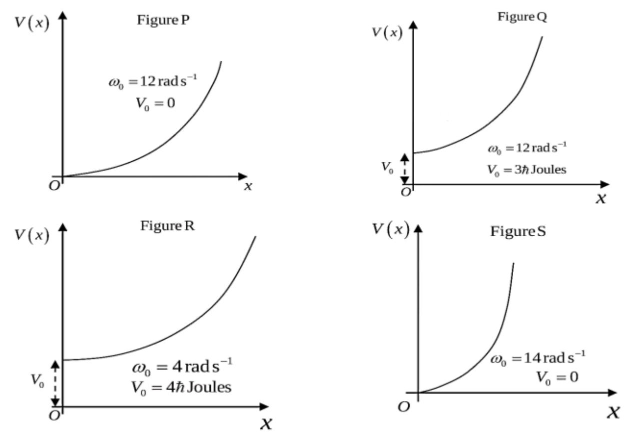
(d) $\sqrt{\dfrac{\hbar}{2m\omega}}\begin{pmatrix}1 & 0 & \sqrt{3}\\ 0 & 0 & 0\\ \sqrt{3} & 0 & 1\end{pmatrix}$

28. A particle of mass m is moving in the potential

$$V(x) = \begin{cases} V_0 + \frac{1}{2}m\omega_0^2 x^2, & x > 0 \\ \infty, & x \le 0 \end{cases}$$

Figures, P, Q, R and S show different combinations of the values of ω_0 and V_0.

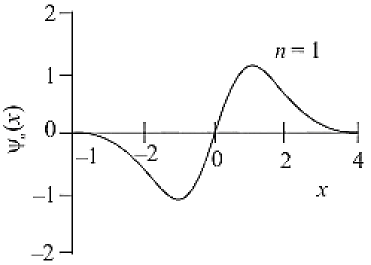
[GATE 2024]



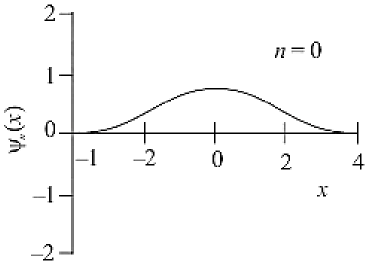
29. If $y_n(x)$ is a solution of the differential equation $y'' - 2xy' - 2ny = 0$ where n is an integer and the prime (') denotes differentiation with respect to x, then acceptable plot(s) of $\psi_n(x) = e^{-x^2} y_n(x)$ is(are)

[GATE 2021]

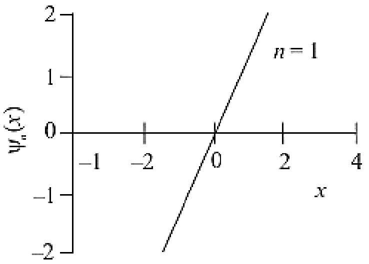
(a)



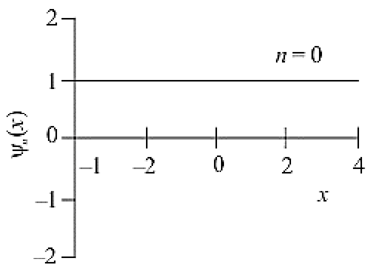
(b)



(c)



(d)



30. The wave function of a spin-less particle of mass m in a one-dimensional potential $V(x)$ is $\psi(x) = A\exp(-\alpha^2 x^2)$ corresponding to an eigenvalue $E_0 = \hbar^2\alpha^2/m$. The potential $V(x)$ is

[GATE 2004]

(a) $2E_0(1 - \alpha^2 x^2)$ (b) $2E_0(1 + \alpha^2 x^2)$

(c) $2E_0\alpha^2 x^2$ (d) $2E_0(1 + 2\alpha^2 x^2$

31. In the first excited state of a one-dimensional harmonic oscillator with angular frequency ω, the energy eigenvalue is given by **[GATE 1997]**

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$

(c) $\frac{3}{2}\hbar\omega$ (d) $2\hbar\omega$

32. Let a particle move in a potential field given by

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

The allowed energies of this particle are

[GATE 2000]

(a) $(n + 1/2)\hbar\omega$ (b) $(2n + 3/2)\hbar\omega$

(c) $(2n + 1/2)\hbar\omega$ (d) $(n + 5/2)\hbar\omega$

33. A particle of mass m is subjected to a potential

$$V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2), -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is………. **[GATE 2014]**

34. The wavefunction for particle is given by the form $e^{-(i\alpha x + \beta)}$, where α and β real constants. In which one of the following potentials $V(x)$, the particle is moving?

[GATE 2024]

(a) $V(x) \propto \alpha^2 x^2$ (b) $V(x) \propto e^{-\alpha x}$

(c) $V(x) = 0$ (d) $V(x) \propto \sin(\alpha x)$

35. A particle of mass m is in a potential $V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and $V(x) = \infty$ for $x \le 0$, where ω is the angular frequency. The ratio of the energies corresponding to the lowest energy level to the next higher level is

[GATE 2025]

(a) $\frac{3}{7}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{3}{5}$

36. Let $|m\rangle$ and $|n\rangle$ denote the energy eigenstates of a one-dimensional simple harmonic oscillator. The position and momentum operators are $\hat{X}$ and $\hat{P}$, respectively. The matrix element $\langle m|\hat{P}\hat{X}|n\rangle$ is non-zero when

[GATE 2025]

(a) $m = n \pm 2$ only

(b) $m = n$ or $m = n \pm 2$

(c) $m = n \pm 3$ only

(d) $m = n \pm 1$ only

❖ **JEST PYQ's**

1. Consider a harmonic oscillator in the state

$$|\psi\rangle = e^{\frac{|\alpha|^2}{2}} e^{\alpha a^{+}} |0\rangle$$

where $|0\rangle$ is the ground state, $a^{+}$is the raising operator and α is a complex number. What is the probability that the harmonic oscillator is in the n-th eigenstate $|n\rangle$?

[JEST 2015]

(a)$e^{-|\alpha^2|}\frac{|\alpha|^{2n}}{n!}$ (b) $e^{-\frac{|\alpha|^2 |\alpha|^n}{2\ \sqrt{n!}}}$

(c)$e^{-|\alpha|^2}\frac{|\alpha|^{n}}{n!}$ (d) $e^{-\frac{|\alpha|^2}{2}}\frac{|\alpha|^{2n}}{n!}$

2. $\phi_0(x)$ and $\phi_1(x)$ are respectively the orthogonal wave functions of the ground and first excited states of a one-dimensional simple harmonic oscillator. Consider the normalized wave function $\psi(x) = c_0\phi_0(x) + c_1\phi_1(x)$, where c_0 and c_1 are real. For what values of c_0 and c_1 will $\langle\psi(x)|x|\psi(x)\rangle$ be maximized?

[JEST 2017]

(a) $c_0 = c_1 = +1\sqrt{2}$

(b) $c_0 = -c_1 = +1\sqrt{2}$

(c) $c_0 = +\sqrt{3}/2, c_1 = +1/2$

(d) $c_0 = +\sqrt{3}/2, c_1 = -1/2$

3. A one-dimensional harmonic oscillator (mass m and frequency ω) is in a state ψ such that the only possible outcomes of an energy measurement are E_0, E_1, or E_2, where E_n is the energy of the n-th excited state. if H is the Hamiltonian of the oscillator, $\langle\psi|H|\psi\rangle = 3\hbar\omega/2$ and $\langle\psi|H^2|\psi\rangle = 11\hbar^2\omega^2/4$, then the probability that the energy measurement yields E_0 is.

[JEST 2018]

(a) 1/2 (b) ¼

(c) 1/8 (d) 0

4. A harmonic oscillator has the following Hamiltonian

$$H_0 = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{x^2}$$

It is perturbed with a potential $V = \lambda\hat{x^4}$. Some of the matrix elements of $\hat{x^2}$ in terms of its expectation value in the ground state are given as follow:

$$\langle 0|\hat{x}^2|0\rangle = C$$
$$\langle 0|\hat{x}^2|2\rangle = \sqrt{2}C$$
$$\langle 1|\hat{x}^2|1\rangle = 3C$$
$$\langle 1|\hat{x}^2|3\rangle = \sqrt{6}C$$

Where $|n\rangle$ is the normalised eigenstate of H_0 corresponding to the eigenvalue $E_n = \hbar\omega(n + 1/2)$. Suppose ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state, respectively. What is the fraction ΔE_1 and ΔE_0

[JEST 2018]

5. A harmonic oscillator has the following Hamiltonian

$$H_0 = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{x^2}$$

It is perturbed with a potential $V = \lambda\hat{x^4}$. Some of the matrix elements of $\hat{x^2}$ in terms of its expectation value in the ground state are given as follow:

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$$\langle 0|\hat{x}^2|2\rangle = \sqrt{2}C$$
$$\langle 1|\hat{x}^2|1\rangle = 3C$$
$$\langle 1|\hat{x}^2|3\rangle = \sqrt{6}C$$

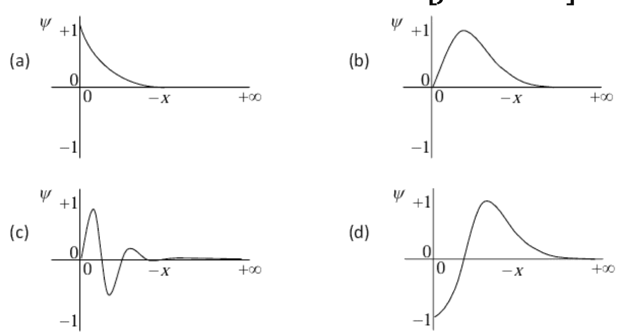
Where $|n\rangle$ is the normalised eigenstate of H_0 corresponding to the eigenvalue $E_n = \hbar\omega(n + 1/2)$. Suppose ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state, respectively. What is the fraction ΔE_1 and ΔE_0 **[JEST 2018]**

6. A quantum particle of mass m is in a one-dimensional potential of the form

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2, & \text{if } x > 0 \\ \infty & \text{if } x \le 0 \end{cases}$$

where ω is a constant. Which one of the following represents the possible ground state wave function of the particle?

[JEST 2019]



7. A one-dimensional harmonic oscillator is in the state

$$|\psi\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} |n\rangle$$

where $|n\rangle$ is the normalized energy eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. Let the expectation value of the Hamiltonian in the state $|\psi\rangle$ be expressed as $\frac{1}{2}\alpha\hbar\omega$. What is the value of α ?

[JEST 2019]

8. Consider the motion of a particle in two dimensions given by the Lagrangian

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{\lambda}{4}(x+y)^2$$

where $\lambda > 0$. The initial conditions are given as $y(0) = 0, x(0) = 42$ meters, $x(0) = y(0) = 0$. What is the value of $x(t) - y(t)$ at $t = 25$ seconds in meters?

[JEST 2019]

9. Consider a quantum particle of mass m moving in a potential

$$V(x,y) = \begin{cases} \frac{1}{2}m\omega^2(x^2+y^2), & \text{for } x>0, y>0 \\ \infty, & \text{otherwise} \end{cases}$$

what is the degeneracy of the energy state $9\hbar\omega$, where $\omega > 0$ measures the strength of the potential?

[JEST 2020]

(a) 4 (b) 2
(c) 10 (d) 5

10. Consider the normalized wave function $\psi = a\psi_0 + b\psi_1$ for a one-dimensional simple harmonic oscillator at some time, where ψ_0 and ψ_1 are the normalized ground state and the first excited state respectively, and a, b are real numbers. For what values of a and b, the magnitude of expectation value of x, i.e. $|\langle x\rangle|$, is maximum?

[JEST 2021]

(a) $a = -b = 1/\sqrt{2}$ (b) $a = b = 1/\sqrt{2}$
(c) $a = 1, b = 0$ (d) $a = 0, b = 1$

11. A particle is in the nth energy eigenstate of an infinite one-dimensional potential well between $x = 0$ and $x = L$. Let P be the probability of finding the particle between $x = 0$ and $x = 1/3$. In the limit $n \to \infty$, the value of P is

[JEST 2021]

(a) 1/9 (b) 2/3
(c) 1/3 (d) $1/\sqrt{3}$

12. The uncertainty Δx in the position of a particle with mass m in the ground state of a harmonic oscillator is $2\hbar/mc$. What is the energy (in units of $10^{-4}\mathrm{mc}^2$) required to excite the system to the first excited state?

[JEST 2021]

13. Consider 5 identical spin $\frac{1}{2}$ particles moving in a 3-dimensional harmonic oscillator potential,

$$V(r) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\omega^2(x^2+y^2+z^2)$$

The degeneracy of the ground state of the system is

[JEST 2022]

(a) 32 (b) 7
(c) 5 (d) 20

14. For a one-dimensional simple harmonic oscillator, for which $|0\rangle$ denotes the ground state, what is the constant β in $\langle 0|e^{ikx}|0\rangle = e^{-\beta\langle 0|x^2|0\rangle}$?

[JEST 2022]

(a) $\beta = 2k^2$ (b) $\beta = k^2$
(c) $\beta = k^2/4$ (d) $\beta = k^2/2$

15. A quantum oscillator with energy levels

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega,\ n = 0,1,2\cdots$$

is in equilibrium at a low enough temperature T so that the occupation of all states with $n \geq 2$ is negligible. What is the mean energy of the oscillator as a function of the inverse temperature $\beta = \left(\frac{1}{k_B T}\right)$?

[JEST 2024]

(a) $\hbar\omega[1-\exp(-\beta\hbar\omega)]$

(b) $\hbar\omega\left[\frac{1}{2}+\frac{1}{1-\exp(\beta\hbar\omega)}\right]$

(c) $\hbar\omega[1+\exp(-\beta\hbar\omega)]$

(d) $\hbar\omega\left[\frac{1}{2}+\frac{1}{1+\exp(\beta\hbar\omega)}\right]$

16. A quantum harmonic oscillator of mass m and angular frequency ω is in the state $|\psi\rangle =$

$\frac{1}{\sqrt{2}}(|287\rangle + |288\rangle)$, where $|n\rangle$ denotes the n^{th} normalized energy eigenstate of the harmonic oscillator. Let

$$L_0 = \sqrt{\frac{\hbar}{m\omega}}$$

denote the oscillator size and $\langle \hat{x} \rangle$ denote the expectation value of the position operator in the state $|\psi\rangle$. What is the value of $\frac{\langle \hat{x} \rangle}{L_0}$? You may use the form of the position operator in terms of the raising and lowering operators:

$$\hat{x} = \frac{L_0}{\sqrt{2}}\left(a + a^{\dagger}\right)$$

[JEST 2024]

17. A quantum mechanical particle in a harmonic oscillator potential has the initial wave function $\Psi_0(x) + \Psi_1(x)$, where $\Psi_0 \& \Psi_1$ are the real wave function in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m = h = \omega = 1$ for oscillator. What is the probability density of finding the particle at x at time $t = \pi$?

[JEST 2013]

(a) $(\Psi_1(x) - \Psi_0(x))^2$

(b) $(\Psi_1(x))^2 - (\Psi_0(x))^2$

(c) $(\Psi_1(x) + \Psi_0(x))^2$

(d) $(\Psi_1(x))^2 + (\Psi_0(x))^2$

18. If $x(t)$ be the position operator at a tie t in the Heisenberg picture for a particle described by the Hamiltonian,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

, what is $e^{i\omega t}\langle |x(t)x(0)|0\rangle$ in units of $\frac{\hbar}{2m\omega}$ where $|0\rangle$ is the ground state?

[JEST 2017]

19. The wave function

$$\psi(x) = A\exp\left(-\frac{b^2x^2}{2}\right)$$

(for real constants A and b) is a normalized eigenfunction of the Schrodinger equation for a particle of mass m and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

[JEST 2019]

(c) $E = \frac{\hbar^2 b^2}{4m}$ (d) $E = \frac{\hbar^2 b^2}{m}$

20. The Hamiltonian of a quantum particle of mass m confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta} - \alpha\right)^2$$

Where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

[JEST 2016]

(a) $\psi_n(\theta) = \frac{e^{\text{in}\,\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$

(b) $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$

(c) $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$

(d) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and

21. A quantum particle is subjected to the potential $V(x) = ax + bx^2$, where a and b are constants. What is the mean position of the particle in the first excited state?

[JEST 2024]

(a) $\frac{a}{b}$ (b) $\frac{a}{2b}$

(c) $-\frac{a}{b}$ (d) $-\frac{a}{2b}$

❖ **TIFR PYQ's**

1. A particle of mass m is placed in the ground state of a one-dimensional harmonic oscillator potential of the form

$$V(x) = \frac{1}{2}kx^2$$

where the stiffness constant k can be varied externally. The ground state wavefunction has the form $\psi(x) \propto \exp\left(-ax^2\sqrt{k}\right)$ where a is a constant. If, suddenly, the parameter k is changed to $4k$, the probability that the particle will remain in the ground state of the new potential is

[TIFR 2011]

(a) 0.47 (b) 0.06

(c) 0.53 (d) 0.67
(e) 0.33 (f) 0.94

2. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

[TIFR 2012]

(a) free particles (b) particles in a box
(c) harmonic oscillators (d) rigid rotators
(e) hydrogenic atoms

3. A harmonic oscillator has the wave function,

$$\psi(x,t) = \frac{1}{5}\left[3\varphi_0(x,t) - 2\sqrt{2}\varphi_1(x,t) + 2\sqrt{2}\varphi_2(x,t)\right]$$

where $\varphi_n(x,t)$ is the eigenfunction belonging to the n-th energy eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. The expectation value $\langle E\rangle$ of energy for the state $\psi(x,t)$ is

[TIFR 2013]

(a) $1.58\hbar\omega$ (b) $0.46\hbar\omega$
(c) $\hbar\omega$ (d) $1.46\hbar\omega$

4. A particle of mass m and charge e is in the ground state of a onedimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

[TIFR 2014]

(a) make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.
(b) make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.
(c) settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.
(d) oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

5. A one-dimensional quantum harmonic oscillator is in its ground state

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

Two experiments, $[A]$ and $[B]$, are performed on the system. In $[A]$, the frequency ω of the oscillator is suddenly doubled, while in [B] the frequency ω is suddenly halved. If p_A and p_B denote the probability in each case that the system is found in its new ground state immediately after the frequency change, which of the following is true?

[TIFR 2015]

(a) $p_A = \sqrt{2}p_B$ (b) $p_A = 2p_B$
(c) $2p_A = p_B$ (d) $p_A = p_B$

6. A two-state quantum system has two observables A and B. It is known that the observable A has eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by

$$|\beta_1\rangle = \frac{3}{5}|\alpha_1\rangle - \frac{4}{5}|\alpha_2\rangle \quad |\beta_2\rangle = \frac{4}{5}|\alpha_1\rangle + \frac{3}{5}|\alpha_2\rangle$$

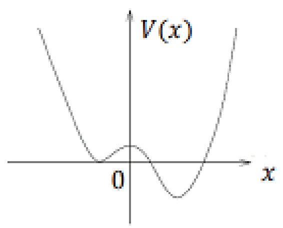
Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

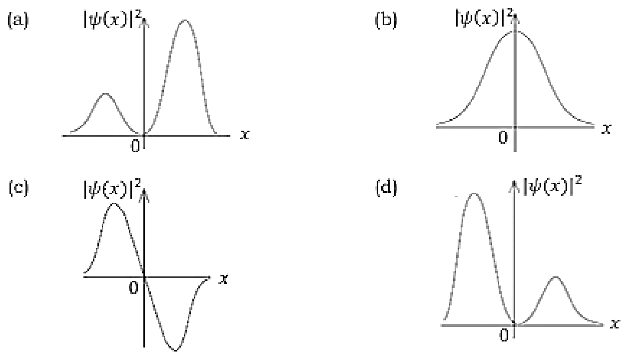
[TIFR 2015]

(a) 0.80 (b) 0.64
(c) 0.60 (d) 0.36

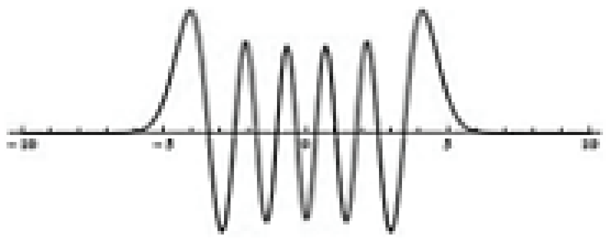
7. A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is

[TIFR 2016]



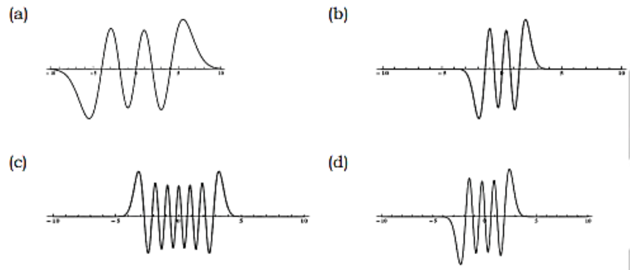


8. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E, as measured from their respective ground state energies. The natural frequency of A is twice that of B.

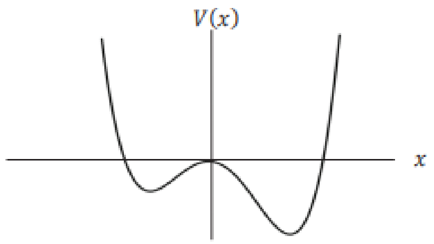


If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A ?

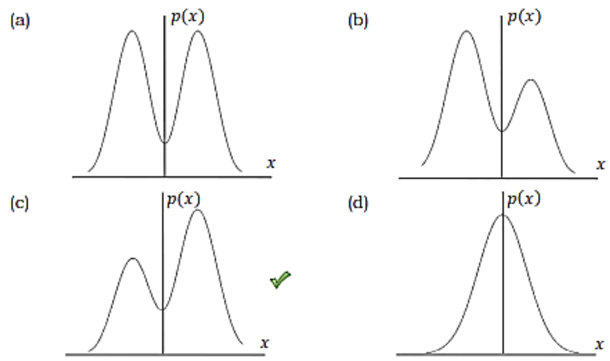
[TIFR 2016]



9. Consider the 1-D asymmetric double-well potential $V(x)$ as sketched below.



The probability distribution $p(x)$ of a particle in the ground state of this potential is best represented by **[TIFR 2017]**



10. A one-dimensional harmonic oscillator of a mass m and natural frequency ω is in the quantum state

$$|\psi\rangle = \frac{1}{3}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$$

at time $t = 0$, where $|n\rangle$ denotes the eigenstate with eigenvalue $\left(n + \frac{1}{2}\right)\hbar\omega$. It follows that the expection value $\langle x\rangle$ of the position operator $\hat{x}$ is

[TIFR 2016]

(a) $x(0)\left[\cos\ \omega t + \frac{1}{\sqrt{3}}\sin\ \omega t\right]$

(b) $x(0)[\cos\ \omega t - \sin\ \omega t]$

(c) $x(0)\left[\cos\ \omega t - \frac{1}{2}\sin\ \omega t\right]$

(d) $x(0)\left[\cos\ \omega t + \frac{1}{2}\sin\ \omega t\right]$

11. A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{4}m\omega^2(5x^2 + 5y^2 + 6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.

[TIFR 2018]

12. At time $t = 0$, the wavefunction of a particle in a harmonic oscillator potential of natural frequency ω is given by

$$\psi(0) = \frac{1}{5}\{3\varphi_0 - 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2\}$$

where $\varphi_n(x)$ denotes the eigenfunction belonging to the n-th eigenvalue of energy. At time $t = \tau$, the wavefunction is found to be

$$\psi(\tau) = -\frac{i}{5}\{3\varphi_0 + 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2\}$$

The minimum value of τ is

[TIFR 2019]

(a) $\frac{\pi}{2\omega}$ (b) $\frac{2\pi}{\omega}$

(c) $\frac{2\pi}{3\omega}$ (d) $\frac{\pi}{\omega}$

13. Three non-interacting particles whose masses are in the ratio 1:4:16 are placed together in the same harmonic oscillator potential $V(x)$.
The degeneracies of the first three energy eigenstates (ordered by increasing energy) will be **[TIFR 2020]**

(a) 1,1,1 (b) 1,1,2

(c) 1,2,1 (d) 1,2,2

14. A particle of mass m is placed in a one-dimensional harmonic oscillator potential

$$V(x) = \frac{1}{2}m\omega^2x^2$$

At $t = 0$, its wavefunction is $\psi(x)$. At $t = 2\pi/\omega$ its wavefunction will be **[TIFR 2020]**

(a) $\psi(x)$ (b) $-\psi(x)$

(c) $-\pi\psi(x)$ (d) $\frac{2\pi}{\omega}\psi(x)$

15. What are the energy eigenvalues for relative motion in one-dimension of a two-body simple quantum harmonic oscillator (each body having mass m) with the following Hamiltonian?

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2(x_1 - x_2)^2$$

[TIFR 2021]

(a) $\left(n + \frac{1}{2}\right)\hbar\omega$ (b) $\sqrt{2}\left(n + \frac{1}{2}\right)\hbar\omega$

(c) $\frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right)\hbar\omega$ (d) $\sqrt{\frac{3}{2}}\left(n + \frac{1}{2}\right)\hbar\omega$

16. Suppose a system is in a normalized state $|\Psi\rangle$ such that $|\Psi\rangle = c(|\varphi_0\rangle + e^{i\theta}|\varphi_1\rangle)$ where $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are the first two normalised eigenstates of a one-dimensional simple harmonic oscillator of frequency ω, and $c > 0$ is a real constant. If the expectation value of the position operator $\hat{x}$ is given by

$$\langle\Psi|\hat{x}|\Psi\rangle = \frac{1}{2}\sqrt{\frac{\hbar}{m\omega}}$$

the value of θ must be

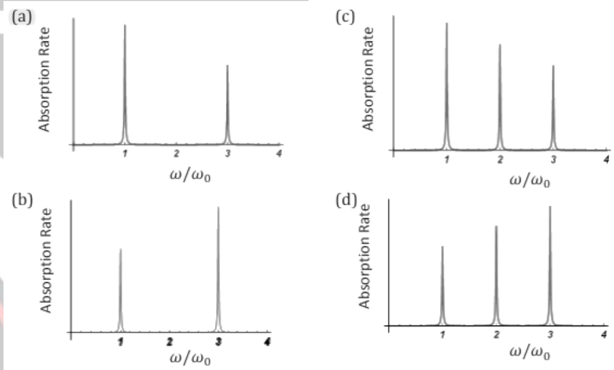
[TIFR 2021]

(a) $3\pi/2$ (b) $\pi/2$

(c) $\pi/4$ (d) π

17. Consider a one-dimensional simple harmonic oscillator with frequency ω_0 in its ground state. An external wave passes through this system, creating a small time-dependent potential of the form $V(x,t) = Ax^3\cos\omega t$, where A and ω are constants. If the absorption rate of the wave is measured as a function of ω, which of the following graphs is the likely result of such a measurement?

[TIFR 2021]



18. A particle of mass m in a three-dimensional potential well has a Hamiltonian of the form

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2y^2 + 2m\omega^2z^2$$

where ω is a constant. If there are two identical spin-1/2 particles in this potential having a total energy the entropy of the system will be
$E = 6\hbar\omega$ **[TIFR 2022]**

(a) $k_B\ln 12$ (b) $k_B\ln 16$

(c) $k_B\ln 14$ (d) $k_B\ln 10$

19. Consider a particle of mass m moving in a one-dimensional potential of the form

$$V(x) = \begin{cases} \frac{1}{2}kx^2 & \text{for } x > 0 \\ \infty & \text{for } x \leq 0 \end{cases}$$

In a quantum mechanical treatment, what is the ground state energy of the particle?

[TIFR 2024]

(a) $\frac{1}{2}\hbar\sqrt{\frac{k}{m}}$ (b) $\frac{3}{2}\hbar\sqrt{\frac{k}{m}}$

(c) $\hbar\sqrt{\frac{k}{m}}$ (d) $\frac{5}{2}\hbar\sqrt{\frac{k}{m}}$

20. The un-normalized energy eigenfunction of a one-dimensional simple quantum harmonic oscillator in dimensionless units ($m=\hbar=\omega=1$) is $\psi_a(x)=(2x^3-3x)e^{-x^2/2}$ Which of the following are two other (un-normalized) eigenfunctions which are closest in energy to ψ_a ?

[TIFR 2024]

(a) $(2x^2-1)e^{-x^2/2}; (4x^4-12x^2+3)e^{-x^2/2}$

(b) $e^{-x^2/2}; (2x^2-1)e^{-x^2/2}$

(c) $xe^{-x^2/2}; (4x^5-20x^3+15x)e^{-x^2/2}$

(d) $(2x^2-1)e^{-x^2/2}; (4x^5+20x^3+15x)e^{-x^2/2}$

21. A two-state quantum system has two observables A and B. It is known that the observable A has eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by

$|\beta_1\rangle=\frac{3}{5}|\alpha_1\rangle-\frac{4}{5}|\alpha_2\rangle\ |\beta_2\rangle=\frac{4}{5}|\alpha_1\rangle+\frac{3}{5}|\alpha_2\rangle$

Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

[TIFR 2015]

(a) 0.80 (b) 0.64

(c) 0.60 (d) 0.36

22. A particle is moving in one dimension under a potential $V(x)$ such that, for large positive values of $x, V(x)\approx kx^\beta$, where $k>0$ and $\beta\geq 1$. If the wavefunction in this region has the form $\psi(x)\sim\exp\left(-x^\lambda\right)$, which of the following is true? **[TIFR 2015]**

(a) $\lambda=\frac{\beta}{2}+1$ (b) $\lambda=\beta$

(c) $\lambda=2\beta-2$ (d) $\lambda=\frac{\beta^2}{2}$

23. A particle of mass m, confined to one dimension x, is in the ground state of a harmonic oscillator potential with a normalized wave function

$$\Psi_0(x)=\left(\frac{2a}{\pi}\right)^{\frac{1}{4}}e^{-ax^2}$$

where $a=m\omega/2\hbar$. Find the expectation value of x^8 in terms of the parameter a. Ans = $105/(256^*a$^4) **[TIFR 2015]**

24. The wave-function Ψ of a particle in a one-dimensional harmonic oscillator potential is given by

$$\Psi=\left(\frac{1}{\pi\ell^2}\right)^{1/4}\left(1+\frac{\sqrt{2}x}{\ell}\right)\exp\left(-\frac{x^2}{2\ell^2}\right)$$

where $\ell=100\mu$m. Find the expectation value of the position x of this particle, in μm.

[TIFR 2018]

25. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wavefunction changes to $\psi(\lambda x,t)$ then the expectation value of kinetic energy T and the potential energy V will change, respectively, to

[TIFR 2018]

(a) λ^2T and V/λ^2 (b) T/λ^2 and V/λ^2

(c) T/λ^2 and λ^2V (d) λ^2T and λ^2V

26. A particle of mass m in a three-dimensional potential well has a Hamiltonian of the form

$$H=\frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{p_z^2}{2m}+\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\omega^2y^2+2m\omega^2z^2$$

where ω is a constant. If there are two identical spin-1/2 particles in this potential having a total energy

the entropy of the system will be

$E=6\hbar\omega$ **[TIFR 2022]**

(a) $k_B\ln\ 14$ (b) $k_B\ln\ 16$

(c) $k_B\ln\ 12$ (d) $k_B\ln\ 10$

27. For a one dimensional quantum harmonic oscillator, at time $t=0$, the particle is in the ground state. What is the expectation value of the position and momentum operator at time t ?

[TIFR 2025]

(a) $\langle x(t)\rangle = \langle p(t)\rangle = 0$

(b) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}}\sin\,\omega t, \langle p(t)\rangle = 0$

(c) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}}\sin\,\omega t, \langle p(t)\rangle = \sqrt{\hbar m\omega}\cos\,\omega t$

(d) $\langle x(t)\rangle = 0, \langle p(t)\rangle = \sqrt{\hbar m\omega}\cos\,\omega t$

❖ Answers key

CSIR-NET				
1. d	2. c	3. c	4. d	5. a
6. b	7. d	8. b	9. a	10. c
11. b	12. b	13. d	14. d	15. d
16. c	17. d	18. b	19. c	20. a
21. **c**	22. d	23. d	24. b	
GATE				
1. c	2.	3. c	4. d	5. c
6. b	7. c	8. b	9. c	10. c
11. a	12. c	13. c	14. d	15. d
16. d	17. 3.25	18. 0	19. c	20. a
21. d	22. a	23. 2.5	24. 0.6	25. a
26. a	27. c	28. bcd	29. b	30. c
31. c	32. b	33. 4	34. c	
JEST				
1. a	2. a	3. b	4. 000	5. 000
6. b	7. 3	8. 42	9. a	10. b
11. c	12. 125	13. d	14. d	15. d
16. 1	17. a	18. 000	19. b	20. a
21. d				
TIFR				
1. F	2. d	3. d	4. b	5. d
6. d	7. a	8. b	9. c	10. a
11.	12. d	13. b	14. b	15. b
16. c	17. a	18. c	19. b	20. a
21. d	22. a	23.	24. 071	25. a
26. a	27. a			

TIFR Q.23- $105/(256\,{}^{*}a^4)$

- **CSIR-NET PYQ's**

1. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{ntm} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by $\psi_{n\ell m}$, where ℓ is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement?

[CSIR JUNE 2011]

(a) There are exactly $(2\ell + 1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

(b) There are $\ell(\ell + 1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

(c) $E_{n\ell m}$ does not depend on ℓ and m for the Coulomb potential.

(d) There is a unique wave function $\psi_{n\ell m}$ and E_{ntm}.

2. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

[CSIR JUNE 2011]

(a) 0.68 (b)0.48

(c) 0.28 (d)0.91

3. Given that the ground state energy of the hydrogen atom is 13.6eV, the ground state energy of positronium (which is a bound state of an electron and a position) is

[CSIR DEC 2011]

(a) +6.8eV (b)6.8eV

(c) −13.6eV (d) −27.2Ev

4. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{21} + 3\psi_{210} + \sqrt{2}\psi^{7}_{21-1}$$

where $\psi_{n/m}$ denotes the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state ψ is:

[CSIR DEC 2012]

(a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$

(c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

5. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to

[CSIR JUNE 2014]

(a) 0.60 (b) 0.90

(c) 0.16 (d) 0.32

6. Suppose that the Coulomb potential of the hydrogen atom is changed by adding an inverse-square term such that the total potential is

$$V(\vec{r}) = -\frac{ze^2}{r} + \frac{g}{r^2}$$

where g is a constant. The energy eigenvalues $E_{m/N}$ in the modified potential

[CSIR JUNE 2016]

(a) depend on n and l, but not on m

(b) depend on n but not on l and m

(c) depend on n and m, but not on l

(d) depend explicitly on all three quantum numbers n, l and m

7. The normalized wavefunction of a particle in three dimensions is given by

$$\psi(r, \theta, \varphi) = \frac{1}{\sqrt{8\pi a^3}} e^{-r/2a}$$

where $a > 0$ is a constant. The ratio of the most probable distance from the origin to the mean distance from the origin, is [You may use $\int_0^\infty dx x^n e^{-x} = n!$].

[CSIR DEC 2017]

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

8. Let the wavefunction of the electron in a hydrogen atom be

$$\psi(\vec{r}) = \frac{1}{\sqrt{6}}\phi_{200}(\vec{r}) + \sqrt{\frac{2}{3}}\phi_{21-1}(\vec{r}) - \frac{1}{\sqrt{6}}\phi_{100}(\vec{r})$$

where $\phi_{nIm}(\vec{r})$ are the eigenstates of the Hamiltonian in the standard notation. The expectation value
of the energy in this state is

[CSIR DEC 2018]

(a) -10.8eV (b) -6.2eV

(c) -9.5eV (d) -5.1eV

9. If the position of the electron in the ground state of a hydrogen atom is measured, the probability that it will be found at a distance $r \geq a_0$ (a_0 being Bohr radius) is nearest to

[CSIR DEC 2018]

(a) 0.91 (b) 0.66

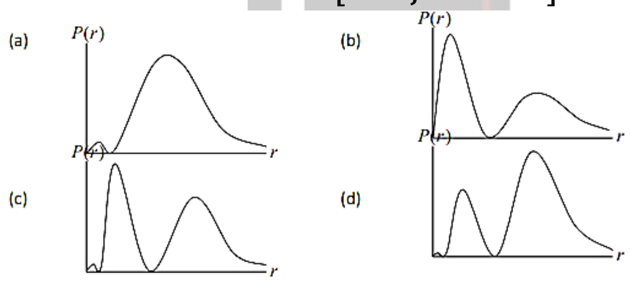
(c) 0.32 (d) 0.13

10. The radial wavefunction of hydrogen atom with the principal quantum number $n = 2$ and the orbital quantum

$$R_{20} = N\left(1 - \frac{r}{2a}\right)e^{-\frac{r}{2a}}$$

where N is the normalized constant. The best schematic representation of the probability density $p(r)$ for the electron to be between r and $r + dr$ is

[CSIR JUNE 2023]



11. A hydrogen atom is in the state

$$|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle$$

where $|\psi_{\text{nlm}}\rangle$ are normalized eigenstates. If $\hat{L}^2$ is measured in this state, the probability of obtaining the value $2\hbar^2$ is

[CSIR JUNE 2024]

(a)$\frac{13}{21}$ (b)$\frac{4}{21}$

(c)$\frac{17}{21}$ (d)$\frac{3}{7}$

❖ GATE PYQ's

1. In spherical coordinates, the wave function describing a state of a system is

$$\psi(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-4/2a_0}\sin\,\theta e^{-i\phi}$$

where a_0 is a constant. Find the parity of the system in this state.

[GATE 2002]

2. In a hydrogen atom, the accidental or Coulomb degeneracy for the $n = 4$ state is

[GATE 2004]

(a) 4 (b) 16

(c) 18 (d) 32

3. In hydro genic states, the probability of finding an electron at $r = 0$ is

[GATE 2006]

(a) zero in state $\phi_{1s}(r)$

(b) non-zero in state $\phi_{1s}(r)$

(c) zero in state $\phi_{2s}(r)$

(d) zero in state $\phi_{2p}(r)$

4. An atomic state of hydrogen is represented by the following wavefunction:

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1 - \frac{r}{2a_0}\right)e^{-r/2a_0}\cos\,\theta$$

where a_0 is a constant. The quantum numbers of the state are

[GATE 2007]

(a) $l = 0, m = 0, n = 1$

(b) $l = 1, m = 1, n = 2$

(c) $l = 1, m = 0, n = 2$

(d) $l = 2, m = 0, n = 3$

5. Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below:

[GATE 2008]

(a) $[L_x, L_y]|\psi_0\rangle = 0$ (b) $J^2|\psi_0\rangle = 0$

(c) $\vec{L}\cdot\vec{S}|\psi_0\rangle \neq 0$ (d) $[S_x, S_y]|\psi_0\rangle = 0$

6. The radial wave function of the electrons in the state of $n = 1$ and $l = 0$ in a hydrogen atom is

$$R_{10} = \frac{2}{a_0^{3/2}} \exp\left(-\frac{r}{a_0}\right), a_0$$

is the Bohr radius. The most probable value of r for an electron is

[GATE 2008]

(a) a_0 (b) $2a_0$

(c) $4a_0$ (d) $8a_0$

7. The normalized ground state wave function of a hydrogen atom is given by

$$\Psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-r/a}$$

where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle \frac{1}{r^2} \right\rangle$ is

[GATE 2011]

(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$

(c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

8. An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

Where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is (Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n+1)!}{\alpha^{n+1}}$)

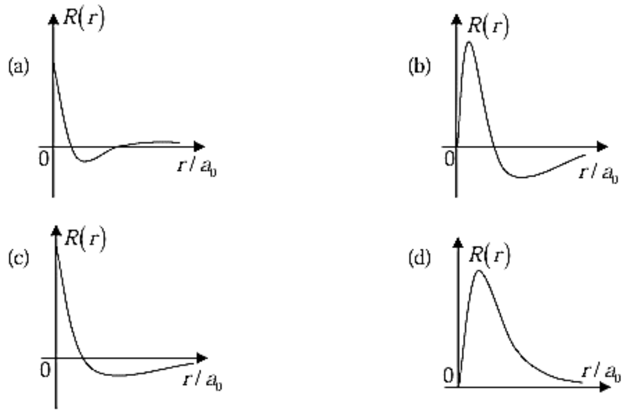
[GATE 2014]

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$

(c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

9. Which one of the following represents the $3p$ radial wave function of hydrogen atom? (a_0 is the Bohr radius)

[GATE 2018]



10. The intrinsic/permanent electric dipole moment in the ground state of hydrogen atom is (a_0 is the Bohr radius)

[GATE 2018]

(a) $-3ea_0$ (b) zero

(c) ea_0 (d) $3ea_0$

11. The radial wave function of a particle in a central potential is give by

$$R(r) = A\frac{r}{a}\exp\left(-\frac{r}{2a}\right)$$

where A is the normalization constant and a is positive constant of suitable dimensions. If γa is the most probable distance of the particle from the force center, the value of γ is

[GATE 2020]

12. The normalized radial wave function of the second excited state of hydrogen atom is

$$R(r) = \frac{1}{\sqrt{24}}\left(a^{-3/2}\right)\frac{r}{a}\left(e^{-r/2a}\right)$$

where a is the Bohr radius and r is the distance from the center of the atom. The distance at which the electron is most likely to be found is $y \times a$. The value of y (in integer) is

[GATE 2021]

13. A system is known to be in a state described by the wave function

$$\psi(\theta, \phi) = \frac{1}{\sqrt{30}}\left(5Y_4^0 + Y_6^0 - 2Y_6^3\right)$$

where $Y_i^m(\theta, \phi)$ are the spherical harmonics the probability of finding the system in a state with $m = 0$ i

(b) zero (b) 2/15

(c) 1/4 (d) 13/15

14. The expectation value of the z-coordinate, $\langle z \rangle$ in the ground state of the hydrogen atom (wavefunction : $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is

[GATE 2005]

(a) a_0 (b) $\frac{a_0}{2}$

(c) $\frac{a_0}{4}$ (d) 0

15. A particle has wavefunction $\psi(x, y, z) = Nze^{-\alpha(x^2+y^2+z^2)}$

where N is a normalization constant and α is a positive constant. In this state, which one of the following options represents the eigenvalues of L^2 and L_z respectively?

Some values of Y_ℓ^m are: $Y_0^0 = \sqrt{\frac{1}{4\pi}}, Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta, Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\phi}$

(a) 0 and 0 (b) $\hbar^2$ and $-\hbar$

(c) $2\hbar^2$ and 0 (d) $\hbar^2$ and $\hbar$

❖ **JEST PYQ's**

1. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been

[JEST 2014]

(a) Less

(b) More

(c) The same

(d) Depends on the electron mass

2. The wave function of a hydrogen atom is given by the following superposition of energy eigen-functions $\psi_{\text{nlu}}(\vec{r})$(n, 1, mare the usual quantum numbers): **[JEST 2016]**

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

(a). $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b). $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$

(c). $\frac{101}{504}$ and $\hbar^2$ (d). $\frac{229}{504}$ and $\hbar^2$

3. If

$$Y_{xy} = \frac{1}{\sqrt{2}}(Y_{22} - Y_{2,-2})$$

where $Y_{l,m}$ are spherical harmonics, then which of the following is true?

[JEST 2016]

(a) Y_{xy} is an eigen function of both L^2 and L_z

(b) Y_{xy} is an eigen function of L^2 but not L_z

(c) Y_{xy} is an eigen function both of L_z but not L^2

(d) Y_{xy} is not an eigen function of either L^2 and L_z

4. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

[JEST 2016]

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) $\frac{3}{2}$

5. What is the binding energy of an electron in the ground state of a He^+ion?

[JEST 2019]

(a) 6.8eV (b) 13.6eV

(c) 27.2eV (d) 54.4Ev

6. Positronium is a short lived bound state of an electron and a positron. The energy difference between the first excited state and ground state of positronium is expected to be around

[JEST 2021]

(a) four times that of the Hydrogen atom

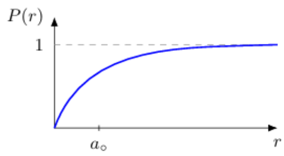
(b) twice that of the Hydrogen atom

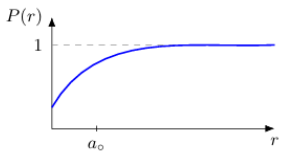
(c) half that of the Hydrogen atom

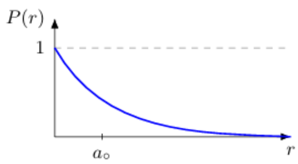
(d) the same as that of the Hydrogen atom

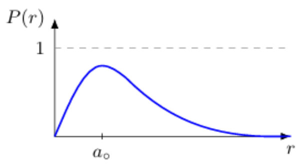
7. The wavefunction of the electron in a Hydrogen atom in a particular state is given by $\pi^{-1/2} a_\circ^{-3/2} \exp(-r/a_\circ)$. Which of the following figures qualitatively depicts the probability ($P(r)$) of the electron to be within a distance r from the nucleus?

[JEST 2022]

A.


B.


C.


D.


8. The radial part of the electronic ground state wave function of the Hydrogen atom is

$$R_{10}(r) = \frac{1}{\sqrt{\pi a_0^3}} \exp\left(-\frac{r}{a_0}\right)$$

where a_0 is the Bohr radius. If $\langle r \rangle$ and r_{mp} denote the expectation value and the maximum probable value of the radial coordinate, respectively, compute $\frac{8}{3}\frac{\langle r \rangle}{r_{mp}}$.

[JEST 2024]

❖ **TIFR PYQ's**

1. The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,\ell,m}(\vec{x})$, where n, ℓ and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\Psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$

The expectation value $\langle E \rangle$ of the energy of this electron, in electron-Volts (eV) will be approximately

[TIFR 2012]

(a) -1.5 (b) -3.7

(c) -13.6 (d) -80.1

(e) +13.6

2. An energy eigenstate of the Hydrogen atom has the wave function

$$\psi_{n\ell m}(r,\theta,\varphi) = \frac{1}{81\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \sin\theta\cos\theta \exp\left[-\left(\frac{r}{3a_0} + i\varphi\right)\right]$$

where a_0 is the Bohr radius. The principal (n), azimuthal (ℓ) and magnetic (m) quantum numbers corresponding to this wave function are

[TIFR 2013]

(a) $n = 3, \ell = 2, m = 1$

(c) $n = 3, \ell = 2, m = -1$

(b) $n = 2, \ell = 1, m = 1$

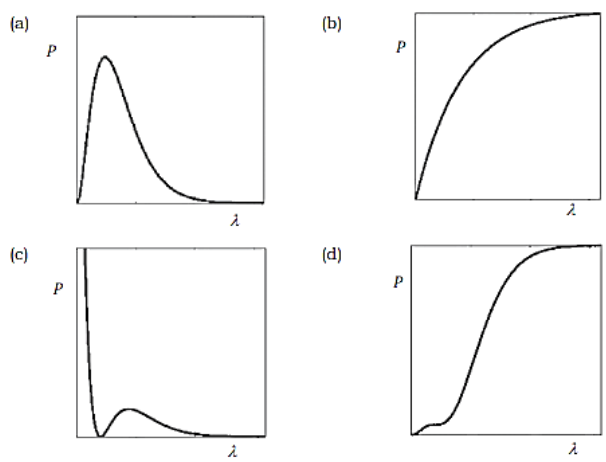
(d) $n = 2, \ell = 1, m = \pm 1$

3. A particle in the $2s$ state of hydrogen has the wave function

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

where r is the radial coordinate w.r.t. the nucleus as origin and a_0 is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius λa_0 centered at the nucleus, is best described by the graph

[TIFR 2014]



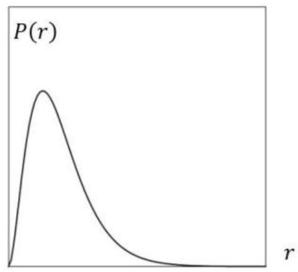
4. An electron is in the $2s$ level of the hydrogen atom, with the radial wave-function

$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

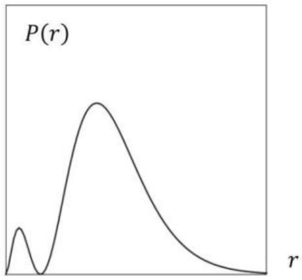
The probability $P(r)$ of finding this electron between distances r to $r+dr$ from the centre is best represented by the sketch

[TIFR 2018]

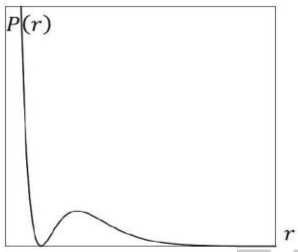
(a)



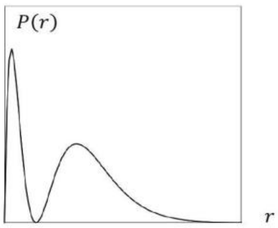
(b)



(c)



(d)



5. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\Psi(\vec{r}) = \frac{1}{\sqrt{10}}\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\psi_{21,-1}(\vec{x})$$

where $\psi_{n\ell m}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number ℓ and magnetic quantum number m.

Neglecting the spin-orbit interaction, the expectation values of $\hat{L}_z$ and $\widehat{L^2}$ for this state are

[TIFR 2019]

(a) $3\hbar/10, 9\hbar^2/5$ (c) $3\hbar/4, 9\hbar^2/25$

(b) $3\hbar/5, 9\hbar^2/10$ (d) $8\hbar/10, 3\hbar^2/5$

6. An electron moves in a hydrogen atom potential in a state | Ψ that has the wave function

$$\Psi(r,\theta,\varphi) = NR_{21}(r)[2iY_1^{-1}(\theta,\varphi) + (2+i)Y_1^0(\theta,\varphi) + 3iY_1^1(\theta,\varphi)]$$

where N is a normalization constant, $R_{nl}(r)$ is the radial wave function and the $Y_l^m(\theta,\varphi)$ are spherical harmonics. The expectation value of $\hat{L}_z$, i.e. the $\hat{z}$-component of the angular momentum operator is

[TIFR 2021]

(a) $\frac{4}{18}\hbar$ (b) $\frac{5}{18}\hbar$

(c) $\frac{9}{18}\hbar$ (d) $\frac{13}{18}\hbar$

❖ Answers key				
CSIR-NET				
1. c	2. a	3. b	4. d	5. d
6. a	7. d	8. d	9. b	10. a
11. a				
GATE				
1. b	2. b	3. acd	4. c	5. a
6. a	7. d	8. d	9. a	10. b
11. 4	12. .4	13. d	14. d	15. c
JEST				
1. a	2. a	3. d	4. b	5. d
6. c	7. a	8. 4		
TIFR				
1. b	2. c	3. d	4. b	5. a
6. b				

❖ **CSIR-NET PYQ's**

1. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}\cdot\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix.
Then the operator $e^{iH/\hbar}$simplifies to

[CSIR JUNE 2011]

(a)$I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\sin\frac{\omega t}{2}$

(b)$I\cos\,\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\sin\,\omega t$

(c)$I\sin\,\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\cos\,\omega t$

(d)$I\sin\,2\omega t + \frac{i\vec{\sigma}\cdot\vec{B}}{B}\cos\,2\omega t$

2. The commutator $[x^2, p^2]$ is

[CSIR JUNE 2012]

(a) 2 in xp (b)$2i\hbar(xp + px)$

(c) $2i\hbar px$ (d) $2i\hbar(xp - px)$

3. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r, θ, φ) ?

[CSIR JUNE 2012]

(a) $-\frac{i\hbar}{\sin^2\theta}\frac{\partial}{\partial\theta}$ (b) $-i\hbar\frac{\partial}{\partial\theta}$

(c) $-\frac{i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\frac{\partial}{\partial\theta}$

4. Given the usual canonical commutation relations, the commutator $[A, B]$ of $A = i(xp_y - yp_x)$ and $\mathrm{B} = (\mathrm{yp_z} + \mathrm{zp_y})$ is:

[CSIR DEC 2012]

(a) $\hbar(\mathrm{xp_z} - \mathrm{p_x z})$ (b) $-\hbar(xp_z - p_x z)$

(c) $h(\mathrm{xp_z} + \mathrm{p_x z})$ (d) $-\hbar(xp_z + p_x z)$

5. If the operators A and B satisfy the commutation relation $[A, B] = I$, where I is the identity operator, then

[CSIR JUNE 2013]

(a) $[e^A, B] = e^A$ (b) $[e^A, B] = [e^B, A]$

(c) $[e^A, B] = [e^{-B}, A]$ (d) $[e^A, B] = 1$

6. If A, B and C are non-zero Hermitian operators, which of the following relations must be false?

[CSIR DEC 2013]

(a) $[A, B] = C$ (b) $AB + BA = C$

(c) $ABA = C$ (d) $A + B = C$

7. Given that

$$\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$$

, the uncertainty Δp_r in the ground state

$$\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

of the hydrogen atom is

[CSIR JUNE 2014]

(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$

(c) $\frac{\hbar}{2a_0}$ (d) $\frac{2h}{a_0}$

8. Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x, p] = i$ in natural units $(\hbar = 1)$. Then the commutator $[x, pe^{-p}]$ is

[CSIR DEC 2014]

(a) $i(1-p)e^{-p}$ (b) $i(1-p^2)e^{-p}$

(c) $i(1-e^{-p})$ (d) ipe^{-p}

9. Suppose the Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is

[CSIR DEC 2014]

(a) $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$ (b) $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$

(c) $-i\hbar\omega x\frac{\partial}{\partial x}$ (d) $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

10. The waveform of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p) = \int \psi(x) e^{-ipx/\hbar} dx$ in the momentum representation. If the action of an operator $\hat{T}$ on $\psi(x)$ is given by $\hat{T}\psi(x) = \psi(x+a)$, where a is a constant, then $\hat{T}\phi(p)$ is given by

[CSIR JUNE 2015]

(a) $-\frac{i}{\hbar} a p \phi(p)$ (b) $e^{-iap/\hbar}\phi(p)$

(c) $e^{+iap/\hbar}\phi(p)$ (d) $\left(1+\frac{i}{\hbar}ap\right)\phi(p)$

11. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter. Then $\frac{d}{dt}\langle V\rangle$, the rate of change of the expression value $\langle V\rangle$ of the potential energy, is

[CSIR JUNE 2015]

(a) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp+px\rangle$

(b) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{2m}\langle p^2\rangle$

(c) $\frac{k}{2m}\langle xp+px\rangle$

(d) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle$

12. Two different sets of orthogonal basis vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ and

$$\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$$

are given for a two-dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

[CSIR JUNE 2015]

(a) $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ (b) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

13. A Hermitian operator $\hat{O}$ has two normalised eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2, respectively. The two states $|u\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u \mid v\rangle = 0$. Which of the following are possible values of θ and ϕ ?

[CSIR DEC 2015]

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$

(d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

14. Consider the operator $a = x + \frac{d}{dx}$ acting on smooth functions of x. The commutator $[a, \cos x]$ is

[CSIR DEC 2016]

(a) $-\sin x$ (b) $\cos x$

(c) $-\cos x$ (d) 0

15. Let

$$a = \frac{1}{\sqrt{2}}(x+ip)$$

and

$$a^\dagger = \frac{1}{\sqrt{2}}(x-ip)$$

be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and h have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle\psi|x|\psi\rangle$ in the state $|\psi\rangle = \exp\left(\lambda a^\dagger - \lambda^* a\right)|0\rangle$, is

[CSIR DEC 2016]

(a) $|\lambda|$ (b) $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$

(c) $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$ (d) $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

16. The Hamiltonian of a two-level quantum system is

$$H = \frac{1}{2}\hbar\omega\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$$

A possible initial state in which the probability of the system being in that quantum state does not change with time, is

[CSIR DEC 2017]

(a) $\begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix}$ (b) $\begin{pmatrix} \cos\frac{\pi}{8} \\ \sin\frac{\pi}{8} \end{pmatrix}$

(c) $\begin{pmatrix} \cos\frac{\pi}{2} \\ \sin\frac{\pi}{2} \end{pmatrix}$ (d) $\begin{pmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{pmatrix}$

17. The product $\Delta x \Delta p$ of uncertainties in the position and momentum of a simple larmonic oscillator of mass m and angular frequency ω in the ground state $|0\rangle$, is $\hbar/2$. The value of the product $\Delta x \Delta p$ in the state $e^{-i\vec{p}l/\hbar}|0\rangle$, where l is a constant and $\hat{p}$ is the momentum operator) is

[CSIR DEC 2018]

(a) $\frac{\hbar}{2}\sqrt{\frac{m\omega l^2}{\hbar}}$ (b) $\hbar$

(c) $\frac{\hbar}{2}$ (d) $\frac{\hbar^2}{m\omega l^2}$

18. Consider the operator $A_x = L_y p_z - L_z p_y$, where L_i and p_i denote, respectively, the components of the angular momentum and momentum operators. The commutator $[A_x, x]$, where x is the x-component of the position operator, is

[CSIR DEC 2018]

(a) $-i\hbar(zp_z + yp_y)$ (b) $-i\hbar(zp_z - yp_y)$

(c) $i\hbar(zp_z + yp_y)$ (d) $i\hbar(zp_z - yp_y)$

19. The operator A has a matrix representation $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ in the basis spanned by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In another basis spanned by

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

the matrix representation of A is

[CSIR JUNE 2019]

(a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$

20. Let $\hat{x}$ and $\hat{p}$ denote position and momentum operators obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. If $|x\rangle$ denotes an eigenstate of $\hat{x}$ corresponding to the eigenvalue x, then $e^{ia\hat{p}/\hbar}|x\rangle$ is

[CSIR DEC 2019]

(a) an eigenstate of $\hat{x}$ corresponding to the eigenvalue x.

(b) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x + a)$.

(c) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x - a)$.

(d) not an eigenstate of $\hat{x}$.

21. A two-state system evolves under the action of the Hamiltonian $H = E_0|A\rangle\langle A| + (E_0 + \Delta)|B\rangle\langle B|$, where $|A\rangle$ and $|B\rangle$ are its two orthonormal states. These states transform to one another under parity i.e., $P|A\rangle = |B\rangle$ and $P|B\rangle = |A\rangle$. If at time $t = 0$ the system is in a state of definite parity $P = 1$, the earliest time t at which the probability of finding the system in a state of parity $P = -1$ is one, is

[CSIR JUNE 2021]

(a) $\frac{\pi\hbar}{2\Delta}$ (b) $\frac{\pi\hbar}{\Delta}$

(c) $\frac{2\pi\hbar}{2\Delta}$ (d) $\frac{2\pi\hbar}{\Delta}$

22. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

[CSIR JUNE 2021]

(a) square of the radial position and z-component of angular momentum (r^2 and L_z)

(b) x-components of linear and angular momenta (p_x and L_x)

(c) y-component of position and z-component of angular momentum (y and L_z)

(d) squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

23. In terms of a complete set of orthonormal basis kets $|\mathrm{n}\rangle, \mathrm{n} = 0, \pm1, \pm2, \cdots$, the Hamiltonian is

$$\mathrm{H} = \sum_{\mathrm{n}} (\mathrm{E}|\mathrm{n}\rangle\langle\mathrm{n}| + \epsilon|\mathrm{n}+1\rangle\langle\mathrm{n}| + \epsilon|\mathrm{n}\rangle\langle\mathrm{n}+1|)$$

where E and ϵ are constants. The state $|\varphi\rangle = \sum_n e^{in\varphi}|n\rangle$ is an eigenstate with energy

[CSIR JUNE 2022]

(a) $E + \epsilon\cos\varphi$ (b) $E - \epsilon\cos$

(c) $E + 2\epsilon\cos\varphi$ (d) $E - 2\epsilon\cos\varphi$

24. The momentum space representation of the Schrödinger equation of a particle in a potential $V(\vec{r})$ is

$$\left(|\mathbf{p}|^2 + \beta\left(\nabla_p^2\right)^2\right)\psi(\mathbf{p},t) = i\hbar\frac{\partial}{\partial t}\psi(\mathbf{p},t)$$

where

$$\left(\boldsymbol{\nabla}_p\right)_i = \frac{\partial}{\partial p_i}$$

, and β is a constant. The potential is (in the following V_0 and a are constants)

[CSIR JUNE 2022]

(a) $V_0 e^{-r^2/a^2}$ (b) $V_0 e^{-r^4/a^4}$

(c) $V_0\left(\frac{r}{a}\right)^2$ (d) $V_0\left(\frac{r}{a}\right)^4$

25. Two operators A and B satisfy the commutation relations $[H,A] = -\hbar\omega B$ and $[H,B] = \hbar\omega A$ where ω is a constant and H is the Hamiltonian of the system. The expectation value $\langle A\rangle_\varphi t = \langle\varphi|A|\varphi\rangle$ in a state φ such that at time $t = 0$ $A_\varphi(0) = 0$ and $B_\varphi(0) = 0$ is

[CSIR JUNE 2023]

(a) $\sin(\omega t)$ (b) $\sinh(\omega t)$

(c) $\cos(\omega t)$ (d) $\cosh(\omega t)$

26. If A and B are hermitian operators and C is an antihermitian operator, then

[CSIR JUNE 2024]

(a) $[[A,B],C]$ is hermitian and $[[A,C],B]$ is anti hermitian

(b) $[[A,B],C]$ and $[[A,C],B]$ are both antihermitian

(c) $[[A,B],C]$ and $[[A,C],B]$ are both Hermitian

(d) $[[A,B],C]$ is antihermitian and $[[A,C],B]$ is Hermitian

27. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,}representsthePoissonbracket, thevalueof $\{A,\{B,C\}\} - \{\{A,B\},C\}$ is given by

[CSIR DEC 2013]

(a) 0 (b) $\{B,\{C,A\}\}$

(c) $\{A,\{C,B\}\}$ (d) $\{\{C,A\},B\}$

28. The operator $A = \sum_{n=0}^{\infty}|n+1\rangle\langle n|$ is defined in terms of the eigenstates $|n\rangle$. of the number operator of the simple Harmonic oscillator. which of the Following is obeyed by A and it's hermitian conjugate A^+?? (I is the Identity operator) **[CSIR DEC 2019]**

(a) $A^+A = 1$ and $AA^+ = I$

(b) $A^+A = 1$ but $AA^+ \neq 1$

(c) $A^+A \neq 1$ and $AA^+ = \hat{I}$

(d) $A^+A \neq I$ and $AA^+ = \hat{I}$

❖ **GATE PYQ's**

1. For any operator A, $i(A^+ - A)$ is

[GATE 2001]

(a) Hermitian (b) anti-Hermitian

(c) unitary (d) orthogonal

2. x and p are two operators which satisfy $[x,p] = i$. The operators X and P are defined as $X = x\cos\phi + p\sin\phi$ and $Y = -x\sin\phi + p\cos\phi$, for ϕ real. Then $[X,Y]$ equals

[GATE 2001]

(a) 1 (b) -1

(c) i (d) $-i$

3. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $[\hat{A},\hat{B}]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $[[\hat{A},\hat{B}],[\hat{B},\hat{A}]]$ is equal to

[GATE 2002]

(a) $\hat{A}\hat{B}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{B}\hat{A}$

(b) $\hat{A}(\hat{A}\hat{B} - \hat{B}\hat{A}) - \hat{B}(\hat{B}\hat{A} - \hat{A}\hat{B})$

(c) zero

(d) $([\hat{A},\hat{B}])^2$

4. A quantum particle is in a state which is the superposition of the eigenstates of the momentum operator $p_x = -i\hbar\frac{\partial}{\partial x}$. If the probability of finding the momentum $\hbar k$ of the particle is 90%, compute its wave function.

[GATE 2002]

5. The wave function of a free particle is given by $\psi(\vec{r}) = Ce^{-(x^2+y^2+z^2)}$, where C is a constant. Compute the momentum space probability density, normalize it to 1 and hence find the value of C.

[GATE 2002]

6. The commutator $[x, P^2]$, where x and P are position and momentum operators respectively, is

[GATE 2003]

(a) $2i\hbar P$

(b) $-i\hbar P$

(c) $2i\hbar xP$

(d) $-2i\hbar xP$

7. The commutator $[L_x, y]$, where L_x is the x-component of the angular momentum operator and y is the y component of the position operator, is equal to

[GATE 2006]

(a) 0

(b) $i\hbar x$

(c) $i\hbar y$

(d) $i\hbar z$

8. If $[x,p] = i\hbar$, the value of $[x^3, p]$ is

[GATE 2007]

(a) $2i\hbar x^2$

(b) $-2i\hbar x^2$

(c) $3i\hbar x^2$

(d) $-3i\hbar x^2$

9. Three operators X, Y and Z satisfy the commutation relations $[X,Y] = i\hbar Z, [Y,Z] = i\hbar X$ and $[Z,X] = i\hbar Y$. The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

[GATE 2007]

(a) $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$

(b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\right\}$

(c) $\left\{0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm\frac{5}{2}, \ldots\right\}$

(d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

10. A finite wave train of an unspecified nature propagates along the positive x axis with a constant speed v and without any change of shape. The differential equation among the four listed below, whose solution it must be, is

[GATE 2008]

(a) $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Psi(x,t) = 0$

(b) $\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Psi(\vec{r},t) = 0$

(c) $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - i\hbar\frac{\partial}{\partial t}\right)\Psi(x,t) = 0$

(d) $\left(\nabla^2 + a\frac{\partial}{\partial t}\right)\Psi(\vec{r},t) = 0$

11. For a physical system, two observables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below:

[GATE 2008]

(a) every eigen state of O_1 must necessarily be an eigen state of O_2

(b) every non-degenerate eigen state of O_1 must necessarily be an eigen state of O_2

(c) when an observation of O_1 is carried out on an arbitrary state $|\Psi\rangle$ of the physical system a subsequent observation of O_2 leads to an unambiguous result

(d) observation of O_1 and O_2 carried out on an arbitrary state $|\Psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observation are made

12. $\hat{A}$ and $\hat{B}$ represent two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ be Hermitian, it is sufficient that

[GATE 2009]

(a) $\hat{B}$ is Hermitian

(b) $\hat{B}$ is anti-Hermitian

(c) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute

(d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ anti-commute

13. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by

[GATE 2011]

(a) $i\hbar\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$

(c) $i\hbar\frac{\partial}{\partial t}$ (d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

14. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators; respectively for a particle. The commentator $[L_x, p_y]$ gives

[GATE 2015]

(a) $-i\hbar p_z$ (b) 0

(c) $i\hbar p_x$ (d) $i\hbar p_z$

15. Which of the following operators is Hermitian?

[GATE 2016]

(a) $\frac{d}{dx}$ (b) $\frac{d^2}{dx^2}$

(c) $i\frac{d^2}{dx^2}$ (d) $\frac{d^3}{dx^3}$

16. If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $[x^2, p^2]$ is

[GATE 2016]

(a) $i\hbar(xp - px)$ (b) $2i\hbar(xp - px)$

(c) $i\hbar(xp + px)$ (d) $2i\hbar(xp + px)$

17. If H is the Hamiltonian for a free particle with mass m, the commutator $[x,[x,H]]$ is

[GATE 2018]

(a) $\hbar^2/m$ (b) $-\hbar^2/m$

(c) $-\hbar^2/(2m)$ (d) $\hbar^2/(2m)$

18. The Hamiltonian operator for a two-level quantum system is $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. If the state of the system at $t = 0$ is given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

then $|\langle\psi(0) \mid \psi(t)\rangle|^2$ at a later time t is

[GATE 2019]

(a) $\frac{1}{2}\left(1 + e^{-(E_1-E_2)t/\hbar}\right)$

(b) $\frac{1}{2}\left(1 - e^{-(E_1-E_2)t/\hbar}\right)$

(c) $\frac{1}{2}(1 + \cos[(E_1 - E_2)t/\hbar])$

(d) $\frac{1}{2}(1 - \cos[(E_1 - E_2)t/\hbar])$

19. In cylindrical coordinates (s, φ, z) which of the following is a Hermitian operator?

[GATE 2022]

(a) $\frac{1}{i}\frac{\partial}{\partial s}$ (b) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$

(c) $\frac{1}{i}\left(\frac{\partial}{\partial s} + \frac{1}{2s}\right)$ (d) $\left(\frac{\partial}{\partial s} + \frac{1}{s}\right)$

❖ **JEST PYQ's**

1. Define $\sigma_x = (f^\dagger + f)$ and $\sigma_y = -i(f^\dagger - f)$, where they σ' are Pauli spin matrices and $f, f^\dagger$ obey anticommutation relation $\{f, f\} = 0, \{f, f^\dagger = 1\}$. Then σ_z is given by

[JEST 2012]

(a) $f^\dagger f - 1$ (b) $2f^\dagger f - 1$

(c) $2f^\dagger f + 1$ (d) $f^\dagger f$

2. The wave function of a free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of

[JEST 2012]

(a) The position operator

(b) The Hamiltonian

(c) The momentum operator

(d) the parity operator

3. The Hermitian conjugate of the operator $\left(-\frac{\partial}{\partial x}\right)$ is

[JEST 2013]

(a) $\frac{\partial}{\partial x}$ (b) $-\frac{\partial}{\partial x}$

(c) $\frac{i\,\partial}{\partial x}$ (d) $-\frac{i\,\partial}{\partial x}$

4. The operator

$$\left(\frac{d}{dx}-x\right)\left(\frac{d}{dx}+x\right)$$

is equivalent to

[JEST 2013]

(a) $\frac{d^2}{dx^2}-x^2$ (b) $\frac{d^2}{dx^2}-x^2+1$

(c) $\frac{d^2}{dx^2}-x\frac{d}{dx}+1$ (d) $\frac{d^2}{dx^2}-2x\frac{d}{dx}-x^2$

5. The operator A and B share all the eigenstates. Then the least possible value of the product of uncertainties $\triangle A \Delta B$ is

[JEST 2014]

(a) $\hbar$

(b) 0

(c) $\frac{\hbar}{2}$

(d) determinant (AB)

6. If a Hamiltonian H is given as $H = |0\rangle\langle 0| - |1\rangle\langle 1| + i(|0\rangle\langle 1| - |1\rangle\langle 0|)$, where $|0\rangle$ and $|1\rangle$ are orthonormal states, the eigenvalues of H are

[JEST 2015]

(a) ± 1 (b) $\pm i$

(c) $\pm\sqrt{2}$ (d) $\pm i\sqrt{2}$

7. The adjoint of a differential operator $\frac{d}{dx}$ acting on a wave function $\psi(x)$ for a quantum mechanical system is:

[JEST 2016]

(a) $\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$

(c) $-\frac{d}{dx}$ (d) $i\hbar\frac{d}{dx}$

8. For operator P and Q, the commutator $[P, Q^{-1}]$ is

[JEST 2016]

(a) $Q^{-1}[P,Q]Q^{-1}$ (b) $-Q^{-1}[P,Q]Q^{-1}$

(c) $Q^{-1}[P,Q]Q$ (d) $-Q[P,Q]Q^{-1}$

9. What is dimension of

$$\frac{\hbar\,\partial\psi}{i\,\partial x}$$

, where ψ is a wave function in two dimensions?

[JEST 2017]

(a) $\mathrm{kgm^{-1}\,s^{-2}}$ (b) $\mathrm{kgm^{-2}}$

(c) $\mathrm{kgm^2\,s^{-2}}$ (d) $\mathrm{kgs^{-1}}$

10. If $x(t)$ be the position operator at a tie t in the Heisenberg picture for a particle described by the Hamiltonian,

$$H = \frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$$

, what is $e^{i\omega t}\langle|x(t)x(0)|0\rangle$ in units of $\frac{\hbar}{2m\omega}$ where $|0\rangle$ is the ground state?

[JEST 2017]

11. If $\psi(x)$ is an infinitely differentiable function, then $\hat{D}\psi(x)$, where the operator

$$\hat{D} = \exp\left(\mathrm{a}x\frac{d}{dx}\right)$$

, is

[JEST 2018]

(a) $\psi(x+a)$ (b) $\psi(ae^a+x)$

(c) $\psi(e^a x)$ (d) $e^a\psi(x)$

12. Consider two canonically conjugate operators $\hat{X}$ and $\hat{Y}$ such that $[\hat{X},\hat{Y}] = i\hbar I$, where I is identity operator. If $\hat{X} = \alpha_{11}\hat{Q}_1+\alpha_{12}\hat{Q}_2, \hat{Y} = \alpha_{21}\hat{Q}_1+\alpha_{22}\hat{Q}_2$, where α_{ij} are complex numbers, and $[\hat{Q}_1,\hat{Q}_2] = zI$, the value of $\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}$ is

[JEST 2018]

(a) $i\hbar z$ (b) $i\hbar/z$

(c) $i\hbar$ (d) Z

13. Consider a 4-dimensional vector space V that is a direct product of two 2-dimensional vector spaces V_1 and V_2. A linear transformation H acting on V is specified by the direct product of linear transformations $\mathrm{H_1}$ and $\mathrm{H_2}$ acting on V_1 and V_2, respectively. In a particular basis,

$$H_1 = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}, H_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$
what is the lowest eigenvalue of H ?

[JEST 2021]

(a) 1 (b) $\frac{3}{2}$

(c) $3-\sqrt{5}$ (d) $\frac{1}{2}(3-\sqrt{5})$

14. If θ and ϕ are respectively the polar and azimuthal angles on the unit sphere, what is $\langle\cos^2(\theta)\rangle$ and $\langle\sin^2(\theta)\rangle$, where $\langle\mathcal{O}\rangle$ denotes the average of $\mathcal{O}$?

[JEST 2022]

(a) $\langle\cos^2(\theta)\rangle = 3/4$ and $\langle\sin^2(\theta)\rangle = 1/4$

(b) $\langle\cos^2(\theta)\rangle = 1/2$ and $\langle\sin^2(\theta)\rangle = 1/2$

(c) $\langle\cos^2(\theta)\rangle = 1/3$ and $\langle\sin^2(\theta)\rangle = 2/3$

(d) $\langle\cos^2(\theta)\rangle = 2/3$ and $\langle\sin^2(\theta)\rangle = 1/3$

15. A and B are 2×2 Hermitian matrices. $|a_1\rangle$ and $|a_2\rangle$ are two linearly independent eigenvectors of A. Consider the following statements:

1.If $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B, then $[A,B]=0$.

2.If $[A,B]=0$, then $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B.Mark the correct option.

[JEST 2024]

(a) Both statements 1 and 2 are true.

(b) Statement 2 is true but statement 1 is false.

(c) Statement 1 is true but statement 2 is false.

(d) Both statements 1 and 2 are false.

❖ TIFR PYQ's

1. Consider a quantum mechanical system with three linear operators $\hat{A}$, $\hat{B}$ and $\hat{C}$, which are related by
$$\hat{A}\hat{B} - \hat{C} = \hat{I}$$
where $\hat{I}$ is the unit operator. If $\hat{A} = d/dx$ and $\hat{B} = x$, then $\hat{C}$ must be

[TIFR 2013]

(a) zero (b) $\frac{d}{dx}$

(c) $-x\frac{d}{dx}$ (d) $x\frac{d}{dx}$

2. It is required to construct the quantum theory of a particle of mass m moving in one dimension x under the influence of a constant force F. The characteristic length-scale in this problem is

[TIFR 2015]

(a) $\frac{\hbar}{mF}$ (b) $\left(\frac{\hbar^2}{mF}\right)^{1/3}$

(c) $\left(\frac{\hbar}{m^2F}\right)^{1/3}$ (d) $\frac{mF}{\hbar^2}$

3. Denote the commutator of two matrices A and B by $[A,B] = AB - BA$ and the anti-commutator by $\{A,B\} = AB + BA$.If $\{A,B\} = 0$, we can write $[A,BC] =$

[TIFR 2017]

(a) $-B[A,C]$ (b) $B\{A,C\}$

(c) $-B\{A,C\}$ (d) $[A,C]B$

4. A system of two spin $-1/2$ particles 1 and 2 has the Hamiltonian $\hat{H} = \epsilon_0 \hat{h}_1 \otimes \hat{h}_2$ where
$$\hat{h}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \hat{h}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
and ϵ_0 is a constant with the dimension of energy. The ground state of this system has energy

[TIFR 2019]

(a) $\sqrt{2}\epsilon_0$ (b) 0

(c) $-2\epsilon_0$ (d) $-4\epsilon_0$

5. Consider $\hat{x}$ and $\hat{p}_x$ as the quantum mechanical position and linear momentum operators with eigenstates $|x\rangle$ and $|p_x\rangle$ and eigenvalues x and p_x, respectively. The eigenvalue of $\hat{x}$ acting on the state $|\psi\rangle = e^{i\hat{p}_x a/2\hbar}|x\rangle$ is

[TIFR 2024]

(a)$x+\frac{a}{2}$ (b) $x-\frac{a}{2}$

(c) $x+a$ (d) $x-a$

6. The momentum operator $i\hbar\frac{d}{dx}$

acts on a wavefunction $\psi(x)$. This operator is Hermitian **[TIFR 2020]**

(a) provided the wavefunction $\psi(x)$ is normalized

(b) provided the wavefunction $\psi(x)$ and derivate $\psi'(x)$ are continuous everywhere

(c) provided the wavefunction $\psi(x)$ vanishes as $x \to \pm\infty$

(d) by its very definition

7. The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be $+1$ is

[TIFR 2013]

(a) $1/\sqrt{2}$ (b) 1

(c) 1/2 (d) ¼

❖ Answers key

CSIR-NET				
1. b	2. b	3. c	4. c	5. a
6. a	7. a	8. a	9. b	10. c
11. a	12. c	13. a	14. a	15. d
16. b	17. c	18. a	19. b	20. c
21. b	22. c	23. c	24. d	25. b
26. b	27. d	28. b		
GATE				
1. a	2. c	3. c	4.	5.
6. a	7. d	8. c	9. c	10. a
11. d	12. c	13. b	14. d	15. b
16. d	17. b	18. c	19. c	
JEST				
1. b	2. d	3. a	4. b	5. b
6. c	7. c	8. b	9. d	10. 0001
11. c	12. b	13. c	14. c	15. c
TIFR				
1. d	2. b	3. c	4. c	5. b
6. c	7. d			

❖ **CSIR-NET PYQ's**

1. If the perturbation $H' = ax$, where a is a constant; is added to the infinite square well potential

$$V(x) = \begin{cases} 0 \text{ for } & 0 \le x \le \pi \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to ground state energy is:

[CSIR JUNE 2011]

(a) $\frac{a\pi}{2}$ (b) $a\pi$

(c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

2. The perturbation $H' = bx^4$, where b is a constant, is added to the one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$. Which of the following denotes the correction to the ground state energy to first order in b ?

[CSIR DEC 2011]

[Hint: The normalized ground state wave function of the one-dimensional harmonic oscillator potential is

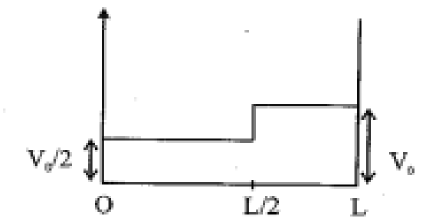
$$\psi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\frac{max^2}{\hbar}}$$

You may use the following integral $\int_{-\infty}^{\infty} x^{2nt} e^{-ax^2} dx = a^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$

(a) $\frac{3b\hbar^2}{4m^2\omega^2}$ (b) $\frac{3b\hbar^2}{2m^2\omega^2}$

(a) $\frac{3b\hbar^2}{2\pi m^2\omega^2}$ (d) $\frac{15b\hbar^2}{4m^2\omega^2}$

3. A constant perturbation as shown in the figure below acts on a particle of mass m confined in a infinite potential well between 0 and L.



The first-order correction to the ground state energy of the particle is

[CSIR DEC 2011]

(a) $\frac{V_0}{2}$ (b) $\frac{3V_0}{4}$

(c) $\frac{V_0}{4}$ (d) $\frac{3V_0}{2}$

4. Consider an electron in a box of lenght L with periodic boundary condition $\psi(x) = \psi(x+L)$. If the electron is in the

$$\psi_k(x) = \frac{1}{\sqrt{L}}e^{jkx}$$

with energy

$$\varepsilon_k = \frac{\hbar^2k^2}{2m},$$

what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x) = V_0\cos\, gx$, where g is an integral multiple of the $2\pi/L$?

[CSIR JUNE 2012]

(a) $V_0^2\varepsilon_g/\varepsilon_k^2$

(b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg}+\frac{1}{g^2-2kg}\right)$

(c) $V_0^2(\varepsilon_k-\varepsilon_g)/\varepsilon_g^2$

(d) $V_0^2/(\varepsilon_k+\varepsilon_g)$

5. The energy eigenvalues of a particle in the potential

$$V(x) = \frac{1}{2}m\omega^2x^2 - ax$$

are

[CSIR DEC 2012]

(a) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$

(b) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega + \frac{a^2}{2m\omega^2}$

(c) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{a^2}{m\omega^2}$

(d) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$

6. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass ' m ' confined in an infinite square well potential

$$V(x)=\begin{cases}0 & -a<x<a\\ \infty & \text{otherwise}\end{cases}$$

The first order correction to the ground state energy of the particle is

[CSIR DEC 2012]

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$

(c) 2ba (d) ba

7. Consider a two-dimensional infinite square will $V(x,y)=\begin{cases}0 & 0<x<a,\ \ 0<y<a\\ \infty & \text{otherwise}\end{cases}$, Its normalized eigenfunctions are

$$\psi_{n_x,n_y}(x,y)=\frac{2}{a}\sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{a}\right)$$

where $n_x, n_y = 1,2,3,$ If a perturbation

$$H'=\begin{cases}V_0 & 0<x<\frac{a}{2},\ 0<y<\frac{a}{2}\\ 0 & \text{otherwise}\end{cases}$$

is applied, then the correction to the energy of the first excited state to order V_0 is

[CSIR JUNE 2013]

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}\left[1\pm\frac{64}{9\pi^2}\right]$

(c) $\frac{V_0}{4}\left[1\pm\frac{16}{9\pi^2}\right]$ (d) $\frac{V_0}{4}\left[1\pm\frac{32}{9\pi^2}\right]$

8. The motion of a particle of mass m in one dimension is described by the Hamiltonian

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda x$$

. What is the difference between the (quantized) encrgies

of the first two levels? (In the following, $\langle x\rangle$ is the expectation value of x in the ground state)

[CSIR DEC 2013]

(a) $\hbar\omega-\lambda\langle x\rangle$ (b) $\hbar\omega+\lambda\langle x\rangle$

(c) $\hbar\omega+\frac{\lambda^2}{2m\omega^2}$ (d) $\hbar\omega$

9. A perturbation $V_{pert}=a\,L^2$ is added to the Hydrogen atom potential. The shift in the energy level of the 2P state, when the effects of spin are neglected up to second order in a, is

[CSIR DEC 2013]

(a) 0 (b) $2a\hbar^2+a^2\hbar^4$

(c) $2a\hbar^2$ (d) $a\hbar^2+\frac{3}{2}a^2\hbar^4$

10. The ground state eigenfunction for the potential $V(x)=-\delta(x)$, where $\delta(x)$ is the delta function, is given by $\psi(x)=Ae^{-\alpha|x|}$, where A and $\alpha>0$ are constants. If a perturbation $H'=bx^2$ is applied, the first order correction to the energy of the ground state will be

[CSIR JUNE 2014]

(a) $\frac{b}{\sqrt{2}\alpha^2}$ (b) $\frac{b}{\alpha^2}$

(c) $\frac{2b}{\alpha^2}$ (d) $\frac{b}{2\alpha^2}$

11. The Hamiltonian H_0 for a three-state quantum system is given by the matrix $H_0=\begin{pmatrix}1&0&0\\0&2&0\\0&0&2\end{pmatrix}$.When perturbed by $H'=\epsilon\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$ where ϵ&1, the resulting shift in the energy eigenvalue $E_0=2$ is

[CSIR DEC 2014]

(a) $\epsilon,-2\in$ (b) $-\epsilon,2\in$

(c) $\pm\epsilon$ (d) $\pm2\in$

12. The ground state energy of a particle of mass m in the potential $V(x)=V_0\cosh\left(\frac{x}{L}\right)$, where L and V_0 are constants (with $V_0\gg\frac{\hbar^2}{2mL^2}$ is approximately

[CSIR DEC 2015]

(a) $V_0+\frac{\hbar}{L}\sqrt{\frac{2V_0}{m}}$ (b) $V_0+\frac{\hbar}{L}\sqrt{\frac{V_0}{m}}$

(c) $V_0+\frac{\hbar}{4L}\sqrt{\frac{V_0}{m}}$ (d) $V_0+\frac{\hbar}{2L}\sqrt{\frac{V_0}{m}}$

13. A hydrogen atom is subjected to the perturbation $V_{per}(r)=\in\cos 2r/a_0$ where a_0 is the Bohr radius. The change in the ground state energy to first order in ϵ is

[CSIR DEC 2015]

(a) $\in/4$ (b) $\in/2$

(c) $-\epsilon/2$ (d) $-\epsilon/4$

14. Consider a particle of mass m in a potential

$$V(x) = \frac{1}{2}m\omega^2x^2 + g\cos\, kx$$

The change in the ground state energy, compared to the simple harmonic potential $\frac{1}{2}m\omega^2x^2$, to first order in g is

[CSIR JUNE 2016]

(a) $g\exp\left(-\frac{k^2\hbar}{2m\omega}\right)$ (b) $g\exp\left(\frac{k^2\hbar}{2m\omega}\right)$

(c) $g\exp\left(-\frac{2k^2\hbar}{m\omega}\right)$ (d) $g\exp\left(-\frac{k^2\hbar}{4m\omega}\right)$

15. A particle of charge q in one dimension is in a simple harmonic potential with angular frequency ω. It is subjected to a time dependent electric ficld $E(t) = Ae^{-(t/\tau)^2}$, where A and τ are positive constants and $\omega\tau \gg 1$. If in the distant past $t \to -\infty$ the particle was in its ground state, the probability that it will be in the first excited state as $t \to +\infty$ is proportional to

[CSIR DEC 2016]

(a) $e^{-\frac{1}{2}(\omega\tau)^2}$ (b) $e^{\frac{1}{2}(\omega\tau)^2}$

(c) 0 (d) $\frac{1}{(\omega\tau)^2}$

16. A constant perturbation H' is applied to a system for time Δt (where $H'\Delta t \ll \hbar$) leading to a transition from a state with energy E_i to another with energy E_f. If the time of application is doubled, the probability of transition will be

[CSIR JUNE 2017]

(a) unchanged (b) doubled

(c) quadrupled (d) halved

17. Two identical bosons of mass m are placed in a one-dimensional potential

$$V(x) = \frac{1}{2}m\omega^2x^2$$

The bosons interact via a weak potential

$$V_{12} = V_0\exp\left[-m\Omega(x_1 - x_2)^2/4\hbar\right]$$

where x_1 and x_2 denote coordinates of the particles. Given that the ground state wavefuntion of the harmonic oscillator is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{\frac{max^2}{2\hbar}}$$

. The ground state energy of the two-bosons system, to the first order in V_0 is

[CSIR June 2013]

(c) $\hbar\omega + V_0\left(1 + \frac{\Omega}{2\omega}\right)^{-1/2}$ (d) $\hbar\omega + V_0\left(1 + \frac{\omega}{\Omega}\right)$

18. The Coulomb potential $V(r) = -e^2/r$ of a hydrogen atom is perturbed by adding $H' = bx^2$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wavefunction is

$$\psi_0 = \frac{1}{\sqrt{\pi a_0^3}}e^{-\pi/40}$$

[CSIR JUNE 2017]

(a) $2ba_0^2$ (b) ba_0^2

(c) $ba_0^2/2$ (d) $\sqrt{2}m_{10}$

19. In the usual notation $|\text{nlm}\rangle$ for the states of a hydrogen like atom, consider the spontaneous transitions $|210\rangle \to |100\rangle$ and $|310\rangle \to |100\rangle$. If t_1 and t_2 are the lifetimes of the first and the second decaying states respectively, then the ratio t_1/t_2 is proportional to

[CSIR JUNE 2017]

(a) $\left(\frac{32}{27}\right)^3$ (b) $\left(\frac{27}{32}\right)^3$

(c) $\left(\frac{2}{3}\right)^3$ (d) $\left(\frac{3}{2}\right)^3$

20. Consider a one-dimensional infinite square well

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 < x < a, \\ \infty & \text{otherwise} \end{cases}$$

If a perturbation

$$\Delta V(x) = \begin{cases} V_0 & \text{for} \quad 0 < x < \frac{a}{3}, \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state, to first order in ΔV, is nearest to

[CSIR DEC 2017]

(a) V_0 (b) $0.16V_0$

(c) $0.2V_0$ (d) $0.33V_0$

21. A particle of mass m is constrained to move in a circular ring to radius R. When a perturbation

$$V' = \frac{a}{R^2}\cos^2\,\phi$$

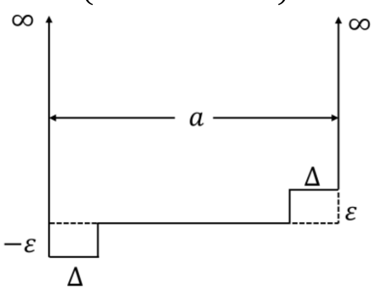
(where a is a real constant) is added, the shift in energy of the ground state, to first ordor in a, is

[CSIR JUNE 2018]

(a) a/R^2 (b) $2a/R^2$

(c) $a/(2R^2)$ (d) $d/(\pi R^2)$

22. The infinite square-well potential of a particle in a box of size a is modified as shown in the figure below (assume $\Delta \ll a$)



The energy of the ground state, compared to the ground state energy before the perturbation was added

[CSIR JUNE 2019]

(a) increases by a term of order ε

(b) decreases by a term of order ε

(c) increases by a term of order ε^2

(d) decreases by a term of order ε^2

23. A charged, spin-less particle of mass m is subjected to an attractive potential

$$V(x, y, z) = \frac{1}{2}k(x^2 + y^2 + z^2)$$

where k is a positive constant. Now a perturbation in the form of a weak magnetic field $\mathrm{B} = B_0\hat{k}$ (where B_0 is a constant) is switched on. Into how many distinct levels will the second excited state of the unperturbed Hamiltonian split?

[CSIR JUNE 2019]

(a) 5 (b) 4

(c) 2 (d) 1

24. A quantum particle in a one-dimensional infinite potential well, with boundaries at 0 and a, is perturbed by adding $H' = \in \delta\left(x - \frac{a}{2}\right)$ to the initial Hamiltonian. The correction to the energies of the ground and the first excited states (to first order in $\in$) are respectively

[CSIR JUNE 2020]

(a) 0 and 0 (b) $2 \in/a$ and 0

(c) 0 and $2 \in/a$ (d) $2 \in/a$ and $2/a$

25. The energies of a two-state quantum system are E_0 and $E_0 + \alpha\hbar$, (where $\alpha > 0$ is a constant) and the corresponding normalized state vectors are $|0\rangle$ and $|1\rangle$, respectively. At time $t = 0$, when the system is in the state $|0\rangle$, the potential is altered by a time independent term V such that

$$\langle 1|V|0\rangle = \frac{\hbar\alpha}{10}$$

The transition probability to the state $|1\rangle$ at times $t << \frac{1}{\alpha}$, is

[CSIR JUNE 2021]

(a) $\frac{\alpha^2 t^2}{25}$ (b) $\frac{\alpha^2 t^2}{50}$

(c) $\frac{\alpha^2 t^2}{100}$ (d) $\frac{\alpha^2 t^2}{200}$

26. The $|3,0,0\rangle$ state (in the standard notation $|n, l, m\rangle$) of the H-atom in the non-relativistic theory decays to the state $|1,0,0\rangle$ via two dipole transitions. The transition route and the corresponding probability are

[CSIR JUNE 2021]

(a) $|3,0,0\rangle \rightarrow |2,1,-1\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{4}$

(b) $|3,0,0\rangle \rightarrow |2,1,1\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{4}$

(c) $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$ and $\frac{1}{3}$

(d) $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$ and $\frac{2}{3}$

27. At time $\mathrm{t} = 0$, a particle is in the ground state of the Hamiltonian

$$\mathrm{H(t)} = \frac{\mathrm{p}^2}{2\mathrm{m}} + \frac{1}{2}\mathrm{m}\omega^2\mathrm{x}^2 +$$

$\lambda \mathrm{x} \sin\frac{\omega \mathrm{t}}{2}$ where λ, ω and m are positive constants. To $\mathrm{O}(\lambda^2)$, the probability that at

$$\mathrm{t} = \frac{2\pi}{\omega}$$

, the particle would be in the first excited state of $\mathrm{H}(\mathrm{t} = 0)$ is

[CSIR JUNE 2022]

(a) $\frac{9\lambda^2}{16\mathrm{m}\hbar\omega^3}$ (b) $\frac{9\lambda^2}{8\mathrm{m}\hbar\omega^3}$

(c) $\dfrac{16\lambda^2}{9m\hbar\omega^3}$ (d) $\dfrac{8\lambda^2}{9m\hbar\omega^3}$

28. To first order in perturbation theory, the energy of the ground state of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 + \frac{\hbar\omega}{\sqrt{512}}\exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

(treating the third term of the Hamiltonian as a perturbation) is **[CSIR JUNE 2022]**

(a) $\frac{15}{32}\hbar\omega$ (b) $\frac{17}{32}\hbar\omega$

(c) $\frac{19}{32}\hbar\omega$ (d) $\frac{21}{32}\hbar\omega$

29. A particle in one dimension is in an infinite potential well between

$$-\frac{L}{2} \le x \le \frac{L}{2}$$

For a perturbation $\varepsilon\cos\left(\frac{\pi x}{L}\right)$ where ε is a small constant, the change in the energy of the ground state, to first order in ε is

$$-\frac{L}{2} \le x \le \frac{L}{2}$$

[CSIR JUNE 2023]

(a) $\dfrac{5\varepsilon}{\pi}$ (b) $\dfrac{10\varepsilon}{3\pi}$

(c) $\dfrac{8\varepsilon}{3\pi}$ (d) $4\dfrac{4\varepsilon}{\pi}$

30. A quantum system is described by the Hamiltonian

$$H = -J\sigma_z + \lambda(t)\sigma_x,$$

where $\sigma_i(i = x, y, z)$ are Pauli matrices, J and λ are positive constants ($J \gg \lambda$) and

$$\lambda(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \lambda & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$$

At $t < 0$, the system is in the ground state. The probability of finding the system in the excited state at $t \gg T$, in the leading order in λ is

[CSIR JUNE 2023]

(a) $\dfrac{\lambda^2}{8J^2}\sin^2\dfrac{JT}{\hbar}$ (b) $\dfrac{\lambda^2}{J^2}\sin^2\dfrac{JT}{\hbar}$

(c) $\dfrac{\lambda^2}{4J^2}\sin^2\dfrac{JT}{\hbar}$ (d) $\dfrac{\lambda^2}{16J^2}\sin^2\dfrac{JT}{\hbar}$

31. Consider a particle in a one-dimensional infinite potential well between $0 \le x \le$ L. If a small perturbation, $V(x) = \lambda\cos\left(\frac{\pi x}{L}\right)$, (where $\lambda \ll 1$) is applied, the firstorder energy correction to the ground state is

[CSIR MARCH 2025]

(a) λ (b) 0

(c) $-\lambda$ (d) 2λ

32. The ground state wavefunction for the hydrogen atom is

$$\psi_0 = \sqrt{\frac{1}{\pi a_0^3}}e^{-\frac{r}{a_0}}, \text{ where } a_0 \text{ is the Bohr radius.}$$

Considering an additional potential H' as a perturbation to the hydrogen atom Hamiltonian, given by

$$H' = \begin{cases} \dfrac{e^2}{4\pi\varepsilon_0}\left[\dfrac{1}{r} - \dfrac{1}{R}\right] & \text{for } 0 < r < R \\ 0 & \text{for } r > R \end{cases}$$

where R is the radius of the proton, $R \ll a_0$. The shift in the ground state energy due to H' is

[CSIR JUNE 2025]

(a) $\left(\dfrac{e^2}{4\pi\varepsilon_0 a_0}\right)\dfrac{4R^2}{3a_0^2}$ (b) $\left(\dfrac{e^2}{4\pi\varepsilon_0 a_0}\right)\dfrac{R}{a_0}$

(c) $-\left(\dfrac{e^2}{4\pi\varepsilon_0 a_0}\right)\dfrac{2R^2}{a_0^2}$ (d) $\left(\dfrac{e^2}{4\pi\varepsilon_0 a_0}\right)\dfrac{2R^2}{3a_0^2}$

❖ **GATE PYQ's**

1. Let $E_n'(n = 0,1,2, \dots \dots \dots .) betheenergyeigenval -$ ues for a particle of mas m placed in an anharmoic potential

$$V(x) = \frac{1}{2}mw^2x^2 + ax^4, (a > 0).$$

Let $E_n = \left(n + \frac{1}{2}\right)$. The according to the first order perturbation theory:

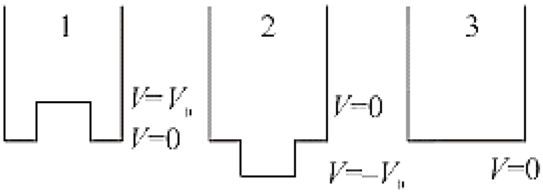
[GATE 1996]

(a) $E_0' = E_0$ (b) $E_0' > E_0$

(c) $E_0' < E_0$ (d) $E_0' < E_0$ for all n

2. Let E_1, E_2, E_3 be the respective ground state energies of the following potentials: which one of the following is correct?

[GATE 2000]



(a) $E_1 < E_2 < E_3$ (b) $E_3 < E_1 < E_2$

(c) $E_2 < E_3 < E_1$ (d) $E_2 < E_1 < E_3$

3. A particle in the ground state of an infinitely deep one-dimensional potential well of width a is subject to a perturbation of the form $V = V_0 \cos^2\left(\frac{\pi x}{a}\right)$
where V_0 is a constant. Find the shift in energy of the particle in the lowest order perturbation theory.

[GATE 2002]

4. The wave function of a one-dimensional harmonic oscillator is

$$\psi_0 = A\exp\left(\frac{-a^2x^2}{2}\right)$$

for the ground state $E_0(\alpha x/10)^4$, the first order change in the ground state energy is:
[Given :
$\left[\Gamma(x+1) = \int_0^\infty tx\exp(-t)dt\right]$

[GATE 2004]

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$

(c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)10^{-4}$

Common data for Q.5, Q. 6 and Q. 7
An unperturbed two-level system has energy eigenvalues E_1 and E_2 and eigen functions $\binom{1}{0}$ and $\binom{0}{1}$ when perturbed, its Hamiltonian is represented by $\begin{pmatrix} E_1 & A \\ A^* & E_2 \end{pmatrix}$

[GATE 2006]

5. The first-order correction to E_1 is:
(a) $4A$ (b) $2A$
(c) A (d) 0

6. The second-order correction to E_1 is
(a) 0 (b) A
(c) $\frac{A^2}{E_2 - E_1}$ (d) $\frac{|A|^2}{E_1 - E_2}$

7. The first-order correction to the eigenfunction $\binom{1}{0}$ is;

(a) $\begin{pmatrix} 0 \\ A^*/(E_1 - E_2) \end{pmatrix}$ (b) $\binom{0}{1}$

(c) $\begin{pmatrix} A^*/(E_1 - E_2) \\ 0 \end{pmatrix}$ (d) $\binom{1}{1}$

8. A particle of mass m is confined in an infinite potential well

$$V(x) = \begin{cases} 0 & \text{if } \quad 0 < x < L \\ \infty & \quad \text{otherwise} \end{cases}$$

It is subjected to a perturbing potential

$$V_p = V_0 \sin\left(\frac{2\pi x}{L}\right)$$

within the well. Let $E^{(1)}$ and $E^{(2)}$ be the correction to the ground state energy in the first and second order in V_0, respectively. Which of the following are true?

[GATE 2010]

(a) $E^{(1)} = 0, E^{(2)} < 0$

(b) $E^{(1)} > 0; E^{(2)} = 0$

(c) $E^{(1)} = 0, E^{(2)}$ depends on the sign of V_0

(d) $E^{(1)} < 0, E^{(2)} < 0$

9. The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

are given by

$$\Psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \text{ where } n = 1,2,3 \dots \dots$$

The particle is subjected to a perturbation

$$V'(x) \quad = V_0 \cos\left(\frac{\pi x}{a}\right) \text{ for } 0 \le x \le \frac{a}{2}$$
$$= 0 \text{ otherwise}$$

The shift in the ground state energy due to perturbation, in the first order perturbation theory is

[GATE 2011]

(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$

(c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

10. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, where $\delta << 1$. The energy eigen values of the perturbed system using the first order perturbation approximation are

[GATE 2012]

(a) 1 and $(1 + 2\delta)$

(b) $(1 + \delta)$ and $(1 - \delta)$

(d) $(1 + \delta)$ and $(1 - 2\delta)$

11. A two-level quantum system has energy eigenvalues E_1 and E_2. A perturbing potential $H' = \lambda\Delta\sigma_x$ is introduced, where Δ is a constant having dimensions of energy, λ is a small dimensionless parameter, and $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
The magnitudes of the first and the second order corrections to E_1 due to H', respectively, are

[GATE 2025]

(a)0 and $\frac{\lambda^2\Delta^2}{|E_1 - E_2|}$

(b)$\frac{|\lambda\Delta|}{2}$ and $\frac{\lambda^2\Delta^2}{|E_1 - E_2|}$

(c)$|\lambda\Delta|$ and $\frac{\lambda^2\Delta^2}{|E_1 - E_2|}$

(d)0 and $\frac{1}{2}\frac{\lambda^2\Delta^2}{|E_1 - E_2|}$

Common Data for Q. 12 and Q. 13:
To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by

$$\delta \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

where δ is a small quantity.

[GATE 2013]

12. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1\sqrt{2}, -1/\sqrt{2}, 0)$

(c) (0,0,1) (d) (1,0,0)

13. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3 + 2\delta, 7 + 2\delta$ (b) $3 + 2\delta, 2 + \delta$

(c) $3, 7 + 2\delta$ (d) $3, 2 + 2\delta$

14. A particle is confined in a one-dimensional potential box with the potential

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < a \\ \infty & \text{otherwise} \end{cases}$$

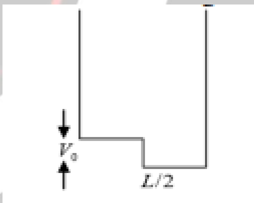
If the particle is subjected to a perturbation, within the box, $W = \beta x$ where β is a small constant, the first order correction to the ground state energy is

[GATE 2014]

(a) 0 (b) $\frac{a\beta}{4}$

(c) $\frac{a\beta}{2}$ (d) $a\beta$

15. A particle is confined in box of length L as shown below



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

[GATE 2015]

(a)$E = \frac{\hbar^2\pi^2}{2mL^2} + V_0$ (b) $E = \frac{\hbar^2\pi^2}{2mL^2} - \frac{V_0}{2}$

(c)$E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$ (d) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$

16. A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E} = \vec{E}_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent is

[GATE 2016]

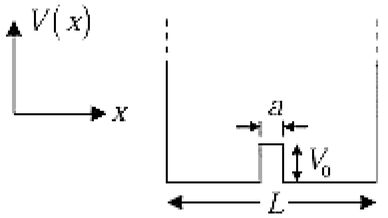
17. The ground state energy of a particle of mass m in an infinite potential well is E_0. It changes to

$E_0(1 + \alpha \times 10^{-3})$, when there is a small potential pump of height

$$V_0 = \frac{\pi^2 \hbar^2}{50mL^2}$$

and width $a = L/100$, as shown in the figure. The value of α is (up to two decimal places).

[GATE 2018]



18. An electric field $\vec{E} = E_0 \hat{z}$ is applied to a hydrogen atom in $n = 2$ excited state. Ignoring spin, the $n = 2$ state is fourfold degenerate, which in the $|l, m\rangle$ basis are given by $|0,0\rangle, |1,1\rangle, |1,0\rangle$ and $|1,-1\rangle$. The H' is the interaction Hamiltonian corresponding to the applied electric field, which of the following matrix elements is non-zero?

[GATE 2019]

(a) $\langle 0,0|H'|0,0\rangle$ (b) $\langle 0,0|H'|1,1\rangle$

(c) $\langle 0,0|H'|1,0\rangle$ (d) $\langle 0,0|H|1,-1\rangle$

19. The Hamiltonian of a system is $H = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & -1 \end{pmatrix}$ with $\varepsilon \ll 1$. The fourth order contribution to the ground state energy of H is $\gamma \varepsilon^4$. The value of γ (rounded off to three decimal places) is

[GATE 2019]

20. Consider the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$ where

$$\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix} \text{ and } \hat{H}'$$

is the time independent perturbation given by

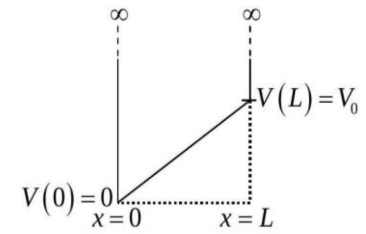
$$\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}, \text{where } k > 0.$$

If, the maximum energy eigenvalues of $\hat{H}$ is $3eV$

corresponding to $E = 2eV$, the value of k (rounded off to three decimal places) in eV is

[GATE 2020]

21. Consider a particle in a one-dimensional infinite potential well with its walls at $x = 0$ and $x = L$. The system is perturbed as shown in the figure.



The first order correction to the energy eigenvalue is

[GATE 2021]

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{3}$

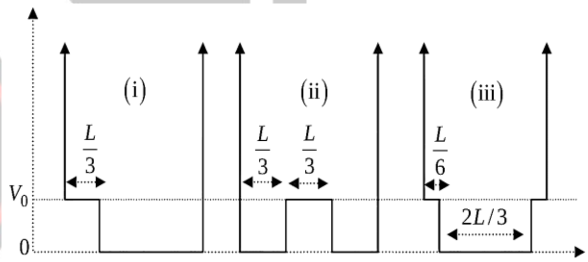
(c) $\frac{V_0}{2}$ (d) $\frac{V_0}{5}$

22. Consider a particle in three different boxes of width L. The potential inside the boxes vary as shown in figures (i), (ii) and (iii) with

$$V_0 < \frac{\hbar^2 \pi^2}{2mL^2}$$

The corresponding ground-state energies of the particle are E_1, E_2 and E_3, respectively. Then

[GATE 2022]



(a) $E_2 > E_1 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_3 > E_1$ (d) $E_3 > E_2 > E_1$

23. A particle of mass m in the $x - y$ plane is confined in an infinite two-dimensional well with vertices $(0,0), (0, L), (L, L), (L, 0)$. The eigen-functions of this particle are

$$\psi_{n_x n_y} = \frac{2}{L} \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right)$$

If perturbation of the form $V = Cxy$, where C is a real constant, is applied, then which of the following statements are correct for the first excited state?

[GATE 2022]

(a) The unperturbed energy is

$$\frac{3\pi^2\hbar^2}{2mL^2}$$

(b) The unperturbed energy is

$$\frac{5\pi^2\hbar^2}{2mL^2}$$

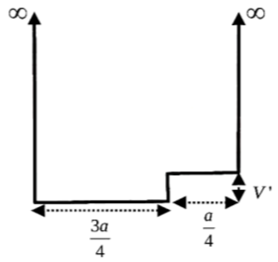
(c) First order energy shift due to the applied perturbation is zero

(d) The shift (δ) in energy due to the applied perturbation is determined by an equation of the form $\begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$, where a and b are real, non-zero constants

24. A particle of mass m in an infinite potential well of width a is subjected to a perturbation,

$$V' = \frac{h^2}{40ma^2}$$

as shown in figure, where h is Planck's constant.



The first order energy shift of the fourth energy eigenstate due to this perturbation is

$$\left(\frac{h^2}{Nma^2}\right)$$

The value of N is (in integer). **[GATE 2024]**

❖ **JEST PYQ's**

1. If Jx, Jy, are angular momentum operators, the eigenvalues of the operator $(J_x + J_y)/\hbar$ are **[JEST 2013]**

(a) real and discrete with rational spacing

(b) real and discrete with irrational spacing

(c) real and continuous

(d) not all real

2. Consider a spin 1/2 particle characterized by the Hamiltonian $H = \omega S_i$. under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by **[JEST 2015]**

(a) $-\frac{g^2}{4\omega}$ (b) $\frac{g^2}{4\omega}$

(c) $-\frac{g^2}{2\omega}$ (d) $\frac{g^2}{2\omega}$

3. A particle of mass m is confined in a potential well given by $V(x) = 0$ for $\frac{-L}{2} < x < \frac{L}{2}L/2$ and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = ax$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct? **[JEST 2015]**

(a) $E_0^{(1)} < 0$ and $E_0^{(2)} > 0$

(b) $E_0^{(1)} > 0$ and $E_0^{(2)} > 0$

(c) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$

(d) $E_0^{(1)} = 0$ and $E_0^{(2)} < 0$

4. Consider a quantum particle of mass m in one dimension in an infinite potential well, i.e., $(x) = 0$ for $\frac{-a}{2} < x < \frac{a}{2}$ and $V(x) = \infty$ for $|x| \geq \frac{a}{2}$. A small perturbation,

$$V'(x) = \frac{2 \in |x|}{a}$$

is added. The change in the ground state energy to $0(\epsilon)$ is: **[JEST 2016]**

(a) $\frac{\epsilon}{2\pi^2}(\pi^2 - 4)$ (b) $\frac{\epsilon}{2\pi^2}(\pi^2 + 4)$

(c) $\frac{\in \pi^2}{2}(\pi^2 + 4)$ (d) $\frac{\epsilon\pi^2}{2}(\pi^2 - 4)$

5. A particle is described by the following Hamiltonian

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2x^2 + \lambda x^4$$

where the quartric term can be treated perturbatively. If ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state respectively, what is the fraction $\Delta E_1/\Delta E_0$? **[JEST 2017]**

6. A harmonic oscillator has the following Hamiltonian

$$H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

It is perturbed with a potential $V = \lambda\hat{x}^4$. Some of the matrix elements of in terms of its expectation value in the ground state are given as follows:

$$\langle 0|\hat{x}^2|0\rangle = C; \langle 0|\hat{x}^2|2\rangle = \sqrt{2}C; \langle 1|\hat{x}^2|1\rangle = 3C; \langle 1|\hat{x}^2|3\rangle = \sqrt{6}C$$

where $|n\rangle$ is the normalized eigenstates of H_0 corresponding to the eigenvlaue $E_n = \hbar\omega(n + 1/2)$. Suppose ΔE_0 and ΔE_1, denote the energy correction $O(\lambda)$ of to the ground state and the first excited state, respectively. What is the fraction $\Delta E_1/E_0$?

[JEST 2018]

7. Consider a two-level quantum system described by the Hamiltonian:

$$H = H_0 + H'$$

where

$$H_0 = \alpha\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ H' = \epsilon\Gamma\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

H' is a small perturbation to the free Hamiltonian $H_0 - \epsilon$ is a small positive dimensionless number, while α, ω and Γ have dimensions of energy and are positive quantities. If we treat this problem perturbatively in the parameter ϵ, which of the following statements about the corrections to ground state energy is true? **[JEST 2023]**

(a) First-order correction is ϵ; second-order correction is

$$-\frac{\epsilon^2\Gamma^2}{2\omega}$$

(b) First-order correction is $\epsilon\Gamma$; second-order correction is 0 .

(c) First-order correction is 0 ; second-order correction is

$$-\frac{\epsilon^2 T^2}{2\omega}$$

(d) First-order correction is 0 ; second-order correction is

$$\frac{\epsilon^2\,\mathrm{T}^2}{2\omega}$$

8. For a one-dimensional simple harmonic oscillator with mass m and angular frequency ω, consider a perturbation λx^4 in the Hamiltonian $(\lambda > 0)$. What is the lowest order correction to the ground state energy?

[The position operator expressed in terms of the raising and lowering operators is $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$.] **[JEST 2025]**

(a) $\frac{3\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$ (b) $\frac{5\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$

(c) $\frac{3\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$ (d) $\frac{5\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$

❖ **TIFR PYQ's**

1. A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics,

[TIFR 2010]

(a) the particle will shoot out of the well and move out towards infinity in one of the two possible directions

(b) the particle will stop oscillating and as time increases it may be found farther and farther away from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

2. A particle of mass ' m ' and charge ' e ' is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

[TIFR 2014]

(a) Make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.

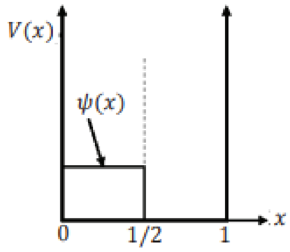
(b) Make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.

(c) Settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.

(d) Oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

3. A particle is confined in a one-dimensional box of unit length, i.e. $L = 1$, i.e. in a potential

$$V(x) = \begin{cases} 0 \text{ if } 0 < x < 1 \\ \infty \text{ otherwise} \end{cases}$$



The energy eigenvalues of this particle are denoted $E_0, E_1, E_2, E_3, \ldots$

In a particular experiment, the wavefunction of this particle, at $t = 0$, is given by

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if } 0 < x < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

If, simultaneously, i.e. at $t = 0$, a measurement of the energy of the particle is made, find $100p_3$, where p_3 is the probability that the measurement will yield the energy E_3.

[TIFR 2016]

4. A quantum mechanical system which has stationary states $|1\rangle, |2\rangle$ and $|3\rangle$, corresponding to energy levels 0eV, 1eV and 2eV respectively, is perturbed by a potential of the form

$$\hat{V} = \varepsilon|1\rangle\langle 3| + \varepsilon|3\rangle\langle 1|$$

where, in eV, $0 < \varepsilon \ll 1$. The new ground state, correct to order ε, is approximately.

[TIFR 2017]

(a) $\left(1 - \frac{\varepsilon}{2}\right)|1\rangle + \frac{\varepsilon}{2}|3\rangle$ (b) $|1\rangle + \frac{\varepsilon}{2}|2\rangle - \varepsilon|3\rangle$

(c) $|1\rangle + \frac{\varepsilon}{2}|3\rangle$ (d) $|1\rangle - \frac{\varepsilon}{2}|3\rangle$

5. A particle of mass m is confined inside a box with boundaries at $x = \pm L$. The ground state and the first excited state of this particle are E_1 and E_2 respectively.

Now a repulsive delta function potential $\lambda\delta(x)$ is introduced at the centre of the box where the constant λ satisfies

$$0 < \lambda \ll \frac{1}{32m}\left(\frac{h}{L}\right)^2$$

If the energies of the new ground state and the new first excited state be denoted as E_1' and E_2' respectively, it follows that

[TIFR 2020]

(a) $E_1' > E_1, E_2' > E_2$ (b) $E_1' = E_1, E_2' = E_2$

(c) $E_1' > E_1, E_2' = E_2$ (d) $E_1' = E_1, E_2' > E_2$

6. The Hamiltonian for a Helium atom is given as $H = H_0 + H_I$, where

$$H_0 = \frac{(p_1^2 + p_2^2)}{2\mu} - \frac{2e^2}{4\pi\epsilon_0 r_1} - \frac{2e^2}{4\pi\epsilon_0 r_2}$$

And

$$H_I = \frac{e^2}{4\pi\epsilon_0 r_{12}}$$

where μ is the reduced mass of the electron, r_1 and r_2 are the distance of the electrons from the nucleus, and r_{12} is the distance between the two electrons. The value of the first ionization potential of the Helium atom is 6eV. What is the correction due to H_I to the ground state energy of the Helium atom, compared to H_0 ?

[TIFR 2024]

(a) 29.8eV (b) $-$29.8eV

(c) 84.2eV (d) $-$2.6eV

7. Consider a particle with mass m in a quantum harmonic oscillator potential with a frequency ω, such that its Hamiltonian is

$$H = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}$$

The Hamiltonian is perturbed by adding a term to the potential $\Delta H = \lambda \sin\, \hat{x}$
where λ is small compared to $\hbar\omega$. The relative change in the ground state energy, to the leading order in $\lambda/(\hbar\omega)$ is given by:

[TIFR 2025]

(a) $O\left(\frac{\lambda^2}{(\hbar\omega)^2}\right)$

(b) $O\left(\frac{\lambda}{(\hbar\omega)}\right)$

(c) $O(1)$

(d) The ground state energy does not change

❖ Answers key				
CSIR-NET				
1. a	2. a	3. b	4. b	5. a
6. d	7. b	8. d	9. c	10. d
11. c	12. d	13. d	14. d	15. a
16. c	17. c	18. b	19. a	20. d
21. c	22. d	23. a	24. b	25. c
26. c	27. d	28. b	29. a	30. b
31. b	32. d			
GATE				
1. b	2. b	3.	4. c	5. d
6. d	7. a	8. a	9. a	10. a
11.	12. c	13. c	14. c	15. d
16. **2**	17. 0.81	18. c	19. 0.125	20. 1
21. **c**	22. a	23. b	24. 160	
JEST				
1. a	2. a	3. d	4. a	5. 5
6. 5	7. c	8. a		
TIFR				
1.	2. b	3. 00	4. d	5. c
6. c	7. a			

❖ CSIR-NET PYQ's

1. In a system consisting of two spin-1/2 particles labeled 1 and 2, let

$$\vec{S}^{(1)} = \frac{\hbar}{2}\vec{\sigma}^{(1)}$$

and

$$\vec{S}^{(2)} = \frac{h}{2}\vec{\sigma}^{(2)}$$

denote the corresponding spin operators. Here $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices.

(A) In the standard basis the matrices for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ respectively,

[CSIR JUNE 2011]

(a) $\frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$

(b) $\frac{\hbar^2}{4}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}$

(d) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & 1 & 0\end{pmatrix}, \frac{\hbar^2}{4}\begin{pmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

(B) These two operators satisfy the relation

(a) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = S_z^{(1)}S_z^{(2)}$

(b) $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = 0$

(c) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = iS_z^{(1)}S_z^{(2)}$

(d) $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = 0$

2. The component along an arbitrary direction n̂, with direction cosines (n_x, n_y, n_z), of the spin of a spin- 1/2 particle is measured. The result is:

[CSIR JUNE 2012]

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$

(c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

3. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix}(1+i)/\sqrt{6}\\ \sqrt{2/3}\end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively.

[CSIR JUNE 2013]

(a) 1/2 and ½ (b) 2/3 and 1/3

(c) 1/4 and ¾ (d) 1/3 and 2/3

4. A spin $-\frac{1}{2}$ particle is in the state

$$\chi = \frac{1}{\sqrt{11}}\begin{pmatrix}1+i\\ 3\end{pmatrix}$$

in the eigenbasis of S^2 and S_x. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are

[CSIR DEC 2013]

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$

(c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

5. Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}]$ is equal to (in the following I is the identity matrix)

[CSIR DEC 2014]

(a) $(\vec{a}\cdot\vec{b})(\sigma_1+\sigma_2+\sigma_3)$ (b) $2i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

(c) $(\vec{a}\cdot\vec{b})I$ (d) $|\vec{a}\parallel\vec{b}|I$

6. The effective spin-spin interaction between the electron spin $\vec{s}_\sigma$ and the proton spin $\vec{s}_p$ in the ground state of the Hydrogen atom is given by $H' = a\vec{s}_e\cdot\vec{s}_p$. As a result of this interaction, the energy levels split by an amount

[CSIR DEC 2014]

(a) $\frac{1}{2}a\hbar^2$ (b) $2a\hbar^2$

(c) $a\hbar^2$ (d) $\frac{3}{2}a\hbar^2$

7. The Hamiltonian for a spin-$\frac{1}{2}$ particle at rest is given by $H = E_0(\sigma_z + \alpha\sigma_x)$, where σ_x and σ_z are Pauli spin matrices and E_0 and α are constants. The eigenvalues of this Hamiltonian are

[CSIR DEC 2015]

(a) $\pm E_0\sqrt{1+\alpha^2}$

(b) $\pm E_0\sqrt{1-\alpha^2}$

(c) E_0 (doubly degenerate)

(d) $E_0\left(1 \pm \frac{1}{2}\alpha^2\right)$

8. The Hamiltonian of a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B}$ is given by $H = -\mu\vec{B}\cdot\vec{\sigma}$, where μ is a real constant and $\bar{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. If $\bar{B} = (B_0, B_0, 0)$ and the spin state at time $t = 0$ is an eigenstate of σ_x, then of the expectation values $\langle\sigma_x\rangle, \langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$

[CSIR JUNE 2018]

(a) Only $\langle\sigma_x\rangle$ changes with time

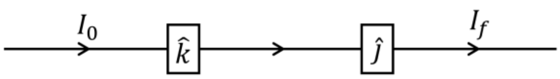
(b) Only $\langle\sigma_y\rangle$ changes with time

(c) Only $\langle\sigma_z\rangle$ changes with time

(d) All three change with time

9. Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x-axis). The directions of their magnetic fields are along the positive z - and y-axes, respectively. Each apparatus only transmits particles with spins aligned in the direction of its magnetic field. If an initially unpolarized beam of spin-$\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $I_0 : I_f$ of the initial and final beams, I

[CSIR JUNE 2018]



(a) 16: 1

(b) 2: 1

(c) 4: 1

(d) 1: 0

10. A system of spin-$\frac{1}{2}$ particles is prepared to be in the eigenstate of σ_z with eigenvalue +1. The system is rotated by an angle of 60° about the x-axis. After the rotation, the fraction of the particles that will be measured to be in the eigenstate of σ_z with eigenvalue +1 is

[CSIR DEC 2018]

(a) 1/3

(b) 2/3

(c) ¼

(d) ¾

11. The Hamiltonian of two interacting particles, one with spin-1 and the other with spin-$\frac{1}{2}$, is given by

$$H = AS_1 \cdot S_2 + B(S_{1x} + S_{2x}),$$

where S_1 and S_2 denote the spin operators of the first and second particles, respectively, and A and B are positive constants. The largest eigenvalue of this Hamiltonian is

[CSIR DEC 2019]

(a) $\frac{1}{2}(A\hbar^2 + 3B\hbar)$

(b) $3A\hbar^2 + B\hbar$

(c) $\frac{1}{2}(3A\hbar^2 + B\hbar)$

(d) $A\hbar^2 + 3B\hbar$

12. Consider the Hamiltonian $\mathrm{H = AI + B\sigma_x + C\sigma_y}$, where A, B and C are positive constants, I is the 2×2 identity matrix and σ_x, σ_y are Pauli matrices. If the normalized eigenvector corresponding to its largest energy eigenvalue is $\frac{1}{\sqrt{2}}\binom{1}{y}$, then y is

[CSIR JUNE 2022]

(a) $\frac{B+iC}{\sqrt{B^2+C^2}}$

(b) $\frac{A-iB}{\sqrt{A^2+B^2}}$

(c) $\frac{A-iC}{\sqrt{A^2+C^2}}$

(d) $\frac{B-iC}{\sqrt{B^2+C^2}}$

13. The Hamiltonian for a spin- 1/2 particle in a magnetic field $\mathbf{B} = B_0\hat{k}$ is given by $H = \lambda\mathbf{S}\cdot\mathbf{B}$, where $\mathbf{S}$ is its spin (in units of $\hbar$) and λ is a constant. If the average spin density is $\langle\mathbf{S}\rangle$ for an ensemble of such non- interacting particles, then $\frac{d}{dt}\langle S_x\rangle$

[CSIR JUNE 2022]

(a) $\frac{\lambda}{\hbar}B_0\langle S_x\rangle$

(b) $\frac{\lambda}{\hbar}B_0\langle S_y\rangle$

(c) $-\frac{\lambda}{\hbar}B_0\langle S_x\rangle$

(d) $-\frac{\lambda}{\hbar}B_0\langle S_y\rangle$

14. A quantum system is described by the Hamiltonian

$$H = JS_z + \lambda S_x$$

where $S_i = \frac{\hbar}{2}\sigma_i$ and $\sigma_i (i = x, y, z)$ are the Pauli matrices. If $0 < \lambda \ll J$, then the leading correction in λ to the partition function of the system at temperature T is

[CSIR DEC 2023]

(a) $\frac{\hbar\lambda^2}{2Jk_BT}\coth\left(\frac{J\hbar}{2k_BT}\right)$

(b) $\frac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\frac{J\hbar}{2k_BT}\right)$

(c) $\frac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\frac{J\hbar}{2k_BT}\right)$

(d) $\frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right)$

15. The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\varphi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\varphi)\chi_+\right].$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}\hbar$. The expectation value of the z component of the total angular momentum in the above state is

[CSIR DEC 2023]

(a) $-\frac{3}{4}\hbar$ (b) $\frac{3}{4}\hbar$

(c) $-\frac{9}{8}\hbar$ (d) $\frac{9}{8}\hbar$

16. An electron is in the spin state $|\psi\rangle = \frac{1}{5}\binom{3i}{4}$ in the $\hat{S}_z$ basis. A measurement of $\hat{S}_x$ is made on this state. The probabilities of getting $\hbar/2$ and $-\hbar/2$ are

[CSIR DEC 2024]

(a) $\frac{1}{3}, \frac{2}{3}$ (b) $\frac{1}{4}, \frac{3}{4}$

(c) $\frac{1}{2}, \frac{1}{2}$ (d) $\frac{3}{7}, \frac{4}{7}$

17. A spin-$\frac{1}{2}$ system is prepared in the initial state

$$|\varphi\rangle = \frac{\sqrt{3}}{2}|\uparrow\rangle + \frac{1}{2}|\downarrow\rangle$$

where $|\uparrow\rangle$ & $|\downarrow\rangle$ are eigenstates of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$ respectively. A measurement of $\hat{S}_z$ is followed by a measurement of $\hat{S}_x$ on the system. What is the probability that the measurement of $\hat{S}_x$ yields a value $+\frac{\hbar}{2}$? **[CSIR JUNE 2025]**

(a) $\frac{1}{2}$ (b) $\frac{2+\sqrt{3}}{4}$

(c) $\frac{2-\sqrt{3}}{4}$ (d) $\frac{3}{8}$

❖ GATE PYQ's

1. A spinless particle moves in a central potential V(r)

[GATE 2001]

(a) The kinetic energy and the potential energy of the particle cannot simultaneously have sharp values

(b) The total energy and the potential energy of the particle can simultaneously have sharp values

(c) The total energy and the square of the orbital angular momentum about the origin cannot simultaneously have sharp values.

(d) The total energy of the particle can have only discrete eigenvalues

2. In a Stern-Gerlach experiment, the magnetic field is in $+z$ direction. A particle comes out of this experiment in $|+\hat{z}\uparrow\rangle$ state. Which of the following statements is true?

[GATE 2002]

(a) The particle has a definite value of the y-component of the spin angular momentum

(b) The particle has a definite value of the square of the spin angular momentum

(c) The particle has a definite value of the x-components of spin angular momentum

(d) The particle has definite values of x-and y-components of spin angular momentum

3. An electron is in a state with spin wave function $\phi_s = \begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix}$ in the S_z representation. What is the probability of finding the z-component of its spin along the $-\hat{z}$ direction?

[GATE 2002]

(a) 0.75 (b) 0.50

(c) 0.35 (d) 0.25

4. A spin half particle is in the state $S_z = \hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by

[GATE 2003]

(a) $0,0,\hbar^2/4,\hbar^2/4$ (b) $0,\hbar^2/4,\hbar^2/4,0$

(c) $0,\hbar^2/4,0,\hbar^2/4$ (d) $\hbar^2/4,\hbar^2/4,0,0$

5. For a spin $-\frac{1}{2}$ particle, the expectation value of $s_x s_y s_z$, where s_x, s_y and s_z are spin operators, is

[GATE 2005]

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$

(c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

6. Which one of the following relations is true for Pauli matrices σ_x, σ_y and σ_z ?

[GATE 2006]

(a) $\sigma_x\sigma_y = \sigma_y\sigma_x$ (b) $\sigma_x\sigma_y = \sigma_z$

(c) $\sigma_x\sigma_y = i\sigma_z$ (d) $\sigma_x\sigma_y = -\sigma_y\sigma_x$

7. For a spin-s particle in the eigen basis of $\vec{S}^2, S_z$ the expectation value $\langle sm|S_x^2|sm\rangle$ is

[GATE 2010]

(a) $\frac{\hbar^2\{s(s+1)-m^2\}}{2}$

(b) $\hbar^2\{s(s+1)-2m^2\}$

(c) $\hbar^2\{s(s+1)-m^2\}$

(d) $\hbar^2 m^2$

8. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here symbols have their usual meanings_________________.

[GATE 2013]

9. Ψ_1 and Ψ_2 are two orthogonal states of a spin 1/2 system. It is given that

$$\Psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix},$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin down states, respectively. When the system is in the state Ψ_2 its probability to be in the spin-up state is……………

[GATE 2014]

10. An operator for a spin- 1/2 particle is given by $\hat{A} = \lambda\vec{\sigma}\cdot\vec{B}$, where

$$\vec{B} = \frac{B}{\sqrt{2}}(\hat{x}+\hat{y}), \vec{\sigma}$$

denotes Pauli matrices and λ is a constant. The eigenvalues of $\hat{A}$ are

[GATE 2015]

(a) $\pm\lambda B/\sqrt{2}$ (b) $\pm\lambda B$

(c) $0, \lambda B$ (d) $0, -\lambda B$

11. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle\vec{s}_1\cdot\vec{s}_2\rangle$ for the ground state is

[GATE 2016]

(a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$

(c) 0 (d) $\frac{1}{4}\hbar^2$

12. σ_x, σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y + \sigma_y\sigma_x$ is equal to

[GATE 2016]

(a) $-3i\sigma_z$ (b) $-i\sigma_z$

(c) $i\sigma_z$ (d) $3i\sigma_z$

13. For the Hamiltonian $H = a_0 I + \vec{b}\cdot\vec{\sigma}$ where $a_0 \in R, \vec{b}$ is a real vector, I is the 2×2 identity matrix, and $\vec{\sigma}$ are the Pauli matrices, the ground state energy is

[GATE 2017]

(a) $|b|$ (b) $2a_0 - |b|$

(c) $a_0 - |b|$ (d) a_0

14. Given the following table,

Group I	Group II
P: Stren-Gerlach experiment	1: wave nature of particles
Q: Zeeman effect	2: Quantization of energy of electrons in the atoms
R: Frank-Hertz experiment	3: Existence of electron spin
S: Davisson-Germer experiment	4: Space quantization of angular momentum

Which one of the following correctly matches the experiments from Group I to their inferences in Group II?

[GATE 2018]

(a) P-2, Q-3, R-4, S-1 (b) P-1, Q-3, R-2, S-4
(c) P-3, Q-4, R-2, S-1 (d) P-2, Q-1, R-4, S-3

15. Electrons with spin in the z-direction ($\hat{z}$) are passed through a Stern-Gerlach (SG) set up with the magnetic field at $\theta = 60°$ from $\hat{z}$. The fraction of electrons that will emerge with their spin parallel to the magnetic field in the SG set up (rounded off to two decimal places) is

[GATE 2019]

$$\left[\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]$$

16. $\hat{S}_x$ denotes the spin operator defined

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Which one of the following is correct?

[GATE 2020]

(a) The eigenstates of spin operator $\hat{S}_x$ are $|\uparrow\rangle_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle_x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) The eigenstates of spin operator $\hat{S}_x$ are $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $|\uparrow\rangle_x$ is $\frac{1}{4}$

(d) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $|\uparrow\rangle_x$ is $\frac{2+\sqrt{3}}{4}$

17. Consider a spin $S = \hbar/2$ particle in the state $|\phi\rangle = \frac{1}{3}\begin{bmatrix} 2+i \\ 2 \end{bmatrix}$. The probability that a measurement finds the state with $S_x = +\hbar/2$ is

[GATE 2021]

(a) 5/18 (b) 11/18

(c) 15/18 (d) 17/18

18. Pauli spin matrices satisfy **[GATE 2022]**

(a) $\sigma_\alpha \sigma_\beta - \sigma_\beta \sigma_\alpha = i \in_{\alpha\beta\gamma} \sigma_\gamma$

(b) $\sigma_\alpha \sigma_\beta - \sigma_\beta \sigma_\alpha = 2i \in_{\alpha\beta\gamma} \sigma_\gamma$

(c) $\sigma_\alpha \sigma_\beta + \sigma_\beta \sigma_\alpha = \in_{\alpha\beta\gamma} \sigma_\gamma$

(d) $\sigma_\alpha \sigma_\beta + \sigma_\beta \sigma_\alpha = 2\delta_{\alpha\beta}$

19. An atom with non-zero magnetic moment has an angular momentum of magnitude $\sqrt{12}\hbar$. When a beam of such atoms is passed through a Stern-Gerlach apparatus, how many beams does it split into?

[GATE 2023]

(a) 3 (b) 7

(c) 9 (d) 25

20. A spin $\frac{1}{2}$ particle is in a spin up state along the x-axis (with unit vector $\hat{x}$) and is denoted as $\left|\frac{1}{2}, \frac{1}{2}\right\rangle_x$. What is the probability of finding the particle to be in a spin up state along the direction $\hat{x}'$, which lies in the xy-plane and makes an angle θ with respect to the positive x-axis, if such a measurement is made?

[GATE 2023]

(a) $\frac{1}{2}\cos^2\frac{\theta}{4}$ (b) $\cos^2\frac{\theta}{4}$

(c) $\frac{1}{2}\cos^2\frac{\theta}{2}$ (d) $\cos^2\frac{\theta}{2}$

21. An electron with mass m and charge q is in the spin up state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ at time $t = 0$. A constant magnetic field is applied along the y-axis, $B = B_0\hat{J}$, where B_0 is a constant. The Hamiltonian of the system is $H = -\hbar\omega\sigma_y$, where $\omega = \frac{qB_0}{2m} > 0$

and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The minimum time after which the electron will be in the spin down state along the x-axis, i.e., $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, is

[GATE 2025]

(a) $\dfrac{\pi}{8\omega}$ (b) $\dfrac{\pi}{4\omega}$

(c) $\dfrac{\pi}{2\omega}$ (d) $\dfrac{\pi}{\omega}$

22. A system of three non-identical spin $\frac{1}{2}$ particles has the Hamiltonian $H = \frac{A}{\hbar^2}(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$, where $\vec{S}_1, \vec{S}_2$ and $\vec{S}_3$ are the spin operators of particles labelled 1,2 and 3 respectively and A is a constant with appropriate dimensions. The set of possible energy eigenvalues of the system is

[GATE 2025]

(a) $0, \dfrac{A}{2}, -A$ (b) $0, \dfrac{A}{2}, -\dfrac{A}{2}$

(c) $0, \dfrac{3A}{2}, -\dfrac{A}{2}$ (d) $0, -\dfrac{3A}{2}, \dfrac{A}{2}$

❖ **JEST PYQ's**

1. Consider a system of two spin $-1/2$ particle with total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$, where S_1 and S_2 are in the terms of Pauli matrices σ_{i}. the spin triplet projection operator is

[JEST 2012]

(a) $\dfrac{1}{4} + S_1 \cdot S_2$ (b) $\dfrac{3}{4} - S_1 \cdot S_2$

(c) $\dfrac{3}{4} + S_1 \cdot S_2$ (d) $\dfrac{1}{4} - S_1 \cdot S_2$

2. Consider a spin- 1/2 particle in the presence of a homogeneous magnetic field of magnitude B long along z-axis which is prepared initially in a state $\left|\psi = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right)$ at time $t = 0$. At what time t will the particle be in the state $-|\psi\rangle$ (μ_B is Bhor magneton)

[JEST 2012]

(a) $t = \dfrac{\pi\hbar}{\mu_B B}$ (b) $t = \dfrac{2\pi\hbar}{\mu_B B}$

(c) $t = \dfrac{\pi\hbar}{2\mu_B B}$ (d) never

3. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with $m = +1,0,-1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement

[JEST 2013]

(a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

4. What are the eigen values of the operator $H = \vec{\sigma} \cdot \vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector

[JEST 2013]

(a) $a_x + a_y$ and a_z (b) $a_x + a_x \pm i a_y$

(c) $\pm(a_x + a_y + a_z)$ (d) $\pm|\vec{a}|$

5. Suppose a spin 1/2 particle is in the state,

$$|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$$

If S_x(x component of the spin angular momentum operator) is measured what is the probability of getting $+\frac{\hbar}{2}$?

[JEST 2014]

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

6. Consider a three-state with energies E, E and $E - 3g$ (where g is a constant) and respective eigenstates

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, |\psi_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

If the system is initially (at $t = 0$) in state

$$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

What is the probability that at a later time ' t ' system will be in state $|\psi_f\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$

[JEST 2014]

(a)0

(b) $\frac{4}{9}\sin^2\left(\frac{3gt}{2\hbar}\right)$

(c) $\frac{4}{9}\cos^2\left(\frac{3gt}{2h}\right)$

(d) $\frac{4}{9}\sin^2\left(\frac{E-3gt}{2\hbar}\right)$

7. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix

$$\left(\frac{1}{\sqrt{10}}\right)\{2, \sqrt{2}, 2i\}$$

in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_z|\psi\rangle$?

[JEST 2016]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

8. If the direction with respect to a right-handed Cartesian coordinate system of the ket vector $|z,+\rangle$ is (0. U. 1), then the direction of the ket vector obtained by application of rotations:

$$\exp\left(-i\sigma_z\frac{\pi}{2}\right)\exp\left(-i\sigma_y\frac{\pi}{4}\right)$$

on the ket $|z,+\rangle$ is (σ_y, σ_z are the Pauli matrices) :

[JEST 2016]

(a) (0,1,0) (b) (1,0,0)

(c) $\frac{(1,1,0)}{\sqrt{2}}$ (d) $\frac{(1,1,1)}{\sqrt{3}}$

9. If

$$\rho = \left[I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z\right]/2$$

, where σ' s are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is

[JEST 2017]

(a) 2^{2017} (b) 2^{-2017}

(c) 1 (d) $\frac{1}{2}$

10. What is the difference between the maximum and the minimum eigenvalues of a system of two electrons whose Hamiltonian is $H + J\vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the corresponding spin angular momentum operators of the two electrons?

[JEST 2018]

(a) J/4 (b) J/2

(c) 3 J/4 (d) J

11. For a spin $\frac{1}{2}$ particle placed in a magnetic field B, the Hamiltonian is $H = -\gamma BS_y = -\omega S_y$, where S_y is the y-component of the spin operator. The state of the system at time $t = 0$ is $|\psi(t=0)\rangle = |+\rangle$, where

$$S_z|\pm\rangle = \pm\frac{\hbar}{2}|\pm\rangle$$

At a letter time t, if S_z measured then what is the probability to get a value $-\frac{\hbar}{2}$?

[JEST 2019]

(a) $\cos^2(\omega t)$ (b) $\sin^2(\omega t)$

(c) 0 (d) $\sin^2\left(\frac{\omega t}{2}\right)$

12. Let $M = 2\mathbb{I} + \sigma_x + i\sigma_y + \sigma_z$ is a 2×2 square matrix, where, σ_α denotes α^{th} Pauli matrix, and $\mathbb{I}$ denotes the 2×2 identity matrix. It is given that $|u\rangle = \binom{1}{0}$ and $|v\rangle = \binom{1}{-1}$ are column vectors. What is the value of $\langle u|\sqrt{M}|v\rangle$?

[JEST 2022]

13. Consider a spin-1 system whose $\hat{S}_z$ eigenstates are given by $|-1\rangle, |0\rangle, |+1\rangle$ corresponding to the eigenvalues $-\hbar, 0, \hbar$. The normalized general state $|\psi\rangle$ of the system can be expressed as

$$|\psi\rangle = c_{-1}|-1\rangle + c_0|0\rangle + c_{+1}|+1\rangle,$$

and c_{-1}, c_0, c_{+1} are complex numbers. Subjected to the condition $\langle\psi|\hat{S}_z|\psi\rangle = 0$, which of the following statements is true?

[JEST 2023]

(a) $|c_{+1}|^2 + 2|c_0|^2 = 1$

(c) $2|c_{-1}|^2 + |c_{+1}|^2 = 1$

(b) $|c_{-1}|^2 + 2|c_0|^2 = 1$

(d) $2|c_{-1}|^2 + |c_0|^2 = 1$

14. Consider the operator $S{\cdot}n$ with eigen kets $|\pm\rangle_{\hat{n}}$ and eigenvalues $\pm\frac{\hbar}{2}$ where $\hat{n}$ is a unit vector and $\boldsymbol{S}$ is the spin operator. A partially polarized beam of spin- $\frac{1}{2}$ particles contains a 25-75 mixture of two pure ensembles, one with $|+\rangle_{\hat{z}}$ and the other with $|+\rangle_{\hat{x}}$ respectively. What is the ensemble average of $\frac{S\cdot\hat{x}}{\hbar}$?

[JEST 2023]

(a) $\frac{1}{3}$ (b) $\frac{3}{8}$

(c) $\frac{1}{4}$ (d) $\frac{3}{16}$

15. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix

$$\left(\frac{1}{\sqrt{10}}\right)\{2, \sqrt{2}, 2i\}$$

in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_z|\psi\rangle$?

[JEST 2016]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

16. A two-level quantum system has the Hamiltonian$H = \hbar\omega_0\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
At $t = 0$, the system is in the state$|\psi(0)\rangle = \binom{1}{0}$ What is the earliest time $t > 0$ at which a measurement of $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ will yield the value -1 with probability one?

[JEST 2024]

(a) Never (b) $\dfrac{2\pi}{\omega_0}$

(c) $\dfrac{\pi}{\omega_0}$ (d) $\dfrac{\pi}{2\omega_0}$

❖ TIFR PYQ's

1. The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix}1/\sqrt{2}\\ 0\\ 1/\sqrt{2}\end{pmatrix}$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{pmatrix}$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be $+1$ is

[TIFR 2013]

(a) $1/\sqrt{2}$ (b) 1

(c) 1/2 (d) ¼

2. Consider the Hamiltonian

$$H = f\vec{\sigma}\cdot\vec{x}$$

Here $\vec{x}$ is the position vector, f is a constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are

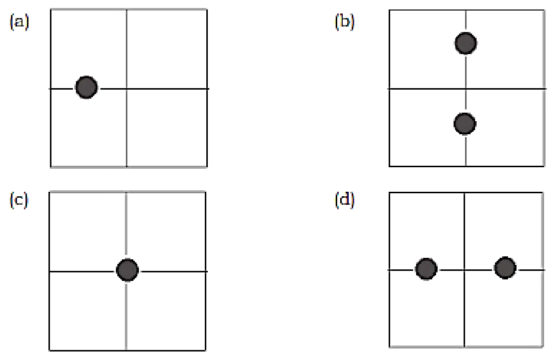
[TIFR 2014]

(a) $f\left(\sqrt{x^2+y^2} \pm z\right)$ (b) $f(x \pm iy)$

(c) $\pm f\sqrt{x^2+y^2+z^2}$ (d) $\pm f(x+y+z)$

3. In a Stern-Gerlach experiment with spin- 1/2 particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another SternGerlach apparatus but with its magnets rotated by 90 degrees counterclockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?

[TIFR 2014]



4. The state $|\Psi\rangle$ of a spin-1 particle is given by

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|1,-1\rangle + |1,0\rangle\exp\frac{i\pi}{3} + |1,1\rangle\exp\frac{2i\pi}{3}\right)$$

where $|S, M_S\rangle$ denote the spin eigenstates with eigenvalues $\hbar^2 S(S+1)$ and $\hbar M_S$ respectively. Find $\langle S_x \rangle$, i.e. the expectation value of the x component of the spin.

[TIFR 2018]

5. The Hamiltonian of a spin- 1/2 particle in a magnetic field $\vec{B}$ is given by $H = -\mu \vec{S} \cdot \vec{B}$, where the components of the spin operator $\vec{S}$ have eigenvalues $\pm\hbar/2$. The spin is pointing in the $+\hat{x}$ direction, when a magnetic field $\vec{B} = B\hat{y}$ is turned on. After a time $t = \pi/2\mu B$, the spin will be pointing along the direction

[TIFR 2021]

(a) $+\hat{z}$ (b) $-\hat{z}$

(c) $-\hat{x}$ (d) $\hat{x} + \hat{z}$

6. Consider an electron with mass m_e, charge $-e$ and spin 1/2, whose spin angular momentum operator is given by $\hat{\vec{S}} = \frac{\hbar}{2}\vec{\sigma}$ This electron is placed in a magnetic field $\vec{\boldsymbol{B}} = B_x \hat{\mathbf{1}} + B_y \hat{\jmath} + B_z \hat{\mathbf{k}}$, where all three components (B_x, B_y, B_z) are nonvanishing. At time $t = 0$, the electron is at rest in the $S_z = \hbar/2$ state. The earliest time when the state of the spin will be orthogonal to the initial state is

[TIFR 2023]

(a) $\dfrac{2m_e}{ge|\vec{\boldsymbol{B}}|}$

(b) infinity, i.e., it will never be orthogonal.

(c) $\dfrac{4m_e}{ge|\vec{B}|}$

(d) dependent on the direction of the magnetic field $\vec{\boldsymbol{B}}$

❖ Answers key				
		CSIR-NET		
1. c/d	2. d	3. d	4. b	5. d
6. a	7. a	8. c	9. c	10. d
11. a	12. a	13. d	14. d	15. b
16. 3	17. b			
		GATE		
1. a	2. b	3. d	4. c	5. a
6. c	7. a	8. 7.6	9. 0.66	10. b
11. b	12. c	13. c	14. b	15. 0.25
16. d	17. d	18. bd	19. b	20. d
		JEST		
1. c	2. a	3. b	4. d	5. c
6. b	7. d	8. b	9. b	10. d
11. d	12. 1.73	13. d	14. b	15. d
16. d				
		TIFR		
1. d	2. c	3. d	4.	5. a
6. b				

❖ **CSIR-NET PYQ's**

1. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as:

[CSIR JUNE 2011]

(a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$
(c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

2. A variational calculation is done with the normalized trial wavefunction

$$\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2)$$

for the one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } |x| \le a \\ \infty & \text{if } |x| > a \end{cases}$$

The ground state energy is estimated to be

[CSIR JUNE 2012]

(a) $\frac{5h^2}{3ma^2}$ (b) $\frac{3h^2}{2ma^2}$

(c) $\frac{3h^2}{5ma^2}$ (d) $\frac{5h^2}{4ma^2}$

3. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction $\psi(x) = Ae^{-bx^2}$ with b as the variational parameter?

[CSIR DEC 2012]

[Hint:

$$\int_{-\infty}^{\infty} x^{2n}e^{-2bx^2}dx = (2b)^{-n-\frac{1}{2}}\Gamma\left(n + \frac{1}{2}\right)]$$

(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$
(c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

4. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\dot{\alpha}(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A\exp(-bx^2)$ is

[Note: $\int_0^{\infty} e^{-t}t^a dt = \Gamma(a + 1)$)

[CSIR JUNE 2013]

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$
(c) $-m\alpha^2/\pi\hbar^2$ (d $-m\alpha^2/\sqrt{5}\pi\hbar^2$

5. Consider a particle of mass m in the potential $V(x) = a|x|, a > 0$. The energy eigen-values $E_n(n = 0,1,2,\dots)$, in the WKB approximation, are

[CSIR DEC 2014]

(a) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{1/3}$

(b) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(c) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)$

(d) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{4/3}$

6. The ground state energy of the attractive delta function potential $V(x) = -b\delta(x)$, where $b > 0$, calculated with the variational trial function

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0, & \text{otherwise,} \end{cases}$$

is

[CSIR DEC 2014]

(a) $-\frac{mb^2}{\pi^2\hbar^2}$ (b) $-\frac{2mb^2}{\pi^2\hbar^2}$

(c) $-\frac{mb^2}{2\pi^2\hbar^2}$ (d) $-\frac{mb^2}{4\pi^2\hbar^2}$

7. The ground state energy of a particle in the potential $V(x) = g|x|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\frac{c}{a^5}}(a^2 - x^2), & x < |a| \\ 0, & x \ge |a| \end{cases}$$

(where g and c are constants) is

[CSIR DEC 2015]

(a) $\frac{15}{16}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\frac{5}{6}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$

(c) $\frac{3}{4}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\frac{7}{8}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$

8. The energy levels for a particle of mass m in the potential $V(x) = \alpha|x|$, determined in the WKB approximation

$$\sqrt{2m}\int_a^b \sqrt{E - V(x)}dx = \left(n + \frac{1}{2}\right)\hbar\pi$$

where a, b are the turning points and $n = 0,1,2,\ldots$), are

[CSIR JUNE 2016]

(a)$E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

(b)$E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

(c)$E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

(d)$E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

9. The ground state energy of a particle of mass m in the potential

$$V(x) = \frac{\hbar^2\beta}{6m}x^4$$

estimated using the normalized trial wavefunction

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

is [Use

$$\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^2 e^{-\alpha x^2} = \frac{1}{2\alpha}$$

and

$$\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^4 e^{-\alpha x^2} = \frac{3}{4\alpha^2}].$$

[CSIR JUNE 2016]

(a) $\frac{3}{2m}\hbar^2\beta^{1/3}$ (b) $\frac{8}{3m}\hbar^2\beta^{1/3}$

(c) $\frac{2}{3m}\hbar^2\beta^{1/3}$ (d) $\frac{3}{8m}\hbar^2\beta^{1/3}$

10. Using the trial function

$$\psi(x) = \begin{cases} A(a^2 - x^2) & ,-a < x < a \\ 0; & ;\text{ otherwise} \end{cases}$$

the ground state energy of a one-dimensional harmonic oscillator is

[CSIR JUNE 2017]

(a)$\hbar\omega$ (b) $\sqrt{\frac{5}{14}}\hbar\omega$

(c)$\frac{1}{2}\hbar\omega$ (d) $\sqrt{\frac{5}{7}}\hbar\omega$

11. The energy eigenvalues E_n of a quantum system in the potential $V = cx^6$ (where $c > 0$ is a constant), for large values of the quantum number n, varies as

[CSIR DEC 2017]

(a) $n^{4/3}$ (b) $n^{3/2}$

(c) $n^{5/4}$ (d) $n^{6/5}$

12. The n-th energy eigenvalue E_n of a one-dimensional Hamiltonian

$$H = \frac{p^2}{2m} + \lambda x^4$$

(where $\lambda > 0$ is a constant) in the WKB approximation, is proportional to

[CSIR JUNE 2018]

(a) $\left(n+\frac{1}{2}\right)^{4/3}\lambda^{1/3}$ (b) $\left(n+\frac{1}{2}\right)^{4/3}\lambda^{2/3}$

(c) $\left(n+\frac{1}{2}\right)^{5/3}\lambda^{1/3}$ (d) $\left(n+\frac{1}{2}\right)^{5/3}\lambda^{2/3}$

13. A one-dimensional system is described by the Hamiltonian

$$H = \frac{p^2}{2m} + \lambda|x|$$

where $\lambda > 0$. The ground state energy varies as a function of λ as

[CSIR DEC 2018]

(a) $\lambda^{5/3}$ (b) $\lambda^{2/3}$

(c) $\lambda^{4/3}$ (d) $\lambda^{1/3}$

14. The Hamiltonian of a particle of mass m in one-dimension is

$$H = \frac{1}{2m}p^2 + \lambda|x|^3$$

where $\lambda > 0$ is constant. If E_1 and E_2, respectively, denote the ground state energies of the particle for $\lambda = 1$ and $\lambda = 2$ (in appropriate units) the ratio $\frac{E_2}{E_1}$ is best approximated by

[CSIR JUNE 2021]

(a) 1.260 (b) 1.414

(c) 1.516 (d) 1.320

15. Using a normalized trial wavefunction $\psi(x) = \sqrt{\alpha}e^{-\alpha|x|}$ (α is a positive real constant) for a particle of mass m in the potential $V(x) =$

$-\lambda\delta(x), (\lambda > 0)$, the estimated ground state energy is

[CSIR JUNE 2024]

(a) $-\dfrac{m\lambda^2}{\hbar^2}$ (b) $\dfrac{m\lambda^2}{\hbar^2}$

(c) $\dfrac{m\lambda^2}{2\hbar^2}$ (d) $-\dfrac{m\lambda^2}{2\hbar^2}$

16. The Hamiltonian of a particle of mass m is given by

$$H = \frac{p^2}{2m} + V(x)$$

with

$$\mathrm{V}(x) = \begin{cases} -\alpha x \text{ for } x \le 0 \\ \ \ \beta x \text{ for } x > 0 \end{cases}$$

where α, β are positive constants. The n^{th} energy eigenvalue E_n obtained using WKB approximation is

$$E_n^{3/2} = \frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2} \pi\left(n - \frac{1}{2}\right) f(\alpha,\beta)\ (n = 1,2,\dots)$$

The function $f(\alpha,\beta)$ is **[CSIR JUNE 2024]**

(a) $\sqrt{\dfrac{\alpha^2\beta^2}{2(\alpha^2+\beta^2)}}$ (b) $\dfrac{\alpha\beta}{\alpha+\beta}$

(c) $\dfrac{\alpha+\beta}{4}$ (d) $\dfrac{1}{2}\sqrt{\dfrac{\alpha^2+\beta^2}{2}}$

❖ JEST PYQ

1. Consider a particle confined by a potential $V(x) = k|x|$, where k is a positive constant. the spectrum $\mathrm{E_n}$ of the system, within the WKB approximation, is proportional to

[JEST 2017]

(a) $\left(n+\frac{1}{2}\right)^{3/2}$ (b) $\left(n+\frac{1}{2}\right)^{2/3}$

(c) $\left(n+\frac{1}{2}\right)^{1/2}$ (d) $\left(n+\frac{1}{2}\right)^{4/3}$

❖ TIFR PYQ's

1. Given a particle confined in a one-dimensional box between $x = -a$ and $x = +a$, a student attempts to find the ground state by assuming a wave-function

$$\psi(x) = \begin{cases} A(a^2 - x^2)^{3/2} & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

The ground state energy E_m is estimated by calculating the expectation value of energy with this trial wave-function. If E_0 is the true ground state energy, what is the ratio E_m/E_0 ?

[TIFR 2018]

2. Consider a particle of mass m in a quartic potential $H = \frac{p^2}{2m} + ax^4$

If we take a variational wavefunction

$\psi(x,\lambda) = e^{-\lambda x^2}$ with $\lambda > 0$ and try to estimate the ground state energy, the value of λ should be chosen as

[You may use the integral

$$\int_{-\infty}^{+\infty} dx(A + Bx^2 + Cx^4)e^{-\lambda x^2} = A\sqrt{\frac{\pi}{\lambda}} + \frac{B}{2}\sqrt{\frac{\pi}{\lambda^3}} + \frac{3C}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

where A, B, C and $\lambda > 0$ are all constants.]

[TIFR 2023]

(a) $\left(\dfrac{5ma}{3\pi^2\hbar^2}\right)^{1/3}$ (b) $\left(\dfrac{3ma}{4\hbar^2}\right)^{1/3}$

(c) $\left(\dfrac{15ma}{8\hbar^2}\right)^{1/3}$ (d) $\left(\dfrac{ma}{2\pi\hbar^2}\right)^{1/3}$

❖ Answers key				
CSIR-NET				
1. b	2. b	3. c	4. c	5. b
6. d	7. a	8. b	9. d	10. b
11. b	12. a	13. b	14. d	15. d
16. b				
JEST PYQ				
1. b				
TIFR				
1.	2. b			

TIFR- Q.1 $\frac{21}{8\pi^2}$

❖ **CSIR-NET PYQ**

1. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

[CSIR DEC 2012]

(a) $14\pi^2\hbar^2/mL^2$
(b) $91\pi^2\hbar^2/mL^2$
(c) $7\pi^2\hbar^2/mL^2$
(d) $3\pi^2\hbar^2/mL^2$

2. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ and $5\varepsilon_0, (\varepsilon_0 > 0)$. Onc system is populated by spin$\frac{1}{2}$ fermions and the other by bosons; What is the value of $E_F - E_B$ where E_p and E_B are the ground state energies of the fermionic and bosonic systems respectively?

[CSIR JUNE 2013]

(a) $6\varepsilon_0$
(b) $2\varepsilon_0$
(c) $4\varepsilon_0$
(d) ε_0

3. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

[CSIR DEC 2013]

(a) 8
(b)4
(c) 3
(d) 2

4. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a.
(Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E = \frac{5\pi^2\hbar^2}{2ma^2}$$

Is **[CSIR JUNE 2014]**

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right) - \sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

5. A particle of mass m is confined in a three-dimensional box by the potential

$$V(x,y,z) = \begin{cases} 0, & 0 \le x,y,z \le a \\ \infty, & \text{otherwise} \end{cases}$$

The number of eigenstates of Hamiltonian with energy

$$\frac{9\hbar^2\pi^2}{2ma^2}$$

is

[CSIR JUNE 2018]

(a) 1
(b) 6
(c) 3
(d) 4

6. Three identical spin $-\frac{1}{2}$ particles of mass m are confined to a one-dimensional box of length L, but are otherwise free. Assuming that they are non-interacting, the energies of the lowest two energy eigenstates, in units of $\frac{\pi^2\hbar^2}{2mL^2}$, are

[CSIR DEC 2018]

(a) 3 and 6
(b) 6 and 9
(c) 6 and 11
(d) 3 and 9

7. Two spin- $\frac{1}{2}$ fermions of mass m are confined to move in a one-dimensional infinite potential well of width L. If the particles are known to be in a spin triplet state, the ground state energy of the system (in units of $\frac{\hbar^2\pi^2}{2mL^2}$) is

[CSIR DEC 2019]

(a) 8
(b) 2
(c) 3
(d) 5

8. Spin $\frac{1}{2}$ fermions of mass m and $4m$ are in a harmonic potential $V(x) = \frac{1}{2}kx^2$. Which configuration of 4 such particles has the lowest value of the ground state energy?

[CSIR JUNE 2020]

(a) 4 particles of mass m

(b) 4 particles of mass $4m$

(c) 1 particle of mass m and 3 particles of mass $4m$

(d) 2 particles of mass m and 2 particles of mass $4m$

9. The energy levels available to each electron in a system of N non-interacting electrons are $E_n = nE_0 n = 0,1,2,\cdots$. A magnetic field, which does not affect the energy spectrum, but completely polarizes the electron spins, is applied to the system. The change in the ground state energy of the system is

[CSIR JUNE 2023]

(a) $\frac{n^2E_0}{2}$ (b) n^2E_0

(c) $\frac{n^2E_0}{8}$ (d) $\frac{n^2E_0}{4}$

10. The number density of a mono-atomic gas is $10^{18}\ \mathrm{m}^{-3}$. The temperature at which the thermal de Broglie wavelength of the atoms (which have mass m) equals the average inter-atomic separation, is

[CSIR JUNE ASSAM 2019]

(a) $\frac{h^2}{2\pi mk_B} \times 10^{12}K$ (b) $\frac{h^2}{2\pi mk_B} \times 10^{10}K$

(c) $\frac{h^2}{2\pi mk_B} \times 10^{9}K$ (d) $\frac{h^2}{2\pi mk_B} \times 10^{11}K$

11. Two non-interacting identical spin $-\frac{1}{2}$ particles, each of mass m, are placed in a two-dimensional infinite square well of side L. The single-particle spatial wavefunction is given by

$$\varphi_{n_x,n_y}(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$$

where n_x and n_y are positive integers. If the particles are in a total spin state $|j = 1, m = 0\rangle$, the lowest possible energy eigenvalue is

[CSIR JUNE 2025]

(a) $\frac{5\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{mL^2}$

(c) $\frac{2\hbar^2\pi^2}{mL^2}$ (d) $\frac{7\hbar^2\pi^2}{2mL^2}$

12. A particle of mass m is in a cubic box of size a. The potential inside the box $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinte outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2h^2}{2ma^2}$, its wave function is:

[CSIR JUNE 2012]

(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a}\sin\frac{5\pi y}{a}\sin\frac{6\pi z}{a}$

(b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a}\sin\frac{4\pi y}{a}\sin\frac{3\pi z}{a}$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a}\sin\frac{8\pi y}{a}\sin\frac{2\pi z}{a}$

(d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a}\sin\frac{2\pi y}{a}\sin\frac{3\pi z}{a}$

13. The Hamiltonian of two interacting particles, one with spin-1 and the other with spin-$\frac{1}{2}$, is given by

$$H = AS_1 \cdot S_2 + B(S_{1x} + S_{2x}),$$

where S_1 and S_2 denote the spin operators of the first and second particles, respectively, and A and B are positive constants. The largest eigenvalue of this Hamiltonian is

[CSIR DEC 2019]

(a) $\frac{1}{2}(A\hbar^2 + 3B\hbar)$ (b) $3A\hbar^2 + B\hbar$

(c) $\frac{1}{2}(3A\hbar^2 + B\hbar)$ (d) $A\hbar^2 + 3B\hbar$

14. The coordinates of two identical particles of spin 0 in one dimension are x_1 and x_2, respectively. Which of the following functions is an admissible wavefunction for the two-particle system?

[CSIR Dec. 2019]

(a) $\frac{1}{2\pi\alpha}e^{-\frac{1}{2a^2}\left(x_1^2+\frac{x_2^2}{4}\right)}$

(b) $\frac{1}{\pi\alpha}e^{i(k_1x_1+k_2x_2)}e^{-\frac{1}{2\alpha^2}(x_1+x_2^2)}$

(c) $\frac{1}{2\pi\alpha}e^{i(k_1x_1+k_2x_2)}e^{-\frac{1}{2\alpha^2}(x_1^2+x_2^2/4)}$

(d) $\frac{1}{21\pi\alpha^5}(x_1-x_2)^2e^{-\frac{1}{2\alpha^2}(x_1^2+x_2^2)}$

15. A one-dimensional box contains one spin 0 particle and four spin $-\frac{1}{2}$ particles. All particles have the same mass, hence the single particle energies are $E_n = n^2E, n = 1,2,3, \ldots$ The energies of the ground and the first excited states of the system of particle energies of the system of particles are, respectively.

[CSIR Dec. 2019]

(a) 10 E and 12 E
(b) 11 E and 20 E
(c) 11 E and 14 E
(d) 10 E and 13 E

16. A system consists of two non-interacting identical spin-$\frac{1}{2}$ particles. The spatial wave functions for the individual particles are given by $\varphi_1(x)$ and $\varphi_2(x)$. Let x_1 and x_2 denote the positions of the particles respectively. The total wave function of the system (not necessarily normalized) can be **[CSIR JUNE 2025]**

(a) $[\varphi_1(x_1)\varphi_2(x_2)-\varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2+|\downarrow\rangle_1|\uparrow\rangle_2]$

(b) $[\varphi_1(x_1)\varphi_1(x_2)+\varphi_2(x_1)\varphi_2(x_2)]|\uparrow\rangle_1|\uparrow\rangle_2$

(c) $\varphi_1(x_1)\varphi_2(x_2)|\uparrow\rangle_1|\uparrow\rangle_2$

(d) $[\varphi_1(x_1)\varphi_2(x_2)-\varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2-|\downarrow\rangle_1|\uparrow\rangle_2]$

❖ **GATE PYQ**

1. Two particles are said to be distinguishable when

[GATE 2001]

(a) the average distance between them is large compared to their de Broglie wavelengths

(b) the average distance between them is small compared to their de Broglie wavelengths

(c) they have overlapping wave packets

(d) their total wave function is symmetric under particle exchange

2. Two spin $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r) = \vec{S}_1\cdot\vec{S}_2V_0(r)$. The contribution of this potential in the singlet and triplet states, respectively, are

[GATE 2004]

(a) $-\frac{3}{2}V_0(r)$and $\frac{1}{2}V_0(r)$

(b) $\frac{1}{2}V_0(r)$and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$and $-\frac{3}{4}V_0(r)$

(d) $=\frac{3}{4}V_0(r)$and $\frac{1}{4}V_0(r)$

Common Data for Q. 3 and Q. 4

The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc.

3. The energy of the lowest state is

[GATE 2004]

(a) zero
(b) $\frac{\hbar^2\pi^2}{2ma^2}$
(c) $\frac{\hbar^2\pi^2}{ma^2}$
(d) $\frac{3\hbar^2\pi^2}{2ma^2}$

4. The degeneracy (including spin) of the level ε_3 is

[GATE 2004]

(a) 2
(b) 4
(c) 6
(d) 8

5. It is necessary to apply quantum statistics to a system of particles if

[GATE 2007]

(a) there is substantial overlap between the wavefunctions of the particles

(b) the mean free path of the particles is comparable to the inter-particle separation

(c) the particles have identical mass and charge

(d) the particles are interacting

6. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by $V(x,y) = 0$ for $-a < x < a$ and $-a < y < a$ $= \infty$ elsewhere

The energy of the first excited state for this particle is given by

[GATE 2012]

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

7. Consider the wave function $\Psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wave function is given by,

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)]$$

Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wave function with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

[GATE 2012]

(a) $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$

(c) $\alpha\alpha$ (d) $\beta\beta$

8. Which one of the following is a fermions?

[GATE 2014]

(a) α particle (b) ${}_4Be^7$ nucleus

(c) Hydrogen atom (d) Deuteron

9. The Pauli matrices for three spin-1/2 particles are $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$, respectively. The dimension of Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \times \vec{\sigma}_2 \times \vec{\sigma}_3$ is

[GATE 2015]

10. Consider a system of eight non-interacting, identical quantum particles of spin- 3/2 in a one dimensional box of length L. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is____________.

[GATE 2015]

11. Let the Hamiltonian for two spin 1/2 particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be

$$H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2(r_1^2 + r_2^2) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1\text{eV}$ and $k = 0.2\text{eV}$. If the ground state has net spin zero, then the energy (in eV) is____________.

[GATE 2015]

12. A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0K$. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is________.

[GATE 2016]

13. For a spin $\frac{1}{2}$ particle, let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote its spin up and spin down states, respectively. If

$$|a\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$$

and

$$|b\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$$

are composite states of two such particles, which of the following statements is true for their total spin S ?

[GATE 2019]

(a) $S = 1$ for $|a\rangle$, and $|b\rangle$ is not an eigenstate of the operator $\hat{S}^2$

(b) $|a\rangle$ is not an eigenstate of the operator $\hat{S}^2$, and $S = 0$ for $|b\rangle$

(c) $S = 0$ for $|a\rangle$, and $S = 1$ for $|b\rangle$

(d) $S = 1$ for $|a\rangle$, and $S = 0$ for $|b\rangle$

14. Three non-interacting bosonic particles of mass m each, are in a one-dimensional infinite potential well of width a. The energy of the third excited state of the system is $x \times \frac{\hbar^2\pi^2}{ma^2}$.

The value of x (in integer) is

[GATE 2021]

15. For a two-nucleon system in spin singlet state, the spin is represented through the Pauli matrices σ_1, σ_2 for particles 1 and 2, respectively. The value of $(\sigma_1 \cdot \sigma_2)$ (in integer) is

[GATE 2021]

16. A system of five identical, non-interacting particles with mass m and spin $\frac{3}{2}$ is confined to a one-dimensional potential well of length L. If the lowest energy of the system is $N\frac{\pi^2\hbar^2}{2mL^2}$, the value of N (in integer) is

17. Consider the wave function $\Psi = \psi(\vec{r}_1,\vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wave function is given by,

$$\psi(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)]$$

Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wave function with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

[GATE 2012]

(a) $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$

(c) $\alpha\alpha$ (d) $\beta\beta$

18. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and Ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

[GATE 2014]

(a) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) - \psi_2(x_2)\psi_2(x_2)]$

(b) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$

(c) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$

(d) $\frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

19. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle\vec{s}_1\cdot\vec{s}_2\rangle$ for the ground state is **[GATE 2016]**

(a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$

(c) 0 (d) $\frac{1}{4}\hbar^2$

❖ JEST PYQ

1. The ground state energy of 5 identical spin- 1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is

[JEST 2012]

(a) $\frac{15}{2}\hbar\omega$ (b) $\frac{13}{2}\hbar\omega$

(c) $\frac{1}{2}\hbar\omega$ (d) $5\hbar\omega$

2. The spatial part of a two- electron state is symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin up -and spin - down states respectively of each particle, the spin - part of the two particle state is.

[JEST 2012]

(a) $|\uparrow\rangle|\uparrow\rangle$

(b) $|\uparrow\rangle|\downarrow\rangle$

(c) $(|\downarrow\rangle|\uparrow\rangle - |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$

(d) $(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$

3. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$ what would be the ground state wave function $\psi(x_1,x_2)$ if both electrons are arranged to have the same spin state?

[JEST 2013]

(a) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) + \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) - \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

4. The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is 2eV. The energy of the quantum mechanical ground state for a system of three non-interacting spin-1/2 particles is

[JEST 2014]

(a) 6eV (b) 10eV

(c) 12eV (d) 16Ev

5. Consider N non-interacting electrons $(N \sim N_A)$ in a box of sides L_x, L_y, L_z Assume that the dispersion relation is $\epsilon(k) = Ck^4$ where C is a constant, the ratio of the ground state energy per particle to the Fermi energy is :

[JEST 2016]

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$

(c) $\frac{3}{5}$ (d) $\frac{5}{7}$

6. Suppose the spin degrees of freedom of a 2-particle system can be described by a 21-dimensional Hilbert Subspace. Which among the following could be the spin of one of the particles?

[JEST 2017]

(a) $\frac{1}{2}$ (b) 3

(c) $\frac{3}{2}$ (d) 2

7. Suppose the spin degree of freedom of two particles (nonzero rest mass and nonzero spin) is described completely by a Hilbert space of dimension twenty one. Which of the following could be the spin of one of the particles?

[JEST 2018]

(a) 2 (b) 3/2

(c) 1 (d) ½

8. Consider a system of 15 non-interacting spin-polarized electrons. They are trapped in a two dimensional isotropic harmonic oscillator potential $V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2)$. The angular frequency ω is such that $\hbar\omega = 1$ in some chosen unit. What is the ground state energy of the system in the same units?

[JEST 2019]

9. The smallest dimension of the Hilbert space in which we can find operators $\hat{x}, \hat{p}$ that satisfy $[\hat{x}, \hat{p}] = i\hbar$ is

[JEST 2021]

(a) 1 (b) 3

(c) 4 (d) ∞

10. A one-dimensional box contains three identical particles in the ground state of the system. Find the ratio of total energies of these particles if they were spin- $\frac{1}{2}$ fermions, to that if they were bosons.

[JEST 2021]

(a) 1 (b) $\frac{14}{3}$

(c) 2 (d) $\frac{1}{3}$

11. Consider eight electrons confined in a 1D box of length d. What is the minimum total energy for the system allowed by Pauli's exclusion principle?

[JEST 2022]

(a) $\frac{30h^2}{md^2}$ (b) $\frac{15h^2}{4md^2}$

(c) $\frac{15h^2}{2md^2}$ (d) $\frac{15h^2}{8md^2}$

12. A quantum mechanical particle of mass m is confined in a one dimensional infinite potential well whose walls are located at $x = 0$ and $x = 1$. The wave function of the particle inside the well is $\psi(x) = \mathcal{N}[x\ln x + (1-x)\ln(1-x)]$ for some normalization constant $\mathcal{N}$. An experimentalist measures the position of the particle on an ensemble of a large number of identical systems in the same state. The mean of the outcomes is found to be $\frac{1}{n}$, where n is an integer. What is n ?

[JEST 2024]

13. Consider 5 identical spin 1/2 particles moving in a 3 -dimensional harmonic oscillator potential,

$$V(r) = \frac{1}{2}m\omega^2r^2 = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$$

The degeneracy of the ground state of the system is **[JEST 2022]**

(a) 20 (b) 7

(c) 5 (d) 32

14. Consider a particle of mass m moving inside a two dimensional square box whose sides are described by the equations $x=0, x=L, y=0, y=L$. What is the lowest eigen value which changes sign under the exchange of x and y ?

[JEST 2012]

(a) $\hbar^2/(mL^2)$ (b) $3\hbar^2/(2mL^2)$

(c) $5\hbar^2/(2mL^2)$ (d) $7\hbar^2/(2mL^2)$

15. What is the difference between the maximum and the minimum eigenvalues of a system of two electrons whose Hamiltonian is $H+J\vec{S}_1\cdot\vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the corresponding spin angular momentum operators of the two electrons?

[JEST 2018]

(a) J/4 (b) J/2

(c) 3 J/4 (d) J

❖ **TIFR PYQ**

1. Two identical non-interacting particles, each of mass m and spin $\frac{1}{2}$, are placed in a onedimensional box of length L. In quantum mechanics, the lowest possible value of the total energy of these two particles is ϵ_0. If, instead, four such particles are introduced into a similar one-dimensional box of length $2L$, then the lowest possible value of their total energy will be

[TIFR 2011]

(a) $2\epsilon_0$ (b) $5\epsilon_0/4$

(c) $3\epsilon_0/2$ (d) ϵ_0

2. 1000 neutral spinless particles are confined in a one-dimensional box of length 100 nm. At a given instant of time, if 100 of these particle have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$, then the number of particles in the left half of the box will be approximately

[TIFR 2015]

(a) 625 (b) 500

(c) 441 (d) 100

3. Consider two spin-1/2 identical particles A and B, separated by a distance r, interacting through a potential$V(r)=\frac{V_0}{r}\vec{S}_A\cdot\vec{S}_B$ where V_0 is a positive constant and the spins are $\vec{S}_{A,B}=\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ in terms of the Pauli spin matrices. The expectation values of this potential in the spin singlet and triplet states are

[TIFR 2016]

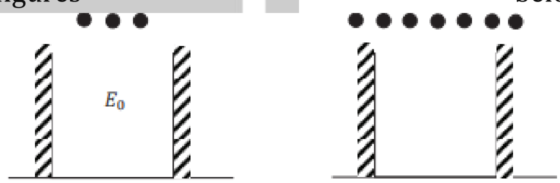
(a) Singlet : $-\frac{V_0}{3r}$, Triplet : $\frac{V_0}{r}$

(b) Singlet : $-\frac{3V_0}{r}$, Triplet : $\frac{V_0}{r}$

(c) Singlet : $\frac{3V_0}{r}$, Triplet : $-\frac{V_0}{r}$

(d) Singlet : $-\frac{V_0}{r}$, Triplet : $\frac{3V_0}{r}$

4. A quantum mechanical system consists of a one-dimensional infinite box, as indicated in the figures below.



3 (three) identical non-interacting spin- 1/2 particles, are first placed in the box, and the ground state energy of the system is found to be $E_0=18\text{eV}$. If 7 (seven) such identical particles are placed in the box, what will be the ground state energy, in units of eV?

[TIFR 2017]

5. A system was formed of three spin $-1/2$ particles A, B and C, respectively and it was prepared in an initial state
$|\psi\rangle=c_1|\uparrow\uparrow\uparrow\rangle+c_2|\uparrow\uparrow\downarrow\rangle+c_3|\uparrow\downarrow\uparrow\rangle+c_4|\uparrow\downarrow\downarrow\rangle+c_5|\downarrow\uparrow\uparrow\rangle+c_6|\downarrow\uparrow\downarrow\rangle+c_7|\downarrow\downarrow\uparrow\rangle+c_8|\downarrow\downarrow\downarrow\rangle$ where the symbols $|\uparrow\rangle$ and $|\downarrow\rangle$ indicate states with $S_z=+1/2$ (spin-up) and $S_z=-1/2$ (spin-down) respectively.
A measurement was made on the system in the initial state and this identified the spin state of the particle A to be $|\downarrow\rangle$ (spin-down). Now the expectation value of $\langle S_z\rangle$ for the particle C could be calculated as

[TIFR 2022]

(a) $\frac{c_5+c_7-c_6-c_8}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

(b) $\dfrac{|c_5|^2+|c_7|^2-|c_6|^2-|c_8|^2}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

(c) $\dfrac{(c_5^*+c_7^*-c_6^*-c_8^*)(c_5+c_7-c_6-c_8)}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

(d) $\dfrac{(c_5+c_7)^*(c_5+c_7)-(c_6+c_8)^*(c_6+c_8)}{|c_5|^2+|c_7|^2+|c_6|^2+|c_8|^2}$

6. Three noninteracting particles whose masses are in the ratio 1: 4: 16 are placed together in the same harmonic oscillator potential $V(x)$. The degeneracies of the first three energy eigenstates (ordered by increasing energy) will be

[TIFR 2017]

(a) 1,1,1 (b) 1,1,2

(c) 1,2,1 (d) 1,2,2

❖ Answers key				
CSIR-NET				
1. a	2. b	3. d	4. a	5. c
6. b	7. d	8. d	9. d	10. b
11. d	12. d	13. a	14. d	15. c
16. a				
GATE				
1. a	2. d	3. d	4. c	5. a
6. d	7. b	8. b	9. 8	10. 5
11. -0.2	12. 24	13. d	14. 6	15. -3
16.	17. b	18. d	19. b	
JEST				
1. b	2. c	3. b	4. c	5. a
6. d	7. c	8. 55	9. d	10. c
11. c	12. 2	13. a	14. c	15. d
TIFR				
1.	2. b	3. b	4. 132	5. b
6. b				

❖ **CSIR-NET PYQ's**

1. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r) = \begin{cases} V_0 & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$ If V_0 is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the Born approximation,

[CSIR JUNE 2012]

(a) increases to four times the original value

(b) increases to twice the original value

(c) decreases to half the original value

(d) decreases to one fourth the original value

2. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential

$$V(r) = \frac{\beta e^{-\mu r}}{r}$$

is given by: (in the following $b = 2k\sin\frac{\theta}{2}, E = \hbar^2 k^2/2m$)

[CSIR JUNE 2013]

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $\frac{2m\beta}{\hbar^2(a^2+b^2)}$

(c) $\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

3. The scattering amplitude $f(\theta)$ for the potential $V(r) = \beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation by (in the following $b = 2k\sin\frac{\theta}{2}$ and

$$E = \frac{\hbar^2 k^2}{2m})$$

[CSIR JUNE 2014]

(a) $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^2}$ (b) $-\frac{4m\beta\mu}{\hbar^2 b^2(b^2+\mu^2)}$

(c) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ (d) $-\frac{4m\beta\mu}{\hbar^2(b^2+\mu^2)^3}$

4. In deep inelastic scattering electrons are scattered off protons to determine if a proton

has any internal structure. The energy of the electron for this must be at least

[CSIR DEC 2014]

(a) 1.25×10^9eV (b) 1.25×10^{12}eV

(c) 1.25×10^6eV (d) 1.25×10^8eV

5. The differential cross-section for scattering by a target is given by

$$\frac{d\sigma}{d\Omega}(\theta,\varphi) = a^2 + b^2\cos^2\theta.$$

If N is the flux of the incoming particles, the number of particles scattered per unit time is

[CSIR JUNE 2015]

(a) $\frac{4\pi}{3}N(a^2+b^2)$ (b) $4\pi N\left(a^2+\frac{1}{6}b^2\right)$

(c) $4\pi N\left(\frac{1}{2}a^2+\frac{1}{3}b^2\right)$ (d) $4\pi N\left(a^2+\frac{1}{3}b^2\right)$

6. The Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2c^2 + m^2c^4$. The classical energy-momentum relation of a particle of charge q in a electromagnetic potential $(\phi, \vec{A})$ is

$$(E - q\phi)^2 = c^2\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + m^2c^4$$

Therefore, the Dirac Hamiltonian for an electron in an electromagnetic field is

[CSIR JUNE 2015]

(a) $c\vec{\alpha}\cdot\vec{p} + \frac{e}{c}\vec{A}\cdot\vec{A} + \beta mc^2 - e\phi$

(b) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi$

(c) $c\left(\vec{\alpha}\cdot\vec{p} + e\phi + \frac{e}{c}|\vec{A}|\right) + \beta mc^2$

(d) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 - e\phi$

7. A particle of energy E scatters off a repulsive spherical potential

$$V(r) = \begin{cases} V_0 & \text{for} \quad r < a \\ 0 & \text{for} \quad r \geq a \end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering, cross-section is

$$\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh ka - 1\right)^2,$$

where

$$k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$$

In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

[CSIR JUNE 2015]

(a) 4 (b) 3

(c) 1 (d) 1/2

8. In the scattering of some elementary cross section σ is found to depend on the total energy E and the fundamental constants h and c using dimensional analysis, the dependence of σ on these quantities is given by

[CSIR DEC 2015]

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$

(c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

9. A particle is scattered by a central potential $V(r)=V_0 r e^{-\mu r}$, where V_0 and μ are positive constants.
If the momentum transfer $\vec{q}$ is such that $q=|\vec{q}| \gg \mu$, the scattering cross-section in the Born approximation, as $q \rightarrow \infty$, depends on q as

[CSIR DEC 2016]

[You may use $\int x^n e^{ax} dx=\frac{d^n}{da^n} \int e^{ax} dx$]

(a) q^{-8} (b) q^{-2}

(c) q^2 (d) q^6

10. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is

(a) $\theta=\frac{\pi}{2}$ (b) $\theta=\pi$

(c) $0<\theta<\frac{\pi}{2}$ (d) $\frac{\pi}{2}<\theta \leq \pi$

11. The dynamics of a free relativistic particle of mass m is governed by the Dirac Hamiltonian $H=c\vec{\alpha} \cdot \vec{p}+\beta mc^2$, where $\vec{p}$ is the momentum operator and $\vec{\alpha}=(\alpha_x, \alpha_y, \alpha_z)$ and β are four 4×4 Dirac matrices. The acceleration operator can be expressed as

[CSIR DEC 2016]

(a) $\frac{2ic}{\hbar}(c\vec{p}-\vec{\alpha} H)$ (b) $2ic^2 \vec{\alpha} \beta$

(c) $\frac{ic}{h} H \ddot{\alpha}$ (d) $-\frac{2ic}{\hbar}(c\vec{p}+\vec{\alpha} H)$

12. Consider the potential

$$U(\vec{r})=\sum_i V_0 a^3 \delta^{(3)}(\vec{r}-\vec{r}_i)$$

where $\vec{r}_c$ are the position vectors of the vertices of a cube of Length ' a ' centred at origin and V_0 is constant. If

$$V_0 a^2 \ll \frac{\hbar^2}{m}$$

, the total scattering cross - section, in the low energy limit is

(a) $16a^2\left(\frac{mv_0 a^2}{\hbar^2}\right)$ (b) $\frac{16a^2}{\pi^2}\left(\frac{mv_0 a^2}{\hbar^2}\right)^2$

(c) $\frac{64a^2}{\pi}\left(\frac{mV_0 a^2}{\hbar}\right)^2$ (d) $\frac{64a^2}{\pi^2}\left(\frac{mV_0 a^2}{\hbar^2}\right)$

13. A phase shift of 30° is observed when a beam of particles of energy 0.1MeV is scattered by a target. When the beam energy is changed, the observed phase shift is 60°. Assuming that only s-wave scattering is relevant and that the cross-section does not change with energy, the beam energy is

[CSIR DEC 2017]

(a) 0.4MeV (b) 0.3MeV

(c) 0.2MeV (d) 0.15MeV

14. The differential scattering cross section $d\sigma/d\Omega$ for the central potential $V(r)=\frac{\beta}{r} e^{-\mu r}$, where β and μ are positive constants, is calculated in the first Born approximation. Its dependence on the scattering angle θ is proportional to (A is a constant below).

[CSIR JUNE 2018]

(a) $\left(A^2+\sin^2 \frac{\theta}{2}\right)$ (b) $\left(A^2+\sin^2 \frac{\theta}{2}\right)^{-1}$

(c) $\left(A^2+\sin^2 \frac{\theta}{2}\right)^{-2}$ (d) $\left(A^2+\sin^2 \frac{\theta}{2}\right)^2$

15. In the particle wave expansion, the differential scattering cross-section is given by

$$\frac{d\sigma}{d(\cos\,\theta)} = \left|\sum_{l} (2l+1)e^{i\delta_l}\sin\,\delta_l P_l(\cos\,\theta)\right|^2$$

where θ is the scattering angle. For a certain neutron-nucleus scattering, it is found that the two lowest phase shifts δ_0 and δ_1 corresponding to s-wave and p-wave, respectively, satisfy $\delta_1 \approx \delta_0/2$. Assuming that the other phase shifts are negligibly small, the differential cross-section reaches if minimum for $\cos\,\theta$ equal to

[CSIR JUNE 2019]

(a) 0 (b) ± 1

(c) $-\frac{2}{3}\cos^2\,\delta_1$ (d) $\frac{1}{3}\cos^2\,\delta_1$

16. The elastic scattering of a charged particle of mass m off an atom can be approximated by the poten tial $V(r) = \frac{\alpha}{r}e^{-ri\hbar}$, where α and R are positive constants. If the wave number of the incoming particle is k and the scattering angle is 2θ, the differential cross-section in the Born approximation in:

[CSIR JUNE 2019]

(a) $\frac{m^2\alpha^2R^4}{4\hbar^4(1+k^2R^2\sin^2\,\theta)}$

(b) $\frac{m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2\,\theta)^2}$

(c) $\frac{2m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2\,2\theta)}$

(d) $\frac{4m^2\alpha^2R^4}{\hbar^4(\mathrm{i}+4k^2R^2\sin^2\,\theta)^2}$

17. The range of the inter-atomic potential in gaseous hydrogen is approximately 5 A. In thermal equilibrium, the maximum temperature for which the atom-atom scattering is dominantly s-wave, is

[CSIR JUNE 2019]

(a) 500 K (b) 100 K

(c) 1 K (d) I mk

18. A particle with incoming wave vector $\vec{k}$, after being scattered by the potential $V(r) = \frac{c}{r^2}$, goes out with wave vector $\vec{k}'$. The differential scattering cross-section, calculated in the first Born approximation, depends on $q = |\vec{k} - \vec{k}'|$, as

[CSIR JUNE 2020]

(a) $1/q^2$ (b) $1/q^4$

(c) $1/q$ (d) $1/q^{3/2}$

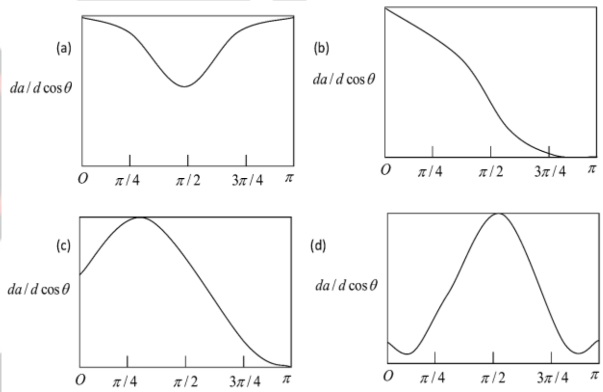
19. In an elastic scattering process at an energy E, the phase shift $\delta_0 \approx 30°, \delta_1 \approx 10°$, while the other phase shifts are zero. The polar angle at which the differential cross section peaks is closest to

[CSIR JUNE 2021]

(a) $20°$ (b) $10°$

(c) 0^0 (d) $30°$

20. The phase shifts of the partial waves in an elastic scattering at energy E are $\delta_0 = 12^0, \delta_1 = 4^0$ and $\delta_{l\geq 2} = 0^0$. The best qualitative depiction of θ dependence of the differential scattering cross-section $\frac{d\sigma}{d\cos\,(\theta)}$ is **[CSIR JUNE 2023]**



21. An incident plane wave with wavenumber k is scattered by a spherically symmetric soft potential. The scattering occurs only in S - and P - waves. The approximate scattering amplitude at angles $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$ are

$$f\left(\theta = \frac{\pi}{3}\right) \simeq \frac{1}{2k}\left(\frac{5}{2} + 3i\right) \text{ and } f\left(\theta = \frac{\pi}{2}\right) \simeq \frac{1}{2k}\left(1 + \frac{3i}{2}\right).$$

Then the total scattering cross-section is closest to **[CSIR DEC 2023]**

(a) $\frac{37\pi}{4k^2}$ (b) $\frac{10\pi}{k^2}$

(c) $\frac{35\pi}{4k^2}$ (d) $\frac{9\pi}{k^2}$

22. In a scattering experiment, a beam of $e^{-}$with an energy of 420 MeV scatters off an atomic nucleus. If the first minimum of the differential cross section is observed at a scattering angle of 45°, the radius of the nucleus (in fermi) is closest to

[CSIR JUNE 2024]

(a) 0.4 (b) 8.0

(c)2.5 (d) 0.8

23. A particle of energy E is scattered off a one-dimensional potential $\lambda\delta(x)$, where λ is a real positive constant, with a transmission amplitude t_+. In a different experiment, the same particle is scattered off another one-dimensional potential $-\lambda\delta(x)$, with a transmission amplitude t_-. In the limit $E \to 0$, the phase difference between $t_+$and $t_-$is

[CSIR JUNE 2024]

(a)$\pi/2$ (b)π

(c) 0 (d) $3\pi/2$

❖ **GATE PYQ's**

1. If σ is the total cross-section and $f(\theta), \theta$ being the angle of scattering, is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction.

[GATE 2002]

(a) $\sigma = |f(\theta)|^2$

(b)$\sigma = \frac{4\pi}{k}|f(\theta = 0)|^2$

(c)$\sigma = \frac{4\pi}{k} \times$ Imaginary part of $[f(\theta = 0)]$

(d)$\sigma = \frac{4\pi}{k}|f(\bar{\theta})|^2$

2. The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresponding Born approximation is known as the first Born approximation. Such an approximation is valid for.

[GATE 2016]

(a) large incident energies and the weak scattering potentials

(b) Large incident energies and strong scattering potential

(c) Small incident energies & weak scattering potential

(d) Small incident particle energies and strong scattering potential

3. Consider the potential $U(r)$ defined as

$$U(r) = -U_0 \frac{e^{-\alpha r}}{r}$$

where α and U_0 are real constants of appropriate dimensions. According to the first Born approximation, the elastic scattering amplitude calculated with $U(r)$ for a (wavevector) momentum transfer q and $\alpha \to 0$, is proportional to

(Useful integral: $\int_0^{\infty} \sin(qr) e^{\alpha r} dr = \frac{q}{\alpha^2 + q^2}$)

[GATE 2021]

(a) q^{-2} (b) q^{-1}

(c) q (d) q^2

4. Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}\text{fm}^{-1}$. Then the Total scattering cross section in unit of fm^2 is

[GATE 2016]

5. A particle is scatted by a spherical symmetric potential. In the centre of mass (cm) frame the wavefunction of particle is $\psi = Ae^{ikz}$ where k is the wavefunction & A is canst. (i) If $f(\theta)$ is the angular wavefunction then in the asymptotic region the scattered coavefunction has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$ (b) $\frac{Af(\theta)e^{-ikr}}{r}$

(c) $\frac{Af(\theta)e^{ikr}}{r^2}$ (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

6. A particle is scatted by a spherical symmetric potential. In the centre of mass (cm) frame the

wavefunction of particle is $\psi = Ae^{ikz}$ where k is the wavefunction & A is canst.
The differential scattering cross section in CM frame is

(a)$\sigma(\theta) = |A|^2 \frac{|f(\theta)|^2}{r^2}$

(b)$\sigma(\theta) = |A|^2|f(\theta)|^2$

(c)$\sigma(\theta) = |f(\theta)|^2$

(a)$\sigma(\theta) = |A||f(\theta)|$

7. A particle is scattered by a central potential. If the dominant contribution to the scattering is from the p-wave, the differential cross section is

(a) isotropic

(b) proportional to $\cos^2 \theta$

(c) stabilized the magnetization

(d) Cause critical magnetic fluctuations

8. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range r_0. Given that,

$$\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$$

where δ is the phase shift, k the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

[GATE 2013]

$(a)\delta \approx -\frac{k}{\gamma} - kr_0$ $(b)\delta \approx -\frac{\gamma}{k} - kr_0$

$(c)\delta \approx \frac{\pi}{2} - kr_0$ $(d)\delta \approx -\frac{\pi}{2} - kr_0$

❖ **JEST PYQ's**

1. In a fixed target elastic scattering experiment, a projectile of mass m, having initial velocity v_0, and impact parameter b, approaches the scatterer. It experiences a central repulsive force $f(r) = \frac{k}{r^2}(k > 0)$. What is the distance of the closest approach d ?

[JEST 2019]

(a)$d = \left(b^2 + \frac{k}{mv_\theta^2}\right)^{\frac{1}{2}}$ (b) $d = \left(b^2 - \frac{k}{mv_0^2}\right)^{\frac{1}{2}}$

(c)$d = b$ (d) $d = \sqrt{\frac{k}{mv_0^2}}$

2. In a fixed target elastic scattering experiment, a projectile of mass m, having initial velocity v_0, and impact parameter b, approaches the scatterer. It experiences a central repulsive force

$$f(r) = \frac{k}{r^2}(k > 0)$$

What is the distance of the closest approach d ?

[JEST 2019]

(a)$d = \left(b^2 + \frac{k}{mv_\theta^2}\right)^{\frac{1}{2}}$ (b) $d = \left(b^2 - \frac{k}{mv_0^2}\right)^{\frac{1}{2}}$

(c)$d = b$ (d)$d = \sqrt{\frac{k}{mv_0^2}}$

❖ **TIFR PYQ's**

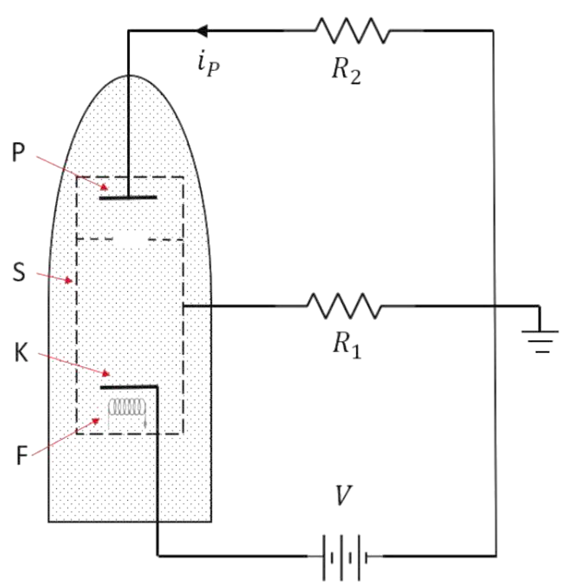
1. In a Rutherford scattering experiment, the number N of particles scattered in a direction θ, i.e. $dN/d\theta$, as a function of the scattering angle θ (in the laboratory frame) varies as

[TIFR 2016]

(a) $\csc^4 \frac{\theta}{2}$ (b) $\csc^2 \frac{\theta}{2} co\,t \frac{\theta}{2}$

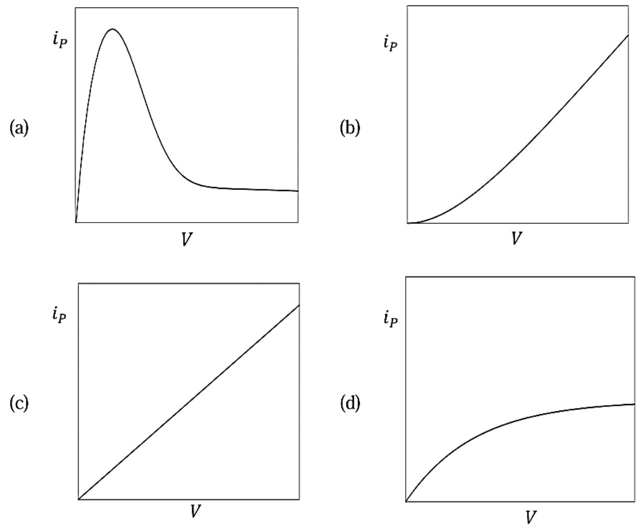
(c) $\csc^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$ (d) $\sec^4 \frac{\theta}{2}$

2. A thyratron consists of a tube filled with Xenon gas which can be used as a high power electrical switch. Electrons are emitted from a cathode K heated by a filament F, and made to accelerate to some energy E by a voltage V applied across the anode plate P.
Electrons that scatter from the Xe atoms get deviated from their path and hit the shield S, which is a conducting envelope that transports the electrons back to ground potential (see figure on the right). The rest of the electrons strike the plate and contribute to



the plate current i_P.
Which of the following graphs of the variation of the plate current i_P with increase in the accelerating voltage V could indicate the wave nature of the electron?

[TIFR 2022]



❖ Answer Key				
CSIR-NET				
1. a	2. b	3. a	4. a	5. d
6. d	7. a	8. c	9. a	10. a
11. a	12. c	13. b	14. c	15. c
16. d	17. c	18. a	19. c	20. b
21. a	22. c	23. b		
GATE				
1. c	2. a	3. a	4. 2	5. a
6. c	7. b			
JEST				
1. a	2. a			
TIFR				
1. b	2. a			