D PHYSICS

CSIR-NET,GATE , ALL SET, JEST, IIT-JAM, BARC

Contact: 8830156303 | 7741947669

PHYSICAL SCIENCE

CLASSICAL MECHANICS

Previous Year Questions [Topic-Wise]

With Answer Key

CSIR-NET/JRF I GATE I JEST I TIFR

❖ CSIR-NET PYQ's

1. The acceleration due to gravity (g) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately:

[CSIR JUNE 2011]

(a) 1.1 (b) 1.3

(c) 2.3 (d) 5.2

2. Two gravitating bodies A and B with masses m_A and m_B, respectively, are moving in circular orbit. Assume that $m_B \gg m_A$ and let the radius of the orbit of body A be R_A. If the body A is losing mass adiabatically, its orbital radius R_A is proportional to

[CSIR JUNE 2011]

(a) $1/m_A$ (b) $1/m_A^2$

(c) m_A (d) m_A^2

3. A constant force F is applied to a relativistic particle of rest mass m. If the particle starts from rest at $t=0$, its speed after a time t is

[CSIR DEC 2011]

(a) Ft/m (b) $c\tanh\left(\frac{\mathrm{Ft}}{\mathrm{mc}}\right)$

(c) $c\left(I - e^{-\mathrm{Ft/mc}}\right)$ (d) $\frac{\mathrm{Fct}}{\sqrt{F^2t^2 + m^2c^2}}$

4. An annulus of mass M made of a material of uniform density has inner and outer radii ' a 'and ' b ' respectively. Its principal moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is:

[CSIR DEC 2011]

(a) $\frac{1}{2}M\frac{(b^4+a^4)}{(b^2-a^2)}$ (b) $\frac{1}{2}M\pi(b^2-a^2)$

(c) $\frac{1}{2}M(b^2-a^2)$ (d) $\frac{1}{2}M(b^2+a^2)$

5. A uniform cylinder of radius r and length ℓ, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is:

[CSIR JUNE 2013]

(a) $V\sqrt{\frac{14\mathrm{r}\ell}{15\mathrm{R}^2}}$ (b) $4\,\mathrm{V}\sqrt{\frac{\mathrm{r}}{15\mathrm{R}}}$

(c) $\frac{4\,\mathrm{V}}{\sqrt{15}}$ (d) $\mathrm{V}\sqrt{\frac{14}{15}}$

6. The number of degrees of freedom of a rigid body in d space-dimensions is

[CSIR JUNE 2013]

(a) 2 d (b) 6

(c) $d(d+1)/2$ (d) d!

7. A particle of mass m is at the stable equilibrium position of its potential energy $V(x) = ax - bx^3$ where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is

[CSIR JUNE 2013]

(a) $(64a^3/9m^2b)^{1/4}$

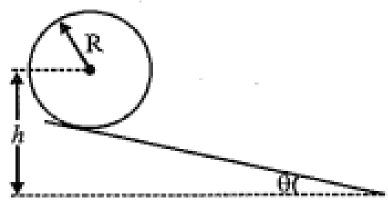
(b) $(64a^3/27m^2b)^{1/4}$

(c) $(16\mathrm{a}^3/27\,\mathrm{m}^2\,\mathrm{b})^{1/4}$

(d) $(3a^3/64m^2b)^{1/4}$

8. A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is

[CSIR DEC 2013]



(a) $\sqrt{g/(h-R)}\tan\,\theta$ (b) $\sqrt{g/(h-R)}$

(c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

9. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

[CSIR DEC 2013]

(a) $2\pi\sqrt{\dfrac{l+R}{g}}$

(b) $\dfrac{2\pi}{\sqrt{g}}(l^2+R^2)^{1/4}$

(c) $2\pi\sqrt{\dfrac{2R^2+2Rl+l^2}{g(R+l)}}$

(d) $\dfrac{2\pi}{\sqrt{g}}(2R^2+2Rl+l^2)^{1/4}$

10. Spherical particles of a given material of density p are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i,e, $F_l = 6\pi\eta R\text{v}$, where η is the viscosity of the medium, R the radius of a particle and v its itstantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass m to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is **[CSIR DEC 2013]**

(a) 8 (b) 1/8

(c) 4 (d) 1/4

11. In a measurement of the viscous drag force experienced by spherical particles in a liquid, the force is found to be proportional to $V^{1/3}$ where V is the measured volume of each particle. If V is measured to be 30 mm^3, with an uncertainty of 2.7 mm^3, the resulting relative percentage uncertainty in the measured force is

[CSIR DEC 2014]

(a) 2.08 (b) 0.09

(c) 6 (d) 3

12. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is **[CSIR DEC 2014]**

(a) 64 km (b) 48 km

(c) 32 km (d) 16 km

13. A particle moves in two dimensions on the ellipse $x^2+4y^2=8$. At a particular instant it is at the point $(x,y)=(2,1)$ and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is

[CSIR JUNE 2015]

(a) -3 (b) -2

(c) 1 (d) 4

14. A particle of unit mass moves in the xy-plane in such a way that $\dot{x}(t)=y(t)$ and $\dot{y}(t)=x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential

[CSIR JUNE 2015]

(a) $\frac{1}{2}(x^2+y^2)$ (b) $\frac{1}{2}(x^2-y^2)$

(c) $x+y$ (d) $x-y$

15. A ball of mass m, initially at rest, is dropped from a height of 5 meters. If the coefficient of restitution is 0.9 , the speed of the ball just before it hits the floor the second time is approximately (take $g=9.8\ \text{m/s}^2$)

[CSIR JUNE 2016]

(a) 9.80 m/s (b) 9.10 m/s

(c) 8.91 m/s (d) 7.02 m/s

16. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values $m=10$ kg, $\gamma=10$ kg/s, and $g\approx 10\ \text{m/s}^2$, the distance travelled (in metres) in time t in seconds, is

[CSIR DEC 2016]

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$

(c) $5t^2-(1-e^t)$ (d) $5t^2$

17. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is

[CSIR DEC 2016]

(a) $\theta=\frac{\pi}{2}$ (b) $\theta=\pi$

(c) $0 < \theta < \frac{\pi}{2}$ (d) $\frac{\pi}{2} < \theta \leq \pi$

18. A ball weighing 100gm, released from a height of 5 m, bounces perfectly elastically off a plate. The collision time between the ball and the plate is 0.5 s. The average force on the plate is approximately

[CSIR JUNE 2017]

(a) 3 N (b) 2 N

(c) 5 N (d) 4 N

19. A solid vertical rod, of length L, and cross-sectional area A, is made of a material of Young's modulus Y. The rod is loaded with a mass M, and as a result, extends by a small amount ΔL in the equilibrium condition. The mass is then suddenly reduced to $M/2$. As a result the rod will undergo longitudinal oscillation with an angular frequency

[CSIR JUNE 2017]

(a) $\sqrt{\frac{2YA}{ML}}$ (b) $\sqrt{\frac{YA}{ML}}$

(c) $\sqrt{\frac{2YA}{M\Delta L}}$ (d) $\sqrt{\frac{YA}{M\Delta L}}$

20. A cyclist, weighing a total of 80 kg with the bicycle, pedals at a speed of 10 m/s. She stops pedalling at an instant which is taken to be $t = 0$. Due to the velocity dependent frictional force, her velocity is found to vary as

$$v(t) = \frac{10}{\left(1 + \frac{t}{30}\right)} \text{m/s}$$

where t is measured in seconds. When the velocity drops to 8 m/s, she starts pedalling again to maintain a constant speed. The energy expended by her in one minute at this (new) speed, is

[CSIR DEC 2017]

(a) 4 kJ (b) 8 kJ

(c) 16 kJ (d) 32 kJ

21. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $\alpha > 0$ is a constant. If the total energy of the particle is E, its time period in a periodic motion is proportional to

[CSIR JUNE 2018]

(a) $E^{-1/3}$ (b) $E^{-1/2}$

(c) $E^{1/3}$ (d) $E^{1/2}$

22. A relativistic particle of mass m and charge e is moving in a uniform electric field of strength b. Starting from rest at $t = 0$, how much time will it take to reach the speed $c/2$?

[CSIR DEC 2018]

(a) $\frac{1}{\sqrt{3}} \frac{mc}{e\varepsilon}$ (b) $\frac{mc}{e\varepsilon}$

(c) $\sqrt{2} \frac{mc}{e\varepsilon}$ (d) $\sqrt{\frac{3}{2}} \frac{mc}{e\varepsilon}$

23. A particle of mass m, moving along the x-direction, experiences a damping force $-\gamma v^2$, where γ is a constant and v is its instantaneous speed. If the speed at $t = 0$ is v_0, the speed at time t is

[CSIR DEC 2018]

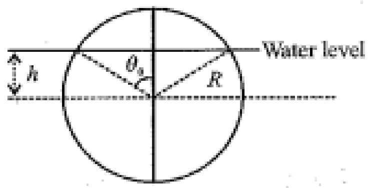
(a) $v_0 e^{-\frac{\gamma v_0 f}{m}}$ (b) $\frac{v_0}{1 + \ln\left(1 + \frac{\gamma v_0 t}{m}\right)}$

(c) $\frac{m v_0}{m + \gamma v_0 t}$ (d) $\frac{2v_0}{1 + e^{\frac{r v_0 t}{m}}}$

24. A solid spherical cork of radius R and specific gravity 0.5 floats on water. The cork is pushed down so that its centre of mass is at a distance h (where $0 < h < R$) below the surface of water, and then released. The volume of the part of the cork above water level is

$$\pi R^3 \left(\frac{2}{3} - \cos\ \theta_0 + \frac{1}{3} \cos^3\ \theta_0\right)$$

, where θ_0 is the angle as shown in the figure.



At the moment of release, the dependence of the upward force on the cork on h is

[CSIR JUNE 2019]

(a) $\frac{h}{R} - \frac{1}{3}\left(\frac{h}{R}\right)^3$ (b) $\frac{h}{R} + \frac{1}{3}\left(\frac{h}{R}\right)^3$

(c) $\frac{h}{R}-\frac{2}{3}\left(\frac{h}{R}\right)^{3}$ (d) $\frac{h}{R}+\frac{2}{3}\left(\frac{h}{R}\right)^{3}$

25. The equation of motion of a forced simple harmonic oscillator is $\ddot{x}+\omega^2 x = A\cos\,\Omega t$, where A is a constant. At resonance $\Omega=\omega$, the amplitude of oscillations at large times.

[CSIR JUNE 2019]

(a) saturates to a finite value

(b) increases with time as $\sqrt{t}$

(c) increases linearly with time

(d) increases exponentially with time

26. An object is dropped on a cushion from a height 10 m above it. On being hit, the cushion is depressel by 0.1 m. Assuming that the cushion provides a constant resistive force, the deceleration of the object after hitting the cushion, in terms of the acceleration due to gravity g, is **[CSIR JUNE 2019]**

(a) 10 g (b) 50 g

(c) 100 g (d) g

27. Following a nuclear explosion, a shock wave propagates radially outwards. Let E be the energy released in the explosion, and ρ be the mass density of the ambient air. Ignoring the temperature of the ambient air, using dimensional analysis, the functional dependence of the radius R of the shock front on E, ρ and the time t is **[CSIR JUNE 2019]**

(a) $\left(\frac{Et^2}{\rho}\right)^{1/5}$ (b) $\left(\frac{\rho}{Et^2}\right)^{1/5}$

(c) $\frac{Et^2}{\rho}$ (d) $E\rho t^2$

28. A ball, initially at rest, is dropped from a height h above the floor bounces again and again vertically.If the co-efficient of restitution between the ball and the floor is 0.5 , the total distance travelled by the ball before it comes to rest is

[CSIR DEC 2019]

(a) $8h/3$ (b) $5h/3$

(c) $3h$ (d) $2h$

29. Falling drops of rain break up and coalesce with each other and finally achieve an approximately spherical shape in the steady state. The radius of such a drop scales with the surface tension σ as

[CSIR JUNE 2020]

(a) $1/\sqrt{\sigma}$ (b) $\sqrt{\sigma}$

(c) σ (d) σ^2

30. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius a is placed on it at a distance r from the centre of the table, its linear velocity will be

[CSIR JUNE 2020]

(a) $-r\omega\hat{r}+a\omega\hat{\theta}$ (b) $r\omega\hat{r}+a\omega\hat{\theta}$

(c) $a\omega\hat{r}+r\omega\hat{\theta}$ (d) 0 (zero)

31. A point mass m, is constrained to move on the inner surface of a paraboloid of revolution $x^2+y^2=az$ (where $a>0$ is a constant). When it spirals down the surface, under the influence of gravity (along $-z$ direction), the angular speed about the z - axis is proportional to

[CSIR JUNE 2020]

(a) 1 (independent of z) (b) z

(c) z^{-1} (d) z^{-2}

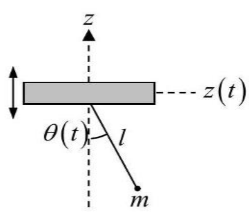
32. Falling drops of rain break up and coalesce with each other and finally achieve an approximately spherical shape in the steady state. The radius of such a drop scales with the surface tension σ as **[CSIR JUNE 2020]**

(a) $1/\sqrt{\sigma}$ (b) $\sqrt{\sigma}$

(c) σ (d) σ^2

33. The fulcrum of a simple pendulum (consisting of a particle of mass m attached to the support by a massless string of length l) oscillates vertically as sin $z(t)=a\sin\,\omega t$, where ω is a constant. The pendulum moves in a vertical plane and $\theta(t)$ denotes its angular position with

[CSIR JUNE 2021]



respect to the z-axis.

If

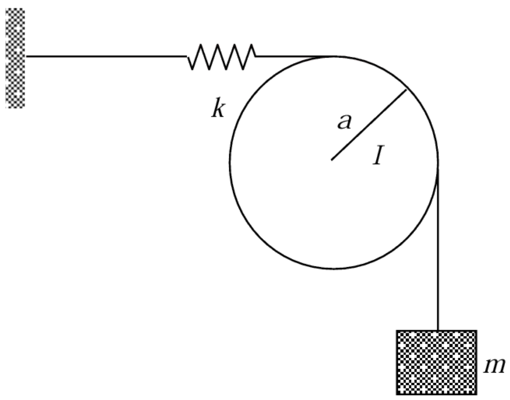
$$\ell \frac{d^2\theta}{dt^2} + \sin\,\theta(g - f(t)) = 0$$

(where g is the acceleration due to gravity) describes the equation of motion of the mass, then $f(t)$ is

(a) $a\omega^2 \cos\,\omega t$ (b) $a\omega^2 \sin\,\omega t$

(c) $-a\omega^2 \cos\,\omega t$ (d) $-a\omega^2 \sin\,\omega t$

34. A wire, connected to a massless spring of spring constant k and a block of mass m, goes around a disc of radius a and moment of inertia I, as shown in the figure. Assume that the spring remains horizontal, the pully rotates freely and there is no slippage between the wire and the pully. The



angular frequency of oscillation of the disc is **[CSIR JUNE 2022]**

(a) $\sqrt{\frac{2\text{ka}^2}{\text{ma}^2 + \text{I}}}$ (b) $\sqrt{\frac{\text{ka}^2}{\text{ma}^2 + \text{I}}}$

(c) $\sqrt{\frac{\text{ka}^2}{\text{ma}^2 + 2\text{I}}}$ (d) $\sqrt{\frac{\text{ka}^2}{2\text{ma}^2 + \text{I}}}$

35. The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height 6 km are T_1 and T_2, respectively. If the radius of earth is approximately 6000 km, then $\frac{(\text{T}_2 - \text{T}_1)}{\text{T}_1}$ is closest to **[CSIR JUNE 2022]**

(a)-10^{-4} (b)-10^{-3}

(c)10 (d)10^{-3}

36. The trajectory of a particle moving in a plane is expressed in polar coordinates (r, θ) by the equation $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$ where the parameters r_0, β and ω are positive. Let v_r and a_r denote the velocity and acceleration, respectively, in the radial direction. For this trajectory **[CSIR JUNE 2023]**

(a) $a_r < 0$ at all times irrespective of the values of the parameters

(b) $a_r > 0$ at all times irrespective of the values of the parameters

(c) $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters

(d) $\frac{dv_r}{dt} > 0$ however, $a_r = 0$ for some choices of parameters

❖ GATE PYQ's

1. A particle of mass M moving in a straight line with speed v collides with a stationary particle of the same mass. In the center of mass coordinate system, the first particle is deflected by 90°. The speed of the second particle, after collision, in the laboratory system will be **[GATE 2002]**

(a) $\frac{v}{\sqrt{2}}$ (b) $\sqrt{2}v$

(c) v (d) $\frac{v}{2}$

2. Two particles of equal mass are connected by an inextensible string of length L. One of the masses is constrained to move on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degrees of freedom of the masses-string system is **[GATE 2002]**

(a) five (b) four

(c) two (d) one

3. An object of mass m rests on a surface with coefficient of static friction μ. Which of the following is NOT correct? **[GATE 2003]**

(a) The force of friction is exactly μmg

(b) The maximum force of friction is μmg

(c) The force of friction is along the surface

(d) The force of friction opposes any effort to move the object

Data for Q. No. 4 to 5
A particle of mass m moving with speed v collides with a stationary particle of equal mass. After the collision, both the particles move. Let θ be the angle between the two velocity vectors

4. If the collision is elastic, then

[GATE 2003]

(a) θ is always less than 90°

(b) θ is always equal to 90°

(c) θ is always greater than 90°

(d) θ cannot be deduced from the given data

5. If the collision is inelastic, then

[GATE 2003]

(a) θ is always less than 90°

(b) θ is always equal to 90°

(c) θ is always greater than 90°

(d) θ could assume any value in the range 0° to 180°

6. If for a system of N particles of different masses $m_1, m_2, \ldots, m_N$ with position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ and corresponding velocities $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_N$, respectively, such that $\sum \vec{v}_i = 0$, then

[GATE 2005]

(a) the total momentum MUST be zero

(b) the total angular momentum MUST be independent of the choice of the origin

(c) the total force on the system MUST be zero

(d) the total torque on the system MUST be zero **(option not correct)**

7. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalized coordinates required to describe the motion of this system is

[GATE 2008]

(a) 1 (b) 2

(c) 4 (d) 6

8. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

[GATE 2014]

(a) $1:1$ (b) $1:\sqrt{3}$

(c) $1:3$ (d) $3:2$

9. A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\ \mathrm{ms}^{-1}$. If the air exerts a frictional force of the form, $f = -kv$, then for $k = 0.05\mathrm{Nm}^{-1}$ s, the velocity (in ms^{-1}) at time $t = 0.2s$ is__________ (upto two decimal places) (use $g = 10\ \mathrm{ms}^{-2}$ and $e = 2.72$)

[GATE 2015]

10. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10gm and 1 cm respectively, the final linear velocity of the cylinder is______ (up to two decimal places)

[GATE 2017

11. If a particle is moving along a sinusoidal curve, the number of degree of freedom of the particle is

12. A projectile of mass 1 kg is launched at an angle of 30° from the horizontal direction at $t = 0$ and takes time T before hitting the ground. If its initial speed is $10\ \mathrm{ms}^{-1}$, the value of the action integral for the entire flight in the units of $\mathrm{kgm}^2\ \mathrm{s}^{-1}$ (rounded off to one decimal place) is [Take $g = 10\ \mathrm{ms}^{-2}$] **[GATE 2019]**

13. A hoop of mass M and radius R rolls without slipping along a straight line on a horizontal surface as shown in the figure. A point mass m slides without friction along thee inner surface of the hoop, performing small oscillations about the mean position. The number of degrees of freedom of the system (in integer) is

[GATE 2021]

❖ JEST PYQ'

1. Consider a uniform distribution of particles with volume density n in a box. The particles have an isotropic velocity distribution with constant magnitude v. The rate at which the particles will be emitted from a hole of area A on one side of this box is

[JEST 2013]

(a) muA

(b) muA/2

(c) muA/4

(d) none of the above

2. If, in a Kepler potential, the pericenter distance of a particle in a parabolic orbit is $\mathbf{r_p}$ while the radius of the circular orbit with the same angular momentum is r_c, then

[JEST 2013]

(a) $r_C = 2r_P$ (b) $r_C = r_P$

(c) $2r_C = r_P$ (d) $r_C = \sqrt{2r_P}$

3. The period of a simple pendulum inside a stationary lift is T. If the lift accelerates downwards with and acceleration g/4, the period of the pendulum will be

[JEST 2013]

(a) T (b) T/4

(c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

4. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum)

[JEST 2013]

(a) $\sqrt{3c}/2$ (b) $3c/4$

(c) $\sqrt{3/5c}$ (d) $c/\sqrt{2}$

5. A small mass **M** hangs from a thin string and can swing like a pendulum. It is attached above the window of a car. When the car is at rest, the string hangs vertically. The angle made by the string with the vertical when the car has a constant acceleration a = 1.2 m/s2 is approximately

[JEST 2013]

(a) 1° (b) 7°

(c) 15° (d) 90°

6. A particle of mass m is thrown upward with velocity v and there is retarding air resistance proportional to the square of the velocity with proportionality constant k. If the particle attains a maximum height after time t, and g is the gravitational acceleration, acceleration, what is the velocity v ? [JEST 2013]

(a) $\sqrt{\frac{k}{g}}\tan\left(\sqrt{\frac{g}{k}}t\right)$ (b) $\sqrt{gk}\tan\left(\sqrt{\frac{g}{k}}t\right)$

(c) $\sqrt{\frac{g}{k}}\tan(\sqrt{gk}t)$ (d) $\sqrt{gk}\tan(\sqrt{gk}t)$

7. Given the fundamental constant $\hbar$ (Planck constant), G (universal gravitation constant) and c (speed of light), which of the following has dimension of length?

[JEST 2014]

(a) $\sqrt{\frac{\hbar G}{c^3}}$ (b) $\sqrt{\frac{\hbar G}{c^5}}$

(c) $\frac{\hbar G}{c^3}$ (d) $\sqrt{\frac{\hbar c}{8\pi G}}$

8. The formula for normal strain in a longitudinal bar is given by $\varepsilon = \frac{F}{AE'}$ where F is normal force applied, A is cross-sectional area of the bar, and E is Young's modulus. If $F = 50 \pm 0.5N, A = 0.2 \pm 0.002\ \mathrm{m}^2$ an $E = 210 \times 10^9 \pm 1 \times 10^9$ Pa, the maximum error in the measurement of strain is

[JEST 2014]

(a) 1.0×10^{-12} (b) 2.95×10^{-11}

(c) 1.22×10^{-9} (d) 1.19×10^{-9}

9. The acceleration experienced by the bob of a simple pendulum is

[JEST 2014]

(a) maximum at the extreme positions

(b) maximum at the lowest (central) positions

(c) maximum at a point between the above two positions

(d) same as all positions

10. Two point objects A and B have masses 1000Kg and 3000Kg respectively. They are initially at rest with a separation equal to 1 m. Their mutual gravitational attraction then draws then together. How far from A's original position will they collide?

[JEST 2014]

(a) 1/3 m (b) 1/2 m

(c) 2/3 m (d) 3/4 m

11. A chain of mass M and length L is suspended vertically with its lower end touching a weighing scale. The chain is released and falls freely onto the scale. Neglecting the size of the individual links, what is the reading of the scale when a length x of the chain has fallen?

[JEST 2015]

(a) $\frac{Mgx}{L}$ (b) $\frac{2Mgx}{L}$

(c) $\frac{3Mgx}{L}$ (d) $\frac{4Mgx}{L}$

12. A particle moving under the influence of a potential $V(r)=\frac{kr^2}{2}$ has a wave function $\psi(r,t)$. If the wave function changes to $\psi(\alpha,r,t)$, the ratio of the average final kinetic energy to the initial kinetic energy will be,

[JEST 2015]

(a) $\frac{1}{\alpha^2}$ (b) α

(c) $\frac{1}{\alpha}$ (d) α^2

13. A spring of force constant k is stretched by x. It takes twice as much work to stretch a second spring by $\frac{x}{2}$. The force constant of the second spring is,

[JEST 2015]

(a) k (b) $2k$
(c) $4k$ (d) $8k$

14. In Millikan's oil drop experiment the electronic charge e could be written as $k\eta^{1.5}$, where κ a function of all experimental parameters with negligible error. If the viscosity of air η is taken to be 0.4% lower than the actual value, what would be the error in the calculated in the calculated value of e ?

[JEST 2015]

(a) 1.5% (b) 0.7%

(c) 0.6% (d) 0.4%

15. A bike stuntman rides inside a well of frictionless surface given by $z=a(x^2+y^2)$, under the action of gravity acting in the negative z-direction. $\vec{g}=-g\hat{z}$. What speed should he maintain to be able to ride at a constant height z_0 without failing down?

[JEST 2015]

(a) $\sqrt{gz_0}$

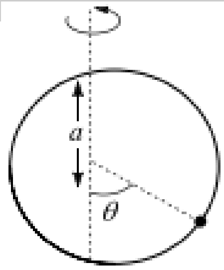
(b) $\sqrt{3gz_0}$

(c) $\sqrt{2gz_0}$

(d) The biker will not be able to maintain a constant height, irrespective of speed.

16. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g<\omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt}=\frac{d^2\theta}{dt^2}=0$)?

[JEST 2016]



(a) $\tan\theta=\frac{\pi g}{\omega^2 a}$ (b) $\sin\theta=\frac{g}{\omega^2 a}$

(c) $\cos\theta=\frac{g}{\omega^2 a}$ (d) $\tan\theta=\frac{g}{\pi\omega^2 a}$

17. In Millikan's oil-drop experiment an oil drop of radius r, mass m and charge $q = \frac{6\pi\eta r v_1}{m}$ is moving upwards with a terminal velocity V_2 due to an applied electric field of magnitude E, where η is the coefficients of viscosity. The acceleration due to gravity is given by:

[JEST 2016]

(a) $q = \dfrac{6\pi\eta r v_1}{m}$ (b) $q = \dfrac{3\pi\eta r v_1}{m}$

(c) $q = \dfrac{6\pi\eta r v_2}{m}$ (d) $q = \dfrac{3\pi\eta r v_2}{m}$

18. A ball of mass 0.1 kg and density 2000 kg/m^3 is suspended by a massless string of length 0.5 m under water having density 1000 kg/m^3. The ball experience a drag force, $\vec{F}_d = -0.2(\vec{v}_b - \vec{v}_\omega)$, where $\vec{v}_b$ and $\vec{v}_\omega$ are the velocities of the ball and water respectively. What will be the frequency of small oscillation for the motion of pendulum, if the water is at rest?

[JEST 2017]

19. A toy car is made from a rectangle block of mass M and four disk wheels of mass m and radius r. The car is attached to a vertical wall by a massless horizontal spring with spring constant k and constrained to move perpendicular to the wall. The coefficient of static friction between the wheels of the car and the floor is μ. The maximum amplitude of oscillations of the car above which the wheels start slipping is

[JEST 2017]

(a) $\dfrac{\mu g(M+2m)(M+4m)}{mk}$

(b) $\dfrac{\mu g(M^2-m^2)}{mk}$

(c) $\dfrac{\mu g(M^2-m^2)}{2mk}$

(d) $\dfrac{\mu g(M+4m)(M+6m)}{2mk}$

20. A bead of mass M slides along a parabolic wire described by $z = 2(x^2+y^2)$. The wire rotates with angular velocity Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$? **[JEST 2017]**

(a) $\sqrt{3g}$ (b) $\sqrt{g}$

(c) $\sqrt{2g}$ (d) $\sqrt{4g}$

21. A ball of mass m starting from rest falls a vertical distance h before striking a vertical spring. Which it compresses by a length δ. What is the spring constant of the spring? (Hint: Measure all the vertical distances from the point where the ball first touches the uncompressed spring, i.e., set this point as the origin of the vertical axis.)

[JEST 2018]

(a) $\dfrac{2mg}{\delta^2}(h+\delta)$ (b) $\dfrac{2mgh}{\delta^3}(h-\delta)$

(c) $\dfrac{2mg}{\delta^2}(h-\delta)$ (d) $\dfrac{2mg}{\delta^2}h$

22. A block of mass M is moving on a frictionless inclined surface of a wedge of mass m under the influence of gravity. The wedge is lying on a rigid frictionless horizontal surface. The configuration can be described using the radius vectors $\vec{r}_1$ and $\vec{r}_2$ shown in the figure. How many constraints are present and what are the types?

[JEST 2018]

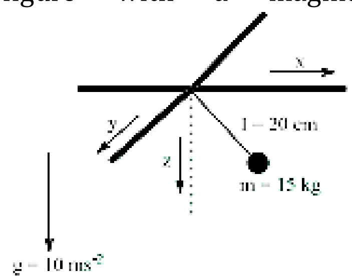
(a) One constraint: holonomic and scleronomous

(b) Two constraints; Both are holonomic; one is scleronomous and eheonomous

(c) Two constraints; Both are scleronomous; one is holonomic and the other is non-holonomic.

(d) Two constraints; Both are holonomic and scleronomous.

23. Consider a simple pendulum in three dimensional space. It consists of a string length $l = 20$ cm and bob mass m $= 15$ kg attached to it as shown in the figure below. The acceleration due to gravity is downwards as shown in the figure with a magnitude $g = 10\ \text{ms}^{-2}$



The pendulum is pulled in the $x-z$ plane to a position where the string makes an angle $\theta = \theta/3$ with the z-axis. It is then released with an angular velocity Ω radians per second about the z-axis. What should be the value of Ω in radians per second so that the angle the string makes with the z-axis does not change with time?

[JEST 2018]

24. A block of mass M rests on a plane inclined at an angle θ with respect to the horizontal. A horizontal force $F = Mg$ is applied to the block. If μ is the static friction between the block and the plane, the range of θ so that the block remains stationary is

[JEST 2018]

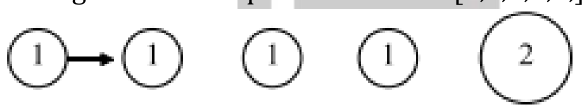
(a) $-\mu \leq \tan\,\theta \leq \mu$

(b) $1-\mu \leq \cos\,\theta \leq 1+mu$

(c) $\dfrac{1-\mu}{1+mu} \leq \tan\,\theta \leq \dfrac{1+\mu}{1-mu}$

(d) $\dfrac{1-\mu}{1+mu} \leq \cot\,\theta \leq \dfrac{1+\mu}{1-mu}$

25. A ball comes in from the left with speed 1 (in arbitrary units) and cause a series of collisions. The other four balls shown the figure are initially at rest. The initial motion is shown below (the number in the circle indicate the object's relative mass). This initial velocities of the balls shown in the figure are represented as [1,0,0,0,0,].



A negative sign means that the velocity is directed to the left. All collisions are elastic. Which of the following indicates the velocities of the balls after all the collisions are completed?

[JEST 2018]

(a) $[-1/2,-1/2,0,0,1/2]$

(b) $[-1/3,0,0,0,2/3]$

(c) $[-1/2,0,0,0,3/4]$

(d) $[-1/2,0,0,0,1/2]$

26. A large cylinder of radius R filled with particles of mass m. the cylinder spins about its axis at an angular speed ω radians per second, providing and acceleration g for the particles at the rim. If the temperature T is constant inside the cylinder, what is the ratio of air pressure P_0 at the axis to the pressure P_c at the rim?

[JEST 2018]

(a) $\exp\left[\dfrac{mgR}{2k_bT}\right]$

(b) $\exp\left[-\dfrac{mgR}{2k_bT}\right]$

(c) $\dfrac{mgR}{2k_bT}$

(d) $\dfrac{2k_bT}{mgR}$

27. Two joggers A and B are running at a steady pace around a circular track. A takes T_A minutes whereas B takes $T_B(>T_A)$ minutes to complete one round. Assuming that they have started together, what will be time taken by A to overtake B for the first time?

[JEST 2019]

(a) $\dfrac{2\pi}{T_A-T_B}$

(b) $\dfrac{1}{T_A}-\dfrac{1}{T_B}$

(c) $\dfrac{1}{T_A+T_B}$

(d) $\left(\dfrac{1}{T_A}-\dfrac{1}{T_B}\right)^{-1}$

28. A bullet with initial speed v_0 is fired at a log of wood. The resistive force by wood on the bullet is given by ηv^α, where $\alpha<1$. What is the time taken to stop the bullet inside the wood log?

[JEST 2019]

(a) $\dfrac{m}{\eta}\dfrac{v_0^{\alpha-1}}{1-\alpha}$

(b) $\dfrac{m}{\eta}\dfrac{v_0^{\alpha+1}}{\alpha+1}$

(c) $\dfrac{m}{\eta}\dfrac{v_0^{1-\alpha}}{1-\alpha}$

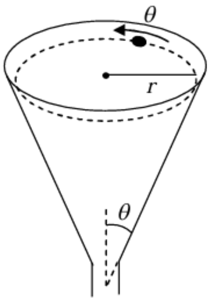
(d) $\dfrac{\eta}{m}\dfrac{v_0^{1-\alpha}}{1-\alpha}$

29. A thin uniform steel chain is 10 m long with a linear mass density of 2 kg m^{-1}. The chain hangs vertically with one end attached to a horizontal axle, having a negligibly small radius compared to its length. What is the work done (in $N-m$) to slowly wind up the chain on to the axle? The acceleration due to gravity is $g = 9.81\ ms^{-1}$.

[JEST 2019]

30. A particle is to slide along the horizontal circular path on the inner surface of the funnel as shown in the figure. The surface of the funnel is frictionless. What must be the speed of the particle (in terms of r and θ) if it is to execute this motion?

[JEST 2020]



(a) $\sqrt{rg\sin\theta}$ (b) $\sqrt{rg\cos\theta}$

(c) $\sqrt{rg\tan\theta}$ (d) $\sqrt{rg\cot\theta}$

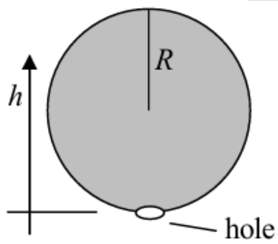
31. Two tuning forks A and B are struck instantaneously to obtain Lissajous figures. The figures go through a complete cycle in 20 s. Fork A is located with wax, so that the cycle period changes to 10 s. If the frequency of fork B is 256.10 Hz, what is the frequency of fork A after loading?

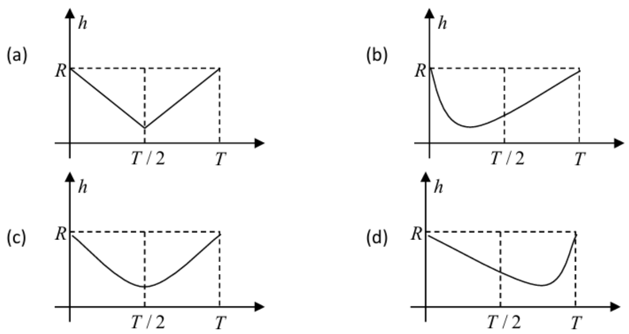
[JEST 2020]

(a) 256.00 Hz (b) 256.05 Hz

(c) 256.15 Hz (d) 256.20 Hz

32. A hollow sphere of radius R, with a small hole at the bottom, is completely filled with a liquid of uniform density (see figure). The liquid drains out of the sphere through the hole at an uniform rate in time T. Which one of the following graphs (a, b, c, d) qualitatively represents the height h of the center of mass (of sphere + liquid inside it), measured from the bottom of the sphere with time? **[JEST 2021]**





33. An aircraft flies over the North pole at a constant speed of 900Km/hr. A small bob is hanging freely from the ceiling of the aircraft. What is the angle (in micro-radians) it makes with the Earth's radial direction? (Take the acceleration due to gravity to be 9.81 m/s^2).

[JEST 2021]

34. For a system of unit mass, the dynamical variables follow the relation $\dot{x}^2 = kx_0^2 + \dot{x}_0^2 - kx^2$ where, x is the position of the system at time t, and x_0 is its initial position. What is the force acting on the system?

[JEST 2022]

(a) $\frac{1}{2}k(x-x_0)^2$ (b) $-k(x-x_0)$

(c) $-\frac{1}{2}k(x-x_0)$ (d) $-kx$

35. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where, k is a constant. The power delivered to the particle by the force acting on it is

[JEST 2022]

(a) $2\pi mk^{\frac{3}{2}}r^2$ (b) mk^2r^2t

(c) $\frac{1}{2}mk^2r^2t$ (d) 0

36. The front-end of a train moving with constant acceleration, passes a pole with velocity u, and its back-end passes the pole with velocity v. With what velocity does the mid-point of this train pass the same pole?

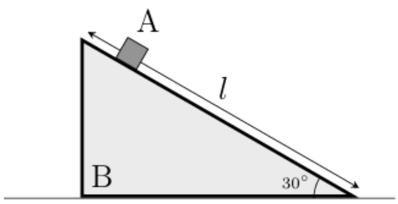
[JEST 2022]

(a) $\frac{uv}{u+v}$ (b) $\frac{1}{2}\sqrt{u^2+v^2}$

(c) $\sqrt{\frac{u^2+v^2}{2}}$ (d) $\frac{u+v}{2}$

37. A small object A of mass m is free to slide on the inclined plane of a triangular block B of mass $2m$ (see figure). Initially both the blocks are motionless. Block A starts sliding under the action of gravity from the highest point of block (b) What is the speed of block B, when block A hits the floor?

[JEST 2022]



(a) $\frac{2}{3}\sqrt{gl}$ (b) $\frac{1}{2}\sqrt{gl}$

(c) $\sqrt{gl}$ (d) $\frac{1}{3}\sqrt{gl}$

38. A circularly polarized laser of power P is incident on a particle of mass m. The particle, which was initially at rest, completely absorbs the incident radiation. The kinetic energy of the particle as a function of time t is given by

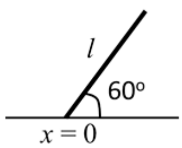
[JEST 2022]

(a) $\frac{1}{2}Pt\left(\frac{Pt}{mc^2}+1\right)$ (b) $\frac{1}{2}Pt\left(\frac{Pt}{mc^2}-1\right)$

(c) $\frac{P^2t^2}{2mc^2}$ (d) $\frac{Pt}{2}$

39. A rod of length $l = 1$ meter is held on a frictionless horizontal surface at an angle of $\theta = 60°$ with the horizontal, as shown in the figure. Take the point of contact of the rod with the horizontal plane as the origin $(x = 0)$. As the support holding the rod is suddenly removed, the rod comes in contact with the horizontal surface. What will be the coordinate of the left end of the rod at the moment of contact?

[JEST 2023]



(a) -0.15 m (b) -0.5 m

(c) -0.2 m (d) -0.25 m

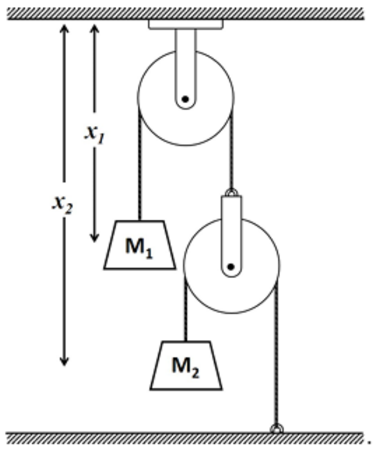
40. A conducting spherical soap bubble of radius R with a wall thickness of $W(\ll R)$ is charged to a potential of V_0. The bubble bursts and becomes a spherical drop with potential V_d. Select the correct value of the ratio [JEST 2023]

$$\eta = \frac{V_d^3 W}{V_0^3 R}$$

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

41. Consider a mass-pulley system as shown in the figure. The heights of the blocks as measured from the ceiling are x_1 and x_2, as shown in the figure.



What is the constraint between x_1 and x_2 ?

[JEST 2024]

(a) They are unconstrained

(b) $x_2 - x_1 =$ constant .

(c) $x_2 + x_1 =$ constant .

(d) $x_2 + 2x_1 =$ constant.

❖ TIFR PYQ;'s

1. heavy mass m is suspended from two identical steel wires of length ℓ, radius r and Young's modulus Y, as shown in the figure below. When the mass is pulled down by a distance $x(x \ll \ell)$ and released, it undergoes elastic oscillations in the vertical direction with a time period

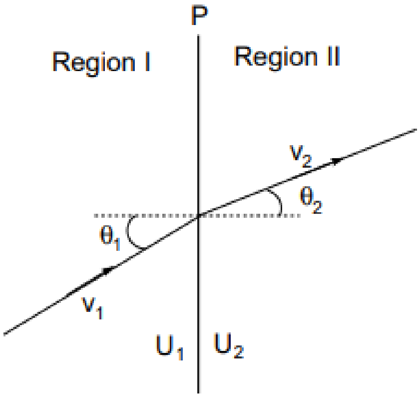
[TIFR 2010]

(a) $\frac{2\pi}{r}\sqrt{\frac{m\ell}{2Y\cos^2\left(\frac{\alpha}{2}\right)}}$ (b) $2\pi\sqrt{\frac{\ell\cos\left(\frac{\alpha}{2}\right)}{g}}$

(c) $\sqrt{\frac{2\pi m\ell}{Yr^2}}$ (d) $\frac{2\pi}{r}\sqrt{\frac{mg\ell}{2Y}}$

2. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from Region I to Region II, where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in Region I and U_2 in Region II.

[TIFR 2011]



Let T_1 be the kinetic energy of the particle in Region I. If the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\,\theta_1/\sin\,\theta_2$ is given by

[TIFR 2011]

(a) $\sqrt{1-T_1/(U_1-U_2)}$ (b) $\sqrt{1+T_1/(U_1-U_2)}$

(c) $\sqrt{1-(U_1+U_2)/T_1}$ (d) $\sqrt{1+(U_1+U_2)/T_1}$

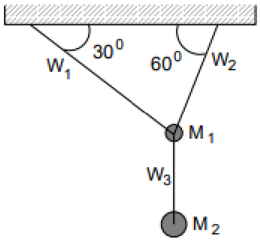
(e) $\sqrt{1-T_1/(U_1-U_2)}$` (f) $\sqrt{1+(U_1-U_2)/T_1}$

3. Two solid spheres S_1 and S_2 of the same uniform density fall from rest under gravity in a viscous medium and, after some time, reach terminal velocities v_1 and v_2 respectively. If the masses of S_1 and S_2 are m_1 and m_2 respectively, and $v_1 = 4v_2$, then the ratio m_1/m_2 is

(a) 1/8 (b) ¼

(c) 4 (d) 8

4. Two masses M_1 and $M_2 (M_1 < M_2)$ are suspended from a perfectly rigid horizontal support by a system of three taut massless wires W_1, W_2 and W_3, as shown in the figure. All the three wires have identical cross-sections and elastic properties and are known to be very strong.



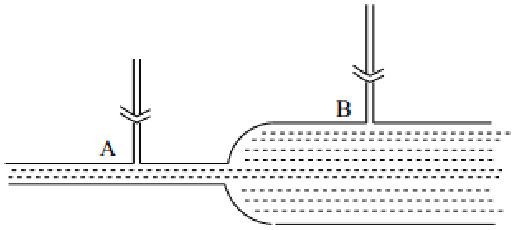
If the mass M_2 is increased gradually, but without limit, we should expect the wires to break in the following order:

[TIFR 2012]

(a) first W_2, then W_1 (b) first W_1, then W_2

(c) first W_2, then W_3 (d) first W_3

5. An ideal liquid of density 1gm/cc is flowing at a rate of 10gm/s through a tube with varying cross-section, as shown in the figure.



Two pressure gauges attached at the points A and B (see figure) show readings of P_A and P_B respectively. If the radius of the tube at the points A and B is 0.2 cm and 1.0 cm respectively, then the difference in pressure $(P_B - P_A)$, in units of dyne cm^{-2}, is closest to

[TIFR 2012]

(a) 100 (b) 120

(c) 140 (d) 160

6. A stone is dropped vertically from the top of a tower of height 40 m. At the same time a gun is aimed directly at the stone from the ground at a horizontal distance 30 m from the base of the tower and fired. If the bullet from the gun is to hit the stone before it reaches the ground, the minimum velocity of the bullet must be, approximately,

[TIFR 2013]

(a) 57.4 ms^{-1} (b) 27.7 ms^{-1}

(c) 17.7 ms^{-1} (d) 7.4 ms^{-1}

7. A particle with time-varying mass

$$m(t) = m_0(1 - t/\tau),$$

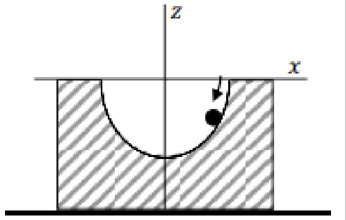
where m_0 and τ are positive constants, moves along the x-axis under the action of a constant positive force F for $0 \le t < \tau$. If the particle is at rest at time $t = 0$, then at time $t = t$, its velocity v will be

[TIFR 2013]

(a) $-\frac{\tau F}{m_0} log\, g\left(1 - \frac{t}{\tau}\right)$ (b) $\frac{Ft}{m_0}\left(1 - \frac{t}{\tau}\right)^{-1}$

(c) $-\frac{Ft}{m_0} log\, g\frac{t}{\tau}$ (d) $\frac{\tau F}{m_0}\left(1 - \frac{t}{\tau}\right)$

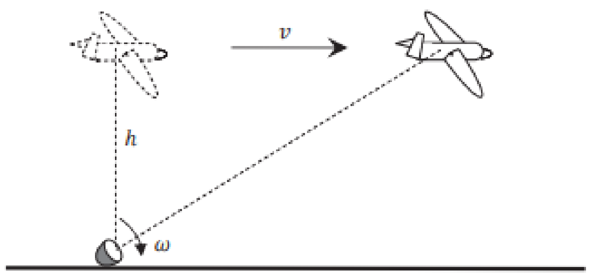
8. A ball of mass m slides under gravity without friction inside a semicircular depression of radius a inside a fixed block placed on a horizontal surface, as shown in the figure. The equation of motion of the ball in the x-direction will be **[TIFR 2013]**



(a) $\ddot{x} = \frac{g}{a}x\sqrt{1 - \frac{x^2}{a^2}}$ (b) $\ddot{x} = -\frac{g}{a}x$

(c) $\ddot{x} = \frac{g}{a}x$ (d) $\ddot{x} = -\frac{g}{a}x\sqrt{1 - \frac{x^2}{a^2}}$

9. The directed beam from a small but powerful searchlight placed on the ground tracks a small plane flying horizontally at a fixed height h above the ground with a uniform velocity v, as shown in the figure below.



If the searchlight starts rotating with an instantaneous angular velocity ω_0 at time $t = 0$ when the plane was directly overhead, then at a later time t, its instantaneous angular velocity $\omega(t)$ is given by

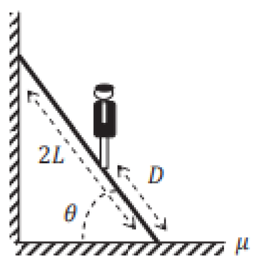
[TIFR 2014]

(a) $\omega_0 \exp(-\omega_0 t)$ (b) $\frac{\omega_0}{1 + \tan \omega_0 t}$

(c) $\frac{\omega_0}{1 + \omega_0^2 t^2}$ (d) $\frac{\omega_0}{1 - \omega_0 t + \frac{1}{2}\omega_0^2 t^2}$

10. A uniform ladder of length $2L$ and mass m leans against a wall in a vertical plane at an angle θ to the horizontal. The floor is rough, having a coefficient of static friction μ. A person of mass M stands on the ladder at a distance D from its base (see figure). If the wall is frictionless, the maximum distance ($D_{\max}$) up the ladder that the person can reach before the ladder slips is

[TIFR 2014]



(a) $2\mu L\left(1 + \frac{m}{M}\right)\tan\,\theta$

(b) $\left\{2\mu\left(1 + \frac{m}{M}\right)\tan\,\theta - \frac{m}{M}\right\}L$

(c) $\mu L \tan\,\theta$

(d) $2\mu L\frac{m}{M}\tan\,\theta$

11. A body of mass m falls from rest at a height h under gravity (acceleration due to gravity g) through a dense medium which provides a resistive force $F = -kv^2$, where k is a constant and v is the speed. It will hit the ground with a kinetic energy

[TIFR 2014]

(a) $\frac{m^2 g}{2k}\exp\left(-\frac{2kh}{m}\right)$

(b) $\frac{m^2 g}{2k}\tanh\frac{2kh}{m}$

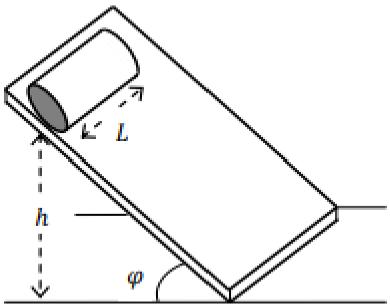
Type equation here.

(c) $\frac{m^2 g}{2k}\left\{1+\exp\left(-\frac{2kh}{m}\right)\right\}$

(d) $\frac{m^2 g}{2k}\left\{1-\exp\left(-\frac{2kh}{m}\right)\right\}$

12. Two cylinders A and B of the same length L and outer radius R were placed at the same height h on an inclined plane at an angle φ with the horizontal (see figure). Starting from rest, each cylinder was allowed to roll down the plane without slipping. It was found that A reached the end of the inclined plane earlier than B. Which of the following possibilities could be true?

[TIFR 2015]



(a) A is hollow and made of copper; B is hollow and made of copper; B is heavier than A.

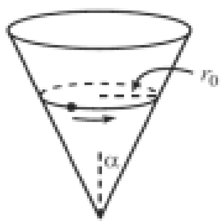
(b) A is solid and made of copper; B is solid and made of aluminium.

(c) A is hollow and made of aluminium; B is solid and made of aluminium.

(d) A is solid and made of copper; B is hollow and made of copper.

13. A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α. If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency ω of this motion will be

[TIFR 2015]

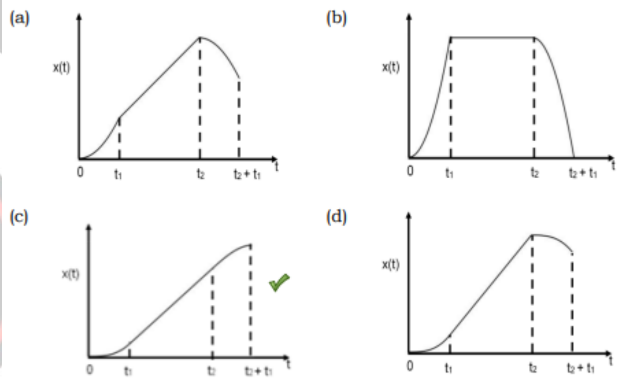


(a) $\sqrt{\frac{g}{r_0 \cos\alpha}}$ (b) $\sqrt{\frac{g}{r_0 \sin\alpha}}$

(c) $\sqrt{\frac{g}{r_0 \cot\alpha}}$ (d) $\sqrt{\frac{g}{r_0 \tan\alpha}}$

14. A car starts from rest and accelerates under a force F increasing linearly in time as $F = at$ where a is a constant. At time $t_1 > 0$, the force F is suddenly switched off. At a later time $t_2 > t_1$, brakes are applied resulting in a force F' whose magnitude increases linearly with time, $F' = -a(t - t_2)$ where a is the same constant as before. Which of the following graphs would best represent the change in the position of the car $x(t)$ with time?

[TIFR 2015]



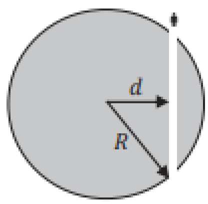
15. An aircraft, which weighs 12000 kg when unloaded, is on a relief mission, carrying 4000 food packets weighing 1 kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second, then the distance between the last two packet drops will be

[TIFR 2016]

(a) 1.5Km (b) 200 m

(c) 150 m (d) 100 m

16.



Imagine that a narrow tunnel is excavated through the Earth as shown in the diagram on the left and that the mass excavated to create the tunnel is extremely small compared to Earth's mass M.
A person falls into the tunnel at one end. at time $t = 0$. Assuming that the tunnel is frictionless, the person will **[TIFR 2016]**
(a) fall straight through, escaping Earth's gravity at time $2\pi\sqrt{R^3/GM}$

(b) describe simple harmonic motion with period $2\pi(d/R)\sqrt{R^3/GM}$

(c) describe simple harmonic motion with period $2\pi\sqrt{(R-d)^3/GM}$

(d) describe simple harmonic motion with period $2\pi\sqrt{R^3/GM}$

17. On a planet having the same mass and diameter as the Earth, it is observed that objects become weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth). **[TIFR 2016]**

18. A ball is dropped vertically from a height H on to a plane surface and permitted to bounce repeatedly along a vertical line. After every bounce, its kinetic energy becomes a quarter of its kinetic energy before the bounce. The ball will come to rest after time

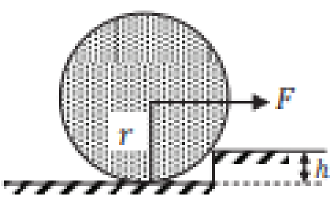
[TIFR 2016]

(a) infinity (b) $(2H/g)^{1/2}$

(c) $2(2H/g)^{1/2}$ (d) $3(2H/g)^{1/2}$

19. A uniform solid wheel of mass M and radius r is halted at a step of height h as shown in the figure. The minimum force F, applied horizontally at the centre of the wheel, necessary to raise the wheel over this step is

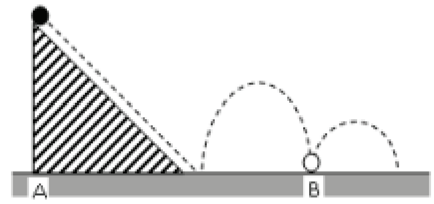
[TIFR 2016]



(a) $Mg\dfrac{\sqrt{h(2r-h)}}{r+h}$ (b) $Mg\dfrac{\sqrt{h(2r+h)}}{r-h}$

(c) $Mg\dfrac{\sqrt{h(r+h)}}{r-h}$ (d) $Mg\dfrac{\sqrt{h(2r-h)}}{r-h}$

20. A small elastic ball of mass m is placed at the apex of a 45° inclined plane as shown in the figure below.



The ball is allowed to slip without friction down the plane (along the dotted line), hit the ground (as shown) and bounce along it. If the height of the inclined plane is h and the coefficient of restitution between the ball and the ground is 0.5 , then the distance AB, as marked on the figure, will be

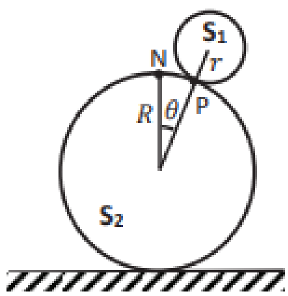
[TIFR 2017]

(a) $3h$ (b) $2h$

(c) $(1+\sqrt{2})h$ (d) $3\sqrt{2}h$

21. A uniform solid sphere S_1 of radius r and mass m is rolling without slipping on top of another sphere S_2 of radius R, as shown in the figure. Initially, S_1 was at rest directly on top of S_2, and then it started rolling down under the influence of gravity. The point of contact P subtends an instantaneous angle θ from the topmost point N of the lower sphere at the centre of the lower sphere.
At what minimum value of θ will the spheres lose contact?

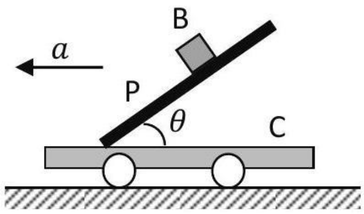
[TIFR 2017]



(a) $\cos^{-1}\dfrac{5}{12}$ (b) $\cos^{-1}\dfrac{5}{13}$

(c) $\cos^{-1}\dfrac{2}{3}$ (d) $\cos^{-1}\dfrac{12}{13}$

22. A small block B of mass m is quickly placed on an inclined plane P, which makes an angle θ with a horizontal cart C, on which P is rigidly fixed (see figure). The coefficient of friction between the block B and the plane C is μ. When the cart stays stationary the block slides down. If the cart C is moving in the horizontal direction with acceleration a, the minimum value



of a for which the block will remain static is

[TIFR 2018]

(a)$g(\cos\theta - \mu\sin\theta)$ (b)$g\dfrac{\tan\theta - \mu}{\mu\tan\theta + 1}$

(c)$g\dfrac{1-\mu\tan\theta}{\mu + \tan\theta}$ (d) $g(\mu - \sin\theta\cos\theta)$

23. Two students A and B try to measure the time period T of a pendulum using the same stopwatch, but following two different methods. Student A measures the time taken for one oscillation, repeats this process $N(\gg 1)$ times and computes the average. On the other hand, Student B just once measures the time taken for N oscillations and divides that number by N. Which of the following statements is true about the errors in T as measured by A and by B ?

[TIFR 2018]

(a) The measurement made by A has a larger error than that made by B.

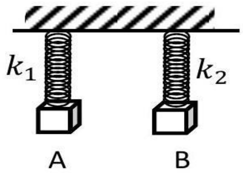
(b) The measurement made by A has a smaller error than that made by B.

(c) A and B will measure the time period with the same accuracy.

(d) It is not possible to determine if the measurement made by A or B has the larger error.

24. Two bodies A and B of equal mass are suspended from two rigid supports by separate massless springs having spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of oscillations of A to that of B is

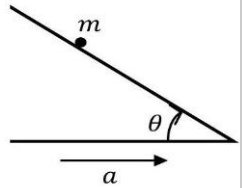
[TIFR 2019]



(a) k_1/k_2 (b) k_2/k_1

(c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$

25. A particle of mass m is placed on an inclined plane making an adjustable angle θ with the horizontal, as shown in the figure. The coefficient of friction between the particle and the inclined plane is μ.
If the inclined plane is moving horizontally with a uniform acceleration $a < g/\mu$ (see figure), the value of θ for which the particle will remain at rest on the plane is **[TIFR 2019]**



(a)$\theta = \tan^{-1}\left(\dfrac{\mu g + a}{g + \mu a}\right)$

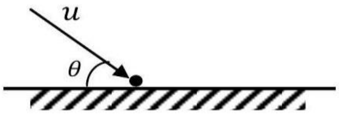
(b)$\theta = -\cot^{-1}\left(\dfrac{\mu a + g}{a + \mu g}\right)$

(c)$\theta = \tan^{-1}\left(\dfrac{\mu g + a}{g - \mu a}\right)$

(d)$\theta = \cot^{-1}\left(\dfrac{\mu a - g}{a + \mu g}\right)$

26. A particle of mass m is bounced on the ground with a velocity u making an angle of θ with the ground. The coefficient of restitution for collisions between the particle and the ground is ε and frictional effects are negligible both on the ground and in the air. The horizontal distance travelled by the particle from the point of initial impact till it begins to slide along the ground is

[TIFR 2019]



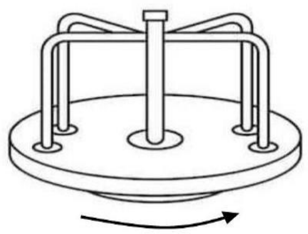
(a) $\frac{u^2}{2g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$ (b) $\frac{u^2}{g}\left(\frac{1}{1-\varepsilon}\right)\sin\theta$

(c) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\tan 2\theta$ (d) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$

27. A roundabout whose rotating base is a heavy uniform disc of radius 2 m and mass 400 kg has a central pillar and handles which are of negligible mass (see figure). The roundabout is set rotating at a steady rate of 20 r.p.m.

Four small children, of mass 10 kg, 20 kg, 30 kg and 40 kg respectively, step gently on to the edge of the roundabout, each with velocity 7.2 km/hr along a tangential direction and cling to the handles. After holding on for some time, the children step gently off the roundabout with the same velocity, but this time in a radial direction. Neglecting all effects of friction and air drag the final rate of rotation of the roundabout will be about

[TIFR 2020]

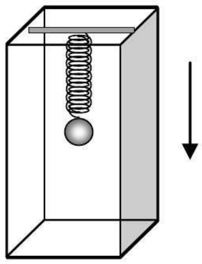


(a) 28 r.p.m (b) 25 r.p.m

(c) 36 r.p.m (d) 21 r.p.m

28. A particle of mass m hangs from a light spring inside a lift (see figure). When the lift is at rest, the mass oscillates in the vertical direction with an angular frequency 2.5rad/s. Now consider the following situation.

The suspended mass is at rest inside the lift which is descending vertically at a speed of 0.5 m/s. If the lift suddenly stops, the



amplitude of oscillations of the mass will be

[TIFR 2020]

(a) 0.20 m (b) 0.25 m

(c) 0.05 m (d) 1.25 m

29. Consider two planets P_1 and P_2 which can be modeled as uniform spheres of radii R_1 and R_2 respectively, and of the same material with the same density and other physical p`roperties. If the maximum possible height of a conical mountain (of the same material) on these planets is denoted by h_1 and h_2 respectively ($h_1 \ll R_1, h_2 \ll R_2$), then the ratio h_1/h_2 is

(a) R_2/R_1 (b) R_1/R_2

(c) $R_2^{2/3}/R_1^{2/3}$ (d) $R_1^{2/3}/R_2^{2/3}$

30. A spherical balloon of radius R is made of a material with surface tension γ and filled with N particles of an ideal gas. If the outside air pressure is P, the pressure P_b inside the balloon is given by

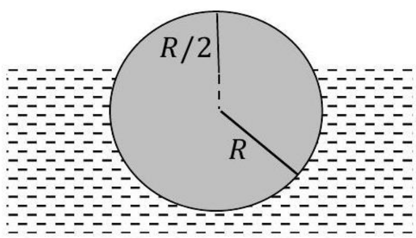
[TIFR 2021]

(a) $P_b = P$ (b) $P_b = P + 2\gamma/R$

(c) $P_b = P - 2\gamma/R$ (d) $P_b = P + 3\gamma/R$A

31. solid homogeneous sphere floats in water with a portion sticking out above the water, as shown in the figure below. The height of the highest point above the water surface is $R/2$ where R is the radius of the sphere. If the density of water is 1 g cm^{-3}, the density of the

[TIFR 2022]



material (in gcm^{-3}) must be

(a) 5/32 (b) 27/32

(c) 13/18 (d) 5/18

32. A pendulum which is suspended from the ceiling of a train has time period T_0 when the train is stationary. When the train moves with a small but steady speed v around a horizontal circular track of radius R, the time period of the pendulum will be

[TIFR 2022]

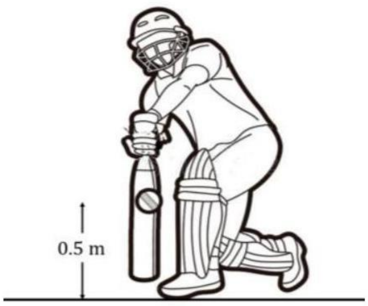
(a) $T_0\left(1+\frac{v^4}{g^2R^2}\right)^{1/4}$

(b) $T_0\left(1+\frac{v^2T_0^2}{4\pi^2R}\right)^{-1/2}$

(c) $T_0\left(1-\frac{v^4}{g^2R^2}\right)^{1/4}$

(d) $T_0\left(1-\frac{v^2T_0^2}{4\pi^2R}\right)^{-1/2}$

33. A cricket ball, bowled by a fast bowler, rises from the pitch at an angle of 30° with a speed of 72 km/hr, then moves straight ahead and, at a height of 0.5 m, strikes the flat surface of the bat held firmly at rest in a horizontal position (see figure). As a result, the ball bounces off elastically, providing a return catch straight back to the bowler.



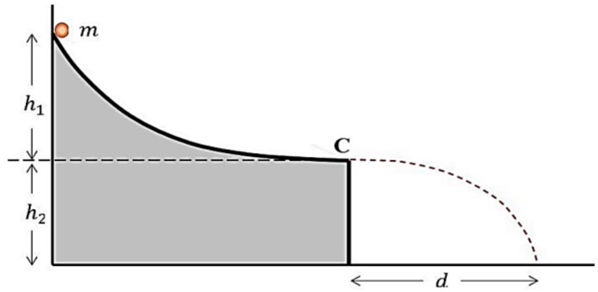
If the coefficient of restitution between the bat and the ball is 0.577 , the acceleration due to gravity is $10\ \mathrm{m\,s^{-2}}$ and air resistance can be neglected, the catch will carry, before hitting the ground, to a distance of approximately

[TIFR 2022]

(a) 37.0 m (b) 19.5 m

(c) 9.5 m (d) 21.0 m

34. A small body of mass m is released from rest at the top of a frictionless curved surface as shown in the figure, and permitted to slide down the curve. At the endpoint C, the tangent to the curve is horizontal. The mass then falls on the ground at a distance d as shown in the figure below when the experiment is carried out on the surface of the Earth. The heights h_1 and h_2 are also shown in the figure.



Suppose the same experiment is repeated on the surface of the Moon, where the acceleration due to gravity is $g' = g/6$, where g is the value on Earth. The corresponding distance d' at which the mass will fall on the ground in the Moon is

[TIFR 2023]

(a) $6d$

(b) d

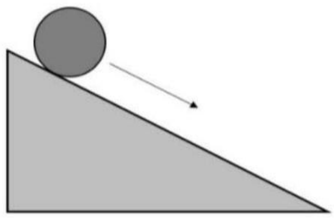
(c) $d\sqrt{h_1/h_2}$

(d) dependent on the shape of the curve

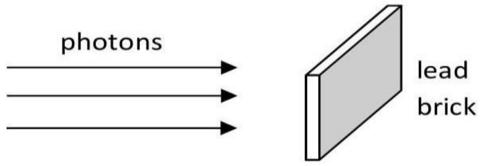
35. A solid cylinder of uniform mass density rolls down a fixed inclined plane without slipping (see figure).
The fraction of the total kinetic energy of the cylinder associated with its rotation about its centre of mass is **[TIFR 2023]**

(a) 1/3 (b) 1/6

(c) ¼ (d) 1/2



36. A beam of photons of 1MeV energy each is shot at a 10 mm thick lead brick (see figure).



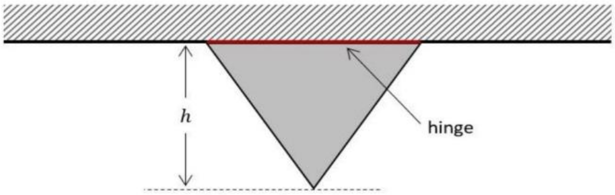
Given that the density of lead is $11.29 \text{ g} - \text{cm}^{-3}$, its atomic mass is 207.2 a.m.u., and also that the interaction cross-section for these photons with a lead atom is 10^{-23} cm^2, the fraction of the incident photons that will cross the brick without losing any energy is

[TIFR 2023]

(a) 28% (b) 72%

(c) 33% (d) 67%

37. A thin equilateral triangular plate of uniform mass density is attached to a fixed horizontal support along one of its sides through a frictionless hinge, as shown in the figure below. The vertical distance between the rod and the lower tip of the plate is h.



If the pointed tip of the plate is displaced (out of the plane of the paper) so that its plane forms a small angle with the vertical plane passing through the rod, the angular frequency ω of the resultant motion is $\omega =$ **[TIFR 2023]**

(a) $\sqrt{\dfrac{2g}{h}}$ (b) $\sqrt{\dfrac{2\sqrt{3}g}{h}}$

(c) $\sqrt{\dfrac{2g}{\sqrt{3}h}}$ (d) $\sqrt{\dfrac{\sqrt{3}g}{2h}}$

38. A particle is executing simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position is x_1 and x_2, the corresponding values of its velocity are v_1 and v_2 respectively. The time period of oscillation is given by

(a) $2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$ (b) $2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_2^2 - v_1^2}}$

(c) $2\pi\dfrac{x_2 - x_1}{v_2 - v_1}$ (d) $2\pi\dfrac{x_2 - x_1}{v_1 - v_2}$

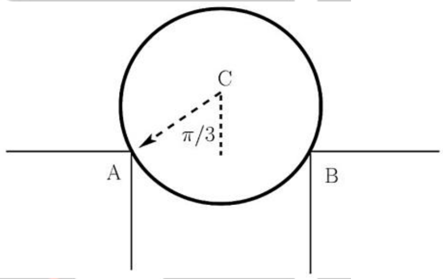
39. Consider an object falling in air. In addition to gravity, it experiences an air resistance force, R, given by $R = bv$, where v is the speed and b is a constant. If the object is dropped from rest ($v = 0$ at $t = 0$), the distance traversed by the object at $t = m/b$ is:

[TIFR 2024]

(a) $\left(\dfrac{m^2 g}{b^2}\right)(e-1)$ (b) $\left(\dfrac{m^2 g}{b^2}\right)\left(1-\dfrac{1}{e}\right)$

(c) $\left(\dfrac{m^2 g}{b^2}\right)\left(\dfrac{1}{e}\right)$ (d) $\left(\dfrac{m^2 g}{b^2}\right)\left(2-\dfrac{1}{e}\right)$

40. A frictionless disk of mass m is balanced at rest on the edges of two platforms at points A and B that are at equal height as shown below. The angle made by the line joining the centre to point A (line CA) with the vertical is $\pi/3$.



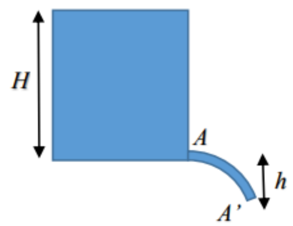
What is the magnitude of the force exerted by point A on the disk? **[TIFR 2024]**

(a) $\dfrac{mg}{\sqrt{3}}$ (b) $\dfrac{mg}{2}$

(c) mg (d) $\dfrac{mg\sqrt{3}}{2}$

41. Water is flowing out of a small horizontal opening of area, A, at the bottom of a tank of height H. The flow is a laminar flow under the influence of gravity. What is the area, A', of the stream transverse to the fluid velocity, at a height h below the opening? (Neglect atmospheric pressure and dissipation effects. The thickness of the stream is negligible compared to H and h.)

[TIFR 2025]



(a) $A\sqrt{\frac{H}{h+H}}$ (b) $A\frac{H}{h+H}$

(c) $A\left(1+\frac{h}{H}\right)$ (d) $A\sqrt{1+\frac{h}{H}}$

Answers key				
CSIR-NET				
1. c	2. b	3. d	4. d	5. d
6. c	7. b	8. c	9. c	10. c
11. d	12. c	13. a	14. a	15. c
16. b	17. a	18. c	19. a	20. b
21. a	22. a	23. c	24. a	25. c
26. c	27. a	28. b	29. a	30. d
31. c	32. b	33. d	34. b	35. d
36. d				
GATE				
1. a	2. b	3. a	4. b	5. a
6.	7. b	8. c	9. 1	10. 3.57
11. 1	12.	13. 2		
JEST				
1. c	2. a	3. c	4. a	5. b
6. c	7. a	8. b	9. a	10. d
11. c	12. d	13. d	14. c	15. c
16. c	17. a	18. 0003	19. d	20. d
21. a	22. a	23. 0	24.	25. b
26. b	27. d	28. c	29. 981	30. d
31. a	32. d	33. 3707	34. d	35. b
36. c	37. d	38. a	39. d	40. a
41. d				
TIFR				
1. d	2.	3. d	4. d	5. b
6. c	7. c	8. d	9. c	10. b
11. d	12.	13. d	14. c	15. c
16. d	17. 85	18. d	19. d	20. b
21. a	22. b	23. a	24. d	25. b
26. d	27. b	28. a	29. a	30. b
31. b	32. d	33.	34. b	35. a
36. b	37. a	38. a	39. c	40. c
41. a				

❖ CSIR-NET PYQ's

1. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by

$$L = \frac{1}{2}m\vec{v}^2 + e\vec{A}\cdot\vec{v} - e\phi$$

, where $\vec{A}$ and ϕ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

[CSIR JUNE 2011]

(a) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$

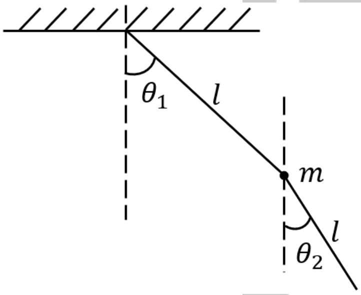
(b) The Hamiltonian of the particle is given by

$$H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A}\cdot\vec{p} + e\phi$$

(c) L remains unchanged under a gauge transformation of the potentials.

(d) Under a gauge transformation of the potentials, L changes by the total time derivative of a function of $\vec{r}$ and t.

2. A double pendulum consists of two point masses m attached by massless strings of length l as shown in the figure: **[CSIR DEC 2011]**



The kinetic energy of the pendulum is :

(a) $\frac{1}{2}$ $\mathrm{m}\ell^2[\dot{\theta}_1^2 + \dot{\theta}_2^2]$

(b) $\frac{1}{2}$ $\mathrm{m}\ell^2[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)]$

(c) $\frac{1}{2}$ $\mathrm{m}\ell^2[\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)]$

(d) $\frac{1}{2}$ $\mathrm{m}\ell^2[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 + \theta_2)]$

3. A particle of mass 'm' moves inside a bowl. If the surface of the bowl is given by the equation

$$z = \frac{1}{2}a(x^2 + y^2)$$

, where a is a constant, the Lagrangian of the particle is:

[CSIR DEC 2011]

(a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 - gar^2)$

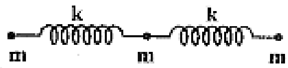
(b) $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2]$

(c) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\dot{\phi}^2 - gar^2)$

(d) $\frac{1}{2}m[(1 + a^2x^2)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2]$

4. Three particles of equal mass 'm' are connected by two identical massless springs of stiffness constant ' k ' as shown in the figure. If x_1, x_2 and x_3 denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:

[CSIR DEC 2012]



(a) $\frac{1}{2}k(x_1^2 + x_2^2 + x_3^2)$

(b) $\frac{1}{2}k[x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$

(c) $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 + 2x_2(x_1 + x_3)]$

(d) $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$

5. The Lagrangian of a particle of mass m moving in one dimensions is given by

$$\mathrm{L} = \frac{1}{2}\mathrm{m}\dot{\mathrm{x}}^2 - \mathrm{bx}$$

where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by: (in the following c_1 and c_2 are constants)

[CSIR JUNE 2013]

(a) $-\frac{b}{2m}t^2 + c_1t + c_2$

(b) $c_1t + c_2$

(c)$c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$

(d)$c_1\cosh\left(\frac{bt}{m}\right)+c_2\sinh\left(\frac{bt}{m}\right)$

6. The number of degrees of freedom of a rigid body in d space-dimensions is

[CSIR JUNE 2013]

(a) 2 d (b) 6

(c) $d(d+1)/2$ (d) d!

7. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by

$$H=\sqrt{p^2+m^2}+V(x)$$

, in units in which the speed of light $c=1$. The corresponding Lagrangian is

[CSIR DEC 2013]

(a) $L=m\sqrt{1+\dot{x}^2}-V(x)$

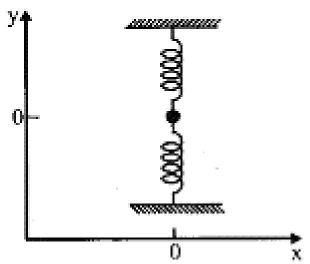
(b) $L=-m\sqrt{1-\dot{x}^2}-V(x)$

(c) $L=\sqrt{1+m\dot{x}^2}-V(x)$

(d)$L=\frac{1}{2}m\dot{x}^2-V(x)$

8. Consider a particle of mass m attached to two identical springs each of length / and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?

[CSIR DEC 2013]



(a)$m\ddot{x}+\frac{kx^3}{l^2}=0$ (b) $m\ddot{x}+kx=0$

(c)$m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

9. The equation of motion of a system described by the time-dependent Lagrangian

$$L=e^{y}\left[\frac{1}{2}m\dot{x}^2-V(x)\right]$$

is

[CSIR DEC 2014]

(a)$m\ddot{x}+\gamma m\dot{x}+\frac{dV}{dx}=0$ (b)$m\ddot{x}+\gamma m\dot{x}-\frac{dV}{dx}=0$

(c)$m\ddot{x}-\gamma m\dot{x}+\frac{dV}{dx}=0$ (d)$m\ddot{x}+\frac{dV}{dx}=0$

10. The Lagrangian of a particle moving in a plane is given in Cartesian coordinates as $L=\dot{x}\dot{y}-x^2-y^2$. In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is

[CSIR DEC 2015]

(a) $\dot{r}\sin\theta+r\dot{\theta}\cos\theta$

(b) $\dot{r}\cos\theta+r\dot{\theta}\sin\theta$

(c) $2\dot{r}\cos 2\theta-r\dot{\theta}\sin 2\theta$

(d) $\dot{r}\sin 2\theta+r\dot{\theta}\cos 2\theta$

11. The Lagrangian of a system moving in three dimensions is

$$L=\frac{1}{2}m\dot{x}_1^2+m(\dot{x}_2^2+\dot{x}_3^2)-\frac{1}{2}kx_1^2-\frac{1}{2}k(x_2+x_3)^2$$

The independent constant(s) of motion is/are

[CSIR JUNE 2016]

(a) energy alone

(b) only energy, one component of the linear momentum and one component of the angular momentum

(c) only energy and one component of the linear momentum

(d) only energy and one component of the angular momentum

12. The parabolic coordinates (ξ,η) are related to the Cartesian coordinates (x,y) by $x=\xi\eta$ and

$$y=\frac{1}{2}(\xi^2-\eta^2)$$

. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is

[CSIR DEC 2016]

(a) $\frac{1}{2}m\left[\dot{\xi}^2+\dot{\eta}^2-\omega^2(\xi^2+\eta^2)\right]$

(b) $\frac{1}{2}m(\xi^2+\eta^2)\left[\left(\dot{\xi}^2+\dot{\eta}^2\right)-\frac{1}{4}\omega^2(\xi^2+\eta^2)\right]$

(c) $\frac{1}{2}m(\xi^2+\eta^2)\left(\dot{\xi}^2+\dot{\eta}^2-\frac{1}{2}\omega^2\xi\eta\right)$

(d) $\frac{1}{2}m(\xi^2+\eta^2)\left(\xi^2+\dot{\eta}^2-\frac{1}{4}\omega^2\right)$

13. The dynamics of a particle governed by the Lagrangian

$$L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2-kx\dot{x}t$$

describes **[CSIR DEC 2016]**

(a) an undamped simple harmonic oscillator

(b) a damped harmonic oscillator with a time varying damping factor

(c) an undamped harmonic oscillator with a time dependent frequency

(d) a free particle

14. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is

$$H=\alpha x^2p+\frac{p^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2x^2$$

where α,β and ω are constant. The corresponding Lagrangian is

[CSIR JUNE 2017]

(a) $\frac{1}{2}(\dot{x}-\alpha x^2)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$

(b) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2-\alpha x^2\dot{x}$

(c) $\frac{1}{2}(\dot{x}^2-\alpha^2x)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$

(d) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2+\alpha x^2\dot{x}$

15. The Hamiltonian of a one-dimensional system is

$$H=\frac{xp^2}{2m}+\frac{1}{2}kx$$

, where m and k are positive constants. The corresponding Euler-Lagrange equation for the system is

[CSIR JUNE 2018]

(a) $m\ddot{x}+k=0$

(b) $m\ddot{x}+2\dot{x}+kx^2=0$

(c) $2mx\ddot{x}-m\dot{x}^2+kx^2=0$

(d) $mx\ddot{x}-2m\dot{x}^2+kx^2=0$

16. Which of the following terms, when added to the Lagrangian $L(x,y,x,y)$ of a system with two degrees of freedom, will not change the equations of motion?

[CSIR DEC 2019]

(a) $x\ddot{x}-y\ddot{y}$ (b) $x\ddot{y}-y\ddot{x}$

(c) $x\dot{y}-y\dot{x}$ (d) $y\dot{x}^2+x\dot{y}^2$

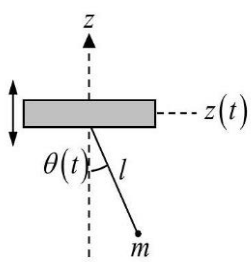
17. A point mass m, is constrained to move on the inner surface of a paraboloid of revolution $x^2+y^2=az$ (where $a>0$ is a constant). When it spirals down the surface, under the influence of gravity (along $-z$ direction), the angular speed about the z - axis is proportional to

[CSIR JUNE 2020]

(a) 1 (independent of z) (b) z

(c) z^{-1} (d) z^{-2}

18. The fulcrum of a simple pendulum (consisting of a particle of mass m attached to the support by a massless string of length l) oscillates vertically as sin $z(t)=a\sin\omega t$, where ω is a constant. The pendulum moves in a vertical plane and $\theta(t)$ denotes its angular position with



respect to the z-axis.

If

$$\ell\frac{d^2\theta}{dt^2}+\sin\,\theta(g-f(t))=0$$

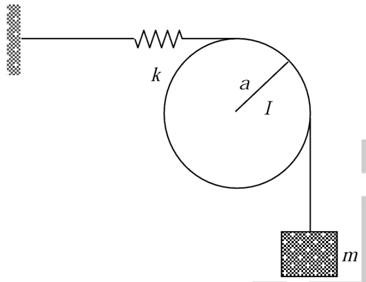
(where g is the acceleration due to gravity) describes the equation of motion of the mass, then $f(t)$ is

[CSIR JUNE 2021]

(a) $a\omega^2\cos\ \omega t$ (b) $a\omega^2\sin\ \omega t$

(c) $-a\omega^2\cos\ \omega t$ (d) $-a\omega^2\sin\ \omega t$

19. A wire, connected to a massless spring of spring constant k and a block of mass m,
goes around a disc of radius a and moment of inertia I, as shown in the figure.
Assume that the spring remains horizontal, the pully rotates freely and there is no slippage between the wire and the pully. The



angular frequency of oscillation of the disc is [CSIR JUNE SEP 2022]

(a) $\sqrt{\frac{2ka^2}{ma^2+I}}$ (b) $\sqrt{\frac{ka^2}{ma^2+I}}$

(c) $\sqrt{\frac{ka^2}{ma^2+2I}}$ (d) $\sqrt{\frac{ka^2}{2ma^2+I}}$

20. The Lagrangian of a system described by three generalized coordinates q_1, q_2 and q_3 is $L = \frac{1}{2}m(\dot{q}_1^2+\dot{q}_2^2)+M\dot{q}_1\dot{q}_2+k\dot{q}_1 q_3$, where m, M and k are positive constants. Then, as a function of time

[CSIR JUNE SEP 2022]

(a)Two coordinates remain constant and one evolves linearly

(b)One coordinates remain constant, one evolves linearly and Third evolves as a quadratic function

(c)One evolves linearly and Two evolves as a quadratic function

(d)Al three evolves linearly

21. The trajectory of a particle moving in a plane is expressed in polar coordinates (r,θ) by the equation $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$ where the parameters r_0, β and ω are positive. Let v_r and a_r denote the velocity and acceleration, respectively, in the radial direction. For this trajectory

[CSIR JUNE 2023]

(a) $a_r < 0$ at all times irrespective of the values of the parameters

(b) $a_r > 0$ at all times irrespective of the values of the parameters

(c) $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters

(d) $\frac{dv_r}{dt} > 0$ however, $a_r = 0$ for some choices of parameters

22. The Hamiltonian of a two particle system is $H = p_1p_2 + q_1q_2$ where q_1 and q_2 are generalized coordinates and p_1 and p_2 are the respective canonical momenta. The Lagrangian of this system is [CSIR JUNE 2023]

(a) $\dot{q}_1\dot{q}_2 + q_1q_2$ (b) $-\dot{q}_1\dot{q}_2 + q_1q_2$

(c) $-\dot{q}_1\dot{q}_2 - q_1q_2$ (d) $\dot{q}_1\dot{q}_2 - q_1q_2$

23. The 1-dimensional Hamiltonian of a classical particle of mass m is

$$H = \frac{p^2}{2m}e^{-x/a} + V(x),$$

where a is a constant with appropriate dimensions. The corresponding Lagrangian is,

[CSIR DEC 2023]

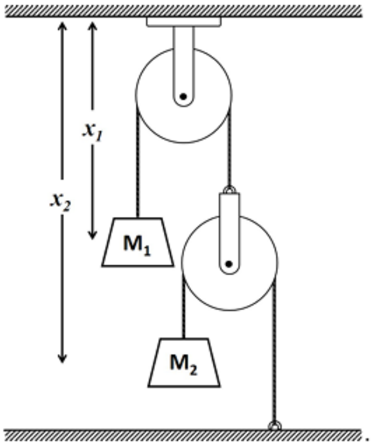
(a) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$

(b) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

(c) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$

(d) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

24. Consider a mass-pulley system as shown in the figure. The heights of the blocks as measured from the ceiling are x_1 and x_2, as shown in the figure.



What is the constraint between x_1 and x_2 ?

[CSIR DEC 2024]

(a) They are unconstrained

(b) $x_2 - x_1 =$ constant .

(c) $x_2 + x_1 =$ constant .

(d) $x_2 + 2x_1 =$ constant.

❖ GATE PYQ's

1. The Lagrangian for the Kepler problem is given by

$$L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{\mu}{r}, (\mu > 0),$$

where (r, θ) denote the polar coordinates and the mass of the particle is unity. Then

[GATE 2001]

(a) $p_\theta = 2r^2\dot{\theta}$

(b) $p_r = 2\dot{r}$

(c) the angular momentum of the particle about the centre of attraction is a constant

(d) the total energy of the particle is time dependent

2. A particle of mass m is constrained to move on the plane curve $xy = C (C > 0)$ under gravity (y axis vertical). The Lagrangian of the particle is given by

[GATE 2001]

(a) $\frac{1}{2}\dot{m}^2x\left(1 + \frac{C^2}{x^4}\right) + \frac{mgC}{x}$

(b) $\frac{1}{2}\dot{m}^2x\left(1 + \frac{C^2}{x^4}\right) - \frac{mgC}{x}$

(c) $\frac{1}{2}\dot{m}^2x\left(1 + \frac{C}{x^2}\right) + \frac{mgC}{x}$

(d) $\frac{1}{2}\dot{m}^2x\left(1 + \frac{C}{x^2}\right) - \frac{mgC}{x}$

3. Consider the Lagrangian

$$L = \frac{q\ln{(\dot{q})^2}}{2} - \lambda q + qf(q)$$

, where $f(q)$ is an arbitrary function and $\lambda > 0$.

[GATE 2001]

(a) Write down the Euler-Lagrange equation of motion

(b) Does the Lagrangian transform covariantly under the transformation $q \to \alpha q$ for α a real constant?

(c) Calculate the Hamiltonian of the system

(d) Is total energy a constant of motion?

4. The Lagrangian of a particle of mass m moving in a plane is given by

$$\mathrm{L} = (1/2)m[v_x^2 + v_y^2] + a(xv_y - yv_x)$$

, where v_x and v_y are velocity components and a is a constant. The canonical momenta of the particle are given by

[GATE 2003]

(a) $p_x = mv_x$ and $p_y = mv_y$

(b) $p_x = mv_x + ay$ and $p_y = mv_y + ax$

(c) $p_x = mv_x - ay$ and $p_y = mv_y + ax$

(d) $p_x = mv_x - ay$ and $p_y = mv_y - ax$

5. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$$

The generalized momenta corresponding to r and θ are given by

[GATE 2004]

(a) $m\dot{r}$ and $mr^2\dot{\theta}$ (b) $m\dot{r}$ and $mr\dot{\theta}$

(c) $m\dot{r}^2$ and $mr^2\dot{\theta}$ (d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

6. If a particle moves outward in a plane along a curved trajectory described by $r = a\theta, \theta = \omega t$, where a and ω are constants, then its

[GATE 2005]

(a) kinetic energy is conserved

(b) angular momentum is conserved

(c) total momentum is conserved

(d) radial momentum is conserved

7. A particle of mass m moves in a potential
$$V(x)=\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\mu v^2$$
, where x is the position coordinate, v is the speed, and ω and μ are constants. The canonical (conjugate) momentum of the particle is

[GATE 2005]

(a) $p=m(1+\mu)v$ (b) $p=mv$

(c) $p=m\mu v$ (d) $p=m(1-\mu)v$

8. Three particles of mass m each situated at $x_1(t), x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is

[GATE 2007]

(a)$$L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2+\frac{k}{2}(x_3-x_2-l)^2$$

(b)$$L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_3-l)^2+\frac{k}{2}(x_3-x_2-l)^2$$

(c)$$L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2+l)^2-\frac{k}{2}(x_3-x_2+l)^2$$

(d)$$L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2-\frac{k}{2}(x_3-x_2-l)^2$$

9. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalized coordinates required to describe the motion of this system is

[GATE 2008]

(a) 1 (b) 2

(c) 4 (d) 6

10. The Lagrangian of a system is given by
$$L=\frac{1}{2}\dot{q}^2+q\dot{q}-\frac{1}{2}q^2$$
. It describes the motion of

[GATE 2008]

(a) a harmonic oscillator

(b) a damped harmonic oscillator

(d) a system with unbounded motion

(c) an anharmonic oscillator

11. The Lagrangian of a free particle in spherical polar coordinates is given by
$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\dot{\phi}^2\sin^2\ \theta\right)$$
. The quantity that is conserved is

[GATE 2009]

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$

(c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}}+\dot{r}\dot{\theta}$

12. The Lagrangian of a particle of mass m moving in one dimension is
$$L=\exp\,(\alpha t)\left[\frac{m\dot{x}^2}{2}-\frac{kx^2}{2}\right]$$
, where α and k are positive constants. The equation of motion of the particle is

[GATE 2009]

(a) $\ddot{x} + \alpha\dot{x} = 0$ (b) $\ddot{x} + \frac{k}{m}x = 0$

(c) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$ (d) $\ddot{x} + \alpha\dot{x} + \frac{k}{m} = 0$

13. A particle is moving under the action of a generalized potential $V(q,\dot{q}) = \frac{(1+\dot{q})}{q^2}$
The magnitude of the generalized force is

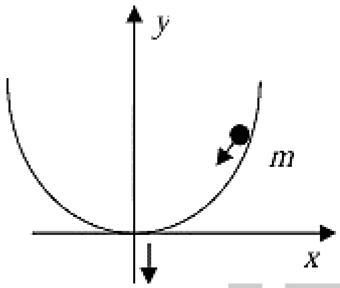
[GATE 2011]

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$

(c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

Statement for Linked Answer Questions 14 and 15:

A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$ axis shown in the figure. Here a is a constant.



14. The Lagrangian for this particle is given by

[GATE 2012]

(a) $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

(b) $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

(d) $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 + mgax^2$

15. The Lagrange's equation of motion of the particle is

[GATE 2012]

(a) $\ddot{x} = 2\text{gax}$

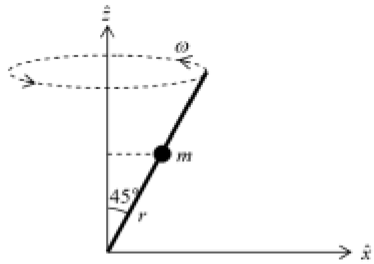
(b) $m(1+4a^2x^2)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

(c) $m(1+4a^2x^2)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

(d) $\ddot{x} = -2gax$

16. A bead of mass m can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, r is the distance of the bead from the origin. The momentum conjugate to r is

[GATE 2014]



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$

(c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

17. The Lagrangian of a system is given by
$$L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$$
, where m, l and g are constants. Which of the following is conserved?

[GATE 2016]

(a) $\dot{\varphi}\sin^2\theta$ (b) $\dot{\varphi}\sin\theta$

(c) $\frac{\dot{\varphi}}{\sin\theta}$ (d) $\frac{\dot{\varphi}}{\sin^2\theta}$

18. A particle ô mass $m = 0.1$ kg is initiālly aat rést āt oorigin. It starts moving with a uniform acceleration $\vec{a} = 10\hat{\imath}\ \text{ms}^{-2}$ at $t = 0$. The action S of the particle, in units of $\text{J} - \text{s}$, at $t = 2$ s is____________ (Give your answer upto two decimal places)

19. If the Lagrangian
$$L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2q^2$$
is modified to
$$L = L_0 + \alpha q\left(\frac{dq}{dt}\right)$$
, which one of the following is true?

[GATE 2017]

(a) both the canonical momentum and equation of motion do not change

(b) canonical momentum changes, equation of motion does not change

(c) canonical momentum does not change, equation of motion changes

(d) both the canonical momentum and equation of motion change

20. Consider the Hamiltonian

$$H(q,p)=\frac{\alpha p^2 q^4}{2}+\frac{\beta}{q^2}$$

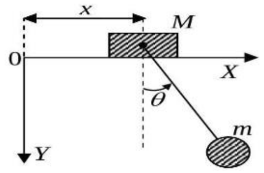
, where α and β are parameters with appropriate dimensions, and q and p are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q,\dot{q})$ is

[GATE 2019]

(a) $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4}-\frac{\beta}{q^2}$ (b) $\frac{2}{\alpha}\frac{\dot{q}^2}{q^4}+\frac{\beta}{q^2}$

(c) $\frac{1}{\alpha}\frac{\dot{q}^2}{q^4}+\frac{\beta}{q^2}$ (d) $-\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4}+\frac{\beta}{q^2}$

21. A uniform block of mass M slides on a smooth horizontal bar. Another mass m is connected to it by an inextensible string of length l of negligible mass, and is constrained to oscillate in the $X-Y$ plane only. Neglect the sizes of the masses. The number of degrees of freedom of the system is two and the generalized coordinates are chosen as x and θ, as shown in the figure.



If p_x and p_θ are the generalized momenta corresponding to x and θ, respectively, then the correct option(s) is (are)

[GATE 2021]

(a) $p_x=(m+M)\dot{x}+ml\cos\,\theta\dot{\theta}$

(b) $p_\theta=ml^2\theta-ml\cos\,\theta\dot{x}$

(c) p_x is conserved

(d) p_θ is conserved

22. The time derivative of a differentiable function $g(q_i,t)$ is added to a Lagrangian $L(q_i,\dot{q}_1,t)$ such that

$$L'=L(q_i,\dot{q}_i,t)+\frac{d}{dt}g(q_i,t)$$

where $q_i,\dot{q}_i,t$ are the generalized coordinates, generalized velocities and time, respectively. Let p_i be the generalized momentum and H the Hamiltonian associated with $L(q_i,\dot{q}_i,t)$. If p_i' and H' are those associated with L', then the correct option(s) is(are) [GATE 2021]

(a) Both L and L' satisfy the Euler-Lagrange's equations of motion

(b) $p_i'=p_i+\frac{\partial}{\partial q_i}g(q_i,t)$

(c) If p_i is conserved, then p_i' is necessarily conserved

(d) $H'=H+\frac{d}{dt}g(q_i,t)$

23. If $(\dot{x}\dot{y}+\alpha xy)$ is a constant of motion of a two-dimensional isotropic harmonic oscillator with Lagrangian

$$L=\frac{m(\dot{x}^2+\dot{y}^2)}{2}-\frac{k(x^2+y^2)}{2}$$

then α is [GATE 2021]

(a) $+\frac{k}{m}$ (b) $-\frac{k}{m}$

(c) $-\frac{2k}{m}$ (d) 0

24. Consider the Lagrangian $L=m\dot{x}\dot{y}-m\omega_0^2xy$. If p_x and p_y denote the generalized momenta conjugate to x and y, respectively, then the canonical equations of motion are

[GATE 2024]

(a) $\dot{x}=\frac{p_x}{m},\dot{p}_x=-m\omega_0^2x,\dot{y}=\frac{p_y}{m},\dot{p}_y=-m\omega_0^2y$

(b) $\dot{x}=\frac{p_x}{m},\dot{p}_x=m\omega_0^2x,\dot{y}=\frac{p_y}{m},\dot{p}_y=m\omega_0^2y$

(c) $\dot{x}=\frac{p_y}{m},\dot{p}_x=-m\omega_0^2y,\dot{y}=\frac{p_x}{m},\dot{p}_y=-m\omega_0^2x$

(d) $\dot{x}=\frac{p_y}{m},\dot{p}_x=m\omega_0^2y,\dot{y}=\frac{p_x}{m},\dot{p}_y=m\omega_0^2x$

❖ JEST PYQ's

1. A double pendulum consists of two equal masses ' m ' suspended by two strings of length l. What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos\,\theta = 1 - \theta^2/2$).

[JEST 2014]

(a) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \omega_1^2\theta_1^2 - \frac{1}{2}\omega_1^2\theta_1^2\right)$

(b) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(c) $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

(b) $L \approx ml^2\left(\frac{1}{2}\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

2. Let us write down the Lagrangian of a system as $L(x, \dot{x}, \ddot{x}) = m x\ddot{x} + kx^2 + cx\dot{x}$ what is the dimension of c ?

[JEST 2014]

(a) MLT^{-3} (b) MT^{-2}

(c) MT (d) ML^2T^{-1}

3. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. which of the following statements is true?

[JEST 2015]

(a) This is a free particle

(b) The particle is experiencing velocity dependent damping

(c) The particle is executing simple harmonic motion

(d) The particle is under constant acceleration

4. A rod of mass m and length l is suspended from two massless vertical springs with a spring constants $k1$ and k_2. What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the ends of the rod?

[JEST 2017]

(a) $\frac{m}{8}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(b) $\frac{m}{2}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - (k_1 + k_2)(x_1^2 + x_2^2)$

(c) $\frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

(d) $\frac{m}{4}(\dot{x}_1^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 - k_2)(x_1^2 + x_2^2)$

5. A possible Lagrangian for a free particle is

[JEST 2017]

(a) $L = \dot{q}^2 - q^2$ (b) $L = \dot{q}^2 - q\dot{q}$

(c) $L = \dot{q}^2 - q$ (d) $L = \dot{q}^2 - \frac{1}{q}$

6. A block of mass M is moving on a frictionless inclined surface of a wedge of mass m under the influence of gravity. The wedge is lying on a rigid frictionless horizontal surface. The configuration can be described using the radius vectors $\vec{r}_1$ and $\vec{r}_2$ shown in the figure. How many constraints are present and what are the types?

(a) One constraint: holonomic and scleronomous

(b) Two constraints; Both are holonomic; one is scleronomous and eheonomous

(c) Two constraints; Both are scleronomous; one is holonomic and the other is non-holonomic.

(d) Two constraints; Both are holonomic and scleronomous.

7. If $\mathrm{F(x,y) = x^2 + y^2 + xy}$, its Legendre transformed function $G(u, \mathrm{v})$, up to a, multiplicative constant, is

[JEST 2018]

(a) $u^2 + v^2 + uv$ (b) $u^2 + v^2 - uv$

(c) $u^2 + v^2$ (d) $(u + v)^2$

8. Consider the Lagrangian

$$L = 1 - \sqrt{1 - q^2} - \frac{q^2}{2}$$

Of a particle executing oscillations whose amplitude is A. if p denotes the momentum of the particle, then $4p^2$ is [JEST 2018]

(a) $(A^2-q^2)(4+A^2-q^2)$

(b) $(A^2+q^2)(4+A^2-q^2)$

(c) $(A^2-q^2)(4+A^2+q^2)$

(d) $(A^2+q^2)(4+A^2+q^2)$

9. Consider the motion of a particle in two dimensions given by the Lagrangian

$$L=\frac{m}{2}(\dot{x}^2+\dot{y}^2)-\frac{\lambda}{4}(x+y)^2$$

where $\lambda>0$. The initial conditions are given as $y(0)=0, x(0)=42$ meters, $x(0)=y(0)=0$. What is the value of $x(t)-y(t)$ at $t=25$ seconds in meters?

[JEST 2019]

❖ TIFR PYQ's

1. The dynamics of a particle of mass m is described in terms of three generalized coordinates ξ,η and φ. If the Lagrangian of the system is

$$L=\frac{1}{8}m\left[(\xi+\eta)\left(\frac{\dot{\xi}^2}{\xi^2}+\frac{\dot{\eta}^2}{\eta^2}\right)+4\xi\eta\dot{\varphi}^2\right]+\frac{1}{8}k(\xi+\eta)^2$$

where k is a constant, then a conserved quantity in the system will be

[TIFR 2011]

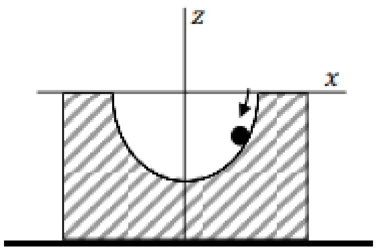
(a) $(m+k)(\dot{\xi}+\dot{\eta})$

(b) $m\xi\eta\dot{\varphi}$

(c) $m(\dot{\xi}^2/\eta^2+\dot{\eta}^2/\xi^2)$

(d) $m(\xi+\eta)(\dot{\xi}/\xi^2+\dot{\eta}/\eta^2)$

2. A ball of mass m slides under gravity without friction inside a semicircular depression of radius a inside a fixed block placed on a horizontal surface, as shown in the figure. The equation of motion of the ball in the x-direction will be [TIFR 2013]



(a) $\ddot{x}=\frac{g}{a}x\sqrt{1-\frac{x^2}{a^2}}$

(c) $\ddot{x}=-\frac{g}{a}x$

(b) $\ddot{x}=\frac{g}{a}x$

(d) $\ddot{x}=-\frac{3g}{a}x\sqrt{1-\frac{x^2}{a^2}}$

3. The Lagrangian of a system described by a single generalized coordinate q is $L=\frac{1}{2}\dot{q}\sin^2 q$

Its Hamiltonian is

[TIFR 2017]

(a) not defined (b) Zero

(a) $-\dot{q}\sin^2 q$ (d) $\dot{q}\left(p-\frac{1}{2}\sin^2 q\right)$

4. A particle of mass m moving in one-dimension x is subjected to the Lagrangian

$$L=\frac{1}{2}m(\dot{x}-\lambda x)^2$$

where λ is a real constant. If it starts at the origin at $t=0$, its motion corresponds to the equation (a is a constant)

[TIFR 2018]

(a) $x=a\exp\lambda t$

(b) $x=a\{1-\exp(-\lambda t)\}$

(c) $x=a\sin\lambda t$

(d) $x=a\sinh\lambda t$

5. The Lagrangian of a system described by generalized coordinates q_1 and q_2 is given by

$$L=\frac{a}{2}(\dot{q}_1^2+\dot{q}_2^2)-\frac{b^2}{\pi}(q_1^2+q_2^2)$$

where a and b are constants. It follows that a conserved quantity in this system is

[TIFR 2020]

(a) $q_1\dot{q}_2-q_2\dot{q}_1$ (b) $q_1\dot{q}_2+q_2\dot{q}_1$

(c) $\frac{q_1\dot{q}_2-q_2\dot{q}_1}{q_1^2+q_2^2}$ (d) $2\pi(q_1^2\dot{q}_2+q_2^2\dot{q}_1)$

6. A particle of mass m moves in a plane (r,θ) under the influence of a force

$$\vec{F} = \frac{mk}{r^3}(x\hat{r} + y\hat{\theta})$$

where $x = r\cos\,\theta$ and $y = r\sin\,\theta$, while k is a constant. The Lagrangian for this system is

[TIFR 2021]

(a) $L = \frac{1}{2}m\left(\frac{\dot{r}^2}{r} + r\dot{\theta}^2 - \frac{kxy}{r}\right)$

(b) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 - \frac{ky}{r^2}\right)$

(c) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 - \frac{kx}{r^2}\right)$

(d) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \frac{kx}{r^2}\right)$

7. A particle of mass m, moving in one dimension x satisfies the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 e^{2kx}$$

where k is a constant. If H is the Hamiltonian of the system, the canonical equations of motion are **[TIFR 2021]**

(a) $\dot{x} = \frac{p}{m}e^{-2kx}, \dot{p} = -2kH$

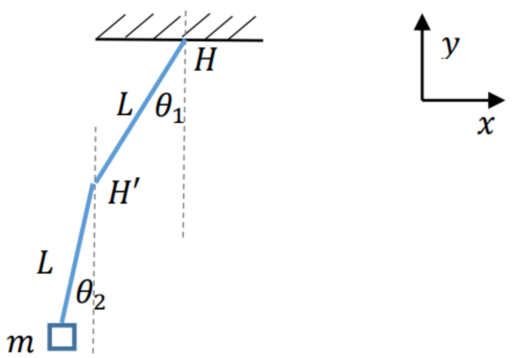
(b) $\dot{x} = \frac{p}{m}e^{2kx}, \dot{p} = -2H$

(c) $\dot{x} = \frac{p}{m}e^{-2kx}, \dot{p} = -\frac{1}{2}kH$

(d) $\dot{x} = \frac{p}{m}e^{-2kx}, \dot{p} = -2H$

8. A composite pendulum consists of two massless rods and a weight m. The two rods are connected by a hinge H'. The other end of the first rod is connected to the ceiling by a hinge H. The rods can move freely about H, H' in the xy plane. What is the Lagrangian of the system?

[TIFR 2025]



(a) $\dfrac{L^2m\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right)}{2} + gLm(\cos\,\theta_1 + \cos\,\theta_2)$

(b) $\dfrac{L^2m\left(\dot{\theta}_1^2 + \dot{\theta}_2^2\right)}{2} + gLm(\cos\,\theta_1 + \cos\,\theta_2)$

(c) $\dfrac{L^2m\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2)\right)}{2} - gLm(\cos\,\theta_1 + \cos\,\theta_2)$

(d) $\dfrac{L^2m\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 - 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right)}{2} - gLm(\cos\,\theta_1 + \cos\,\theta_2)$

❖ Answer Key				
CSIR-NET PYQ				
1. d	2. b	3. d	4. d	5. a
6. c	7. b	8. a	9. a	10. d
11. a	12. b	13. d	14. a	15. c
16. b	17. c	18. d	19. b	20. a
21. d	22. d	23. a	24. d	
GATE PYQ				
1. c	2. b	3. b	4. c	5. a
6. d	7. d	8.	9. b	10. a
11. c	12. d	13. c	14. b	15. b
16. a	17. a	18. 26.65	19. b	20. a
21. a	22. a,b	23. a	24. c	
JEST PYQ				
1. b	2. c	3. a	4. c	5. b
6. d	7. b	8. a	9. 42	
TIFR PYQ				
1.	2. d	3. a	4. d	5. a
6. c	7. a	8. a		

❖ **CSIR-NET PYQ's**

1. The Hamiltonian of a particle of unit mass moving in the xy-plane is given to be:

$$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$$

in suitable units. The initial values are given to be $(x(0), y(0)) = (1,1)$ and

$$(p_x(0), p_y(0)) = \left(\frac{1}{2}, -\frac{1}{2}\right)$$

. During the motion, the curves traced out by the particles in the xy plane and the $p_p p_y$-plane are **[CSIR JUNE 2011]**

(a) Both straight lines

(b) A straight line and a hyperbola respectively

(c) A hyperbola an ellipse, respectively

(d) Both hyperbolas

2. If the Lagrangian of a particle moving in one dimension is given by

$$L = \frac{\dot{x}^2}{2x} - V(x)$$

, the Hamiltonian is: **[CSIR JUNE 2012]**

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$

(c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$

3. The Hamiltonian of a simple pendulum consisting of a mass ' m ' attached to a massless string of

length l is

$$\mathrm{H} = \frac{\mathrm{p}_\theta^2}{2\,\mathrm{m}\ell^2} + \mathrm{mg}\ell(1 - \cos\,\theta)$$

. If L denotes the Lagrangian, the value of $\frac{\mathrm{dL}}{\mathrm{dt}}$ is : **[CSIR DEC 2012]**

(a) $-\frac{2\,\mathrm{g}}{\ell}\mathrm{p}_\theta \sin\,\theta$ (b) $-\frac{g}{\ell}p_\theta \sin\,2\theta$

(c) $\frac{\mathrm{g}}{\ell}\mathrm{p}_\theta \cos\,\theta$ (d) $\ell p_0^2 \cos\,\theta$

4. A system is governed by the Hamiltonian

$$\mathrm{H} = \frac{1}{2}(\mathrm{p_x} - \mathrm{ay})^2 + \frac{1}{2}(\mathrm{p_y} - \mathrm{bx})^2$$

where a and b are constants and $\mathrm{p_x}, \mathrm{p_y}$ are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved? **[CSIR JUNE 2013]**

(a) $a = -3, b = 2$ (b) $\mathrm{a} = 3, \mathrm{b} = -2$

(c) $a = 2, b = -3$ (d) $\mathrm{a} = -2, \mathrm{b} = 3$

5. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H = \sqrt{p^2 + m^2} + V(x)$, in units in which the speed of light $c = 1$. The corresponding Lagrangian is **[CSIR DEC 2013]**

(a) $L = m\sqrt{1 + \dot{x}^2} - V(x)$

(b) $L = -m\sqrt{1 - \dot{x}^2} - V(x)$

(c) $L = \sqrt{1 + m\dot{x}^2} - V(x)$

(d) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

6. A particle moves in a potential

$$V = x^2 + y^2 + \frac{z^2}{2}$$

. Which component (s) of the angular momentum is / are constant (s) of motion? **[CSIR DEC 2013]**

(a) none (b) $\mathrm{L_x}, \mathrm{L_y}$ and $\mathrm{L_z}$

(c) only $\mathrm{L_x}$ and $\mathrm{L_y}$ (d) only $\mathrm{L_z}$

7. A particle of mass m and coordinate q has the Lagrangian

$$L = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$$

where λ is a constant. The Hamiltonian for the system is given by **[CSIR JUNE 2014]**

(a) $\frac{\dot{p}^2}{2m} + \frac{\lambda q p^2}{2m^2}$ (b) $\frac{p^2}{2(m - \lambda q)}$

(c) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m - \lambda q)^2}$ (d) $\frac{pq}{2}$

8. If the Lagrangian of a dynamical system in two dimensions is

$$L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$$

, then its Hamiltonian is **[CSIR JUNE 2015]**

(a) $H=\frac{1}{m}p_xp_y+\frac{1}{2m}p_y^2$

(b) $H=\frac{1}{m}p_xp_y+\frac{1}{2m}p_x^2$

(c) $H=\frac{1}{m}p_xp_y-\frac{1}{2m}p_y^2$

(d) $H=\frac{1}{m}p_xp_y-\frac{1}{2m}p_x^2$

9. The Lagrangian of a system moving in three dimensions is

$$L=\frac{1}{2}m\dot{x}_1^2+m(\dot{x}_2^2+\dot{x}_3^2)-\frac{1}{2}kx_1^2-\frac{1}{2}k(x_2+x_3)^2$$

The independent constant(s) of motion is/are

[CSIR JUNE 2016]

(a) energy alone

(b) only energy, one component of the linear momentum and one component of the angular momentum

(c) only energy and one component of the linear momentum

(d) only energy and one component of the angular momentum

10. The Hamiltonian of a system with generalized coordinate and momentum (q,p) is $H=p^2q^2$. A solution of the Hamiltonian equation of motion is (in the following A and B are constants)

[CSIR JUNE 2016]

(a) $p=Be^{-2At},q=\frac{A}{B}e^{2At}$

(b) $p=Ae^{-2At},q=\frac{A}{B}e^{-2At}$

(c) $p=Ae^{At},q=\frac{A}{B}e^{-At}$

(d) $p=2Ae^{-A^2t},q=\frac{A}{B}e^{A^2t}$

11. A particle in two dimensions is in a potential $V(x,y)=x+2y$. Which of the following (apart from the total energy of the particle) is also a constant of motion?

[CSIR DEC 2016]

(a) p_y-2p_x (b) $\dot{p}_x-2p_y$

(c) p_x+2p_y (d) p_y+2p_x

12. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is

$$H=\alpha x^2p+\frac{p^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2x^2$$

where α,β and ω are constant. The corresponding Lagrangian is

[CSIR JUNE 2017]

(a) $\frac{1}{2}(\dot{x}-\alpha x^2)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$

(b) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2-\alpha x^2\dot{x}$

(c) $\frac{1}{2}(\dot{x}^2-\alpha^2x)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$

(d) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2+\alpha x^2\dot{x}$

13. A Hamiltonian system is described by the canonical coordinate q and canonical momentum p. A new coordinate Q is defined as $Q(t)=q(t+\tau)+p(t+\tau)$, where t is the time and τ is a constant, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted time. The new canonical momentum $P(t)$ can be expressed as

[CSIR JUNE 2017]

(a) $p(t+\tau)-q(t+\tau)$

(b) $p(t+\tau)-q(t-\tau)$

(c) $\frac{1}{2}[p(t-\tau)-q(t+\tau)]$

(d) $\frac{1}{2}[p(t+\tau)-q(t+\tau)]$

14. The Hamiltonian of a one-dimensional system is

$$H=\frac{xp^2}{2m}+\frac{1}{2}kx$$

, where m and k are positive constants. The corresponding Euler-Lagrange equation for the system is

[CSIR JUNE 2018]

(a) $m\ddot{x}+k=0$

(b) $m\ddot{x}+2\dot{x}+kx^2=0$

(c) $2mx\ddot{x}-m\dot{x}^2+kx^2=0$

(d) $mx\ddot{x}-2m\dot{x}^2+kx^2=0$

15. The Hamiltonian of a classical one-dimensional harmonic oscillator is

$$H=\frac{1}{2}(p^2+x^2)$$

, in suituble units. The total time derivative of the dynamical variable $(p+\sqrt{2}x)$ is

[CSIR DEC 2018]

(a) $\sqrt{2}p-x$ (b) $p-\sqrt{2}x$

(c) $p+\sqrt{2}x$ (d) $x+\sqrt{2}p$

16. The Hamiltonian of a system with two degrees of freedom is $H=q_1p_1-q_2p_2+aq_1^2$, where $a>0$ is a constant. The function $q_1q_2+\lambda p_1p_2$ is a constant of motion only if λ is

[CSIR DEC 2019]

(a) 0 (b) 1

(c) $-a$ (d) a

17. The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length l is

$$\mathrm{H}=\frac{\mathrm{p}_\theta^2}{2\,\mathrm{m}\ell^2}+\mathrm{mg}\ell(1-\cos\,\theta)$$

. If L denotes the Lagrangian, the value of $\frac{\mathrm{dL}}{\mathrm{dt}}$ is :

[NET]

(a) $-\frac{2g}{\ell}p_\theta\sin\,\theta$ (b) $-\frac{g}{\ell}p_\theta\sin\,2\theta$

(c) $\frac{g}{\ell}p_\theta\cos\,\theta$ (d) $\ell\mathrm{p}_\theta^2\cos\,\theta$

18. The Hamiltonian of a two particle system is $H=p_1p_2+q_1q_2$ where q_1 and q_2 are generalized coordinates and p_1 and p_2 are the respective canonical momenta. The Lagrangian of this system is

[CSIR JUNE 2023]

(a) $q_1q_2+q_1q_2$ (b) $-q_1q_2+q_1q_2$

(c) $-q_1q_2-q_1q_2$ (d) $q_1q_2-q_1q_2$

19. The 1-dimensional Hamiltonian of a classical particle of mass m is

$$H=\frac{p^2}{2m}e^{-x/a}+V(x),$$

where a is a constant with appropriate dimensions. The corresponding Lagrangian is,

[CSIR DEC 2023]

(a) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2e^{x/a}-V(x)$

(b) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2e^{-x/a}-V(x)$

(c) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2e^{x/a}-V(x)$

(d) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2e^{-x/a}-V(x)$

20. A Lagrangian is given by

$$L=\frac{1}{2}m(\dot{x}^2+\dot{y}\dot{z}+\dot{z}^2)-\alpha(2x+3y+z).$$

The conserved momentum is

[CSIR DEC 2023]

(a)$m[2\dot{x}+\dot{z}]$ (b)$m[2\dot{x}+\dot{y}+\dot{z}]$

(c)$m\left[\dot{x}+\frac{3}{2}\dot{y}+\frac{1}{2}\dot{z}\right]$ (d) $m[2\dot{x}+3\dot{z}]$

❖ **GATE PYQ's**

1. Hamilton canonical equations of motion for a conservative system are

[GATE 2002]

(a) $-\frac{dq_i}{dt}=\frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt}=\frac{\partial H}{\partial q_i}$

(b) $\frac{dp_i}{dt}=\frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt}=\frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt}=\frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt}=\frac{\partial H}{\partial q_i}$

(d) $\frac{dq_i}{dt}=\frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt}=\frac{\partial H}{\partial q_i}$

2. The Hamiltonian corresponding to the Lagrangian $L=a\dot{x}^2+b\dot{y}^2-kxy$ is

[GATE 2004]

(a) $\frac{p_x^2}{2a}+\frac{p_y^2}{2b}+kxy$ (b) $\frac{p_x^2}{4a}+\frac{p_y^2}{4b}-kxy$

(c) $\frac{p_x^2}{4a}+\frac{p_y^2}{4b}+kxy$ (d) $\frac{p_x^2+p_y^2}{4ab}+kxy$

3. The Hamiltonian of a particle is $H=\frac{p^2}{2m}+pq$, where q is the generalized coordinate and p is the corresponding canonical momentum. The Lagrangian is

[GATE 2007]

(a) $\frac{m}{2}\left(\frac{dq}{dt}+q\right)^2$

(b) $\frac{m}{2}\left(\frac{dq}{dt}-q\right)^2$

(c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2+q\frac{dq}{dt}-q^2\right]$

(d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2-q\frac{dq}{dt}+q^2\right]$

4. Three particles of mass m each situated at $x_1(t), x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length l. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is

[GATE 2007]

(a) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2+\frac{k}{2}(x_3-x_2-l)^2$

(b) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_3-l)^2+\frac{k}{2}(x_3-x_2-l)^2$

(c) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2+l)^2-\frac{k}{2}(x_3-x_2+l)^2$

(d) $L=\frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2+\left(\frac{dx_2}{dt}\right)^2+\left(\frac{dx_3}{dt}\right)^2\right]-\frac{k}{2}(x_1-x_2-l)^2-\frac{k}{2}(x_3-x_2-l)^2$

5. he Lagrangian of a particle of mass m is

$$L=\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]-\frac{V}{2}(x^2+y^2)+W\sin\,\omega t$$

, where V, W and ω are constants.

The conserved quantities are **[GATE 2007]**

(a) energy and z-component of linear momentum only

(b) energy and z-component of angular momentum only

(c) z-components of both linear and angular momenta only

(d) energy and z-components of both linear and angular momenta

6. The Lagrangian of a free particle in spherical polar coordinates is given by

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\phi^2\sin^2\,\theta\right)$$

. The quantity that is conserved is

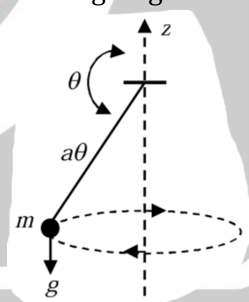
[GATE 2009]

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$

(c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}}+\dot{r}\dot{\theta}$

7. A particle of mass m is attached to a fixed point O by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure

The Lagrangian of the particle is



$$L(\theta,\phi)=\frac{1}{2}ma^2\left(\dot{\theta}^2+\sin^2\,\theta\dot{\phi}^2\right)-mga\cos\,\theta$$

Where θ and ϕ are the polar angles. The Hamiltonian of the particle is

[GATE 2012]

(a) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\,\theta}\right)-mga\cos\,\theta$

(b) $H=\frac{1}{2ma^2}\left(p_\theta^2+\frac{p_\phi^2}{\sin^2\,\theta}\right)+mga\cos\,\theta$

(c) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\phi^2\right)-mga\cos\,\theta$

(d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

8. The Lagrangian for a particle of mass m at a position r moving with a velocity $\vec{v}$ is given by

$$L = \frac{m}{2}\vec{v}^2 + C\vec{r}\cdot\vec{v} - V(r)$$

, where $V(r)$ is a potential and C is a constant . If $\vec{p}_c$ is the canonical momentum, then its Hamiltonian is given by

[GATE 2015]

(a) $\frac{1}{2m}(\vec{p}_c + C\vec{r})^2 + V(r)$

(b) $\frac{1}{2m}(\vec{p}_c - C\vec{r})^2 + V(r)$

(c) $\frac{p_c^2}{2m} + V(r)$

(d) $\frac{1}{2m}p_c^2 + C^2r^2 + V(r)$

9. The Lagrangian of a system is given by

$$L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$$

, where m, l and g are constants.
Which of the following is conserved?

[GATE 2016]

(a) $\dot{\varphi}\sin^2\theta$
(b) $\dot{\varphi}\sin\theta$
(c) $\frac{\dot{\varphi}}{\sin\theta}$
(d) $\frac{\dot{\varphi}}{\sin^2\theta}$

10. The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r}_1$ and $\vec{r}_2$ having velocities $\vec{v}_1$ and $\vec{v}_2$ is given by

$$H = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2}\hat{z}\cdot(\vec{r}_1\times\vec{r}_2)$$

, where C is a constant. Which one of the following statements is correct?

[GATE 2015]

(a) the total energy and total momentum are conserved

(b) only the total energy is conserved

(c) the total energy and the z-component of the total angular momentum are conserved

(d) the total energy and total angular momentum are conserved

11. The Hamiltonian for a particle of mass m is

$$H = \frac{p^2}{2m} + kqt$$

where q and p are the generalized coordinate and momentum, respectively, t is time and k is a constant. For the initial condition, $q = 0$ and $p = 0$ at $t = 0, q(t) \propto t^\alpha$. The value of α is

[GATE 2019]

12. Consider the Hamiltonian

$$H(q,p) = \frac{\alpha p^2q^4}{2} + \frac{\beta}{q^2}$$

, where α and β are parameters with appropriate dimensions, and q and p are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q,\dot{q})$ is

[GATE 2019]

(a) $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} - \frac{\beta}{q^2}$
(b) $\frac{2}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$
(c) $\frac{1}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$
(d) $-\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$

13. A small disc is suspended by a fiber such that it is free to rotate about the fiber axis (see figure). For small angular deflections, the Hamiltonian for the disc is given by

$$H = \frac{p_\theta^2}{2I} + \frac{1}{2}\alpha\theta^2$$

where I is the moment of inertia and α is the restoring torque per unit deflection. The disc is subjected to angular deflections (θ) due to thermal collisions from the surrounding gas at temperature T and p_θ is the momentum conjugate to θ. The average and the rootmean-square angular deflection, θ_{avg} and θ_{rms}, respectively are

[GATE 2020]

(a) $\theta_{\text{avg}} = 0$ and $\theta_{\text{rms}} = \left(\frac{k_BT}{\alpha}\right)^{3/2}$

(b) $\theta_{\text{avg}} = 0$ and $\theta_{\text{rms}} = \left(\frac{k_BT}{\alpha}\right)^{1/2}$

(c) $\theta_{\text{avg}} \neq 0$ and $\theta_{rms} = \left(\frac{k_BT}{\alpha}\right)^{1/2}$

(d) $\theta_{\text{avg}} \neq 0$ and $\theta_{rms} = \left(\frac{k_BT}{\alpha}\right)^{3/2}$

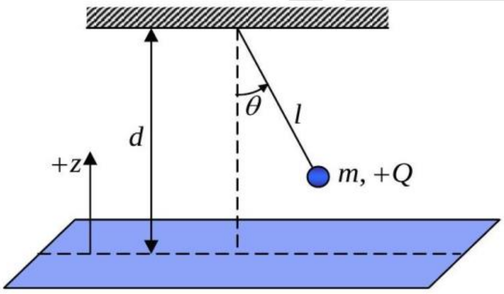
14. Consider the Lagrangian
$$L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$$
, where a, b and c are constants. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

[GATE 2020]

(a) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$ (b) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$

(c) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$ (d) $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

15. Consider a point charge $+Q$ of mass m suspended by a massless, inextensible string of length l in free space (permittivity ε_0) as shown in the figure. It is placed at a height $d(d > l)$ over an infinitely large, grounded conducting plane. The gravitational potential energy is assumed to be zero at the position of the conducting plane and is positive above the plane.



if θ represents the angular position and p_θ its corresponding canonical momentum, then the correct Hamiltonian of the system is

[GATE 2021]

(a) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

(b) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

(c) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

(d) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

16. The time derivative of a differentiable function $g(q_i, t)$ is added to a Lagrangian $L(q_i, \dot{q}_1, t)$ such that
$$L' = L(q_i, \dot{q}_i, t) + \frac{d}{dt} g(q_i, t)$$
where $q_i, \dot{q}_i, t$ are the generalized coordinates, generalized velocities and time, respectively. Let p_i be the generalized momentum and H the Hamiltonian associated with $L(q_i, \dot{q}_i, t)$. If p_i' and H' are those associated with L', then the correct option(s) is(are)

[GATE 2021]

(a) Both L and L' satisfy the Euler-Lagrange's equations of motion

(b) $p_i' = p_i + \frac{\partial}{\partial q_i} g(q_i, t)$

(c) If p_i is conserved, then p_i' is necessarily conserved

(d) $H' = H + \frac{d}{dt} g(q_i, t)$

17. A particle of mass m is free to move on a frictionless horizontal two dimensional (r, θ) plane, and is acted upon by a force $\vec{F} = -\frac{k}{2r^3}\hat{r}$ with k being a positive constant. If p_r and p_θ are the generalized momenta corresponding to r and θ respectively, then what is the value of $\frac{dp_r}{dt}$?

[GATE 2023]

(a) $\frac{p_\theta^2 - 2mk}{2mr^3}$ (b) $\frac{2p_\theta^2 - mk}{mr^3}$

(c) $\frac{p_\theta^2 - 2mk}{mr^3}$ (d) $\frac{2p_\theta^2 - mk}{2mr^3}$

18. Consider two non-identical spin $\frac{1}{2}$ particles labelled 1 and 2 in the spin product state
$$\left|\frac{1}{2}, \frac{1}{2}\right\rangle \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$
. The Hamiltonian of the system is
$$H = \frac{4\lambda}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2$$
where $\vec{S}_1$ and $\vec{S}_2$ are the spin operators of particles 1 and 2 , respectively, and λ is a constant with appropriate dimensions. What is the expectation value of H in the above state?

[GATE 2023]

(a) $-\lambda$ (b) -2λ

(c) λ (d) 2λ

19. The non-relativistic Hamiltonian for a single electron atom is

$$H_0 = \frac{p^2}{2m} - V(r)$$

where $V(r)$ is the Coulomb potential and m is the mass of the electron. Considering the spinorbit interaction term

$$H' = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\vec{L}\cdot\vec{S}$$

added to H_0, which of the following statement is/are true?

[GATE 2024]

(a) H' commutes with L^2

(b) H' commutes with L_z and S_z

(c) For a given value of principal quantum number n and orbital angular momentum quantum number l, there are $2(2l+1)$ degenerate eigenstates of H_0

(d) H_0, L^2, S^2, L_z and S_z have a set of simultaneous eigenstates

❖ JEST PYQ's

1. Consider a Hamiltonian system with a potential energy function given by $V(x) = x^2 - x^4$. which of the following is correct?

[JEST 2014]

(a) The system has one stable point

(b) The system has two stable points

(c) The system has three stable points

(d) The system has four stable points

2. $(Q_1, Q_2, \mathrm{P}_1, \mathrm{P}_2)$ And $(q_1, \mathrm{q}_2, \mathrm{P}_1, \mathrm{P}_2)$ are two sets of canonical coordinates, where Q_i and q_i are the coordinates and P_i and p_i are the corresponding conjugate momenta. If $P_1 = q_2$ and $P_2 = p_1$, then which of the following relations is true?

[JEST 2017]

(a) $Q_1 = q_1, Q_2 = P_2$ (b) $Q = p_2, Q_2 = q_1$

(c) $Q_1 = -p_2, Q_2 = q_1$ (d) $Q_1 = q_1, Q_2 = -P_2$

3. The Hamiltonian for a particle of mass m is given by

$$H = \frac{(p - \alpha q)^2}{(2m)}$$

, where α is a non-zero constant. Which one of the following equations is correct?

[JEST 2020]

(a) $p = m\dot{q}$ (b) $\alpha\dot{p} = \dot{q}$

(c) $\ddot{q} = 0$ (d) $L = \frac{1}{2}m\dot{q}^2 - \alpha q\dot{q}$

4. For a system of unit mass, the dynamical variables follow the relation $\dot{x}^2 = kx_0^2 + \dot{x}_0^2 - kx^2$ where, x is the position of the system at time t, and x_0 is its initial position. What is the force acting on the system?

[JEST 2022]

(a) $\frac{1}{2}k(x - x_0)^2$ (b) $-k(x - x_0)$

(c) $-\frac{1}{2}k(x - x_0)$ (d) $-kx$

5. The Lagrangian of a particle of unit mass is given by

$$L = \frac{1}{2}(\dot{x}^2 - x^2 + 2x\dot{x})$$

. The Hamiltonian of this system is given by

[JEST 2022]

(a) $\frac{1}{2}(p^2 + x^2)$ (b) $\frac{1}{2}p^2 - px + x^2$

(c) $\frac{1}{2}(p - x)^2$ (d) $\frac{1}{2}p^2 + px - x^2$

6. The Lagrangian of a particle of unit mass is given by

$$L = \frac{1}{2}(\dot{x}^2 - x^2 + 2x\dot{x})$$

. The Hamiltonian of this system is given by

[JEST 2022]

(a) $\frac{1}{2}(p^2 + x^2)$ (b) $\frac{1}{2}p^2 - px + x^2$

(c) $\frac{1}{2}(p - x)^2$ (d) $\frac{1}{2}p^2 + px - x^2$

❖ TIFR PYQ's

1. The dynamics of a particle of mass m is described in terms of three generalized coordinates ξ, η and φ. If the Lagrangian of the system is

$$L=\frac{1}{8}m\left[(\xi+\eta)\left(\frac{\dot{\xi}^2}{\xi^2}+\frac{\dot{\eta}^2}{\eta^2}\right)+4\xi\eta\dot{\varphi}^2\right]+\frac{1}{8}k(\xi+\eta)^2$$

where k is a constant, then a conserved quantity in the system will be

[TIFR 2011]

(a) $(m+k)(\dot{\xi}+\dot{\eta})$

(b) $m\xi\eta\dot{\varphi}$

(c) $m\left(\dot{\xi}^2/\eta^2+\dot{\eta}^2/\xi^2\right)$

(d) $m(\xi+\eta)\left(\dot{\xi}/\xi^2+\dot{\eta}/\eta^2\right)$

2. A dynamical system with two degrees of freedom, has generalized coordinates q_1 and q_2, and kinetic energy

$$T=\lambda\dot{q}_1\dot{q}_2$$

If the potential energy is $V(q_1,q_2)=0$, the correct form of the Hamiltonian for this system is **[TIFR 2012]**

(a) p_1p_2/λ

(b) $\lambda\dot{q}_1\dot{q}_2$

(c) $(p_1\dot{q}_1+p_2\dot{q}_2)/2$

(d) $(p_1q_2+p_2q_1)/2$

3. If a central force acting on a particle of mass m is given by $F(r)=-\frac{k}{r^2}$ where r is the distance of the particle from the origin and k is a positive constant, the Hamiltonian for the system, in spherical polar coordinates, will have the form

[TIFR 2013]

(a) $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\varphi}^2\right)+\frac{k}{r}$

(b) $\frac{p_r^2}{2m}+\frac{p_\theta^2}{2mr^2}+\frac{p_\varphi^2\csc^2\theta}{2mr^2}+\frac{k}{r}$

(c) $\frac{1}{2mr^2}\left(r^2p_r^2+p_\theta^2+p_\varphi^2+2mkr\right)$

(d) $\frac{p_r^2}{2m}+\frac{p_\theta^2}{2m}+\frac{p_\varphi^2}{2m}+\frac{k}{r}$

4. The Conservation Principles for energy, linear momentum and angular momentum arise from the necessity that

[TIFR 2014]

(a) the laws of physics should not involve infinite quantities.

(b) internal forces on a body should cancel out, by Newton's (third) law of action and reaction.

(c) physical measurements should be independent of the origin and orientation of the coordinate system.

(d) the laws of physics should be independent of the state of rest or motion of the observer.

5. The Hamiltonian of a dynamical system is equal to its total energy, provided that its Lagrangian

[TIFR 2018]

(a) does not contain velocity-dependent terms.

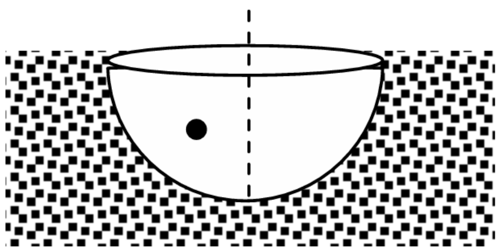
(b) is separable in generalized coordinates and velocities.

(c) does not have terms which explicitly depend on the coordinates.

(d) has no explicit time dependence.

6. A bead of mass m slides under the influence of gravity along the frictionless interior of a hemispherical cup of radius a sunk vertically into the ground with its open side level with the ground (see sketch on the right). In terms of spherical polar coordinates (θ,φ) set up with the centre of the upper circle as origin, the Hamiltonian H for this system will be

[TIFR 2019]



(a) $H=\frac{m}{2}\left(a^2\dot{\theta}^2+a^2\dot{\varphi}^2\csc^2\theta\right)+2mga\sin^2\frac{\theta}{2}$

(b) $H=\frac{m}{2}\left(a^2\dot{\theta}^2+a^2\dot{\varphi}^2\sin^2\theta\right)+2mga\sin^2\frac{\theta}{2}$

(c)$H=\frac{1}{2ma^2}\left(p_\theta^2+p_\varphi^2\sin^2\,\theta\right)+2mga\sin^2\,\frac{\theta}{2}$

(d) $H=\frac{1}{2ma^2}\left(p_\theta^2+p_\varphi^2\csc^2\,\theta\right)+2mga\sin^2\,\frac{\theta}{2}$

7. A system is composed of a large number of non-interacting classical particles moving in two dimensions, which individually obey the Hamiltonian

$$\frac{p_x^2+p_y^2}{2m}+\frac{1}{2}m\omega^2(x^2+y^2)$$

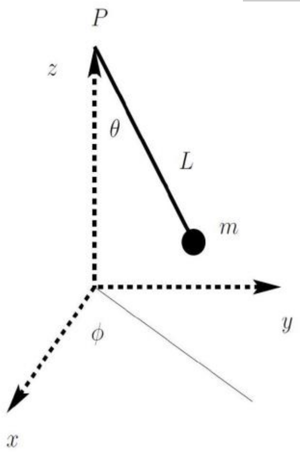
and the system is connected to a heat bath at a temperature T.
The probability of finding a particle within a radius R from the origin is given by

[TIFR 2020]

(a)$1-\exp\left(-\frac{m\omega^2R^2}{2T}\right)$ (b)$\exp\left(-\frac{m\omega^2R^2}{2T}\right)$

(c)$\mathrm{erp}\left(\sqrt{\frac{m}{2T}}\omega R\right)$ (d)$1-\frac{m\omega^2R^2}{2T}$

8. A particle of mass m is attached to a massless string of length L. The other end of the string is fixed at a point P as shown in the figure. m moves under gravity and the tension of the string. The motion of the string is described using the generalized coordinates θ and ϕ which change with time. θ is the polar angle made by the string with the vertical and ϕ is the azimuthal angle made by the projection of the string on the xy plane. The conjugate momenta to the variables (θ,ϕ) are (p_θ,p_ϕ), respectively.



Assuming that the string is tight throughout the motion, the Hamiltonian for the system is given by: **[TIFR 2024]**

(a) $\frac{1}{2m}\left[p_\theta^2+p_\phi^2\mathrm{cosec}^2\;\theta\right]-2mgL\sin^2\left(\frac{\theta}{2}\right)$

(b) $\frac{1}{2m}\left[p_\theta^2+p_\phi^2\mathrm{cosec}^2\;\theta\right]+2mgL\sin^2\left(\frac{\theta}{2}\right)$

(c) $\frac{1}{2m}\left[p_\theta^2+p_\phi^2\sin^2\;\theta\right]+2mgL\sin^2\left(\frac{\theta}{2}\right)$

(d) $\frac{1}{2m}\left[p_\theta^2+p_\phi^2\sin^2\;\theta\right]-2mgL\sin^2\left(\frac{\theta}{2}\right)$

Answer Key				
CSIR-NET PYQ				
1. d	2. a	3. a	4. d	5. b
6. d	7. b	8. c	9. b	10. a
11. a	12. a	13. d	14. c	15. a
16. a	17.	18. d	19. a	20. b
GATE PYQ				
1. d	2. c	3. b	4. a	5. c
6. c	7. b	8. b	9. a	10. b
11. 3	12. a	13. b	14. a	15. d
16. a,b	17. d	18. a	19. acd	
JEST PYQ				
1. a	2. c	3. c	4. d	5. b
6. b				
TIFR PYQ				
1. b	2. a	3. b	4. c	5. d
6. d	7. a	8. b		

❖ CSIR-NET PYQ's

1. The Poisson bracket $\{|r|, |p|\}$ has the value

[CSIR JUNE 2012]

(a) $|r||p|$ (b) $\hat{r}, \hat{p}$

(c) 3 (d) 1

2. A system is governed by the Hamiltonian $H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$ where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

[CSIR JUNE 2013]

(a) $a = -3, b = 2$ (b) $a = 3, b = -2$

(c) $a = 2, b = -3$ (d) $a = -2, b = 3$

3. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,}representsthePoissonbracket, thevalueof$\{A,\{B,C\}\} - \{\{A,B\},C\}$ is given by

[CSIR DEC 2013]

(a) 0 (b) $\{B,\{C,A\}\}$

(c) $\{A,\{C,B\}\}$ (d) $\{\{C,A\},B\}$

4. The coordinates and momenta $x_i, p_i (i = 1,2,3)$ of a particle satisfy the canonical Poisson bracket relations $\{x_i, p_j\} = \delta_{ij}$. If $C_1 = x_2p_3 + x_3p_2$ and $C_2 = x_1p_2 - x_2p_1$ are constants of motion, and if $C_3 = \{C_1, C_2\} = x_1p_3 + x_3p_1$, then

[CSIR JUNE 2014]

(a) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = C_2$

(b) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = -C_2$

(c) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$

(d) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$

5. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresponding time-independent generating function is

[CSIR JUNE 2014]

(a) $\frac{P}{q^2}$ (b) q^2P

(c) q^2/P (d) qP^2

6. The expression $\sum_{i,j,k-1}^{3} \epsilon_{jk} \{x_i, \{p_j, L_k\}\}$ (where ϵ_{jk} is the Levi-Civita symbol, $\vec{x}, \vec{p}, \vec{L}$ are the position, momentum and angular momentum respectively, and $\{A, B\}$ represents the Poisson bracket of A and B) simplifies to

[CSIR DEC 2014]

(a) 0 (b) 6

(c) $\vec{x} \cdot (\vec{p} \times \vec{L})$ (d) $\vec{x} \times \vec{p}$

7. A mechanical system is described by the Hamiltonian

$$H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$$

As a result of the canonical transformation generated by $F(q,Q) = -\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes

[CSIR DEC 2014]

(a) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}Q^2$

(b) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}P^2$

(c) $\frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$

(d) $\frac{1}{2m}Q^2P^4 + \frac{m\omega^2}{2}P^{-2}$

8. Let q and p be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical?

[CSIR JUNE 2015]

$A: Q_1 = \frac{1}{\sqrt{2}}q^2$ and $P_1 = \frac{1}{\sqrt{2}}p^2$

$B: Q_2 = \frac{1}{\sqrt{2}}(p+q)$ and $P_2 = \frac{1}{\sqrt{2}}(p-q)$

(a) neither A nor B (b) both A and B

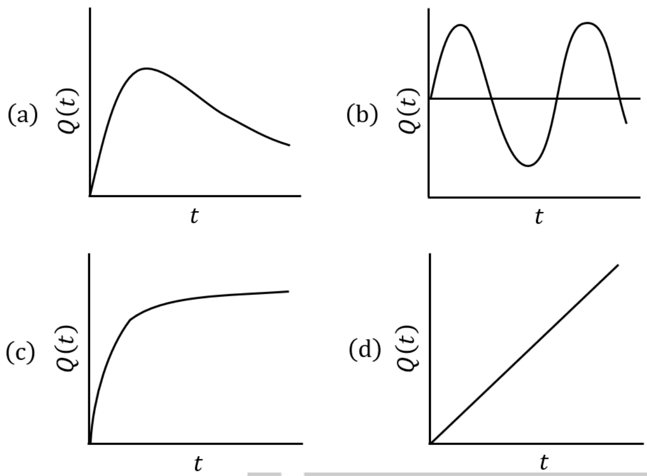
(c) only A (d) only B

9. A canonical transformation $(p,q) \to (P,Q)$ is performed on the Hamiltonian

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 q^2$$

via the generating function $F = \frac{1}{2} m\omega q^2 \cot\, Q$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on t ?

[CSIR DEC 2015]



10. A canonical transformation $(q,p) \to (Q,P)$ is made through the generating function $F(q,P) = q^2 P$ on the Hamiltonian

$$H(q,p) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4} q^4$$

, where α and β are constants. The equations of motion for (Q,P) are

[CSIR JUNE 2016]

(a) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\beta Q$

(b) $\dot{Q} = \frac{4P}{\alpha}$ and $P = \frac{-\beta Q}{2}$

(c) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\frac{2P^2}{Q} - \beta Q$

(d) $\dot{Q} = \frac{2P}{\alpha}$ and $\dot{P} = -\beta Q$

11. A particle in two dimensions is in a potential $V(x,y) = x + 2y$. Which of the following (apart from the total energy of the particle) is also a constant of motion?

[CSIR DEC 2016]

(a) $p_y - 2p_x$ (b) $\dot{p}_x - 2p_y$

(c) $p_x + 2p_y$ (d) $p_y + 2p_x$

12. A Hamiltonian system is described by the canonical coordinate q and canonical momentum p. A new coordinate Q is defined as $Q(t) = q(t+\tau) + p(t+\tau)$, where t is the time and τ is a constant, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted time. The new canonical momentum $P(t)$ can be expressed as

[CSIR JUNE 2017]

(a) $p(t+\tau) - q(t+\tau)$

(b) $p(t+\tau) - q(t-\tau)$

(c) $\frac{1}{2}[p(t-\tau) - q(t+\tau)]$

(d) $\frac{1}{2}[p(t+\tau) - q(t+\tau)]$

13. Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator

$$\left[\frac{1}{2m}p^2 + \beta x^2, \frac{1}{m}p^2 + \gamma x^2\right],$$

where β and γ are constants, is zero if

[CSIR DEC 2017]

(a) $\gamma = \beta$ (b) $\gamma = 2\beta$

(c) $\gamma = \sqrt{2\beta}$ (d) $2\gamma = \beta$

14. Let (x,p) be the generalized coordinate and momentum of a Hamiltonian system. If new variables (X,P) are defined by $X = x^\alpha \sinh(\beta p)$ and $P = x^\gamma \cosh(\beta p)$, where α, β and γ are constants, then the conditions for it to be a canonical transformation, are

[CSIR DEC 2017]

(a) $\alpha = \frac{1}{2\beta}(\beta+1)$ and $\gamma = \frac{1}{2\beta}(\beta-1)$

(b) $\beta = \frac{1}{2\gamma}(\alpha+1)$ and $\gamma = \frac{1}{2\alpha}(\alpha-1)$

(c) $\alpha = \frac{1}{2\beta}(\beta-1)$ and $\gamma = \frac{1}{2\beta}(\beta+1)$

(d) $\beta = \frac{1}{2\gamma}(\alpha-1)$ and $\gamma = \frac{1}{2\alpha}(\alpha+1)$

15. The Hamiltonian of a classical one-dimensional harmonic oscillator is

$$H = \frac{1}{2}(p^2 + x^2)$$

, in suitube units. The total time derivative of the dynamical variable $(p+\sqrt{2}x)$ is

[CSIR DEC 2018]

(a) $\sqrt{2}p - x$ (b) $p - \sqrt{2}x$

(c) $p + \sqrt{2}x$ (d) $x + \sqrt{2}p$

16. The generator of the infinitesimal canonical transformation $q \to q' = (1+\varepsilon)q$ and $p \to p' = (1-\varepsilon)p$ is

[CSIR DEC 2019]

(a) $q+p$ (b) qp

(c) $\frac{1}{2}(q^2 - p^2)$ (d) $\frac{1}{2}(q^2+p^2)$

17. The Hamiltonian of a system with two degrees of freedom is $H = q_1p_1 - q_2p_2 + aq_1^2$, where $a > 0$ is a constant. The function $q_1q_2 + \lambda p_1p_2$ is a constant of motion only if λ is

[CSIR DEC 2019]

(a) 0 (b) 1

(c) $-a$ (d) a

18. A canonical transformation from the phase space coordinates (q,p) to (Q,P) is generated by the function

$$\psi(p,Q) = \frac{p^2}{2\omega}\tan\, 2\pi Q.$$

where ω is a positive constant. The function $\psi(p,Q)$ is related to $F(q,Q)$ by the Legendre transform $\psi = pq - F$, where F is defined by $dF = pdq - PdQ$. If the solution for (P,Q) is

$$P(t) = \frac{\omega}{4\pi}t^2, Q(t) = Q_0 = \text{ constant},$$

where t is time, then the solution for (p,q) variables can be written as **[CSIR DEC 2023]**

(a) $p = \frac{\omega t}{2\pi}\cos\, 2\pi Q_0, q = \frac{t}{2\pi}\sin\, 2\pi Q_0$

(b) $p = -\frac{\omega t}{2\pi}\cos\, 2\pi Q_0, q = \frac{t}{2\pi}\sin\, 2\pi Q_0$

(c) $p = \frac{\omega t}{2\pi}\sin\, 2\pi Q_0, q = \frac{t}{2\pi}\cos\, 2\pi Q_0$

(d) $p = -\frac{\omega t}{2\pi}\sin\, 2\pi Q_0, q = \frac{t}{2\pi}\cos\, 2\pi Q_0$

19. For the transformation $x \to X = \frac{\alpha p}{x}, p \to P = \beta x^2$ between conjugate pairs of a coordinate and its momentum, to be canonical, the constants α and β must satisfy

[CSIR JUNE 2023]

(a) $1 + \frac{1}{2}\alpha\beta = 0$ (b) $1 - \frac{1}{2}\alpha\beta =$

(c) $1 + 2\alpha\beta = 0$ (d) $1 - 2\alpha\beta = 0$

20. For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

If $H(q,p)$ is the Hamiltonian of the system and $A(p,q) = \frac{1}{\sqrt{2}}(p+iq)$, the Poisson bracket $\{A,H\}$ is **[CSIR JUNE 2024]**

(a) $i\,A$ (b) A^*

(c) $-iA^*$ (d) $-i\,A$

❖ GATE PYQ's

1. If Poisson bracket $[q, f(p)] = \alpha f(p)$, where α is a scalar then $f(p)$ is equal to

[GATE 2001]

(a) $e^{\alpha p}$ (b) $e^{-\alpha p}$

(c) $\alpha e^{-\alpha p}$ (d) αe^{-p}

2. The value of the Poisson bracket $[\vec{a}\cdot\vec{r}, \vec{b}\cdot\vec{p}]$, where $\vec{a}$ and $\vec{b}$ are constant vectors, is

[GATE 2004]

(a) $\vec{a}\vec{b}$ (b) $\vec{a} - \vec{b}$

(c) $\vec{a} + \vec{b}$ (d) $\vec{a}\cdot\vec{b}$

3. For the given transformations (i) $Q = p, P = -q$ and (ii) $Q = p, P = q$, where p and q are canonically conjugate variables, which one of the following statement is true?

[GATE 2006]

(a) Both (i) and (ii) are canonical

(b) Only (i) is canonical

(c) Only (ii) is canonical

(d) Neither (i) nor (ii) is canonical

4. For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

. If

$$A(p,q) = \frac{p + iq}{\sqrt{2}}$$

and $H(p,q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p,q)H(p,q)\}$ is given by

[GATE 2008]

(a) $iA(p,q)$ (b) $A^*(p,q)$

(c) $-iA^*(p,q)$ (d) $-iA(p,q)$

5. If p and q are the position and momentum variables, which one of the following is not a canonical transformation?

[GATE 2009]

(a) $Q = \alpha q$ and $P = \frac{1}{\alpha}p$, for $\alpha \neq 0$

(b) $Q = \alpha q + \beta p$ and $P = \beta p + \alpha p$ for α, β real and $\alpha^2 - \beta^2 = 1$

(c) $Q = p$ and $P = q$

(d) $Q = p$ and $P = -q$

Statement for Linked Answer Questions 6 and 7 :

The Lagrangian for a simple pendulum is given by :

$$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos\ \theta)$$

6. Hamilton's equations are then given by

[GATE 2010]

(a)$\dot{p}_\theta = -mgl\sin\ \theta;\ \dot{\theta} = \frac{p_\theta}{ml^2}$

(b)$\dot{p}_\theta = mgl\sin\ \theta;\ \dot{\theta} = \frac{p_\theta}{ml^2}$

(c)$\dot{p}_\theta = -m\ddot{\theta};\ \dot{\theta} = \frac{p_\theta}{m}$

(d)$\dot{p}_\theta = \left(\frac{g}{l}\right)0; \dot{\theta} = \frac{p_0}{ml}$

7. The Poisson bracket between θ and $\dot{\theta}$ is

[GATE 2010]

(a)$\{\theta, \dot{\theta}\} = 1$ (b) $\{\theta, \dot{\theta}\} = \frac{1}{ml^2}$

(c)$\{\theta, \dot{\theta}\} = \frac{1}{m}$ (d) $\{\theta, \dot{\theta}\} = \frac{g}{l}$

8. Let (p,q) and (P,Q) be two pairs of canonical variables. The transformation

$$Q = q^\alpha \cos\ (\beta p)$$
$$P = q^\alpha \sin\ (\beta p)$$

Is canonical for

[GATE 2011]

(a) $\alpha = 2, \beta = 1/2$ (b) $\alpha = 2, \beta = 2$

(c) $\alpha = 1, \beta = 1$ (d) $\alpha = 1/2, \beta = 2$

9. Given that the linear transformation of generalized coordinate q and the corresponding momentum p,

$$Q = q + 4ap$$
$$P = q + 2p$$

Is canonical, the value of the constant a is______________. **[GATE 2014]**

10. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

[GATE 2014]

(a) $\dot{q} = \{q, H\}; \dot{p} = \{p, H\}$

(b) $\dot{q} = \{H, q\}; \dot{p} = \{H, p\}$

(c) $\dot{q} = \{H, p\}; \dot{p} = \{H, q\}$

(d) $\dot{q} = \{p, H\}; \dot{p} = \{q, H\}$

11. The Poisson bracket $[x, xp_y + yp_x]$ is equal to

[GATE 2017]

(a) $-x$ (b) y

(c) $2p_x$ (d) p_y

12. For the transformation

$Q = \sqrt{2q}e^{-1+2\alpha}\cos\ p, P = \sqrt{2q}e^{-\alpha-1}\sin\ p$

(where α is a constant) to be canonical, the value of α is **[GATE 2018]**

13. Consider a transformation from one set of generalized coordinate and momentum (q,p) to another set (Q,P) denoted by,

$$Q = pq^s;\ P = q^r$$

where s and r are constants. The transformation is canonical if

[GATE 2019]

(a) $s = 0$ and $r = 1$

(b) $s = 2$ and $r = -1$

(c) $s = 0$ and $r = -1$

(d) $s = 2$ and $r = 1$

14. Let p be the momentum conjugate to the generalized coordinate q. If the transformation

$$Q = \sqrt{2}q^m \cos\, p$$
$$P = \sqrt{2}q^m \sin\, p$$

is canonical, then $m =$ **[GATE 2020]**

15. If $\vec{a}$ and $\vec{b}$ are constant vectors, $\vec{r}$ and $\vec{p}$ are generalized positions and conjugate momenta, respectively, then for the transformation $Q = \vec{a}\cdot\vec{p}$ and $P = \vec{b}\cdot\vec{r}$ to be canonical, the value of $\vec{a}\cdot\vec{b}$ (in integer) is

[GATE 2021]

16. If $(\dot{x}\dot{y} + axy)$ is a constant of motion of a two-dimensional isotropic harmonic oscillator with Lagrangian

$$L = \frac{m(\dot{x}^2 + \dot{y}^2)}{2} - \frac{k(x^2 + y^2)}{2}$$

then a is **[GATE 2022]**

(a) $+\frac{k}{m}$

(b) $-\frac{k}{m}$

(c) $-\frac{2k}{m}$

(d) 0

17. A system with time independent Hamiltonian $H(q,p)$ has two constants of motion $f(q,p)$ and $g(q,p)$. Then which of the following Poisson brackets are always zero?

[GATE 2022]

(a) $\{H, f + g\}$

(b) $\{H, \{f, g\}\}$

(c) $\{H + f, g\}$

(d) $\{H, H + fg\}$

18. If $F_1(Q,q) = Qq$ is the generating function of a canonical transformation from (p,q) to (P,Q), then which one of the following relations is correct?

[GATE 2024]

(a) $\frac{p}{P} = \frac{Q}{q}$

(b) $\frac{P}{p} = \frac{Q}{q}$

(c) $\frac{p}{P} = -\frac{Q}{q}$

(d) $\frac{P}{p} = -\frac{Q}{q}$

19. Let $\rho(\vec{p},\vec{q},t)$ be the phase space density of an ensemble of a system. The Hamiltonian of the system is $H(\vec{p},\vec{q})$. If $\{A,B\}$ denotes the Poisson bracket of A and B, then $\frac{d\rho}{dt} = 0$ implies

[GATE 2024]

(a) $\frac{\partial\rho}{\partial t} = 0$

(b) $\frac{\partial\rho}{\partial t} \propto \{\rho, H\}$

(c) $\frac{\partial\rho}{\partial t} \propto \left\{\rho, \frac{\vec{p}\cdot\vec{q}}{2}\right\}$

(d) $\frac{\partial\rho}{\partial t} \propto \left\{\rho, \frac{\vec{q}\cdot\vec{q}}{2}\right\}$

❖ JEST PYQ's

1. If the coordinate q and the momentum p from a canonical pair (q,p), which one of the sets given below also forms a canonical?

[JEST 2012]

(a) $(q, -p)$

(b) (q^2, p^2)

(c) $(p, -q)$

(d) $(q^2, -p^2)$

2. If the Poisson bracket $\{x, p\} = -1$, then the Poisson bracket $\{x^2 + p, p\}$ is

[JEST 2013]

(a) $-2x$

(b) $2x$

(c) 1

(d) -1

3. (Q_1, Q_2, P_1, P_2) And (q_1, q_2, P_1, P_2) are two sets of canonical coordinates, where Q_i and q_i are the coordinates and P_i and p_i are the corresponding conjugate momenta. If $P_1 = q_2$ and $P_2 = p_1$, then which of the following relations is true?

[JEST 2017]

(a) $Q_1 = q_1, Q_2 = P_2$

(b) $Q = p_2, Q_2 = q_1$

(c) $Q_1 = -p_2, Q_2 = q_1$

(d) $Q_1 = q_1, Q_2 = -P_2$

4. (Q_1, Q_2, P_1, P_2) And (q_1, q_2, P_1, P_2) are two sets of canonical coordinates, where Q_i and q_i are the coordinates and P_i and p_i are the corresponding conjugate momenta. If $P_1 = q_2$ and $P_2 = p_1$, then which of the following relations is true?

[JEST 2017]

(a) $Q_1 = q_1, Q_2 = P_2$

(b) $Q = p_2, Q_2 = q_1$

(c) $Q_1 = -p_2, Q_2 = q_1$ (d) $Q_1 = q_1, Q_2 = -P_2$

5. If (q, p) is a canonically conjugate pair, which of the following is not a canonically conjugate pair?

[JEST 2018]

(a) $(q^2, pq^{-1}/2)$

(b) $(q^2, -qp^{-1}/2)$

(c) $(pq^{-1} - q^2)$

(d) $(f(p), -q/f'(p))$, where $f'(p)$ is the derivative of $f(p)$ with respect to p.

6. The coordinate q and the momentum p of a particle satisfy

$$\frac{dq}{dt} = p, \frac{dp}{dt} = -3q - 4p$$

If $A(t)$ is the area of any region of points moving in the (q, p)-space, then the ratio $A(t)/A(0)$ is

[JEST 2018]

(a) 1 (b) exp $(-3t)$

(c) exp $(-4t)$ (d) exp $(-3t/4)$

7. Consider the following transformation of the phase space coordinates $(q, p) \to (Q, P)$

$$Q = q^a \cos\, bpP = q^a \sin\, bp$$

For what values of a and b will the transformation be canonical?

[JEST 2019]

(a) 1,1 (b) $\frac{1}{2}, \frac{1}{2}$

(c) $2, \frac{1}{2}$ (d) $\frac{1}{2}, 2$

8. The Hamiltonian of a classical particle is given by

$$H(p, q) = \frac{p^2}{2m} + \frac{kq^2}{2}$$

Given $F(p, q, t) = \ln(p + im\omega q) - i\alpha\omega t$ is a constant of motion (where $\omega = \sqrt{\frac{k}{m}}$). What is the value of α ?

[JEST 2020]

(a) 2π (b) 0

(c) 1 (d) π

9. A particle of mass m is subject to the potential $V(x, y, t) = K(x^2 + y^2)$, where (x, y) are the Cartesian coordinates of the particle and K is a constant. Which one of the following quantities is a constant of motion?

[JEST 2021]

(a) $\dot{y}x + \dot{x}y$ (b) $\dot{y}x - \dot{x}y$

(c) $\dot{y} + \dot{x}$ (d) $\dot{y}y + \dot{x}x$

10. Let q and p be the canonical phase space coordinates of a system, where q is the generalized coordinate and p is the generalized momentum. Let us make a transformation of the generalized coordinate as $Q = q^2$. Which of the following functions is canonically conjugate to Q ?

[JEST 2024]

(a) p^2 (b) $\frac{p}{q}$

(c) $\frac{p}{2q}$ (d) $\frac{p^2}{2q^2}$

❖ TIFR PYQ's

1. In a system with two degrees of freedom, if (p, q) are the canonical coordinates, then which of the following transformations to (P, Q) is canonical?

[TIFR 2015]

(a) $P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}\frac{2q}{p}$

(b) $P = \frac{1}{2}(p^2 + q^2), Q = \cot^{-1}\frac{p}{q}$

(c) $P = \frac{1}{2}(p^2 + q^2), Q = \sin^{-1}\frac{q}{2p}$

(d) $P = \frac{1}{2}(p^2 + q^2), Q = \cos^{-1}\frac{p}{q}$

2. Consider the Hamiltonian for a one-dimensional classical oscillator

$$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$$

A canonical transformation to variables (P, Q) via the generating function

$$F = \frac{m\omega q^2}{2}\cot\, Q$$

leads to which of the following Hamiltonians in the new coordinates? **[TIFR 2023]**

(a) $H = 2\omega P$ (b) $H = P^2 + \omega^2 Q^2$

(c) $H = \omega P$ (d) $H = 2\omega Q$

3. The Lagrangian of a system described by generalized coordinates q_1 and q_2 is given by

$$L = \frac{a}{2}(\dot{q}_1^2 + \dot{q}_2^2) - \frac{b^2}{\pi}(q_1^2 + q_2^2)$$

where a and b are constants. It follows that a conserved quantity in this system is

[TIFR 2020]

(a) $q_1\dot{q}_2 - q_2\dot{q}_1$ (b) $q_1\dot{q}_2 + q_2\dot{q}_1$

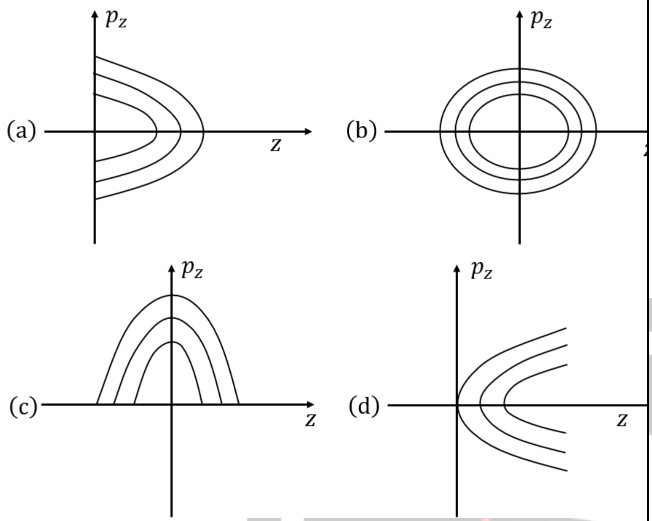
(c) $\dfrac{q_1\dot{q}_2 - q_2\dot{q}_1}{q_1^2 + q_2^2}$ (d) $2\pi(q_1^2\dot{q}_2 + q_2^2\dot{q}_1)$

Answers key				
CSIR-NET				
1. b	2. d	3. d	4. d	5. b
6. b	7. d	8. d	9. d	10. b
11. a	12. d	13. b	14. c	15. a
16. b	17. a	18. a	19. c	20. a
GATE				
1. b	2. d	3. b	4. a	5. c
6. a	7. b	8. d	9. 2.25	10. a
11. b	12. 2	13. b	14. 0.5	15. 1
16. a	17. abd	18. c	19. b	
JEST				
1. c	2. a	3. c	4. c	5. c
6. c	7. d	8. c	9. b	10. c
TIFR				
1. b	2. c	3. a		

❖ **CSIR-NET PYQ's**

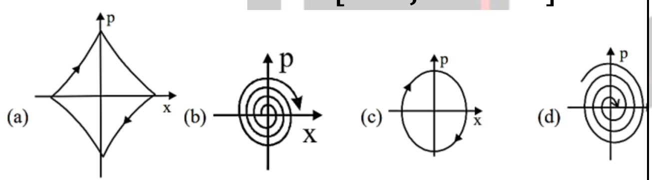
1. The trajectory on the zp_z-plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at $z = 0$ is given by approximately by (neglect friction):

[CSIR DEC 2011]



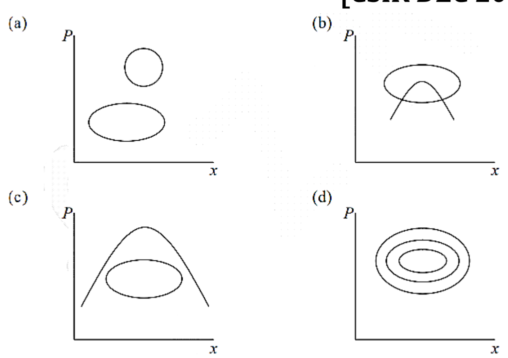
2. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?

[CSIR JUNE 2012]



3. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)?

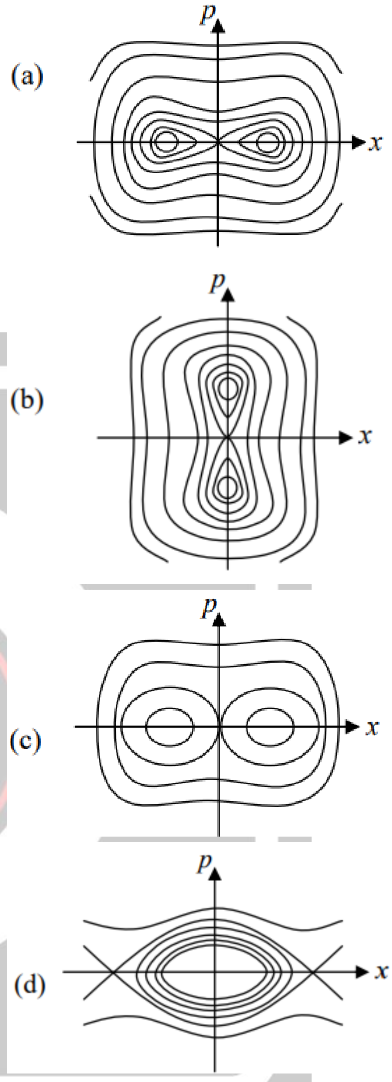
[CSIR DEC 2012]



4. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4?$$

[CSIR JUNE 2015]



5. The Lagrangian of a free relativistic particle (in one-dimension) of mass m is given by $L = -m\sqrt{1 - \dot{x}^2}$, where $\dot{x} = dx/dt$. If such a particle is acted upon by a constant force in the direction of its motion, the phase space trajectories obtained from the corresponding Hamiltonian are

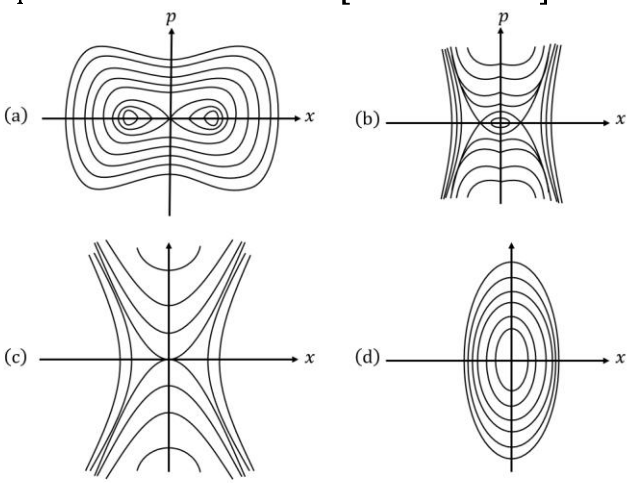
[CSIR JUNE 2017]

(a) ellipses (b) cycloids

(c) hyperbolas (d) parabolas

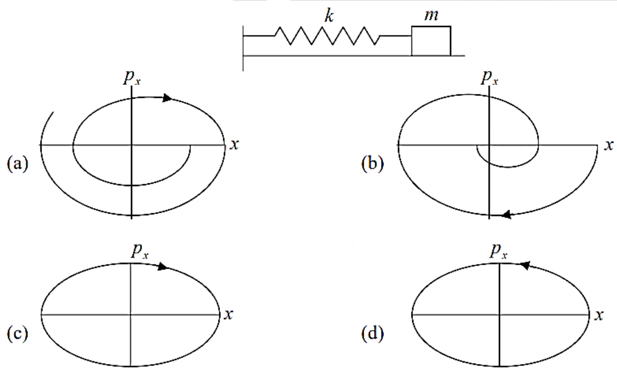
6. A particle moves in one dimension in a potential $V(x) = -k^2x^4 + \omega^2x^2$, where k and ω are constants. Which of the following curves best

describes the trajectories of this system in phase space? **[CSIR DEC 2017]**



7. A block of mass m, attached to a spring, oscillates horizontally on a surface. The co-efficient of friction between the block and the surface is μ. Which of the following trajectories best describes the motion of the block in the phase space (xp_x-plane)?

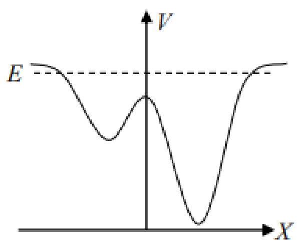
[CSIR DEC 2019]

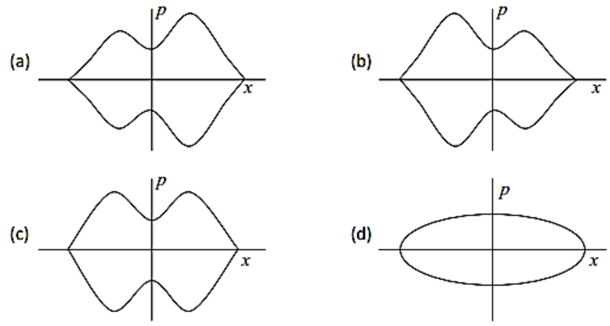


8. Consider a particle with total energy E moving in one dimension in a potential $V(x)$ as shown in the figure below.

Which of the following figures best represents the orbit of the particle in the phase space?

[CSIR JUNE 2020]



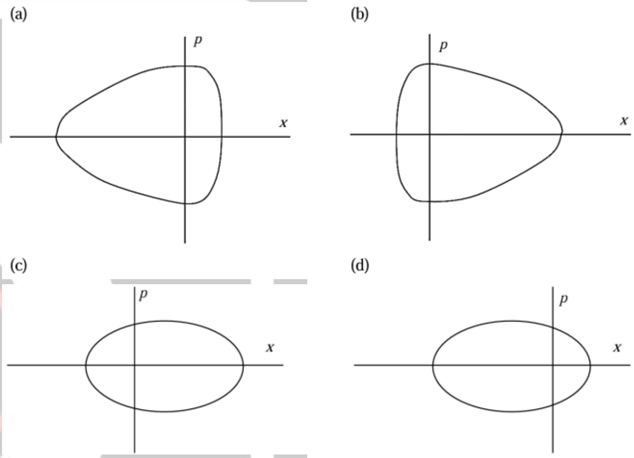


9. The Lagrangian of a particle in one dimension is

$$L = \frac{m}{2}\dot{x}^2 - ax^2 - V_0 e^{-10x}$$

where a and V_0 are positive constants. The best qualitative representation of a trajectory in the phase space is

[CSIR JUNE SEP 2022]



10. The electric and magnetic fields in an inertial frame are $\mathbf{E} = 3a\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ and $\mathbf{B} = \frac{5a}{c}\hat{\mathbf{k}}$, where a is a constant. A massive charged particle is released from rest. The necessary and sufficient condition that there is an inertial frame, where the trajectory of the particle is a uniform-pitched helix, is

[CSIR JUNE SEP 2022]

(a) $1 < a < \sqrt{2}$ (b) $-1 < a < 1$

(c) $-a^2 > 1$ (d) $a^2 > 2$

11. The evolution of the dynamical variables $x(t)$ and $p(t)$ is given by

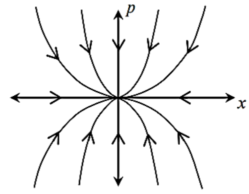
$\dot{x} = ax$

$\dot{p} = -p$

where a is a constant. The trajectory in (x, p) space for $-1 < a < 0$ is best described by

[CSIR JUNE 2024]

1.



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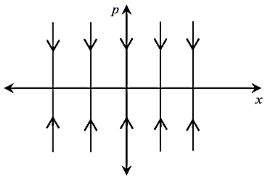

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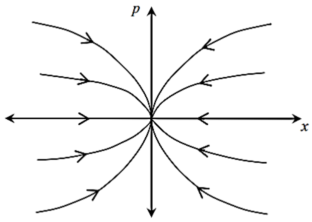

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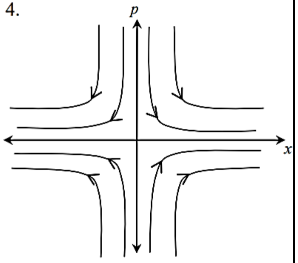
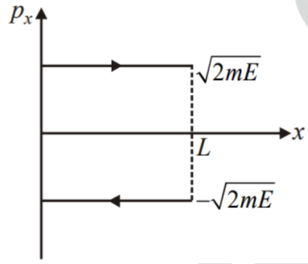


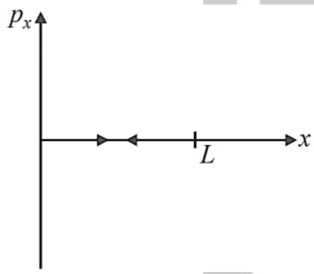
❖ GATE PYQ's

1. The phase diagram of a free particle of mass m kinetic energy E, moving in one-dimensional box with perfectly elastic walls at $x = 0$ and $x = L$, is given by **[GATE 2006]**

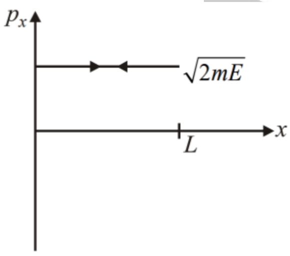
(a)



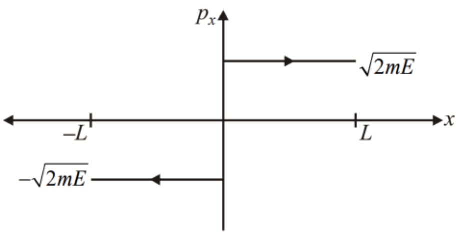
(b)



(c)

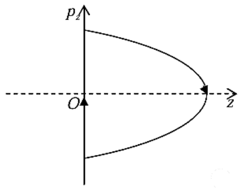


(d)

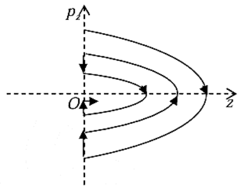


2. An inelastic ball of mass m has been thrown vertically upwards from the ground at $z = 0$. The initial kinetic energy of the ball is E. The phase trajectory of the ball after successive bouncing on the ground is

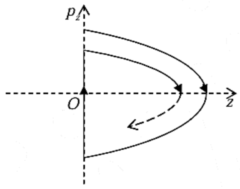
(a)



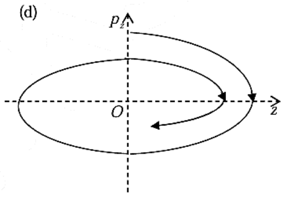
(b)



(c)



(d)

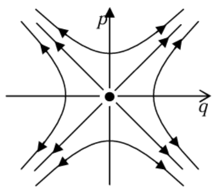


3. The Hamiltonian of a particle of mass m is given by

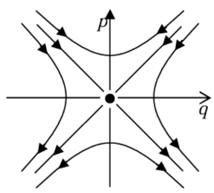
$$H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$$

. Which one of the following figures describes the motion of the particle in phase space? **[GATE 2014]**

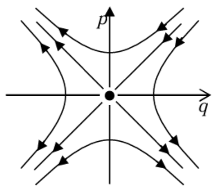
(a)



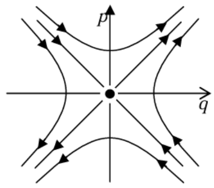
(b)



(c)



(d)

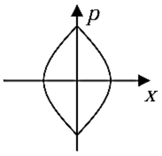


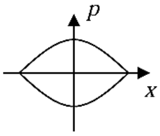
4. The phase space trajectory of an otherwise free particle bouncing between two hard walls elastically in one dimensions is a **[GATE 2017]**

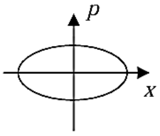
(a) straight line (b) parabola

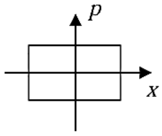
(c) rectangle (d) circle

5. A particle moves in one dimension under a potential $V(x) = \alpha|x|$ with some non-zero total energy. Which one of the following best describes the particle trajectory in the phase space? **[GATE 2018]**

(a) 

(b) 

(c) 

(d) 

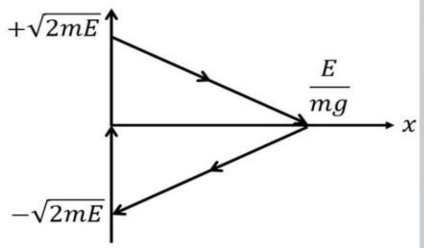
6. A ball bouncing off a rigid floor is described by the potential energy function

$$V(x) = mgx \quad \text{for } x > 0$$
$$= \infty \quad \text{for } x \leq 0$$

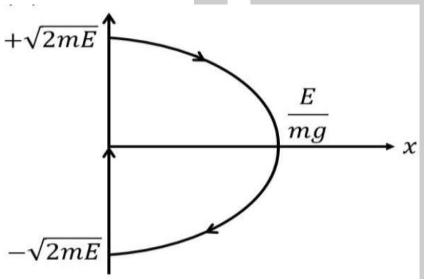
Which of the following schematic diagrams best represents the phase space plot of the ball?

[GATE 2019]

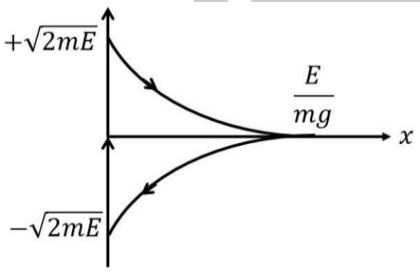
(a)



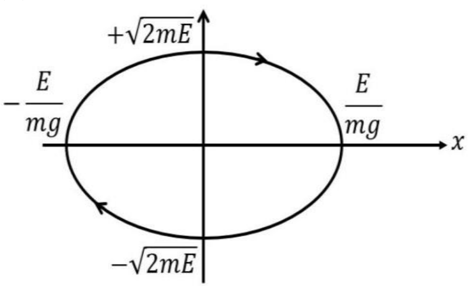
(b)



(c)



(d)



❖ JEST PYQ's

1. The coordinate q and the momentum p of a particle satisfy

$$\frac{dq}{dt} = p, \frac{dp}{dt} = -3q - 4p$$

If $A(t)$ is the area of any region of points moving in the (q, p)-space, then the ratio $A(t)/A(0)$ is

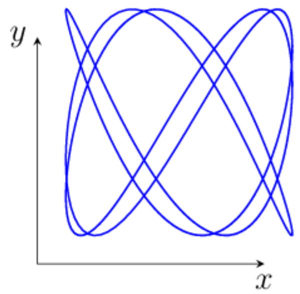
[JEST 2018]

(a) 1 (b) exp $(-3t)$

(c) exp $(-4t)$ (d) exp $(-3t/4)$

2. The trajectory of a particle which undergoes simple harmonic motion on a plane is shown in the figure. The ratio of the frequencies for the motion along x and y directions is given by

[JEST 2022]



(a) $\frac{4}{5}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{3}{5}$

Answers key				
CSIR-NET				
1. a	2. d	3. b	4. a	5. c
6. b	7. b	8. a	9. b	10. c
11. a				
GATE				
1. a	2. c	3. d	4. c	5. a
6. b				
JEST				
1. c	2. d			

❖ CSIR-NET PYQ's

1. A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is:

[CSIR JUNE 2011]

(a) $\sqrt{8b}$
(b) $\sqrt{8a}$
(c) $\sqrt{8a/b}$
(d) $\sqrt{8b/a}$

2. Consider the motion of a classical particle in a one-dimensional double-well potential $V(x) = \frac{1}{4}(x^2-2)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then

[CSIR JUNE 2012]

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$

(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$

(c) the particle will switch between the right and left wells

(d) the particle will approach the bottom of the right well and settle there

3. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

[CSIR DEC 2012]

(a) $\frac{\rho g}{\rho_0 H}$
(b) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$
(c) $\sqrt{\frac{\rho g}{\rho_0 H}}$
(d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

4. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

[CSIR JUNE 2014]

(a) 0° to the vertical and $\sqrt{3}T$

(b) 30° to the vertical and T/2

(c) 60° to the vertical and $T/\sqrt{2}$

(d) 0° to the vertical and $T/\sqrt{3}$

5. The time evolution of a one-dimensional dynamical system is described by

$$\frac{dx}{dt} = -(x+1)(x^2-b^2)$$

If this has one stable and two unstable fixed points, then the parameter ' b ' satisfies

[CSIR JUNE 2014]

(a) $0 < b < 1$
(b) $b > 1$
(c) $b < -1$
(d) $b = 2$

6. A particle of mass m is moving in the potential

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$

where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is

[CSIR DEC 2014]

(a) $\sqrt{a/m}$
(b) $\sqrt{2a/m}$
(c) $\sqrt{3a/m}$
(d) $\sqrt{6a/m}$

7. A particle of mass m moves in the one-dimensional potential $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$ where $\alpha, \beta > 0$. One of the equilibrium points is $x = 0$. The angular frequency of small oscillations about the other equilibrium point is

[CSIR JUNE 2015]

(a) $\frac{2\alpha}{\sqrt{3m\beta}}$
(b) $\frac{\alpha}{\sqrt{m\beta}}$
(c) $\frac{\alpha}{\sqrt{12m\beta}}$
(d) $\frac{\alpha}{\sqrt{24m\beta}}$

8. The Lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$$

where m and k are positive constants. The frequencies of its normal modes are

[CSIR DEC 2015]

(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{2m}(13 \pm \sqrt{73})}$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$

9. A solid vertical rod, of length L, and cross-sectional area A, is made of a material of Young's modulus Y. The rod is loaded with a mass M, and as a result, extends by a small amount ΔL in the equilibrium condition. The mass is then suddenly reduced to $M/2$. As a result the rod will undergo longitudinal oscillation with an angular frequency

[CSIR JUNE 2016]

(a) $\sqrt{\frac{2YA}{ML}}$ (b) $\sqrt{\frac{YA}{ML}}$

(c) $\sqrt{\frac{2YA}{M\Delta L}}$ (d) $\sqrt{\frac{YA}{M\Delta L}}$

10. The dynamics of a particle governed by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$$

describes

[CSIR DEC 2016]

(a) an undamped simple harmonic oscillator

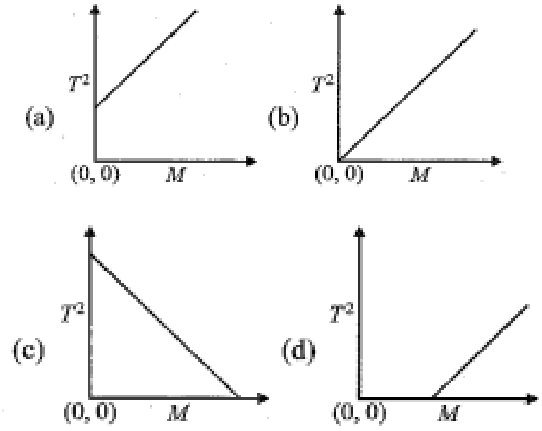
(b) a damped harmonic oscillator with a time varying damping factor

(c) an undamped harmonic oscillator with a time dependent frequency

(d) a free particle

11. The spring constant k, of a spring of a mass m_s, is determined experimentally by loading the spring with mass M and recording the time period T, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is

[CSIR DEC 2017]



a

12. A monoatomic gas of volume V is in equilibrium in a uniform vertical cylinder, the lower end of which is closed by a rigid wall and the order by a frictionless piston. The piston is pressed lightly and released. Assume that the gas is a poor conductor of heat and the cylinder and piston are perfectly insulating. If the cross-sectional area of the cylinder is A, the angular frequency of small oscillations of the piston about the point of equilibrium, is

[CSIR DEC 2017]

(a) $\sqrt{\frac{5gA}{(3V)}}$ (b) $\sqrt{\frac{4gA}{(3V)}}$

(c) $\frac{5}{3}\sqrt{\frac{gA}{V}}$ (d) $\sqrt{\frac{7gA}{(5V)}}$

13. A particle of mass m, kept in a potential

$$V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$$

, [where k and λ are positive constants], undergoes small oscillations about an equilibrium point. The frequency of oscillations is

[CSIR JUNE 2018]

(a) $\frac{1}{2\pi}\sqrt{\frac{2\lambda}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{\lambda}{m}}$

14. The time period of a particle of mass m, undergoing small oscillations around $x = 0$, in the potential $V = V_0 \cosh\left(\frac{x}{L}\right)$, is

[CSIR DEC 2018]

(a) $\pi\sqrt{\frac{mL^2}{V_0}}$ (b) $2\pi\sqrt{\frac{mL^2}{2V_0}}$

(c) $2\pi\sqrt{\frac{mL^2}{V_0}}$ (d) $2\pi\sqrt{\frac{2mL^2}{V_0}}$

15. The time evolution of a coordinate x of a particle is described by the equation:

$$\frac{d^4x}{dt^4} + 2\Omega^2\frac{d^2x}{dt^2} + (\Omega^4 - A^4)x = 0$$

For $\Omega > A$, the particle will

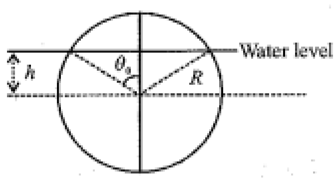
[CSIR JUNE 2019]

(a) eventually come to rest at the origin
(b) eventually drift to infinity ($|x| \to \infty$)
(c) oscillate about the origin
(d) eventually come to rest at Ω/A or $-\Omega/A$

16. A solid spherical cork of radius R and specific gravity 0.5 floats on water. The cork is pushed down so that its centre of mass is at a distance h (where $0 < h < R$) below the surface of water, and then released. The volume of the part of the cork above water level is

$$\pi R^3\left(\frac{2}{3} - \cos\,\theta_0 + \frac{1}{3}\cos^3\,\theta_0\right)$$

, where θ_0 is the angle as shown in the figure.



At the moment of release, the dependence of the upward force on the cork on h is

[CSIR JUNE 2019]

(a) $\frac{h}{R} - \frac{1}{3}\left(\frac{h}{R}\right)^3$ (b) $\frac{h}{R} + \frac{1}{3}\left(\frac{h}{R}\right)^3$

(c) $\frac{h}{R} - \frac{2}{3}\left(\frac{h}{R}\right)^3$ (d) $\frac{h}{R} + \frac{2}{3}\left(\frac{h}{R}\right)^3$

17. The equation of motion of a forced simple harmonic oscillator is $\ddot{x} + \omega^2 x = A\cos\,\Omega t$, where A is a constant. At resonance $\Omega = \omega$, the amplitude of oscillations at large times.

[CSIR JUNE 2019]

(a) saturates to a finite value

(b) increases with time as $\sqrt{t}$

(c) increases linearly with time

(d) increases exponentially with time

18. The fixed points of the time evolution of a one-variable dynamical system described by $y_{t+1} = 1 - 2y_t^2$ are 0.5 and -1 . The fixed points 0.5 and -1 are **[CSIR DEC 2019]**

(a) Both stable

(b) Both unstable

(c) Unstable and stable, respectively

(d) Stable and unstable, respectively

19. Two coupled oscillators in a potential

$$V(x,y) = \frac{1}{2}kx^2 + 2xy + \frac{1}{2}ky^2 (k > 2)$$

can be decoupled into two independent harmonic oscillators (coordinates: x', y') by means of an appropriate transformation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = S\begin{pmatrix} x \\ y \end{pmatrix}$$

. The transformation matrix S is

[CSIR JUNE 2020]

(a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & 1 \\ 1 & -\frac{1}{\sqrt{2}} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

20. The velocity $v(x)$ of a particle moving in one dimension is given by $v(x) = v_0 \sin\left(\frac{\pi x}{x_0}\right)$, where v_0 and x_0 are positive constants of appropriate dimensions. If the particle is initially at $x/x_0 = \epsilon$, where $|\in| \ll 1$, then, in the long time, it

[CSIR JUNE 2020]

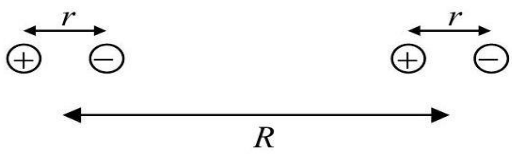
(a) Executes an oscillatory motion around $x = 0$

(b) Tends towards $x = 0$

(c) Tends towards $x = x_0$

(d) Executes an oscillatory motion around $x = x_0$

21. A linear diatomic molecule consists of two identical small electric dipoles with an equilibrium separation R, which is assumed to be a constant. Each dipole has charges $\pm q$ of mass m separated by r when the molecule is at equilibrium. Each dipole can execute simple harmonic motion of angular frequency ω.



Recall that the interaction potential between two dipoles of moments p_1 and p_2, separated by $R_{12} = R_{12}\hat{n}$ is $\left(p_1 \cdot p_2 - 3(p_1 \cdot \hat{n})(p_2 \cdot \hat{n})\right)/(4\pi\epsilon_0 R_{12}^3)$.
Assume that $R \gg r$ and let

$$\Omega^2 = \frac{q^2}{4\pi\epsilon_0 m R^3}$$

. The angular frequencies of small oscillations of the diatomic molecule are **[CSIR FEB 2022]**

(a) $\sqrt{\omega^2 + \Omega^2}$ and $\sqrt{\omega^2 - \Omega^2}$

(b) $\sqrt{\omega^2 + 3\Omega^2}$ and $\sqrt{\omega^2 - 3\Omega^2}$

(c) $\sqrt{\omega^2 + 4\Omega^2}$ and $\sqrt{\omega^2 - 4\Omega^2}$

(d) $\sqrt{\omega^2 + 2\Omega^2}$ and $\sqrt{\omega^2 - 2\Omega^2}$

22. A particle of mass m moves in a potential that is

$$V = \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2)$$

in the coordinates of a non-inertial frame F. The frame F is rotating with respect to an inertial frame with an angular velocity $\hat{k}\Omega$, where $\hat{k}$ is the unit vector along their common z-axis. The motion of the particle is unstable for all angular frequencies satisfying

[CSIR JUNE 2021]

(a) $(\Omega^2 - \omega_1^2)(\Omega^2 - \omega_2^2) > 0$

(b) $(\Omega^2 - \omega_1^2)(\Omega^2 - \omega_2^2) < 0$

(c) $(\Omega^2 - (\omega_1 + \omega_2)^2)(\Omega^2 - |\omega_1 - \omega_2|^2)$

(d) $(\Omega^2 - (\omega_1 + \omega_2)^2)(\Omega^2 - |\omega_1 - \omega_2|^2) < 0$

23. The Lagrangian of a system of two particles is

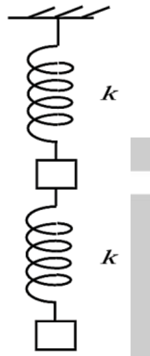
$$L = \frac{1}{2}\dot{x}_1^2 + 2\dot{x}_2^2 - \frac{1}{2}(x_1^2 + x_2^2 + x_1 x_2)$$

. The normal frequencies are best approximated by **[CSIR JUNE SEP 2022]**

(a) 1.2 and 0.7 (b) 1.5 and 0.5

(c) 1.7 and 0.5 (d) 1.0 and 0.4

24. A system of two identical masses connected by identical springs, as shown in the figure, oscillates along the vertical direction.



The ratio of the frequencies of the normal modes is

[CSIR JUNE 2023]

(a) $\sqrt{3-\sqrt{5}} : \sqrt{3+\sqrt{5}}$ (b) $3-\sqrt{5} : 3+\sqrt{5}$

(c) $\sqrt{5-\sqrt{3}} : \sqrt{5+\sqrt{3}}$ (d) $5-\sqrt{3} : 5+\sqrt{3}$

25. A particle of unit mass subjected to the 1-dimensional potential

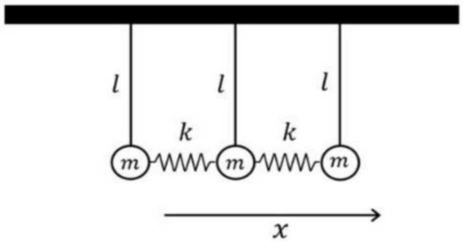
$$V(x) = \frac{2\alpha}{x^3} - \frac{3\beta}{x^2}$$

executes small oscillations about its equilibrium position, where α and β are positive constants with appropriate dimensions. The time period of small oscillations is **[CSIR DEC 2023]**

(a) $\frac{\pi\alpha^2}{\sqrt{6\beta^5}}$ (b) $\frac{\pi\alpha^2}{\sqrt{3\beta^5}}$

(c) $\frac{2\pi\alpha^2}{\sqrt{3\beta^5}}$ (d) $\frac{2\pi\alpha^2}{\sqrt{6\beta^5}}$

26. Three identical simple pendulum (of mass m and equilibrium string length l) are attached together by springs of spring constant k, as shown in the figure. **[CSIR JUNE 2024]**



The frequencies of small oscillations are given by

$$\sqrt{\frac{g}{l}}, \sqrt{\frac{k}{m}+\frac{g}{l}}, \sqrt{\frac{3k}{m}+\frac{g}{l}}$$

. The normal modes (without normalisation) corresponding to these frequencies respectively are

(a) $(1,1,1), (1,0,1), (1,-2,1)$

(b) $(1,1,1), (1,0,-1), (1,2,1)$

(c) $(1,1,1), (1,0,-1), (1,-2,1)$

(d) $(1,2,1), (1,0,-1), (1,1,1)$

27. A linear molecule is modelled as two atoms of equal mass m placed at coordinates x_1 and x_2, connected by a spring of spring constant k. The molecule is moving in one dimension under an additional external potential

$$V(x_1, x_2) = \frac{1}{2} m\omega_0^2 (x_1^2 + x_2^2)$$

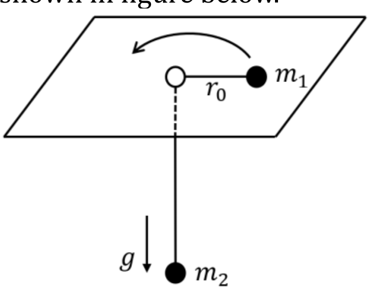
. If one frequency of molecular vibration is ω_0, the other frequency is

[CSIR JUNE 2024]

(a) $\sqrt{\omega_0^2 - \frac{k}{m}}$ (b) $\sqrt{\omega_0^2 + \frac{k}{m}}$

(c) $\sqrt{\omega_0^2 + \frac{2k}{m}}$ (d) $\sqrt{\omega_0^2 - \frac{2k}{m}}$

28. Two particles of masses m_1 and m_2 are connected by a massless thread of length ℓ as shown in figure below.



The particle of mass m_1 on the plane undergoes a circular motion with radius r_0 and angular momentum L. When a small radial displacement ε (where $\varepsilon < r_0$) is applied, its radial coordinate is found to oscillate about r_0. The frequency of the oscillations is

[CSIR JUNE 2019]

(a) $\sqrt{(m_2 g}$ (b) $\sqrt{\frac{7m_2 g}{(m_1+m_2)r_0}}$

(c) $\sqrt{\frac{3m_2 g}{\left(m_1+\frac{m_2}{2}\right)r_0}}$ (d) $\sqrt{\frac{3m_2 g}{(m_1+m_2)r_0}}$

29. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

[CSIR-NET]

(a) $2\pi\sqrt{\frac{l+R}{g}}$

(b) $\frac{2\pi}{\sqrt{g}}(l^2+R^2)^{1/4}$

(c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$

(d) $\frac{2\pi}{\sqrt{g}}(2R^2+2Rl-l^2)^{1/4}$

❖ GATE PYQ's

1. The Lagrangian for a three particles system is given by:

$$L = \frac{1}{2}(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2) - a^2(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3),$$

where a is real. Then one of the normal coordinates has a frequency ω given by

[GATE 2001]

(a) $\omega^2 = a^2$ (b) $\omega^2 = a^2/2$

(c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

2. A mass m is connected on either side with a spring each of spring constant k_1 and k_2. The free ends of springs are tied to rigid supports. The displacement of the mass is x from equilibrium position. Which one of the following is TRUE?

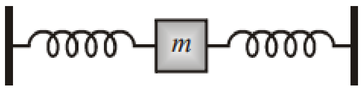
[GATE 2004]

(a) The force acting on the mass is $-(k_1k_2)^{1/2}x$

(b) The angular momentum of the mass is zero about the equilibrium point and its Lagrangian is

$$\frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1+k_2)x^2$$

(c) The total energy of the system is $\frac{1}{2}m\dot{x}^2$



(d) The angular momentum of the mass is $mx\dot{x}$ and the Lagrangian of the system is

$$\frac{m}{2}\dot{x}^2 + \frac{1}{2}(k_1+k_2)x^2$$

3. A hoop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

[GATE 2008]

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{4g}}$

(c) $2\pi\sqrt{\frac{R}{g}}$ (d) $2\pi\sqrt{\frac{9R}{7g}}$

4. The Lagrangian of a diatomic molecule is given by

$$L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}x_1x_2$$

, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k > 0$. The normal frequencies are

[GATE 2009]

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$

(c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm\left(\frac{k}{2m}\right)^{1/2}$

5. A classical particle is moving in an external field $V(x,y,z)$ which is invariant under the following infinitesimal transformations

$$x \to x' = x + \delta x$$
$$y \to y' = y + \delta y$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix} = R_z\begin{pmatrix} x \\ y \end{pmatrix}$$

Where R_z is the matrix corresponding to rotation about the z-axis. The conserved quantities are (the symbols have their usual meaning)

[GATE 2009]

(a) p_x, p_z, L_z (b) p_x, p_y, L_z, E

(c) p_y, L_z, E (d) p_y, p_z, L_x, E

6. A particle is placed in region with the potential

$$V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$$

, where $k, \lambda > 0$. Then

[GATE 2010]

(a) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium

(b) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium

(c) $x = 0$ and $x = \frac{k}{\lambda}$ are points of unstable equilibrium

(d) there are no points of stable of unstable equilibrium

7. Two bodies of mass m and $2m$ are connected by a spring constant k. The frequency of the normal mode is

[GATE 2011]

(a) $\sqrt{\frac{3k}{2m}}$ (b) $\sqrt{\frac{k}{m}}$

(c) $\sqrt{\frac{2k}{3m}}$ (d) $\sqrt{\frac{k}{2m}}$

8. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

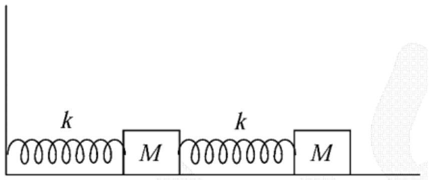
[GATE 2012]

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π

9. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is

[GATE 2013]



(a) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$

(c) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$ (d) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$

10. Two masses m and 3 m are attached to the two ends of a massless spring with force constant K. If $m = 100$ g and $K = 0.3$Nlm, then the natural angular frequency of oscillation is Hz

[GATE 2014]

11. A particle of mass m is in a potential given by

$$V(r) - \frac{a}{r} + \frac{a r_0^2}{3r^3},$$

Where a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

[GATE 2014]

(a)$2\pi\sqrt{\frac{m r_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{m r_0^3}{a}}$

(c)$2\pi\sqrt{\frac{2m r_0^3}{a}}$ (d) $4\pi\sqrt{\frac{m r_0^3}{a}}$

12. Two blocks are connected by a spring of spring constant k. One block has mass m and the other block has mass 2 m. If the ratio $k/m = 4s^{-2}$, the angular frequency of vibration ω of the two block spring system in S^{-1} is _______ (Give your answer upto two decimal places)

[GATE 2016]

13. Two identical masses of 10 gm each are connected by a mass less spring of spring constant 1 N/m. The non-zero angular eigen frequency of the system is rad/s (up to two decimal places).

[GATE 2017]

14. In the context of small oscillations, which one of the following does NOT apply to the normal coordinates?

[GATE 2018]

(a) Each normal coordinate has an eigen-frequency associated with it

(b) The normal coordinates are orthogonal to one another

(c) The normal coordinates are all independent

(d) The potential energy of the system is a sum of squares of the normal coordinates with constant coefficients

15. The potential energy of a particle of mass m is given by

$U(x) = a\sin(k^2 x - \pi/2),\ a > 0,\ k^2 > 0$

The angular frequency of small oscillations of the particle about $x = 0$ is

[GATE 2020]

(a)$k^2\sqrt{\frac{2a}{m}}$ (b) $k^2\sqrt{\frac{a}{m}}$

(c)$k^2\sqrt{\frac{a}{2m}}$ (d) $2k^2\sqrt{\frac{a}{m}}$

16. A piston of mass m is fitted to an airtight horizontal cylindrical jar. The cylinder and piston have identical unit area of cross-section. The gas inside the jar has volume V and is held at pressure $P = P_{\text{atmosphere}}$. The piston is pushed inside the jar very slowly over a small distance. On releasing, the piston performs an undraped simple harmonic motion of low frequency. Assuming that the gas is ideal and no heat is exchanged with the atmosphere, the frequency of the small oscillations is proportional to **[GATE 2022]**

(a) $\sqrt{\frac{P}{\gamma m V}}$ (b) $\sqrt{\frac{P\gamma}{Vm}}$

(c) $\sqrt{\frac{P}{mV^{\gamma-1}}}$ (d) $\sqrt{\frac{\gamma P}{mV^{\gamma-1}}}$

17. The equation of motion for the forced simple harmonic oscillator is

$$\ddot{x}(t) + \omega^2 x(t) = F\cos(\omega t)$$

where $x(t=0) = 0$ and $\dot{x}(t=0) = 0$. Which one of the following options is correct?

[GATE 2024]

(a) $x(t) \propto t\sin(\omega t)$ (b) $x(t) \propto t\cos(\omega t)$

(c) $x(t) = \infty$ (d) $x(t) \propto e^{\omega t}$

❖ JEST PYQ's

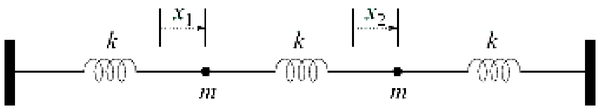
1. For small angular displacement (i.e., sin ≈ θ), a simple pendulum oscillates harmonically. For larger displacements, the motion

[JEST 2012]

(a) Becomes a periodic
(b) remains periodic with the same period
(c) Remains periodic with a higher period
(d) remains periodic with a lower period.

2. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$ corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$ respectively.

[JEST 2016]



At t = 0, the displacements are $x_1 = A, x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. the displacement of the second particle at time t is given by:

[JEST 2016]

(a)x_2(t) $= \frac{A}{2}\left(\cos(\omega_0 t) + \cos(\sqrt{3}\omega_0 t)\right)$

(b)x_2(t) $= \frac{A}{2}\left(\cos(\omega_0 t) - \cos(\sqrt{3}\omega_0 t)\right)$

(c)x_2(t) $= \frac{A}{2}\left(\sin(\omega_0 t) - \sin(\sqrt{3}\omega_0 t)\right)$

(d)x_2(t) $= \frac{A}{2}\left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}}\sin(\sqrt{3}\omega_0 t)\right)$

3. A particle of mass 1 kg is undergoing small oscillation about the aquarium point in the potential

$$V(x) = \frac{1}{2x^{12}} - \frac{1}{x^6}$$

for x > 0 meters. The time period (in seconds) of the oscillation is

[JEST 2018]

(a) $\pi/2$ (b) $\pi/3$

(c) 1.0 (d) π

4. Consider the Lagrangian

$$L = 1 - \sqrt{1 - \dot{q}^2} - \frac{q^2}{2}$$

Of a particle executing oscillations whose amplitude is A. if p denotes the momentum of the particle, then 4p^2 is

[JEST 2018]

(a) $(A^2 - q^2)(4 + A^2 - q^2)$

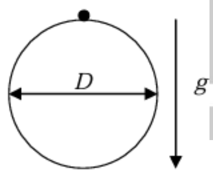
(b) $(A^2 + q^2)(4 + A^2 - q^2)$

(c) $(A^2 - q^2)(4 + A^2 + q^2)$

(d) $(A^2 + q^2)(4 + A^2 + q^2)$

5. A hoop of diameter D is pivoted at the topmost point on the circumference as shown in the figure. The acceleration due to gravity g is acting downwards. What is the time period of small oscillations in the plane of the hoop?

[JEST 2019]



(a)$2\pi\sqrt{\frac{D}{2g}}$ (b) $2\pi\sqrt{\frac{5D}{6g}}$

(c)$2\pi\sqrt{\frac{D}{2g}}$ (d) $2\pi\sqrt{\frac{2D}{g}}$

6. A particle of mass m moves in a one-dimensional potential $V(x) = F_0|x|$, where F_0 is a positive constant. Given the initial conditions, $x(0) = x_0 > 0$ and $\dot{x}(0) = 0$, which one of the following statements is correct? **[JEST 2020]**
(a) The particle undergoes simple harmonic

motion about the origin with frequency

$$\omega = 2\pi\sqrt{\frac{F_0}{mx_0}}$$

(b) The angular frequency of oscillations of the particle is

$$\omega = \frac{1}{2}\pi\sqrt{\frac{F_0}{2mx_0}}$$

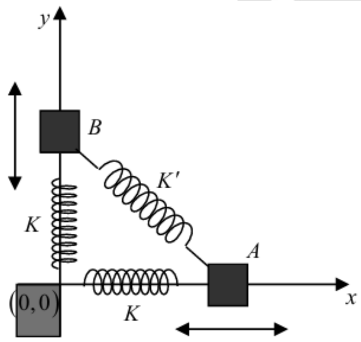
(c) The particle begins from rest and is accelerated along the positive x-axis such that

$$x(t) = \frac{x_0 + F_0 t^2}{2m}$$

(d) The angular frequency of oscillations of the particle is independent of its mass

7. Two equal masses A and B are connected to a fixed support at the origin by two identical springs with spring constant K and the same unstretched length L. They are also connected to each other by a spring with spring constant K' and unstretched length $\sqrt{2}L$. The equilibrium position, with all springs unstretched, is shown in the figure. If A is constrained to move only along the x axis and B is constrained to move only along the y axis, then the angular frequencies ω_1, ω_2 of the normal modes are

[JEST 2021]



(a) $\omega_1 = \sqrt{\frac{K}{m}}, \omega_2 = \sqrt{\frac{K+K'}{m}}$

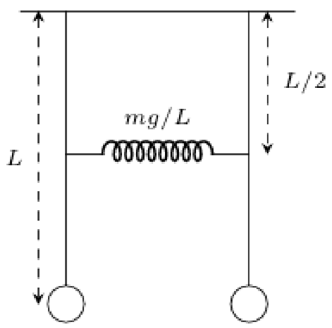
(b) $\omega_1 = \sqrt{\frac{K}{m}}, \omega_2 = \sqrt{\frac{2K'}{m}}$

(c) $\omega_1 = \sqrt{\frac{2K}{m}}, \omega_2 = \sqrt{\frac{K+K'}{m}}$

(d) $\omega_1 = \sqrt{\frac{K}{m}}, \omega_2 = \sqrt{\frac{K+K'}{m}}$

8. Two identical simple pendula of length L are connected by a spring at a height of $L/2$ as shown in the figure. Assuming the spring constant is mg/L, where m is the mass of the bob and g is the acceleration due to gravity, what is the ratio of the highest to lowest eigenfrequencies of the system?

[JEST 2022]



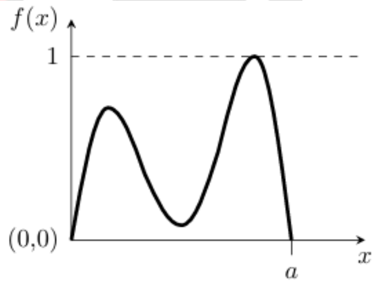
(a) $\sqrt{3}$

(b)

(c) $\sqrt{2}$

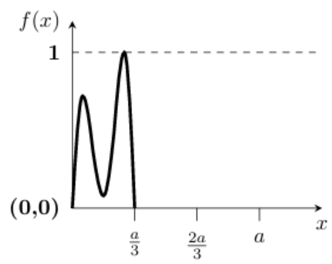
(d) $\sqrt{3/2}$

9. The function $f(x)$ shown below has non-zero values only in the range $0 < x < a$.

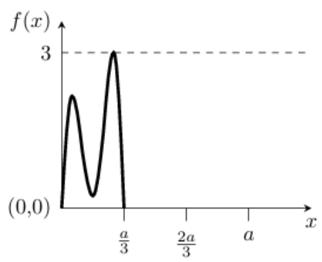


Which of the following figure represents $f(3x)$?

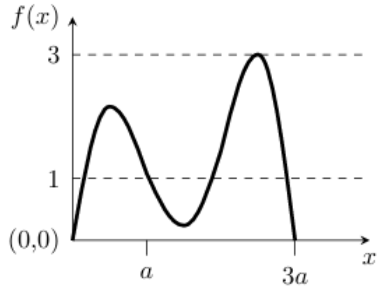
[JEST 2022]

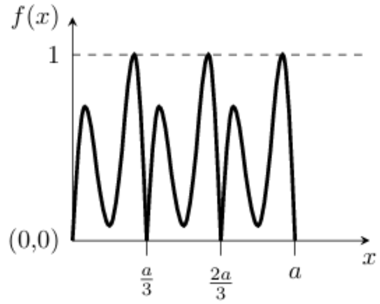


A.



B.

C.


D.


10. The force experienced by a mass confined to move along the x-axis is of the form $F(x) = -k_1x - k_2x^n$ where x is the displacement of the mass from $x = 0, k_1$ and k_2 are positive constants and n is a positive integer. For small displacements, the motion of the mass remains symmetric about $x = 0$ **[JEST 2023]**

(a) when n is any positive integer.

(c) only when $n = 1$.

(b) when n is an odd positive integer.

(d) when n is an even positive integer.

11. Two particles of mass m and $4m$ confined to move along the x-axis are subjected to the force $F(x) = -kx$. At time $t = 0$, the smaller mass m starts from rest at $x_1(t = 0) = A$ and the larger mass $4m$ starts from rest at $x_2(t = 0) = -A$. The point on the x-axis where the first collision between the two particles occurs is: **[JEST 2023]**

(a)$x = \frac{A}{2}$ (b) $x = -\frac{A}{2}$.

(c)$x = -\frac{A}{4}$ (d) $x = 0$.

12. Consider a Hamiltonian system with a potential energy function given by $V(x) = x^2 - x^4$. Which of the following is correct? **[JEST 2016]**

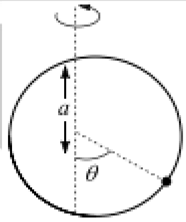
(a) The system has one stable point

(b) The system has two stable points

(c) The system has three stable points

(d) The system has four stable points

13. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)? **[JEST 2017]**



(a)tan $\theta = \frac{\pi g}{\omega^2 a}$ (b) sin $\theta = \frac{g}{\omega^2 a}$

(c)cos $\theta = \frac{g}{\omega^2 a}$ (d) tan $\theta = \frac{g}{\pi\omega^2 a}$

14. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$? **[JEST 2017]**

(a) $\sqrt{3g}$ (b) $\sqrt{g}$

(c) $\sqrt{2g}$ (d) $\sqrt{4g}$

15. Consider two coupled harmonic oscillators of mass m each. The Hamiltonian describing the oscillators is

$$\hat{H} = \frac{\hat{P}_1^2}{2m} + \frac{\hat{P}_2^2}{2m} + \frac{1}{2}m\omega^2\left(\hat{x}_1^2 + \hat{x}_2^2 + (\hat{x}_1^2 - \hat{x}_2)^2\right)$$

The eigenvalues of $\hat{H}$ are given by (with n_1 and n_2 being non-negative integers) **[JEST 2018]**

(a) $E_{n1,n2} = \hbar\omega(n_1 + n_2 + 1)$

(b)$E_{n1,n2} = \hbar\omega\left(n_1 + \frac{1}{2}\right) + \frac{1}{\sqrt{3}}\hbar\omega\left(n_2 + \frac{1}{2}\right)$

(c) $E_{n1,n2} = \hbar\omega\left(n_1 + \frac{1}{2}\right) + \sqrt{3}\hbar\omega\left(n_2 + \frac{1}{2}\right)$

(d) $E_{n1,n2} = \frac{1}{\sqrt{3}}\hbar\omega(n_1 + n_2 + 1)$

❖ TIFR PYQ's

1. A particle of mass m moves in one dimension under the influence of a potential energy

$$V(x) = -a\left(\frac{x}{\ell}\right)^2 + b\left(\frac{x}{\ell}\right)^4$$

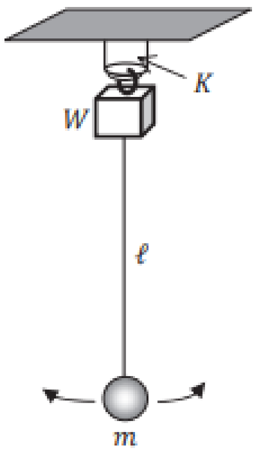
where a and b are positive constants and ℓ is a characteristic length. The frequency of small oscillations about a point of stable equilibrium is

[TIFR 2013]

(a) $\frac{1}{2\pi\ell}\sqrt{\frac{b}{m}}$ (b) $\frac{1}{\pi\ell}\sqrt{\frac{a^2}{mb}}$

(c) $\frac{1}{\pi\ell}\sqrt{\frac{a}{m}}$ (d) $\frac{2b}{\pi\ell}\sqrt{\frac{1}{ma}}$

2. A weight W is suspended from a rigid support by a hard spring with stiffness constant K. The spring is enclosed in a hard plastic sleeve, which prevents horizontal motion, but allows vertical oscillations (see figure). A simple pendulum of length ℓ with a bob of mass $m(mg \ll W)$ is suspended from the weight W and is set oscillating in a horizontal line with a small amplitude. After some time has passed, the weight W is observed to be oscillating up and down with a large amplitude (but not hitting the sleeve).



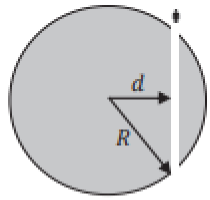
It follows that the stiffness constant K must be

[TIFR 2014]

(a) $K = \frac{4W}{\ell}$ (b) $K = \frac{2W}{\ell}$

(c) $K = \frac{W}{\ell}$ (d) $K = \frac{W}{2\ell}$

3.



Imagine that a narrow tunnel is excavated through the Earth as shown in the diagram on the left and that the mass excavated to create the tunnel is extremely small compared to Earth's mass M.
A person falls into the tunnel at one end. at time $t = 0$. Assuming that the tunnel is frictionless, the person will

[TIFR 2016]

(a) fall straight through, escaping Earth's gravity at time $2\pi\sqrt{R^3/GM}$

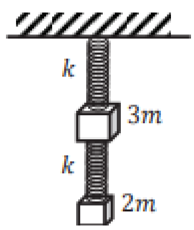
(b) describe simple harmonic motion with period $2\pi(d/R)\sqrt{R^3/GM}$

(c) describe simple harmonic motion with period $2\pi\sqrt{(R-d)^3/GM}$

(d) describe simple harmonic motion with period $2\pi\sqrt{R^3/GM}\nabla$

4. Two masses $3m$ and $2m$ are suspended vertically by identical massless springs, each of stiffness constant k. The mass $2m$ is suspended from the mass $3m$ and the mass $3m$ is suspended from a rigid support, as shown in the figure. If only vertical motion is permitted, the frequencies of small oscillations of this system are

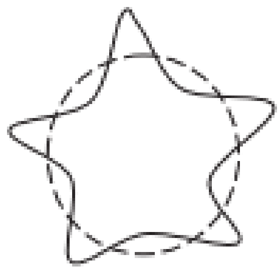
[TIFR 2017]



(a) $\sqrt{\frac{k}{m}}, \sqrt{\frac{k}{6m}}$ (b) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{k}{3m}}$

(c) $\sqrt{\frac{k}{m}}, \sqrt{\frac{3k}{2m}}$ (d) $\sqrt{\frac{2k}{3m}}, \sqrt{\frac{3k}{2m}}$

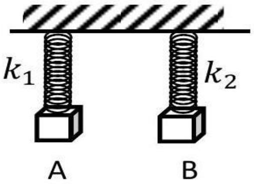
5. In outer space, where the effects of gravity can be neglected, a drop of liquid assumes a spherical shape. However, when disturbed it undergoes shape oscillations (see figure). The frequency v of oscillation of a drop depends on its equilibrium radius, its density and the surface tension.
What would be the numerical value of the ratio $v_{\mathrm{Hg}}/v_{\mathrm{H_2O}}$ of the frequencies of oscillation between a drop of mercury (Hg) and a drop of water (H_2O) of the same equilibrium radius ? You may use the following data: **[TIFR 2017]**



Liquid	Density in **gm cm^3**	Surface tension in **N m^{-1}**
water	1.0	0.073
mercury	13.6	0.487

6. Two bodies A and B of equal mass are suspended from two rigid supports by separate massless springs having spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of oscillations of A to that of B is

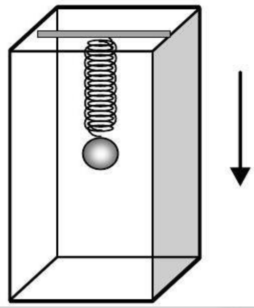
[TIFR 2019]



(a) k_1/k_2 (b) k_2/k_1

(c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$

7. A particle of mass m hangs from a light spring inside a lift (see figure). When the lift is at rest, the mass oscillates in the vertical direction with an angular frequency 2.5rad/s. Now consider the following situation.
The suspended mass is at rest inside the lift which is descending vertically at a speed of 0.5 m/s. If the lift suddenly stops, the

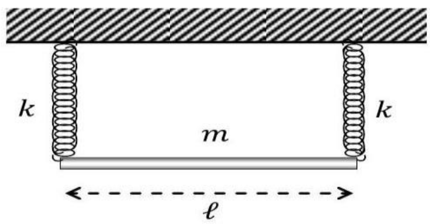


amplitude of oscillations of the mass will be **[TIFR 2020]**

(a) 0.20 m (b) 0.25 m

(c) 0.05 m (d) 1.25 m

8. A uniform rod of length ℓ and mass m is suspended horizontally from a rigid support by two identical massless springs, each with stiffness constant k, as sketched below.



If the springs can move only in the vertical direction, the frequency of small oscillations of the rod about equilibrium is given by

[TIFR 2020]

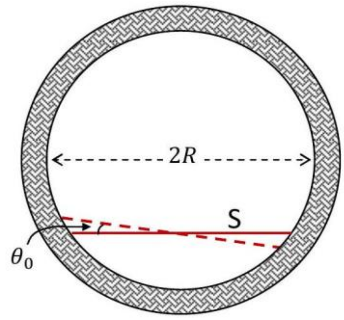
(a) $\sqrt{2k/m}$ and $\sqrt{6k/m}$

(b) $\sqrt{2k/m}$ and $\sqrt{2\pi k/m}$

(c) $\sqrt{\pi k/2m}$ and $\sqrt{6k/m}$

(d) $\sqrt{k/m}$ and $\sqrt{2\pi k/m}$

9. A stick S of uniform density of mass M, length L, and negligible width, is constrained to move such that its two ends always stay on the inside of a fixed vertical, circular ring of inner radius R, as shown below.



If the stick S is displaced by a small angle θ_0 from its equilibrium position and then allowed to oscillate freely, the angular frequency ω of oscillations will be
[Ignore the friction between the stick and the ring.] **[TIFR 2021]**

(a) $\left(\frac{4g}{2R^2 - L^2}\right)^{1/2}\left(R^2 - \frac{L^2}{4}\right)^{1/4}$

(b) $\left(\frac{4g}{6R^2 - L^2}\right)^{1/2}\left(R^2 - \frac{L^2}{4}\right)^{1/4}$

(c) $\left(\frac{6g}{3R^2 - L^2}\right)^{1/2}\left(R^2 - \frac{L^2}{4}\right)^{1/4}$

(d) $\left(\frac{6g}{6R^2 - L^2}\right)^{1/2}\left(R^2 - \frac{L^2}{4}\right)^{1/4}$

10. A system with two generalized coordinates (q_1, q_2) is described by the Lagrangian

$$L = m\left(\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2 + \frac{3}{2}\dot{q}_2^2\right) - k\left(\frac{3}{2}q_1^2 + 2q_1q_2 + q_2^2\right)$$

where m is the mass, and k is a constant.
This system can execute oscillations with two possible time periods **[TIFR 2022]**

(a) $T = 2\pi\sqrt{\frac{2m}{k}}$ and $T = 2\pi\sqrt{\frac{m}{2k}}$

(b) $T = 2\pi\sqrt{\frac{m}{2k}(5 - 2\sqrt{6})}$ and $T = 2\pi\sqrt{\frac{m}{2k}(5 + 2\sqrt{6})}$

(c) $T = \pi\sqrt{\frac{m}{k}(1 - \sqrt{15})}$ and $T = \pi\sqrt{\frac{m}{k}(1 + \sqrt{15})}$

(d) $T = 2\pi\sqrt{\frac{2m}{3k}}$ and $T = 2\pi\sqrt{\frac{3m}{2k}}$

11. A pendulum which is suspended from the ceiling of a train has time period T_0 when the train is stationary. When the train moves with a small but steady speed v around a horizontal circular track of radius R, the time period of the pendulum will be **[TIFR 2022]**

(a) $T_0\left(1 - \frac{v^2T_0^2}{4\pi^2R}\right)^{-1/2}$

(b) $T_0\left(1 + \frac{v^2T_0^2}{4\pi^2R}\right)^{-1/2}$

(c) $T_0\left(1 - \frac{v^4}{g^2R^2}\right)^{-1/4}$

(d) $T_0\left(1 + \frac{v^4}{g^2R^2}\right)^{1/4}$

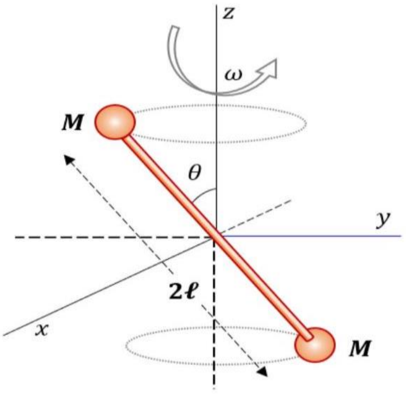
12. A simple pendulum is oscillating freely in the vertical plane. If the string is shortened very slowly to half its length, the angular amplitude $\theta_{\max}$ will change by a factor **[TIFR 2023]**

(a) $2^{3/4}$ (b) $\sqrt{2}$

(c) 2 (d) Does not change.

13. A dumbbell consists of two small spherical masses M each, connected by a thin massless rod of length 2ℓ.

This dumbbell is centered at the origin, and is rotating about the z-axis with a uniform angular velocity ω, making an angle θ with the z-axis (see figure).
Neglecting effects due to gravity, at the instant when the dumbbell is wholly in the yz-plane (as shown in the figure), the magnitude of torque about the origin will be **[TIFR 2022]**



(a) $M\ell^2\omega^2 \sin 2\theta$ (b) $2M\ell^2\omega^2 \sin^2 \theta$

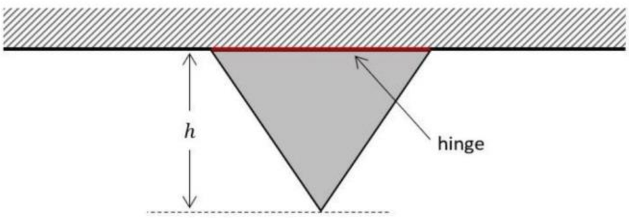
(c) $2M\ell^2\omega^2 \cos^2 \theta$ (d) zero

14. A particle is executing simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position is x_1 and x_2, the corresponding values of its velocity are v_1 and v_2 respectively. The time period of oscillation is given by **[TIFR 2023]**

(a) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$ (b) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_2^2 - v_1^2}}$

(c) $2\pi\frac{x_2 - x_1}{v_2 - v_1}$ (d) $2\pi\frac{x_2 - x_1}{v_1 - v_2}$

15. A thin equilateral triangular plate of uniform mass density is attached to a fixed horizontal support along one of its sides through a frictionless hinge, as shown in the figure below. The vertical distance between the rod and the lower tip of the plate is h.



If the pointed tip of the plate is displaced (out of the plane of the paper) so that its plane forms a small angle with the vertical plane passing through the rod, the angular frequency ω of the resultant motion is $\omega =$ **[TIFR 2023]**

(a) $\sqrt{\frac{2g}{h}}$ (b) $\sqrt{\frac{2\sqrt{3}g}{h}}$

(c) $\sqrt{\frac{2g}{\sqrt{3}h}}$ (d) $\sqrt{\frac{\sqrt{3}g}{2h}}$

16. Consider a diatomic molecule with two atoms of masses $m_1 = 1$ a.m.u. and $m_2 = 8$ a.m.u. which are separated by a distance r and bound by an effective interaction potential of the form

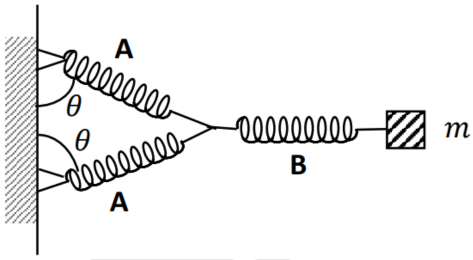
$$V(r) = \epsilon\left(\frac{a^4}{4r^4} - \frac{b^2}{2r^2}\right)$$

where $\epsilon = 4 \times 10^{-18}$ J, $a = b = 1$Å and 1 a.m.u. $\approx 1.6 \times 10^{-27}$ kg. Making a small oscillations approximation, the transition frequency corresponding to the vibrational spectra of the molecule is approximately **[TIFR 2023]**

(a) 1.2×10^{14} Hz (b) 0.4×10^{14} Hz

(c) 7.5×10^{14} Hz (d) 3.6×10^{14} Hz

17. Consider a mass m connected to a network of massless springs shown in the figure below.



The spring constant of spring A is k_A, and that of spring B is k_B. The springs are shown in a relaxed position, and the angle θ in this position is $\pi/3$. The mass is displaced horizontally by a small distance. What is the angular frequency of small oscillations of m ? (Ignore gravity and friction.) **[TIFR 2024]**

(a) $\sqrt{(3k_Ak_B)/[m(2k_B + 3k_A)]}$

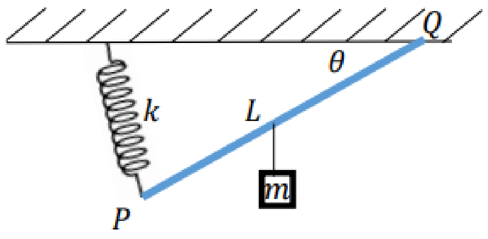
(b) $\sqrt{(k_Ak_B)/[m(k_B + k_A)]}$

(c) $\sqrt{(2k_Ak_B)/[m(k_B + 2k_A)]}$

(d) $\sqrt{(\sqrt{3}k_Ak_B)/[m(k_B + \sqrt{3}k_A)]}$

18. A massless rigid rod of length L is suspended with an ideal spring of spring constant k at one end P, and by a hinge on the other end, Q. The rest length of the spring is zero. A mass m is suspended from the mid-point of the rod. This results in tilting of the rod by angle θ. What is the angle θ ?

[TIFR 2025]



(a) $\tan^{-1}\left(\frac{mg}{2kL}\right)$ (b) $\sin^{-1}\left(\frac{mg}{2kL}\right)$

(c) $\cos^{-1}\left(\frac{mg}{kL}\right)$ (d) $\sec^{-1}\left(\frac{mg}{kL}\right)$

Answers key				
CSIR-NET				
1. b	2. b	3. d	4. c	5. b
6. b	7. b	8. a	9. a	10. d
11. a	12. a	13. c	14. c	15. c
16. a	17. c	18. b	19. b	20. c
21. c	22. b	23. d	24. a	25. d
26. c	27. c	28. d	29. c	
GATE				
1. c	2. b	3. a	4. d	5. b
6. b	7. a	8. b	9. a	10. 0.32
11. a	12. 2.45	13. 14.14	14. b	15. b
16. b	17. a			
JEST				
1. c	2. b	3. b	4. a	5. c
6. b	7. a	8. d	9. a	10. b
11. b	12. a	13. c	14. d	15. c
TIFR				
1. b	2. a	3. d	4. a	5. 0.37
6. d	7. a	8. a	9. d	10. a
11. a	12. a	13. a	14. a	15. a
16. a	17. a	18. a		

❖ CSIR-NET PYQ's

1. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by I_1, I_2 and I_3 is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity $(\omega_1, \omega_2, \omega_3)$ are

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3, \dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3, \dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\omega_2$$

[CSIR JUNE 2011]

(A) The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are

(a) $(1,-1,0), (-1,0,1)$ and $(0,-1,1)$

(b) $(1,1,0), (1,0,1)$ and $(0,1,1)$

(c) $(1,0,0), (0,1,0)$ and $(0,0,1)$

(d) $(1,1,1), (-1,-1,-1)$ and $(0,0,0)$

(B) The constants of motion are

(a) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$

(b) $I_1\omega_1^2 + I_2 w_2^2 + I_3\omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

(c) $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$ and $\omega_1 + \omega_2 + \omega_3$

(d) $\omega_1^2 + \omega_2^2 + \omega_3^2$ and $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

2. Two masses, m each, are placed at the points $(x, y) = (a, a)$ and $(-a, -a)$, and two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$.
The principal moments of inertia of the system are

[CSIR DEC 2015]

(a) $2ma^2, 4ma^2$ (b) $4ma^2, 8ma^2$

(c) $4ma^2, 4ma^2$ (d) $8ma^2, 8ma^2$

3. Two bodies of equal mass ' m ' are connected by a massless rigid rod of length ' l ' lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is

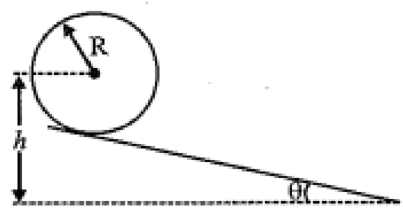
[CSIR DEC 2012]

(a) $m\ell^2\omega/4$ (b) $m\ell^2\omega/2$

(c) $m\ell^2\omega$ (d) $2\dot{m}\ell^2\omega$

4. A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is

[CSIR DEC 2012]



(a) $\sqrt{g/(h-R)}\tan\,\theta$ (b) $\sqrt{g/(h-R)}$

(c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

5. A uniform cylinder of radius r and length ℓ, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is:

[CSIR JUNE 2013]

(a) $V\sqrt{\frac{14r\ell}{15R^2}}$ (b) $4\,V\sqrt{\frac{r}{15R}}$

(c) $\frac{4\,V}{\sqrt{15}}$ (d) $V\sqrt{\frac{14}{15}}$

6. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is **[CSIR DEC 2013]**

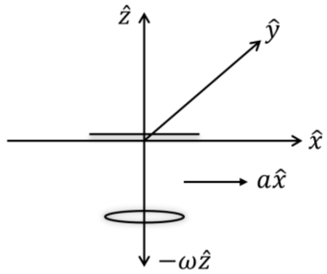
(a) $2\pi\sqrt{\frac{l+R}{g}}$

(b) $\frac{2\pi}{\sqrt{g}}(l^2 + R^2)^{1/4}$

(c) $2\pi\sqrt{\dfrac{2R^2+2Rl+l^2}{g(R+l)}}$

(d) $\dfrac{2\pi}{\sqrt{g}}(2R^2+2Rl+l^2)^{1/4}$

7. A disc of mass m is free to rotate in a plane parallel to the xy-plane with an angular velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.



The car accelerates in the positive x-direction with an acceleration $a > 0$. Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?

[CSIR DEC 2017]

(a) only the x and the z coordinates change

(b) only the y and the z coordinates change

(c) only the x and the y coordinates change

(d) all the three coordinates change

8. Earth may be assumed to be an axially symmetric freely rotating rigid body. The ratio of the principal moments of inertia about the axis of symmetry and an axis perpendicular to it is 33:32. If T_0 is the time taken by earth to make one rotation around its axis of symmetry, then the time period of precession is closest to

[CSIR JUNE SEP 2022]

(a) $33T_0$ (b) $33T_0/2$

(c) $32T_0$ (d) $16T_0$

9. A uniform circular diss on the xy plane with its canter at the origin has a moment of inertia I_0 about the x-axis. If the disc is set in rotation about the origin with an angular velocity $\omega = \omega_0(\hat{j}+\hat{k})$ the direction of its angular momentum is along

[CSIR JUNE 2023]

(a) $-\hat{\imath}+\hat{j}+\hat{k}$ (b) $-\hat{\imath}+\hat{j}+2\hat{k}$

(c) $\hat{j}+2\hat{k}$ (d) $\hat{j}+\hat{k}$

10. A one-dimensional rigid rod is constrained to move inside a sphere such that its two ends are always in contact with the surface. The number of constraints on the Cartesian coordinates of the endpoints of the rod is

[CSIR JUNE 2023]

(a) 3 (b) 5

(c) 2 (d) 4

11. A uniform plane square sheet of mass m is centered at the origin of an inertial frame. The sheet is rotating about an axis passing through the origin. At an instant when all its vertices lie on x and y axes, the angular momentum is $\vec{L} = I_0\omega_0(2\hat{\imath}+\hat{j}+2\hat{k})$, where I_0 is the moment of inertia about the x axis. At this instant, the angular velocity of the sheet is

[CSIR-JUNE 2024]

(a) $(2\hat{\imath}+\hat{j}+2\hat{k})\omega_0$ (b) $(2\hat{\imath}+\hat{j}+\hat{k})\omega_0$

(c) $(2\hat{\imath}+\hat{j})\omega_0$ (d) $(\hat{\imath}+\hat{j})\omega_0$

❖ GATE PYQ's

1. A square lamina OABC of side l and negligible thickness is lying in the XOY plane of a Cartesian coordinate system such that O is at the origin and the sides OA and OC are along the positive X and Y directions respectively. Calculate the moment of inertia tensor and the directions of the three principal moments. The mass of the lamina is m.

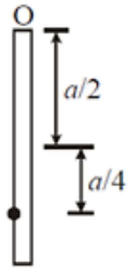
[GATE 2001]

2. A uniform thin circular disc of mass M and radius R lies in the X − Y plane with its centre at the origin. Find the moments of inertia tensor. What are the values of the principal moments of inertia? Find the principal axes.

[GATE 2002]

3. A particle of mass m is attached to a thin uniform rod of length a and mass 4 m. The distance of the particle from the center of mass of the rod is $a/4$. The moment of inertia of the combination about an axis passing through O normal to the rod is

[GATE 2004]



(a) $\frac{64}{48}ma^2$ (b) $\frac{91}{48}ma^2$

(c) $\frac{27}{48}ma^2$ (d) $\frac{51}{48}ma^2$

4. A circular hoop of mass M and radius a rolls without slipping with constant angular speed ω along the horizontal x-axis in the xy-plane. When the centre of the hoop is at a distance $d = \sqrt{2}a$ from the origin, the magnitude of the total angular momentum of the hoop about the origin is

[GATE 2005]

(a) $Ma^2\omega$ (b) $\sqrt{2}Ma^2\omega$

(c) $2\text{Ma}^2\omega$ (d) $3Ma^2\omega$

5. Two solid spheres of radius R and mass M each are connected by a thin rigid rod of negligible mass. The distance between the centres is $4R$. The moment of inertia about an axis passing through the centre of symmetry and perpendicular to the line joining the spheres is

[GATE 2005]

(a) $\frac{11}{5}MR^2$ (b) $\frac{22}{5}MR^2$

(c) $\frac{44}{5}MR^2$ (d) $\frac{88}{5}MR^2$

6. A particle of mass 2 kg is moving such that at time t, its position, in metre, is given by $\vec{r}(t) = 5\hat{\imath} - 2t^2\hat{\jmath}$. The angular momentum of the particle at $t = 2$ s about the origin, in kgm^2 s^{-1}, is

[GATE 2006]

(a) $-40\hat{k}$ (b) $-80\hat{k}$

(c) $80\hat{k}$ (d) $40\hat{k}$

7. A system of four particles is in xy-plane. Of these, two particles each of mass m are located at (1,1) and $(-1,-1)$. The remaining two particles each of mass $2m$ are located at $(-1,1)$ and $(1,-1)$. The xy component of the moment of inertia tensor of this system of particles is

[GATE 2006]

(a) 10 m (b) $-10m$

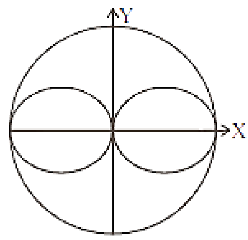
(c) $2m$ (d) $-2m$

8. The moment of inertia of a uniform sphere of radius r about an axis passing through its centre is given by

$$\frac{2}{5}\left(\frac{4\pi}{3}r^5\rho\right)$$

. A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radius $R/2$ hollowed out of it, as shown in the figure. The moment of inertia of the resulting body about the Y axis is

[GATE 2007]



(a) $\frac{\pi\rho R^5}{4}$ (b) $\frac{5\pi\rho R^5}{12}$

(c) $\frac{7\pi\rho R^5}{12}$ (d) $\frac{3\pi\rho R^5}{4}$

9. A rigid body is rotating about its Centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, the rate of change of its kinetic energy is

[GATE 2008]

(a) $\frac{1}{2}\vec{\tau}\cdot\vec{\omega}$ (b) $\frac{1}{2}\vec{L}\cdot\vec{\omega}$

(c) $\frac{1}{2}(\vec{\tau}\vec{\omega} + \vec{L}\vec{\alpha})$ (d) $\frac{1}{2}\vec{L}\cdot\vec{\alpha}$

10. The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of inertia about an axis

$$\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

is

[GATE 2008]

(a) 6 (b) 5

(c) 2 (d) 8/3

11. A heavy symmetrical top is rotating about its own axis of symmetry (the z-axis). If I_1, I_2 and I_3 are the principal moments of inertia along x, y and z axes respectively then

[GATE 2011]

(a) $I_2 = I_3; I_1 \neq I_2$ (b) $I_1 = I_3; I_1 \neq I_2$

(c) $I_1 = I_2; I_1 \neq I_3$ (d) $I_1 \neq I_2 \neq I_3$

12. Two uniform thin rods of equal length, L, and masses M_1 and M_2 are joined together along the length. The moment of inertia of the combined rod of length $2L$ about an axis passing through the mid-point and perpendicular to the length of the rod is

[GATE 2012]

(a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$

(c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

13. A uniform circular disk of radius R and mass M is rotating with angular speed ω about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is

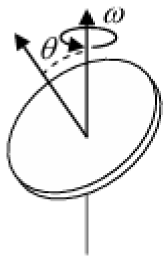
[GATE 2013]

(a) $\frac{\sqrt{3}}{4}\omega MR^2$ (b) $\frac{\sqrt{3}}{8}\omega MR^2$

(c) $\frac{\sqrt{7}}{8}\omega MR^2$ (d) $\frac{\sqrt{7}}{4}\omega MR^2$

14. A uniform circular disc of mass m and radius R is rotating with angular speed ω about an axis passing through its centre and making an angle $\theta = 30°$ with the axis of the disc. If the kinetic energy of the disc is $\alpha m\omega^2 R^2$, the value of α is (up to two decimal places).

[GATE 2018]



15. A hoop of mass M and radius R rolls without slipping along a straight line on a horizontal surface as shown in the figure. A point mass m slides without friction along thee inner surface of the hoop, performing small oscillations about the mean position. The number of degrees of freedom of the system (in integer) is

[GATE 2021]

16. A symmetric top has principal moments of inertia $I_1 = I_2 = \frac{2\alpha}{3}, I_3 = 2\alpha$ about a set of principal axes 1,2,3 respectively, passing through its center of mass, where α is a positive constant. There is no force acting on the body and the angular speed of the body about the 3-axis is $\omega_3 = \frac{1}{8}$ rad/s. With what angular frequency in rad/s does the angular velocity vector $\vec{\omega}_1$ precess about the 3 -axis?

[GATE 2023]

(a) 2 (b) 3

(c) 5 (d) 7

❖ JEST PYQ's

1. A cylindrical shell of mass in has an outer radius b and an inner radius a. The moment of inertia of the shell about the axis of the cylinder is:

[JEST 2016]

(a) $\frac{1}{2}m(b^2 - a^2)$ (b) $\frac{1}{2}m(b^2 + a^2)$

(c) $m(b^2 + a^2)$ (d) $m(b^2 - a^2)$

2. What is the change in the kinetic energy of rotation of the earth if its radius shrinks by 1% ? Assume that the mass remains the same and the density is uniform.

[JEST 2019]

(a) increases by 1% (b) increases by 2%
(c) decreases by 1% (d)decreases by 2%

3. A small insect of mass m is sitting on the rim of a uniform circular horizontal disk of radius R and mass M. The system is rotating at a constant angular velocity ω_i about a frictionless vertical axis passing through the center of the disk. The insect started to crawl towards the center of the disk. Assume $\frac{M}{m} = 10$, and let the final angular velocity of the system, when the insect reaches the centre of the disk be ω_f. What is the value of $\frac{100\omega_f}{\omega_i}$?

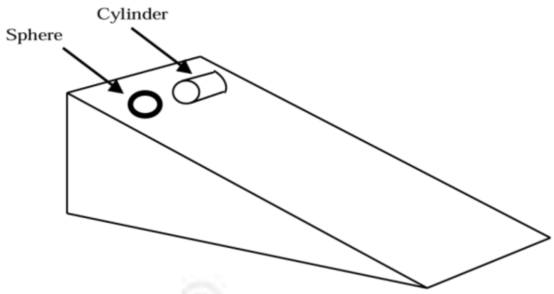
[JEST 2020]

4. A cleaning machine presses a circular mop of radius $R = 30$ cm vertically down on a floor with a total force $F = 25$ N and rotates it with a constant angular speed about the vertical axis passing through the centre of mop. If the force is distributed uniformly over the mop and if the coefficient of friction between the mop and the floor is $\mu = 0.25$, what is the value of torque in $N -$ cm applied by the machine on the mop?

[JEST 2020]

5. A solid sphere and a solid cylinder, both of uniform mass density, start rolling down without slipping from rest from the same height along an inclined plane (see figure). Which one of the following statements is correct?

[JEST 2021]



(a) The sphere would reach the bottom faster.

(b) The cylinder would reach the bottom faster.

(c) The heavier one would reach the bottom faster if both have identical radii.

(d) Both the objects would reach the bottom at the same time if their radii are identical.

6. If $\vec{x}_A$ and $\vec{x}_B$ are the position vectors of two points on a rigid body, which one of the following is NOT necessarily true?

[JEST 2021]

(a) $\ddot{\vec{x}}_A - \ddot{\vec{x}}_B = 0$

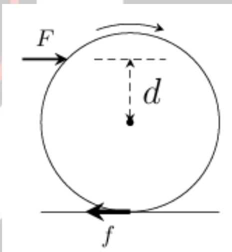
(b) $(\vec{x}_A - \vec{x}_B) \cdot (\dot{\vec{x}}_A - \dot{\vec{x}}_B) = 0$

(c) $(\vec{x}_A - \vec{x}_B) \cdot (\ddot{\vec{x}}_A - \ddot{\vec{x}}_B) + |\dot{\vec{x}}_A - \dot{\vec{x}}_B|^2 = 0$

(d) $\frac{d}{dt}|\vec{x}_A - \vec{x}_B| = 0$

7. A cylinder of radius R is constrained to roll without slipping on a horizontal plane under the action of a constant force F applied d distance above the axis of the cylinder. In the process, it experiences a frictional force f at the point of contact (see figure). For what value of d, the magnitude of f is minimum?

[JEST 2022]

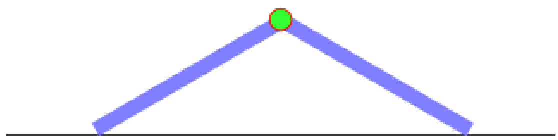


(a) R (b) $R/2$

(c) $-R/2$ (d) $-R$

8. Two uniform rods of length 1 m are connected to a friction-less hinge (a) The hinge is held at a height and the other ends of the rods rests on a friction-less plane, such that the angle between the rods is $2\pi/3$. If the hinge is released from the rest, what is the speed of the hinge when it hits the floor? [Acceleration due to gravity is 9.81 ms^{-2}]

[JEST 2022]



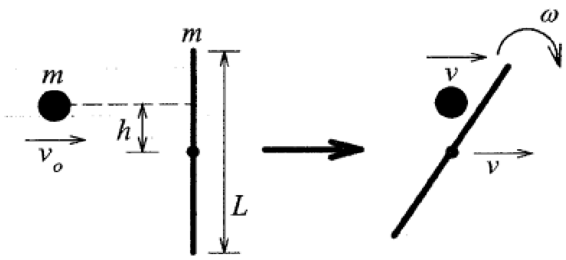
9. A cylindrical rigid block has principal moments of inertia I about the symmetry axis and $2I$ about each of the perpendicular axes passing through the center of mass. At some instant, the components of angular momentum about the center of mass in the body-fixed principal axis frame is (l, l, l), with $l > 0$. What is the cosine of the angle between the angular momentum and the angular velocity? **[JEST 2024]**

(a) $\frac{2}{\sqrt{6}}$ (b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5}{3\sqrt{3}}$

❖ TIFR PYQ's

1. A mass m travels in a straight line with velocity v_0 perpendicular to a uniform stick of mass m and length L, which is initially at rest. The distance from the centre of the stick to the path of the travelling mass is h (see figure). Now the travelling mass m collides elastically with the stick, and the centre of the stick and the mass m are observed to move with equal speed v after the collision. Assuming that the travelling mass m can be treated as a point mass, and the moment of inertia of the stick about its center is $I = \frac{mL^2}{12}$, it follows that the distance h must be **[TIFR 2010]**

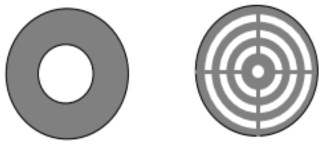


(a) $\frac{L}{2}$ (b) $\frac{L}{4}$

(c) $\frac{L}{\sqrt{6}}$ (d) $\frac{L}{\sqrt{3}}$

(e) $\frac{L}{3}$ (f) zero

2. A scientist is given two heavy spheres made of the same metal, which have the same diameter and weight, and is asked to distinguish the spheres, without damaging them in any way. Though the spheres look identical, one of them is actually a hollow spherical shell, while the other is a set of concentric shells mounted on four thin rods of the same metal (see figure).



To make this distinction, the scientist must perform an experiment where each sphere is **[TIFR 2011]**

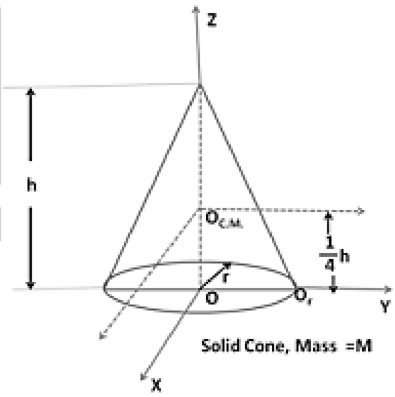
(a) set rotating under the action of a constant torque

(b) made into the bob of a long simple pendulum and set oscillating

(c) immersed fully in a non-corrosive liquid and then weighed

(d) given the same electric charge Q and the potential is measured

3. Consider the uniform solid right cone depicted in the figure on the right. This cone has mass M and a circular base of radius r. If the moment of inertia of the cone about an axis parallel to the X axis passing through the centre of mass $O_{C.M}$ (see figure) is given by $\frac{3}{80}M(4r^2 + h^2)$



then the moment of inertia about another axis parallel to the X axis, but passing through the point O_r (see figure), is **[TIFR 2013]**

(a) $\frac{3}{80}M(4r^2 + h^2)$

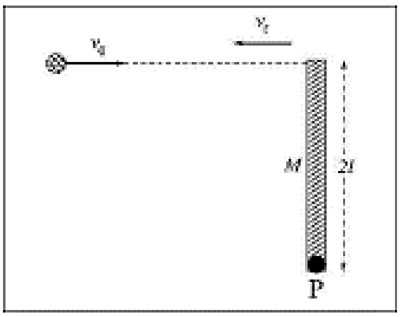
(b) $\frac{1}{20}M(23r^2 + 2h^2)$

(c) $\frac{3}{40}M(2r^2+h^2)$

(d) $\frac{1}{30}M(15r^2+4h^2)$

4. A thin uniform rod of length $2l$ and mass M is pivoted at one end P on a horizontal plane (see figure). A ball of mass $m \ll M$ and speed v_0 strikes the free end of the rod perpendicularly and bounces back with velocity v_f along the original line of motion as shown in the fig. If the collision is perfectly elastic the magnitude of v_f is

[TIFR 2015]

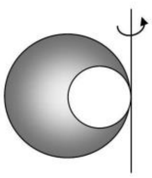


(a) $\frac{M-4m}{M+4m}v_0$ (b) $\frac{M+4m}{M-4m}v_0$

(c) $\frac{M-3m}{M+3m}v_0$ (d) $\frac{M+3m}{M-3m}v_0$

5. The three-dimensional object sketched on the right is made by taking a solid sphere of uniform density (shaded) with radius R, and scooping out a spherical cavity (unshaded) as shown, which has diameter R.
If this object has mass M, its moment of inertia about the tangential axis passing through the point where the spheres touch (as shown in the figure) is

[TIFR 2019]

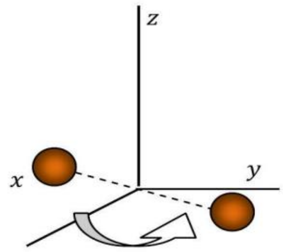


(a) $\frac{3}{16}MR^2$ (b) $\frac{62}{35}MR^2$

(c) $\frac{31}{70}MR^2$ (d) $\frac{31}{20}MR^2$

6. Consider a diatomic molecule of oxygen which is rotating in the xy-plane about the z axis. The z axis passes through the centre of the molecule and is perpendicular to its length. At room temperature, the average separation between the two oxygen atoms is 1.21×10^{-10} m (the atoms are treated as point masses). The molar mass of oxygen is 16gm/mol. If the angular velocity of the molecule about the z axis is 2×10^{12} rad/s, its rotational kinetic energy will be closest to

[TIFR 2021]

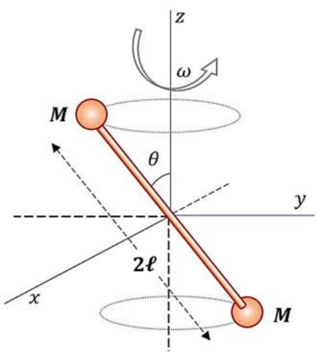


(a) 1.95×10^{-22} Joule (b) 7.78×10^{-22} Joule

(c) 15.56×10^{-22} Joule (d) 3.89×10^{-22} Joule

7. A dumbbell consists of two small spherical masses M each, connected by a thin massless rod of length 2ℓ. This dumbbell is centered at the origin, and is rotating about the z-axis with a uniform angular velocity ω, making an angle θ with the z-axis (see figure). Neglecting effects due to gravity, at the instant when the dumbbell is wholly in the yz-plane (as shown in the figure), the magnitude of torque about the origin will be

[TIFR 2022]

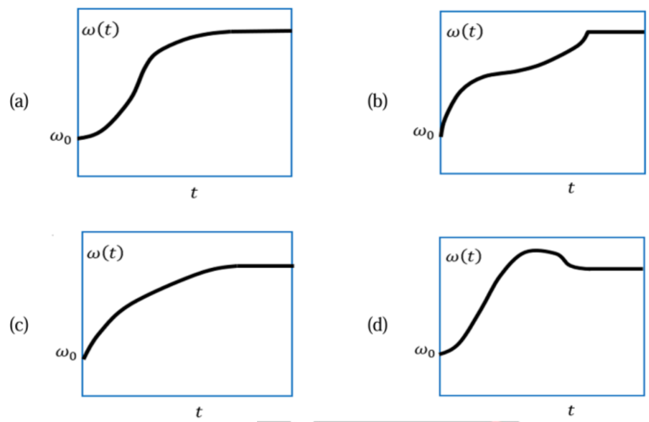


(a) zero (b) $2M\ell^2\omega^2\sin^2\theta$

(c) $2M\ell^2\omega^2\cos^2\theta$ (d) $M\ell^2\omega^2\sin 2\theta$

8. A hollow metal sphere filled with a thick, highly viscous oil is rotating about a vertical axis with an initial angular velocity ω_0. However, there is a small hole at the bottom of this sphere, through which drops of oil are leaking out vertically at a steady rate. The variation of the angular velocity $\omega(t)$ of the sphere with time t is best represented graphically by

[TIFR 2022]



Answers key				
CSIR-NET				
1. c/b	2. b	3. b	4. c	5. d
6. c	7. d	8. c	9. c	10. a
11. b				
GATE				
1.	2.	3. b	4. c	5. c
6. b	7. c	8. b	9. c	10. b
11. c	12. c	13. c	14. 0.21	15. 2
16. a				
JEST				
1. b	2. b	3. 0120	4. 0125	5. a
6. a	7. b	8. 1.92	9. b	
TIFR				
1.	2. a	3. b	4. c	5. d
6. d	7. d	8. a		

❖ CSIR-NET PYQ

1. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass ' m ' horizontally with speed ' v '. The acceleration of the bullet, in the reference frame of the shooter, is **[CSIR JUNE 2012]**

(a) $2v\Omega$ to his right (b) $2v\Omega$ to his left

(c) $v\Omega$ to his right (d) $v\Omega$ to his left

2. A turn-table is rotating with a constant angular velocity ω_0. In the rotating frame fixed to the turn: table, a particle moves radially outwards at a constant speed v_0. The acceleration of the particle in the $r\theta$-coordinates, as seen from an inertial frame, the origin of which is at the centre of the turt: table, is **[CSIR JUNE 2019]**

(a) $-r\omega_0^2\hat{r}$ (b) $2r\omega_0^2\hat{r}+v_0\omega_0\hat{\theta}$

(c) $r\omega_0^2\hat{r}+2v_0\omega_0\hat{\theta}$ (d) $-r\omega_0^2\hat{r}+2v_0\omega_0\hat{\theta}$

3. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius a is placed on it at a distance r from the centre of the table, its linear velocity will be

(a) $-r\omega\hat{r}+a\omega\hat{\theta}$ (b) $r\omega\hat{r}+a\omega\hat{\theta}$

(c) $a\omega\hat{r}+r\omega\hat{\theta}$ (d) 0 (zero)

4. A particle of mass m moves in a potential that is

$$V=\frac{1}{2}m(\omega_1^2x^2+\omega_2^2y^2+\omega_3^2z^2)$$

in the coordinates of a non-inertial frame F. The frame F is rotating with respect to an inertial frame with an angular velocity $\hat{k}\Omega$, where $\hat{k}$ is the unit vector along their common z-axis. The motion of the particle is unstable for all angular frequencies satisfying **[CSIR FEB 2022]**

(a) $(\Omega^2-\omega_1^2)(\Omega^2-\omega_2^2)>0$

(b) $(\Omega^2-\omega_1^2)(\Omega^2-\omega_2^2)<0$

(c) $(\Omega^2-(\omega_1+\omega_2)^2)(\Omega^2-|\omega_1-\omega_2|^2)>0$

(d) $(\Omega^2-(\omega_1+\omega_2)^2)(\Omega^2-|\omega_1-\omega_2|^2)<0$

❖ GATE PYQ

1. A rigid frictionless rod rotates anticlockwise in a vertical plane with angular velocity $\vec{\omega}$. Abead of mass m moves outward along the rod with constant velocity $\vec{u}_0$. The bead will experience a coriolis force **[GATE 2004]**

(a) $2mu_0\omega\hat{\theta}$ (b) $-2mu_0\omega\hat{\theta}$

(c) $4mu_0\omega\hat{\theta}$ (d) $-mu_0\omega\hat{\theta}$

2. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed ω. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If r is the distance of the mass from the axis of rotation and v is its speed then the magnitude of the Coriolis force is **[GATE 2005]**

(a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$

(c) $mv\omega$ (d) $2mv\omega$

3. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If $\vec{F}_i$ and $\vec{F}_{co}$ denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then **[GATE 2015]**

(a) $\vec{F}_c$ is radially outward and $\vec{F}_{co}=\vec{F}_c$

(b) $\vec{F}_c$ is radially inward and $\vec{F}_{co}=-2\vec{F}_c$

(c) F_c is radially outward and $F_{C_0}=-2F_c$

(d) $\vec{F}_c$ is radially outward and $\vec{F}_{co}=2\vec{F}_c$

❖ JEST PYQ

1. Two objects of unit mass are thrown up vertically with a velocity of 1 ms^{-1} at latitudes 45^0 N and 45°S, respectively. The angular velocity of the rotation of Earth is given to be $7.29\times10^{-5}\ \mathrm{s}^{-1}$. In which direction will the objects deflect when they reach their highest point (due to Coriolis force)? Assume zero air resistance. **[JEST 2019]**

(a) to the east in Northern hemisphere and west in Southern Hemisphere

(b) to the west in Northern hemisphere and east in Southern Hemisphere

(c) to the east in both hemispheres

(d) to the west in both hemispheres

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where, k is a constant. The power delivered to the particle by the force acting on it is

[JEST 2022]

(a) $2\pi mk^{\frac{3}{2}}r^2$ (b) mk^2r^2t

(c) $\frac{1}{2}mk^2r^2t$ (d) 0

Answer Key					
CSIR NET					
1. a	2. d	3. d	4. b		
GATE					
1. b	2. d	3. d			
JEST					
1. d	2. b				

❖ CSIR-NET PYQ's

1. The Hamiltonian of a classical particle moving in one dimension is

$$H = \frac{p^2}{2m} + \alpha q^4$$

where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E \le E_0$ the available volume of phase space depends on E_0 as

[CSIR DEC 2014]

(a) $E_0^{3/4}$

(b) E_0

(c) $\sqrt{E_0}$

(d) is independent of E_0

2. The energy of a one-dimensional system, govern med by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^{2n}$$

, where k and n are two positive constants, is E_0. The time period of oscillation τ satisfies

[CSIR JUNE 2017]

(a)$\tau \propto k^{-1/n}$

(b) $\tau \propto k^{-1/2n} E_0^{\frac{1-n}{2n}}$

(c)$\tau \propto k^{-1/2n} E_0^{\frac{n-2}{2n}}$

(d) $\tau \propto k^{-1/t} E_0^{\frac{1+n}{2n}}$

3. Consider a set of particles which interact by a pair potential $V = ar^6$, where r is the inter-particle separation and $a > 0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is

[CSIR DEC 2017]

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

4. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $\alpha > 0$ is a constant. If the total energy of the particle is E, its time period in a periodic motion is proportional to

[CSIR JUNE 2018]

(a) $E^{-1/3}$

(b) $E^{-1/2}$

(c) $E^{1/3}$

(d) $E^{1/2}$

5. The motion of a particle in one-dimension is described by the Lagrangian

$$L = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 - x^2\right)$$

in suitable units. The value of the action along the classical path from $x = 0$ at $t = 0$ to $x = x_0$ at $t = t_0$, is

[CSIR DEC 2018]

(a) $\frac{x_0^2}{2\sin^2 t_0}$

(b) $\frac{1}{2}x_0^2 \tan t_0$

(c) $\frac{1}{2}x_0^2 \cot t_0$

(d) $\frac{x_0^2}{2\cos^2 t_0}$

6. A particle of mass m moves in one dimension in the potential $V(x) = kx^4$, $(k > 0)$. At time $t = 0$, the particle starts from rest at $x = A$. For bounded motion, the time period of its motion is

[CSIR JUNE 2019]

(a) proportional to $A^{-1/2}$

(b) proportional to A^{-1}

(c) independent of A

(d) not well-defined (the system is chaotic)

7. A particle in one dimension executes oscillatory motion in a potential $V(x) = A|x|$, where $A > 0$ is a constant of appropriate dimension. If the time period T of its oscillation depends on the total energy E as E^{α}, then the value of α is

[CSIR JUNE 2021]

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

8. A particle in one dimension executes oscillatory motion in a potential $V(x) = A|x|$, where $A > 0$ is a constant of appropriate dimension. If the time period T of its oscillation depends on the total energy E as E^{α}, then the value of α is

[CSIR FEB 2022]

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

❖ GATE PYQ's

1. Consider the motion of a particle along the x-axis in a potential $V(x) = F|x|$. Its ground state energy E_0 is estimated using the uncertainty principle. Then E_0 is proportional to

[GATE 2019]

(a) $F^{\frac{1}{3}}$ (b) $F^{\frac{1}{2}}$

(c) $F^{\frac{2}{5}}$ (d) $F^{\frac{2}{3}}$

2. A projectile of mass 1 kg is launched at an angle of 30° from the horizontal direction at $t = 0$ and takes time T before hitting the ground. If its initial speed is $10\ \mathrm{ms}^{-1}$, the value of the action integral for the entire flight in the units of $\mathrm{kgm}^2\ \mathrm{s}^{-1}$ (rounded off to one decimal place) is [Take $g = 10\ \mathrm{ms}^{-2}$] [GATE 2019]

3. A particle of mass 1 kg is released from a height of 1 m above the ground. When it reaches the ground, what is the value of Hamilton's action for this motion in Js ? (g is the acceleration due to gravity; take gravitation potential to be zero on the ground)

[GATE 2022]

(a) $-\frac{2}{3}\sqrt{2g}$ (b) $\frac{5}{3}\sqrt{2g}$

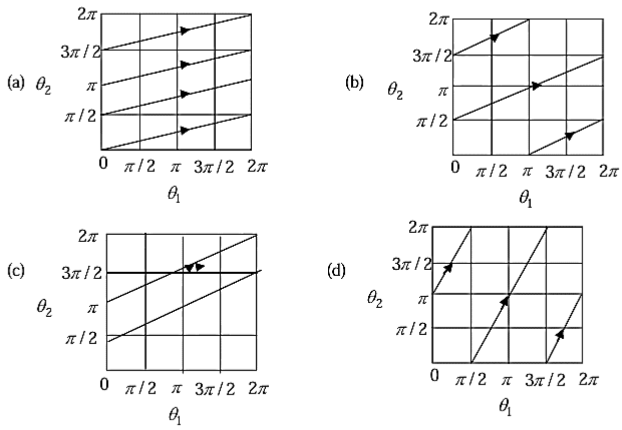
(c) $3\sqrt{2g}$ (d) $-\frac{1}{3}\sqrt{2g}$

4. In the action-angle variables $(I_1, I_2, \theta_1, \theta_2)$ consider the Hamiltonian $H = 4I_1I_2$ and $0 \leq \theta_1, \theta_2 < 2\pi$. Let

$$\frac{I_1}{I_2} = \frac{1}{2}$$

. Which of the following are possible plots of the trajectories with different initial conditions in $\theta_1 - \theta_2$ plane?

[GATE 2022]



❖ JEST PYQ's

1. A classical particle with total energy E moves under the influence of a potential $V(x,y) = 3x^3 + 2x^2y + 2xy^2 + y^3$. The average potential energy, calculated over a long time is equal to,

[JEST 2015]

(a) $\frac{2E}{3}$ (b) $\frac{E}{3}$

(c) $\frac{E}{5}$ (d) $\frac{2E}{5}$

2. A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2 + y^2)$, under the action of gravity acting in the negative z-direction. $\vec{g} = -g\hat{z}$. What speed should he maintain to be able to ride at a constant height z_0 without failing down?

[JEST 2015]

(a) $\sqrt{gz_0}$

(b) $\sqrt{3gz_0}$

(c) $\sqrt{2gz_0}$

(d) The biker will not be able to maintain a constant height, irrespective of speed.

3. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

[JEST 2017]

(a) $\sqrt{3g}$ (b) $\sqrt{g}$

(c) $\sqrt{2g}$ (d) $\sqrt{4g}$

4. Consider a particle with total energy E is oscillating in a potential $U(x) = A|x|^n$ with $A > 0$ and $n > 0$ in one dimension. Which one of the following gives the relation between the time-period of oscillation T and the total energy E :

[JEST 2020]

(a) $T \propto E^{1/n-1/2}$ (b) $T \propto E^0$

(c) $T \propto E^n$ (d) $T \propto E^{1/n}$

5. The action corresponding to the motion of a particle in one dimension is:

$$S = \int_{t_i}^{t_f} dt \left[\frac{1}{2}m\dot{x}^2 - V(x) + \alpha x\ddot{x} + \beta x\dot{x}\right]$$

where m is the mass of the particle, α, β are constants, and $V(x)$ is a potential which is a function of x. The position and velocity are held fixed at the end points of the trajectory. The equation of motion of the particle is

[JEST 2023]

(a) $(2\alpha + m)\ddot{x} - \frac{dV}{dx} = 0$

(c) $(2\alpha - m)\ddot{x} - \beta\dot{x} - \frac{dV}{dx} = 0$

(b) $(2\alpha - m)\ddot{x} + \beta\dot{x} - \frac{dV}{dx} = 0$

(d) $(2\alpha - m)\ddot{x} - \frac{dV}{dx} = 0$

6. A classical system has the following action:

$$S = \int (\dot{q}^2 + \alpha q\dot{q} + \beta q^2\dot{q})dt,$$

where q is the generalized coordinate, and α and β are constants. Which of the following statements is true about the dynamics of the system?

[JEST 2024]

(a) The dynamics depends on the ratio $\frac{\alpha}{\beta}$.

(b) The dynamics depends only on α.

(c) The dynamics depends only on β.

(d) The dynamics is independent of α and β.

❖ TIFR PYQ's

1. The Hamiltonian of a particle of charge q and mass m in an electromagnetic field is given by

$$H = \frac{1}{2m}|\vec{p} - q\vec{A}(\vec{x},t)|^2 + q\varphi(\vec{x},t)$$

where $(\varphi, \vec{A})$ are the electromagnetic potentials. Clearly this Hamiltonian changes under a gauge transformation

$$\varphi \to \varphi - \frac{\partial\chi}{\partial t} \quad \vec{A} \to \vec{A} + \vec{\nabla}\chi$$

where $\chi(\vec{x},t)$ is a gauge function. Nevertheless the motion of the particle is not affected because

[TIFR 2018]

(a) the Lagrangian does not change under the gauge transformation.

(b) the motion of the particle is correctly described only in the Lorenz gauge.

(c) the action of the particle changes only by surface terms which do not vary.

(d) the Lorentz force is modified to balance the effect of the gauge transformation.

2. A simple pendulum is oscillating freely in the vertical plane. If the string is shortened very slowly to half its length, the angular amplitude $\theta_{\max}$ will change by a factor **[TIFR 2023]**

(a) $2^{3/4}$ (b) $\sqrt{2}$

(c) 2 (d) Does not change.

3. A simple pendulum is oscillating freely in the vertical plane. If the string is shortened very slowly to half its length, the angular amplitude $\theta_{\max}$ will change by a factor

[TIFR 2023]

(a) $2^{3/4}$ (b) $\sqrt{2}$

(c) 2 (d) Does not change.

❖ Answer Key				
CSIR-NET				
1. a	2. b	3. c	4. a	5. c
6. b	7. b	8. b		
GATE				
1. d	2. 33.3	3. d	4. b,c	
JEST				
1. d	2. c	3. d	4. a	5. d
6. d				
TIFR				
1. c	2. a	3. a		

❖ **CSIR-NET PYQ's**

1. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are M_t, M_π and zero respectively.

[CSIR JUNE 2011]

(A) The energy of π^- is:

(a) $\frac{(M_\tau^2 - M_\pi^2)c^2}{2M_\tau}$ (b) $\frac{(M_r^2 + M_\pi^2)c^2}{2M_r}$

(c) $(M_r - M_\pi)c^2$ (d) $\sqrt{M_\tau M_\pi} c^2$

(B) The velocity is π^- is:

(a) $\frac{(M_\tau^2 - M_\pi^2)c}{M_\tau^2 + M_\pi^2}$ (b) $\frac{(M_r^2 - M_\pi^2)c}{M_r^2 - M_\pi^2}$

(c) $\frac{M_\pi c}{M_\tau}$ (d) $\frac{M_\tau c}{M_\pi}$

2. A constant force F is applied to a relativistic particle of rest mass m. If the particle starts from rest at $t = 0$, its speed after a time t is

[CSIR DEC 2011]

(a) Ft/m (b) $\text{ctanh}\left(\frac{\text{Ft}}{\text{mc}}\right)$

(c) $c\left(I - e^{-\text{Ft/mc}}\right)$ (d) $\frac{\text{Fct}}{\sqrt{F^2t^2 + m^2c^2}}$

3. Two events, separated by a (spatial) distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8c (where the speed of light $c = 3 \times 10^8$ m/s) is:

[CSIR JUNE 2012]

(a) 60 s (b) 40 s

(c) 20 s (d) 0 s

4. What is the proper time interval between the occurrence of two events if in one inertial frame the events are separated by 7.5×10^5 m and occur 6.5 s apart?

[CSIR JUNE 2012]

(a) 6.50 s (b) 6.00 s

(c) 5.75 s (d) 5.00 s

5. If a Higgs boson of mass m_H with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is

[CSIR JUNE 2012]

[Note: The invariant mass of a system of two particles, with four-momenta p_1 and p_2 is $(p_1 + p_2)^2$]

(a) βm_H (b) m_H

(c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$

6. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass ' m '. Then

[CSIR DEC 2012]

(a) $\frac{dE}{dp} = \text{constant}$ (b) $p = mv$

(c) $v = \frac{cp}{\sqrt{p^2 + m^2c^2}}$ (d) $E = mc^2$

7. A binary star system consists of two stars S_1 and S_2, with masses m and $2m$ respectively separated by a distance ' r '. If both S_1 and S_2 individually follow circular orbits around the centre of the mass with intantaneous speeds v_1 and v_2 respectively, the ratio of speeds v_1/v_2 is:

[CSIR DEC 2012]

(a) $\sqrt{2}$ (b) 1

(c) ½ (d) 2

8. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum)

[CSIR JUNE 2013]

(a) $\left(1 - \frac{u^2}{c^2}\right)^{1/2}$ (b) $\left(1 - \frac{u^2}{c^2}\right)^{-1/2}$

(c) $\left(1 - \frac{u^2}{c^2}\right)$ (d) $\left(1 - \frac{u^2}{c^2}\right)^{-1}$

9. A light source is switched on and off at a constant frequency f. An observer moving with a velocity

u with respect to the light source will observe the freuqency of the switching to be

[CSIR JUNE 2014]

(a) $f\left(1-\frac{u^2}{c^2}\right)^{-1}$ (b) $f\left(1-\frac{u^2}{c^2}\right)^{-1/2}$

(c) $f\left(1-\frac{u^2}{c^2}\right)$ (d) $f\left(1-\frac{u^2}{c^2}\right)^{1/2}$

10. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is:

[CSIR DEC 2014]

(a) $v=c\sqrt{1-\frac{mc^2}{E}}$

(b) $v=\sqrt{\frac{2E}{m}}\left(1+\frac{mc^2}{E}\right)$

(c) $v=c\sqrt{1-\left(\frac{mc^2}{R}\right)^2}$

(d) $v=c\left(1+\frac{mc^2}{E}\right)$

11. Consider three inertial frames of reference A, B, and C. The frame B moves with a velocity $c/2$ with respect to A and C moves with a velocity $c/10$ with respect to B in the same direction. The velocity of C as measured in A is

[CSIR JUNE 2015]

(a) $\frac{3c}{7}$ (b) $\frac{4c}{7}$

(c) $\frac{c}{7}$ (d) $\frac{\sqrt{3}c}{7}$

12. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, the charge per unit length (as measured by the moving observer) is

[CSIR JUNE 2015]

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$ (b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$

(c) $\frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{0}{1\left(1-\frac{v^2}{v^2}\right)}$

13. Consider a particle of mass m moving with a speed v. If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then the value of $(T_R-T_N)/T_R$ for $v=0.01\text{c}$, is

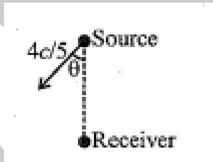
[CSIR DEC 2015]

(a) 1.25×10^{-5} (b) 5.0×10^{-5}

(c) 7.5×10^{-5} (d) 1.0×10^{-4}

14. A distant source, emitting radiation of frequency ω, moves with a velocity $4c/5$ in a certain direction with respect to a receiver (as shown in the figure).

[CSIR DEC 2015]



The upper cut-off frequency of the receiver is $3\omega/2$. Let θ be the angle as shown. For the receiver to detect the radiation, θ should at least be

[CSIR DEC 2015]

(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{3}{4}\right)$

(c) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (d) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

15. Let (x,t) and (x',t') be the coordinate systems used by the observers O and O', respectively. Ob. server O' moves with a velocity $\text{v}=\beta c$ along their common positive x-axis. If $x_+=x+ct$ and $x_-=x-ct$ are the linear combinations of the coordinates, the Lorentz transformation relating O and O' takes the form

[CSIR JUNE 2016]

(a) $x'_+=\frac{x_--\beta x_+}{\sqrt{1-\beta^2}}$ and $x'_-=\frac{x_+-\beta x_-}{\sqrt{1-\beta^2}}$

(b) $x'_+=\sqrt{\frac{1+\beta}{1-\beta}}x_+$ and $x'_-=\sqrt{\frac{1-\beta}{1+\beta}}x_-$

(c) $x'_+ = \dfrac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$ and $x'_- = \dfrac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$

(d) $x'_+ = \sqrt{\dfrac{1-\beta}{1+\beta}}\, x_+$ and $x'_- = \sqrt{\dfrac{1+\beta}{1-\beta}}\, x_-$

16. For a particle of energy E and momentum p (in a frame F), the rapidity y is defined as
$$y = \frac{1}{2}\ln\left(\frac{E+p_3c}{E-p_3c}\right)$$
. In a frame F' moving with velocity $v = (0,0,\beta c)$ with respect to F, the rapidity y' will be

[CSIR JUNE 2016]

(a) $y' = y + \frac{1}{2}\ln(1-\beta^2)$

(b) $y' = y - \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$

(c) $y' = y + \ln\left(\frac{1+\beta}{1-\beta}\right)$

(d) $y' = y + 2\ln\left(\frac{1+\beta}{1-\beta}\right)$

17. A relativistic particle moves with a constant velocity v with respect to the laboratory frame. In time τ, measured in the rest frame of the particle, the distance that it travels in the laboratory frame is

[CSIR DEC 2016]

(a) $v\tau$

(b) $\dfrac{c\tau}{\sqrt{1-\frac{v^2}{c^2}}}$

(c) $v\tau\sqrt{1-\frac{v^2}{c^2}}$

(d) $\dfrac{v\tau}{\sqrt{1-\frac{v^2}{c^2}}}$

18. Consider a radioactive nucleus that is travelling at a speed $c/2$ with respect to the lab frame. It emits γ-rays of frequency v_0 in its rest frame. There is a stationary detector (which is not on the path of the nucleus) in the lab. If a γ-ray photon is emitted when the nucleus is closest to the detector, its observed frequency at the detector is

[CSIR DEC 2016]

(a) $\frac{\sqrt{3}}{2}v_0$

(b) $\frac{1}{\sqrt{3}}v_0$

(c) $\frac{1}{\sqrt{2}}v_0$

(d) $\sqrt{\frac{2}{3}}v_0$

19. An inertial observer sees two events E_1 and E_2 happening at the same location but 6μ apart in time. Another observer moving with a constant velocity v (with respect to the first one) sees the same events to be 9μ s apart. The spatial distance between the events, as measured by the second observer, is approximately

[CSIR JUNE 2017]

(a) 300 m

(b) 1000 m

(c) 2000 m

(d) 2700 m

20. A light signal travels from a point A to a point B, both within a glass slab that is moving with uniform velocity (in the same direction as the light) with speed 0.3c with respect to an external observer. If the refractive index of the slab is 1.5 , then the observer will measure the speed of the signal as

[CSIR DEC 2017]

(a) $0.67c$

(b) $0.81c$

(c) $0.97c$

(d) c

21. Two particles A and B move with relativistic velocities of equal magnitude v, but in opposite directions, along the x-axis of an inertial frame of reference. The magnitude of the velocity of A, as seen from the rest frame of B, is

[CSIR JUNE 2018]

(a) $\dfrac{2v}{\left(1-\frac{v^2}{c^2}\right)}$

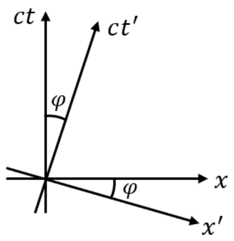
(b) $\dfrac{2v}{\left(1+\frac{v^2}{c^2}\right)}$

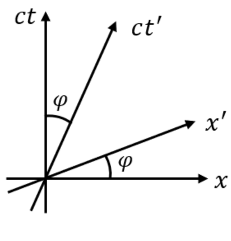
(c) $2v\sqrt{\dfrac{c-v}{c+v}}$

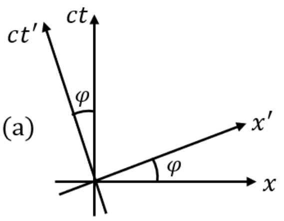
(d) $\dfrac{2v}{\sqrt{1-\frac{v^2}{c^2}}}$

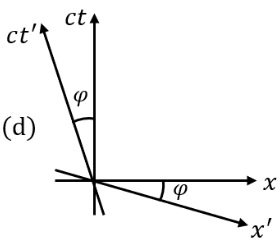
22. An inertial frame K' moves with a constant speed v with respect to another inertial frame K along their common x-axis in the positive x-direction. Let (x, ct) and (x', ct') denote the space-time coor dinates in the frame K and K', respectively. Which of the following space-time diagrams correctly describes the t'-axis ($x' = 0$ line) and the x'-axis ($t' = 0$ line) in the $x - ct$ plane ? (In the following figures tan $\varphi = v/c$).

[CSIR JUNE 2018]

(a) 

(b) 

(a) 

(d) 

23. A relativistic particle of mass m and charge e is moving in a uniform electric field of strength b. Starting from rest at $t = 0$, how much time will it take to reach the speed $c/2$?

[CSIR DEC 2018]

(a) $\frac{1}{\sqrt{3}}\frac{mc}{e\varepsilon}$ (b) $\frac{mc}{e\varepsilon}$

(c) $\sqrt{2}\frac{mc}{e\varepsilon}$ (d) $\sqrt{\frac{3}{2}\frac{mc}{e\varepsilon}}$

24. Consider the decay $A \rightarrow B + C$ of a relativistic spin-$\frac{1}{2}$ particle A. Which of the following statements is true in the rest frame of the particle A ?

[CSIR DEC 2018]

(a) The spin of both B and C may be ½

(b) The sum of the masses of B and C is greater than the mass of A

(c) The energy of B is uniquely determined by the masses of the particles

(d) The spin of both B and C may be integral

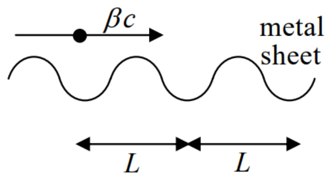
25. An inertial observer A at rest measures the electric and magnetic field $E = (\alpha, 0,0)$ and $B = (\alpha, 0,2\alpha)$ in a region, where α is a constant. Another inertial observer B, moving with a constant velocity with respect to A, measures the field as $E' = (E'_x, \alpha, 0)$ and $B' = (\alpha, B'_y, \alpha)$. Then, in units $c = 1, E'_x$ and B'_y are given, respectively, by

[CSIR JUNE 2019]

(a) -2α and α (b) 2α and $-\alpha$

(c) α and -2α (d) $-\alpha$ and 2α

26. A point charge is moving with a uniform velocity βC along the positive x-direction, parallel to and very close to a corrugated metal sheet (see the figure below).
The wavelength of the electromagnetic radiation received by an observer along the direction of motion is



(a) $\frac{1}{\beta}\sqrt{1 - \beta^2}$ (b) $L\sqrt{1 - \beta^2}$

(c) $L\beta\sqrt{1 - \beta^2}$ (d) L

27. Following a nuclear explosion, a shock wave propagates radially outwards. Let E be the energy released in the explosion, and ρ be the mass density of the ambient air. Ignoring the temperature of the ambient air, using dimensional analysis, the functional dependence of the radius R of the shock front on E, ρ and the time t is

[CSIR DEC 2019]

(a) $\left(\frac{Et^2}{\rho}\right)^{1/5}$ (b) $\left(\frac{\rho}{Et^2}\right)^{1/5}$

(c) $\frac{Et^2}{\rho}$ (d) $E\rho t^2$

28. The fixed points of the time evolution of a one-variable dynamical system described by $y_{t+1} = 1 - 2y_t^2$ are 0.5 and -1 . The fixed points 0.5 and -1 are

[CSIR DEC 2019]

(a) Both stable

(b) Both unstable

(c) Unstable and stable, respectively

(d) Stable and unstable, respectively

29. A heavy particle of rest mass M while moving along the positive z-direction, decays into two identical light particles with rest mass m (where $M > 2m$). The maximum value of the momentum that any one of the lighter particles can have in a direction perpendicular to the $z-$ direction, is

[CSIR JUNE 2020]

(a) $\frac{1}{2}C\sqrt{M^2 - 4m^2}$ (b) $\frac{1}{2}C\sqrt{M^2 - 2m^2}$

(c) $C\sqrt{M^2 - 4m^2}$ (d) $\frac{1}{2}MC$

30. A monochromatic source emitting radiation with a certain frequency moves with a velocity v away from a stationary observer A. It is moving towards another observer B (also at rest) along a line joining the two. The frequencies of the radiation recorded by A and B are ν_A and ν_B, respectively. If the ratio $\frac{\nu_B}{\nu_A} = 7$, then the value of $\frac{v}{c}$ is

[CSIR JUNE 2021]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{\sqrt{3}}{2}$

31. A particle of rest mass m is moving with a velocity $v\hat{k}$, with respect to an inertial frame S. The energy of the particle as measured by an observer S′, who is moving with a uniform velocity $u\hat{\imath}$ with respect to S (in terms of $\gamma_u = 1/\sqrt{1 - u^2/c^2}$ and $\gamma_v = 1/\sqrt{1 - v^2/c^2}$) is

[CSIR JUNE SEP 2022]

(a) $\gamma_u\gamma_v m(c^2 - uv)$

(b) $\gamma_u\gamma_v mc^2$

(c) $\frac{1}{2}(\gamma_u + \gamma_v)mc^2$

(d) $\frac{1}{2}(\gamma_u + \gamma_v)m(c^2 - uv)$

32. The coordinates of the following events in an observer's inertial frame of reference are as follows:

Event 1: $t_1 = 0, x_1 = 0$: A rocket with uniform velocity $0.5c$ crosses the observer at origin along x axis

Event 2: $t_2 = T, x_2 = 0$: The observer sends a light pulse towards the rocket

Event 3: t_3, x_3 : The rocket receives the light pulse

The values of t_3, x_3 respectively are

[CSIR DEC 2023]

(a) $2T, cT$ (b) $2T, \frac{c}{2}T$

(c) $\frac{\sqrt{3}}{2}T, \frac{2}{\sqrt{3}}cT$ (d) $\frac{2}{\sqrt{3}}T, \frac{\sqrt{3}}{2}cT$

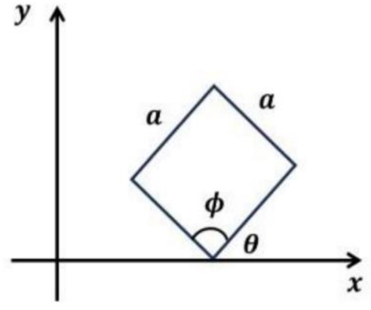
33. The charge density and current of an infinitely long perfectly conducting wire of radius a, which lies along the z-axis, as measured by a static observer are zero and a constant I, respectively. The charge density measured by an observer, who moves at a speed $v = \beta c$ parallel to the wire along the direction of the current, is

[CSIR JUNE 2023]

(a) $-\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (b) $-\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

(c) $\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (d) $\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

34. A square plate of dimension $a \times a$ makes an angle $\theta = \pi/4$ with the x axis in its rest frame (S) as shown in the figure.



It is moving with a speed $v = \sqrt{\frac{2}{3}}c$ along the x axis with respect to an observer S' (where c is the speed of light in vacuum). The value of the interior angle ϕ indicated in the figure (which is obviously $\pi/2$ in the frame S), as measured in S' is **[CSIR JUNE 2024]**

(a) $\frac{\pi}{3}$
(b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{6}$
(d) $\frac{4\pi}{3}$

35. A particle of mass $\frac{1GeV}{c^2}$ and its antiparticle, both moving with the same speed v, produce a new particle X of mass $\frac{10\text{GeV}}{c^2}$ in a head-on collision. The minimum value of v required for this process is closest to

(a) $0.83c$
(b) $0.93c$
(c) $0.98c$
(d) $0.88c$

❖ GATE PYQ's

1. An observer S sees a stream of particles moving along the x-axis. Each particle has a velocity $-u\hat{\imath}(u > 0)$. The observer S sees N particle per unit time cross the point $x = 0$. Another observer S' has a velocity $v\hat{\imath}(v > 0)$ with respect to S. (The x'-axis of S' coincides with the x-axis of S). Treating the problem relativistically, answer the following: **[GATE 1991]**

(a) What is n, the number of particles per unit length on the x-axis, as determined by S ?
(b) What is the velocity of each particle as seen by S'' ?
(c) What is the number of particles per unit length on the x'-axis, as determined by S' ?
(d) Hence, determine N', the number of particles per unit time passing $x' = 0$, as seen by S'.

2. A meson, in its rest frame, has an average life time of $2.21 \times 10^{-6} s$.
If mesons formed high in the atmosphere travel with a speed of $0.99c$, where c is the speed of light, find the average distance travelled by the meson before decaying. **[GATE 1995]**

3. A rod of length L, always parallel to the x-axis of frame S, moves with a speed u in the y-direction with respect to S-frame. The ends of the rod are at $(0, ut)$ and (L, ut) in the xy-plane of S. Another frame S' is moving with respect to S along their common xx' axes with speed v so that the usual Lorentz transformations are valid. Find the tagent of the angle which the rod makes with the x'-axis as observed in frame S'. **[GATE 1996]**

4. A particle of rest mass $m_\circ$, moving with a speed of $0.8c$, collides with another particle of rest mass 5 m/9, which is at rest. A completely inelastic collision occurs, in which the two masses stick to each other after collision. Determine
(a) the velocity of the combined mass (in units of c, the velocity of light) after collision,
(b) the rest mass of the combine. **[GATE 1998]**

5. Kinetic energy of a relativistic particle of rest mass m moving with speed v is **[GATE 1997]**

(a) $\frac{1}{2}mv^2$
(b) $\frac{mc^2}{\sqrt{1 - v^2/c^2}}$
(c) $\frac{mc^2}{\sqrt{1 - v^2/c^2}} = mc^2$
(d) $\frac{1}{2}m(v^2 - c^2)$

6. The momentum of an electron (mass m) which has the same kinetic energy as its rest mass energy is (c is velocity of light) **[GATE-1998]**

(a) $\sqrt{3}mc$
(c) mc
(b) $\sqrt{2}mc$
(d) $mc/\sqrt{2}$

7. A circle of radius 5 m lies at rest in $x - y$ plane in the laboratory. For an observer moving with a uniform velocity V along the y direction, the circle appears to be an ellipse with an equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

The speed of the observer in terms of the velocity of light c is **[GATE-1998]**

(a) 9c/25
(b) $3c/5$
(c) $4c/5$
(d) $16c/25$

8. A relativistic particle of mass m and velocity $c/2\hat{z}$ is moving towards a wall. The wall is moving with velocity $c/3\hat{z}$. The velocity of the particle after it suffers an elastic collision is $v\hat{z}$ with v equal to **[GATE-1999]**

(a) $c/2$
(b) $c/5$

(c) $c/7$ (d) $c/15$

9. Which of the following equations is relativistically invariant? (α, β, γ and δ are constants of suitable dimensions)

[GATE 2001]

(a) $\frac{\partial \phi(x,t)}{\partial t} = \alpha \frac{\partial^2 \phi(x,t)}{\partial x^2}$

(b) $\frac{\partial^2 \phi(x,t)}{\partial t^2} = \beta \frac{\partial^2 \phi(x,t)}{\partial x^2}$

(c) $\frac{\partial^2 \phi(x,t)}{\partial t^2} = \gamma \frac{\partial \phi(x,t)}{\partial x}$

(d) $\frac{\partial \phi(x,t)}{\partial t} = \delta \frac{\partial^3 \phi(x,t)}{\partial x^3}$

10. The homogeneity of time leads to the law of conservation of

[GATE 2002]

(a) linear momentum

(b) angular momentum

(c) energy

(d) parity

11. An electron is accelerated from rest by 10.2 million volts. The percent increase in its mass is

[GATE 2002]

(a) 20,000 (b) 2,000

(c) 200 (d) 20

12. Two events, 10^{-7} s apart in time, take place at two points 50 m apart on the X-axis. Find the speed of an observer moving along the X-axis who observes the two events simultaneously. What will be the spatial separation between these two events as seen by this observer?

[GATE 2002]

13. Two events are separated by a distance of 6×10^5 km and the first event occurs 1sec before the second event. The interval between the two events

[GATE 2003]

(a) is time-like

(b) is light-like (null)

(c) is space-like

(d) cannot be determined from the information given

14. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (c is the speed of light in vacuum)

[GATE 2003]

(a) $c/3$ (b) $\sqrt{2}c/3$

(c) $c/2$ (d) $\frac{\sqrt{3}}{2}c$

15. An electron gains energy so that its mass becomes $2m_0$. Its speed is

[GATE 2004]

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$

(c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

16. Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because

[GATE 2005]

(a) the mass is not conserved

(b) the energy is not conserved

(c) the momentum is not conserved

(d) the charge is not conserved

17. A car is moving with constant linear acceleration a along horizontal x-axis. A solid sphere of mass M and radius R is found rolling without slipping on the horizontal floor of the car in the same direction as seenfrom an inertial frame outside the car. The acceleration of the sphere in the inertial frame is

[GATE 2005]

(a) $\frac{a}{7}$ (b) $\frac{2a}{7}$

(c) $\frac{3a}{7}$ (d) $\frac{5a}{7}$

18. A rod of length l_0 makes an angle θ_0 with the y-axis in its rest frame, while the rest frame moves to the right along the x-axis with relativistic speed v with respect to the lab frame. If

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

the angle θ in the lab frame is

[GATE 2005]

(a) $\theta = \tan^{-1}(\gamma \tan\ \theta_0)$

(b) $\theta = \tan^{-1}(\gamma \cot\ \theta_0)$

(c) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\tan\ \theta_0\right)$

(d) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\cot\ \theta_0\right)$

19. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is its rest mass. The linear momentum of the particle is

[GATE 2006]

(a) $2m_0c$ (b) $\frac{2m_0}{\sqrt{3}}$

(c) m_0c (d) $\frac{m_0c}{\sqrt{3}}$

Statement for Linked Answer Questions 20 & 21:

In the laboratory frame, a particle P of rest mass m0 is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q , moving in the negative x direction with a speed of $\frac{2c}{5}$.

20. The speed of the particle P in the rest frame of the particle Q is **[GATE 2007]**

(a) $\frac{7c}{95}$ (b) $\frac{13c}{85}$

(c) $\frac{3c}{5}$ (d) $\frac{63c}{95}$

21. The energy of the particle P in the rest frame of the particle Q is

[GATE 2007]

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$

(c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

22. The rate of a clock in a spaceship "Aakash gang" is observed from earth to be 5/13 of the rate of the clocks on earth. If both Aakashganga and Suryashakti are moving in the same direction relative to someone on earth, then the speed of Aakashganga relative to Suryashakti is

[GATE 2008]

(a) $\frac{12}{13}c$ (b) $\frac{4}{5}c$

(c) $\frac{8}{17}c$ (d) $\frac{5}{6}c$

23. Assuming the mean life time of a muon (in its rest frame) to be 2×10^{-6} s, its life time in the laboratory frame, when it is moving with a velocity 0.95c is

[GATE 2009]

(a) 6.4×10^{-6} s (b) 0.62×10^{-6} s

(c) 2.16×10^{-6} s (d) $0.9 \times 10^{-6}s$

24. For the set of all Lorentz transformations with velocities along the x-axis, consider the two statements given below:
P: If L is a Lorentz transformation then, L^{-1} is also a Lorentz transformation.
Q: If L_1 and L_2 are Lorentz transformations then, L_1L_2 is necessarily a Lorentz transformation
Choose the correct option

[GATE 2010]

(a) P is true and Q is false

(b) both P and Q are true

(c) both P and Q are false

(d) P is false and Q is true

25. A π^0 meson at rest decays into two photons, which move along the x-axis. They are both detected simultaneously after a time, $t = 10$ s. In an inertial frame moving with a velocity $V = 0.6c$ in the direction of one of the photons, the time interval between the two detections is

[GATE 2010]

(a) 15 s (b) 0 s

(c) 10 s (d) 20 s

26. Two particles, each of rest mass m collide head on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is

[GATE 2011]

(a) 5 m/4 (b) $2m$

(c) $5m/2$ (d) 25 m/8

27. A rod of proper length l_0 oriented parallel to the x-axis moves with speed $2c/3$ along the x-axis in the Sframe, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

[GATE 2012]

(a) $0.35l_0$ (b) $0.48l_0$

(c) $0.87l_0$ (d) $0.97l_0$

28. A rod of proper length l_0 oriented parallel to the x-axis moves with speed $2c/3$ along the x-axis in the Sframe, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

[GATE 2012]

(a) $0.35l_0$ (b) $0.48l_0$

(c) $0.87l_0$ (d) $0.97l_0$

29. Consider the decay of a pion into a muon and an anti-neutrino $\pi^{-1} \rightarrow \mu^{-1} + \bar{v}_\mu$ in the pion rest frame.

$m_\pi = 139.6 \text{MeV/c}^2, m_\mu = 105.7 \text{MeV/c}^2, m_v \approx 0$ The energy (in MeV) of the emitted neutrino to the nearest integer is________________.

30

30. An electron is moving with a velocity of $0.85c$ in the same direction as that of moving photon. The relative velocity of the electron with respect to photon is

[GATE 2013]

(a) c (b) $-c$

(c) $0.15c$ (d) $-0.15c$

31. The relativistic form of Newton's second law of motion is

[GATE 2013]

(a) $F = \dfrac{mc}{\sqrt{c^2 - v^2}} \dfrac{dv}{dt}$

(b) $F = \dfrac{m\sqrt{c^2 - v^2}}{c} \dfrac{dv}{dt}$

(c) $F = \dfrac{mc^2}{c^2 - v^2} \dfrac{dv}{dt}$

(d) $F = m \dfrac{c^2 - v^2}{c^2} \dfrac{dv}{dt}$

32. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

[GATE 2014]

(a) 1: 1 (b) 1: $\sqrt{3}$

(c) 1: 3 (d) 3: 2

33. Which one of the following quantities is invariant under Lorentz transformation?

[GATE 2014]

(a) Charge density (b) Charge

(c) Current (d) Electric field

34. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z axis. Their velocities are given

[GATE 2015]

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$

(b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$

(d) $\vec{v}_1 = (0.6c)\hat{z}; \vec{v}_2 = (-0.8c)\hat{z}$

35. In an inertial frame S, two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6c\hat{y}$ with respect to S are given by **[GATE 2015]**

(a) $ct_A' = 0; ct_B' = -3/2$

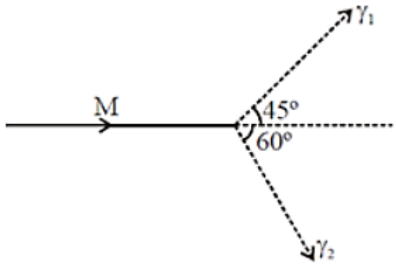
(b) $ct_A' = 0; ct_B' = 0$

(c) $ct_A' = 0; ct_B' = 3/2$

(d) $ct_A' = 0; ct_B' = 1/2$

36. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of m_0c, where c is the speed of light in vacuum, is___________ (Given your answer upto two decimal places) **[GATE 2016]**

37. A particle of rest mass M is moving along the positive x-direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is 1GeV and the energy of γ_2 is 0.82GeV. The value of M (in units of GeV/c^2) is_____ (Give your answer upto two decimal places) **[GATE 2016]**



38. In an inertial frame of reference S, an observer finds two events occurring at the same time at coordinates $x_1 = 0$ and $x_2 = d$. A different inertial frame S' 'moves with velocity v with respect to S along the positive x-axis. An observer in S' 'also notices these two events and finds them to occur at times t_1' and t_2' and at positions x_1' and x_2', respectively. If $\Delta t' = t_2' - t_1', \Delta x' = x_2' - x_1'$ and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, which of the following statements is true? **[GATE 2016]**

(a) $\Delta t' = 0, \Delta x' = \gamma d$

(b) $\Delta t' = 0, \Delta x' = d/\gamma$

(c) $\Delta t' = -\gamma v d/c^2, \Delta x' = \gamma d$

(d) $\Delta t' = -\gamma v d/c^2, \Delta x' = d/\gamma$

39. An object travels along the x-direction with velocity c/2 in a frame O. An observer in a frame O′ sees the same ohject travelling with velocity $c/4$. The relative velocity of Ω' with respect to Ω in units of c is (up to two decimal places) **[GATE 2017]**

40. A spaceship is travelling with a velocity of $0.7c$ away from a space station. The spaceship ejects a probe with a velocity $0.59c$ opposite to its own velocity. A person in the space station would see the probe moving at a speed X_c, where the value of X is (up to three decimal places). **[GATE 2018]**

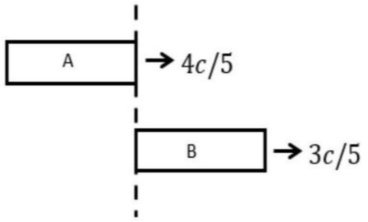
41. An interstellar object has speed v at the point of its shortest distance R from a star of much larger mass M. Given $v^2 = 2GM/R$, the trajectory of the object is **[GATE 2018]**

(a) circle (b) ellipse

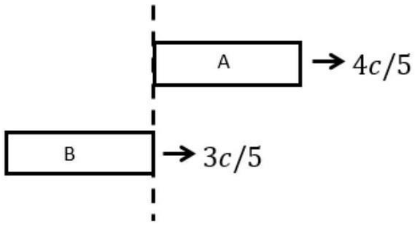
(c) parabola (d) hyperbola

42. Two spaceships A and B, each of the same rest length L, are moving in the same direction with speeds $\frac{4c}{5}$ and $\frac{3c}{5}$, respectively, where c is the speed of light. As measured by B, the time taken by A to completely overtake B [see figure below] in units of L/c (to the nearest integer) is **[GATE 2019]**

(i)



(ii)



43. Two events, one on the earth and the other one on the Sun, occur simultaneously in the earth's frame. The time difference between the two events as seen by an observer in a spaceship moving with velocity $0.5c$ in the earth's frame along the line joining the earth to the Sun is Δt, where c is the speed of light. Given that light travels from the Sun to the earth in 8.3 minutes in the earth's frame, the value of $|\Delta t|$ in minutes (rounded off to two decimal places) is

(Take the earth's frame to be inertial and neglect the relative motion between the earth and the sun)

[GATE 2019]

44. Two observers O and O' observe two events P and Q. The observers have a constant relative speed of $0.5c$. In the units, where the speed of light, c, is taken as unity, the observer O obtained the following coordinates:

Event $P: x = 5,\ y = 3,\ z = 5,\ t = 3$

Event $Q: x = 5,\ y = 1,\ z = 3,\ t = 5$

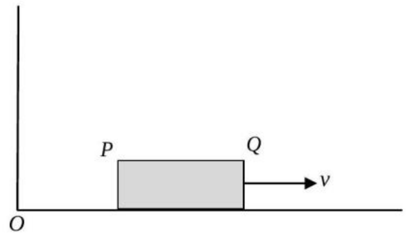
The length of the space-time interval between these two events, as measured by O', is L. The value of $|L|$ (in integer) is **[GATE 2021]**

45. Two identical particles of rest mass m_0 approach each other with equal and opposite velocity $v = 0.5c$, where c is the speed of light. The total energy of one particle as measured in the rest frame of the other is $E = \alpha m_0 c^2$. The value of α is (Round off to two decimal places)

[GATE 2022]

46. A rod PQ of proper length L lies along the x-axis and moves towards the positive x direction with speed $v = \frac{3c}{5}$ with respect to the ground (see figure), where c is the speed of light in vacuum. An observer on the ground measures the positions of P and Q at different times t_P and t_Q respectively in the ground frame, and finds the difference between them to be $\frac{9L}{10}$. What is the value of $t_Q - t_P$?

[GATE 2023]



(a) $\frac{L}{3c}$ (b) $\frac{L}{5c}$

(c) $\frac{L}{6c}$ (d) $\frac{2L}{3c}$

47. An inertial observer sees two spacecrafts S and T flying away from each other along x axis with individual speed $0.5c$, where c is the speed of light. The speed of T with respect to S is

[GATE 2024]

(a) $\frac{4}{5}c$ (b) $\frac{4}{3}c$

(c) c (d) $\frac{2}{3}c$

❖ JEST PYQ's

1. A binary system consists of two stars of equal mass ' m ' orbiting each other in a circular orbit under the influence of gravitational forces. The period of the orbit is τ. At $t = 0$, the motion is stopped and the stars are allowed to fall to towards each other. After what time t, expressed in terms of τ, do they collide? The following integral may be useful

$$(x = r^{1/2})\int \frac{x^2\,dx}{\sqrt{x^2 - a^2}} = \frac{x}{2}\sqrt{x^2 - a^2} + \frac{\alpha}{2}\sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$$

[JEST 2012]

(a) $\sqrt{2\tau}$ (b) $\frac{\tau}{\sqrt{2}}$

(c) $\frac{\tau}{2\sqrt{2}}$ (d) $\frac{\tau}{4\sqrt{2}}$

2. In a certain inertial frame two light pulses are emitted at point 5 km apart and separated in time by $5\mu s$. An observer moving at a speed V along the line joining these points notes that the pulses are simultaneous. Therefore V is

[JEST 2012]

(a) 0.7c (b) 0.8c

(c) 0.3c (d) 0.9c

3. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum p. If c dentes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v is.

[JEST 2013]

(a) $\frac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\frac{(E-vp)}{\sqrt{1-(v/c)^2}}$

(c) $\frac{(E+vp)}{[1-(v/c)^2]^2}$ (d) $\frac{(E-vp)}{[1-(v/c)^2]^2}$

4. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum) **[JEST 2013]**

(a) $\sqrt{3c}/2$ (b) $3c/4$

(c) $\sqrt{3/5c}$ (d) $c/\sqrt{2}$

5. Under a Galilean transformation, the coordinates and momenta of any particle/system transform as: $t'=t, \vec{r}'=\vec{r}+\hat{v}$ and $\vec{p}'=\vec{p}+m\vec{v}$ and $\vec{v}$ is the velocity of the boosted frame with respect to the original frame. A unitary operator carrying out these transformations for a system having total mass M, total momentum $\vec{P}$ and centre of mass coordinate $\vec{X}$ is

[JEST 2013]

(a) $e^{jM\hat{v}\cdot\bar{X}/h}e^{j\vec{v}\cdot\vec{P}/h}$

(b) $e^{iM\hat{V}\cdot\hat{X}/h}e^{-i\vec{v}\cdot\vec{P}/h}e^{-iN^2\lfloor(2h)}$

(d) $e^{it\vec{P}\cdot\dot{P}/h}e^{MMv^2t(2h)}$

6. A light beam is propagating through a block of glass with index of refraction n. If glass is moving at constant velocity v in the same direction as the beam, the velocity of the light in the glass block as measured by an observer in the laboratory is approximately

[JEST 2013]

(a) $u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$

(b) $u=\frac{c}{n}-v\left(1-\frac{1}{n^2}\right)$

(c) $u=\frac{c}{n}+v\left(1+\frac{1}{n^2}\right)$

(d) $u=\frac{c}{n}$

7. The coordinate transformation $x'=0.8x+0.6y, y'=0.6x-0.8y$ represents

[JEST 2013]

(a) A translation.

(b) a proper rotation.

(c) A reflection.

(d) None of the above.

8. A k meson (with a rest mass of 494MeV) at rest decays into a muon (with a rest mass of 106MeV) and a neutrino. The energy of the neutrino, which can be taken to be massless, is approximately

[JEST 2013]

(a) 120MeV (b) 236MeV

(c) 300MeV (d) 388MeV

9. In a certain inertial frame two light pulses are emitted, a distance 5 km apart and separated by $5\mu s$. An observer who is traveling, parallel to the line joining the points where the pulses are emitted, at a velocity V with respect to this frame notes that the pulses are simultaneous. Therefore V is

[JEST 2014]

(a) 0.7c (b) 0.8c

(c) 0.3c (d) 0.9c

10. A monochromatic wave propagates in a direction making an angle $60°$ with the x-axis in the reference frame of source. The source moves at speed $v = \frac{4c}{5}$ towards the observer. The direction of the (cosine of angle) wave as seen by the observer is

[JEST 2014]

(a) $\cos\ \theta' = \frac{13}{14}$ (b) $\cos\ \theta' = \frac{3}{14}$

(c) $\cos\ \theta' = \frac{13}{6}$ (d) $\cos\ \theta' = \frac{1}{2}$

11. The distance of a star from the Earth is 4.25 light years, as measured from the Earth. A space ship travels from Earth to the star at a constant velocity in 4.25 years, according o the clock on the ship. The speed of the space ship in units of the speed of light is,

[JEST 2015]

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

12. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t = 0$ at the Sun and another event occurs on Earth at $t = 1$ minute. The velocity of the inertial frame in which both these events are simultaneous is:

[JEST 2016]

(a) $c/8$ with the velocity vector pointing from Earth to Sun

(b) $c/8$ with the velocity vector pointing from Sun to Earth

(c) The events can never be simultaneous - no such frame exists

(d) $c\sqrt{1-\left(\frac{1}{8}\right)^2}$ with velocity vector pointing from Sun to Earth

13. A person on Earth observes two rockets A and B directly approaching each other with speeds 0.8c and 0.6c respectively. At a time when the distance between the rockets is observed to be 4.2×10^8 m, the clocks of the rockets and the Earth are synchronized to $t = 0$ s. the time of collision (in seconds) of the two rockets as measured in rockets A's frame is $x/10$. What is x ?

[JEST 2018]

14. An optical line of wavelength 5000Å in the spectrum of light from a star is found to be redshifted by an amount of 2Å. Let v be the velocity at which the star is receding. Ignoring relativistic effects, what is the value of $\frac{c}{v}$?

[JEST 2019]

15. A spaceship moves away from Earth with a relativistic speed v and fires a shuttle craft in the forward direction at a speed v relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at a speed v relative to the Earth?

JEST 2021]

(a) $3v$ (b) $\frac{3v}{\sqrt{1-\frac{v^2}{c^2}}}$

(c) $\left(\frac{3+v^2/c^2}{1+3v^2/c^2}\right)v$ (d) $\frac{2v}{1+v^2/c^2}+v$

16. A binary star system consists of two starts with same mass M revolving about a common centre of mass in a circular orbit with velocities much smaller than the speed of light, $c = 3.0 \times 10^8$ m/s. The axis of the plane of rotation is perpendicular to our line of sight. The wavelength of a particular spectral line from one of the stars is observed to change with a period of 2.40×10^5 seconds. If the ratio of maximum to minimum wavelength of the line is 1.0022 , the distance between the stars (in 10^9 m) to the nearest integer, is **[JEST 2021]**

17. A beam of light is propagating through a block of glass with refractive index n. The block is moving with a constant speed v in the direction opposite to that of the beam. The speed of the light beam in the block as measured by an observer in the laboratory frame is:

[JEST 2023]

(a) $\frac{c}{2n}$ (b) $\frac{c}{n}$

(c) $\dfrac{c}{2n}$ (d) $\dfrac{c-nv}{n-\frac{v}{c}}$

18. A rocket is moving in free space with speed $\frac{c}{2}$. After a fuel tank is gently detached, the rocket is found to be moving with a speed $\frac{c}{4}$ with respect to the detached fuel tank. What is the final speed of the rocket in the original frame of reference?

[JEST 2023]

(a) $\frac{2}{7}c$ (b) $\frac{2}{3}c$

(c) $\frac{3}{4}c$ (d) $\frac{4}{5}c$

19. A stationary body explodes into two fragments, each of rest mass m. The two fragments move apart at speeds ηc (where c is the speed of light and $0<\eta<1$) relative to the original body. The rest mass of the original body is

[JEST 2023]

(a) $2m\sqrt{1-\eta^2}$ (b) $2m(1-\eta^2)$

(c) $\dfrac{2m}{\sqrt{1-\eta^2}}$ (d) $2m$

20. Two trains, each having proper length L_0 are moving towards each other with the same speed v but in opposite directions as measured by an observer in an inertial frame. What is the length of one of the trains as measured by an observer in the other train?

[JEST 2024]

(a) $L_0\left(\dfrac{c^2-v^2}{c^2+v^2}\right)$ (b) $L_0\sqrt{\left(\dfrac{c^2-v^2}{c^2+v^2}\right)}$

(c) $L_0\sqrt{1-\dfrac{v^2}{4c^2}}$ (d) $L_0\sqrt{1-\dfrac{v^2}{c^2}}$

❖ TIFR PYQ's

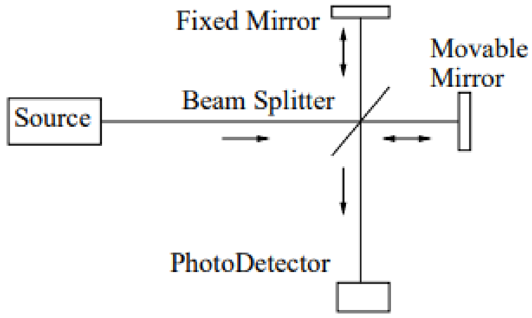
1. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass $M+\Delta$. If the transition from the ground state to the excited state proceeds by the absorption of a photon, the photon frequency in the laboratory frame (where the atom is initially at rest) is

[TIFR 2010]

(a) $\dfrac{\Delta c^2}{h}$ (b) $\dfrac{\Delta c^2}{h}\left(1+\dfrac{\Delta}{2M}\right)$

(c) $\dfrac{Mc^2}{h}$ (d) $\dfrac{\Delta c^2}{h}\left(1-\dfrac{\Delta}{2M}\right)$

2. The Michelson interferometer in the figure below can be used to study properties of light emitted by distant sources.



A Source S_1, when at rest, is known to emit light of wavelength 632.8 nm. In this case, if the Movable Mirror is translated through a distance d, it is seen that 99,565 interference fringes pass across the Photo-Detector. For another Source S_2, moving at an uniform speed of $1.5\times10^7\ \text{m s}^{-1}$ towards the interferometer along the straight line joining it to the Beam Splitter, one sees 100,068 interference fringes pass across the Photo-Detector for the same displacement d of the Movable Mirror. It follows that S_2, in its own rest frame, must be emitting light of wavelength

[TIFR 2011]

(a) 661.9 nm (b) 662.8 nm

(c) 598.9 nm (d) 631.2 nm

(e) 599.6 nm (f) 628.0 nm

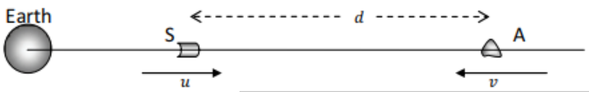
3. A high-velocity missile, travelling in a horizontal line with a kinetic energy of 3.0 Gigajoules (GJ), explodes in flight and breaks into two pieces A and B of equal mass. One of these pieces (A) flies off in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 G.J. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of

[TIFR 2012]

(a) 30° (b) 33°24′

(c) 45° (d) 60°

4. A spaceship S blasts off from the Earth. After some time, Earth station informs the crew that they have settled into a constant velocity $0.28c$ radially outward from the Earth, but unfortunately they are on a headon collision course with an asteroid A at a distance of 15 light-minutes coming in towards the Earth along the same radius (see figure below).



Instruments on-board the spaceship immediately estimate the speed of the asteroid to have a constant value 0.24c. It follows that the maximum time (in minutes) available to the crew to evacuate the ship before the collision is

[TIFR 2013]

(a) 60 (b) 30

(c) 29 (d) 63

5. A particle P, of rest mass M and energy E, suddenly decays into two particles A and B of rest masses m_A and m_B respectively, and both particles move along the straight line in which P was moving. A possible energy E_A of the particle A will be **[TIFR 2013]**

(a) $\frac{E}{2}\left\{1+\left(\frac{m_A-m_B}{M}\right)^2\right\}$

(b) $\frac{E}{2}\left\{1-\left(\frac{m_A^2-m_B^2}{M^2}\right)\right\}$

(c) $\frac{E}{2}\left\{1+\left(\frac{m_A+m_B}{M}\right)^2\right\}$

(d) $\frac{E}{2}\left\{1+\left(\frac{m_A^2-m_B^2}{M^2}\right)\right\}$

6. The Conservation Principles for energy, linear momentum and angular momentum arise from the necessity that

[TIFR 2014]

(a) the laws of physics should not involve infinite quantities.

(b) internal forces on a body should cancel out, by Newton's (third) law of action and reaction.

(c) physical measurements should be independent of the origin and orientation of the coordinate system.

(d) the laws of physics should be independent of the state of rest or motion of the observer.

7. Cosmic ray muons generated at the top of the Earth's atmosphere decay according to the radioactive decay law

$$N(t) = N(0)\exp\left(-\frac{0.693t}{T_{1/2}}\right)$$

where $N(t)$ is the number of muons at time t, and $T_{1/2} = 1.52\mu s$ is the proper half-life of the muon. Immediately after generation, most of these muons shoot down towards the Earth's surface. Some of these muons decay on the way, but their interaction with the atmosphere is negligible.
An observer on the top of a mountain of height 2.0 km above mean sea level detects muons with the speed $0.98c$ over a period of time and counts 1000 muons. The number of muons of the same speed detected by an observer at mean sea level in the same period of time would be

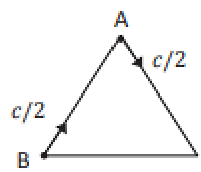
[TIFR 2014]

(a) 232 (b) 539

(c) 839 (d) 983

8. In the laboratory frame, two observers A and B are moving along the sides of an equilateral triangle with equal speeds $c/2$, as shown in the figure.

[TIFR 2014]



The speed of B as measured by A will be

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{4}{3\sqrt{3}}c$

(c) $\frac{\sqrt{13}}{7}c$ (d) $\frac{\sqrt{5}}{3}c$

9. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory? (c = velocity of light in vacuum)

[TIFR 2015]

(a) $0.93c$ (b) 0.98c

(c) $0.96c$ (d) $0.9c$

10. A collimated beam of pions originate from an accelerator and propagates in vacuum along a long straight beam pipe. The intensity of this beam was measured in the laboratory after a distance of 75 m and found to have dropped to one-fourth of its intensity at the point of origin. If the proper halflife of a pion is 1.77×10^{-8} s, the speed of the pions in the beam, as measured in the laboratory, must be

[TIFR 2015]

(a) $0.99c$ (b) $0.98c$

(c) 0.97c (d) $0.96c$

11. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory? (c = velocity of light in vacuum)

[TIFR 2015]

(a) $0.93c$ (b) 0.98c

(c) $0.96c$ (d) $0.9c$

12. In a moving car, the wheels will skid if the brakes are applied too suddenly. This is because

[TIFR 2016]

(a) the inertia of the car will carry it forward.

(b) the momentum of the car must be conserved.

(c) the impulsive retarding force exceeds the limiting force of static friction.

(d) the kinetic friction will suddenly get converted to static friction.

13. An aircraft, which weighs 12000 kg when unloaded, is on a relief mission, carrying 4000 food packets weighing 1 kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second, then the distance between the last two packet drops will be

[TIFR 2016]

(a) 1.5Km (b) 200 m

(c) 150 m (d) 100 m

14. In a futuristic scenario, two spaceships, A and B, are running a race, where they start from the same point (marked START) but fly in opposite directions at constant speeds close to the speed of light. An observer fixed at the starting point observes that they both cross the points marked END, which are equidistant from the starting point, at the same time. Afterwards this observer receives messages from both spaceships.

Which of the following could be true?

[TIFR 2016]

(a) Both A and B agree that A won the race.

(b) A and B both claim to have won the race.

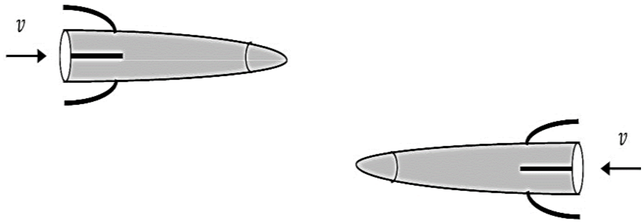
(c) Both A and B agree that they crossed the end point simultaneously.

(d) A thinks B won the race while B thinks A won the race.

15. Cosmic ray muons, which decay spontaneously with proper lifetime 2.2μs, are produced in the atmosphere, at a height of 5 km above sea level. These move straight downwards at 98% of the speed of light. Find the percent ratio $100 \times (N_A/N_B)$ of the number of muons measured at the top of two

mountains A and B, which are at heights 4,848 m and 2,682 m respectively above mean sea level.
[TIFR 2017]

16. From an observational post E on the Earth, two ballistic missiles, each of rest length ℓ from nosetip to tail-end, are observed to fly past each other, with the same uniform relativistic speed $c/2$, in opposite directions, as shown below.



What is the time taken for the tail-end of one of the missiles to cross the tail-end of the other missile, as measured from the post E ?
[TIFR 2018]

17. If the velocity of the Earth in its orbit is v, find $\delta E/E$, where E is the translational (nonrelativistic) kinetic energy of the Earth and δE is its relativistic correction to the lowest order in v/c. **[TIFR 2018]**

18. On a compact stellar object the gravity is so strong that a body falling from rest will soon acquire a velocity comparable with that of light. If the force on this body is $F = mg$ where m is the relativistic mass and g is a constant, the velocity of this falling body will vary with time as

[TIFR 2019]

(a) $v = \dfrac{c}{1 - \dfrac{2c}{gt}}$

(b) $v = \dfrac{2c}{\pi} \tan \dfrac{gt}{c}$

(c) $v = c\tanh \dfrac{gt}{c}$

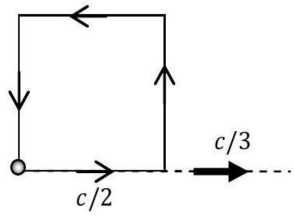
(d) $v = c\left\{1 - \exp\left(-\dfrac{gt}{c}\right)\right\}$

19. A particle of rest mass $\sqrt{3}$ g emerges from a gun with a velocity $v = c/4$. If the rest mass of the gun is 1 kg, its approximate speed of recoil will be **[TIFR 2020]**

(a) $c/1000$ (b) $c/2236$

(c) $c/1732$ (d) $c/2309$

20. In the laboratory frame, a particle at rest starts moving with a speed $c/2$ from one corner of a square (see figure) and traverses the four sides of the square so that it returns to its original position. At each corner, it changes direction without any change in speed.



If the entire square now moves with a speed $c/3$ in the laboratory frame, as indicated in the figure, the speed of the particle (in the laboratory frame) when it returns to its original position will be

[TIFR 2020]

(a) $\dfrac{2\sqrt{2}c}{15}$ (b) $\dfrac{c}{5}$

(c) $\dfrac{2\sqrt{2}c}{3}$ (d) $\dfrac{c}{5\sqrt{3}}$

21. In a futuristic scenario, an inter-planetary meeting is arranged on planet Pegasus XIV, which is 10 light years away from the Earth. The team of representatives from Earth would like to take a spaceship to Pegasus XIV that has a one-hour flight time according to the watches of the passengers. The Team Leader from Earth would like to leave from the Earth half-an-hour later, but taking a spaceship that has a half-an hour flight time according to the watches of the passengers. If the time taken for acceleration may be neglected for both spaceships, which of the following statements is correct?
[TIFR 2021]

(a) The Team Leader would reach Pegasus XIV at the same time as the committee members.

(b) The Team Leader would reach Pegasus XIV before the committee members.

(c) The Team Leader would reach Pegasus XIV after the committee members.

(d) The situation described in the problem is not possible by the laws of physics.

22. A planet is moving around a star of mass M_0 in a circular orbit of radius R. The star starts to lose its mass very slowly (adiabatically), and after some time, it reaches a mass $M (M < M_0)$. If the motion of the planet is still circular at that time, the radius of its orbit will become

[TIFR 2021]

(a) $R\left(\frac{M_0}{M}\right)^{1/2}$ (b) $R\left(\frac{M}{M_0}\right)^{2}$

(c) $R\left(\frac{M_0}{M}\right)^{2}$ (d) $R\left(\frac{M}{M_0}\right)$

23. An observer O, moving with relativistic speed v away from a fixed plane mirror M in a line perpendicular to the mirror surface, sends a pulse of light of wavelength λ towards the mirror.

The wavelength of the light reflected back to the observer will be **[TIFR 2022]**

(a) $\lambda\left(\frac{c+v}{c-v}\right)$ (b) $\lambda\left(\frac{c+2v}{c-2v}\right)$

(c) $\lambda\sqrt{\frac{c-v}{c+v}}$ (d) $\lambda\left(\frac{c-2v}{c+2v}\right)$

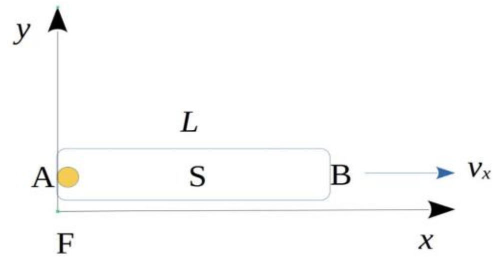
24. A faint star is known to emit light of a given frequency at an average rate of 10 photons per minute. An astronomer plans to measure this rate using a photon-counting detector. If she wants to measure the rate to an accuracy of 5%, approximately how long should be the exposure time? **[TIFR 2023]**

(a) 1 hour (b) 40 minutes

(c) 20 minutes (d) 10 minutes

25. Consider a spaceship S of length L is moving relativistically in the x direction with a speed v_x relative to an inertial reference frame F as shown in the figure. In S, a light bulb is placed at the left end (point A) and a detector is placed at the right end (point B). What is the time taken for light to travel from A to B in the reference frame F ?

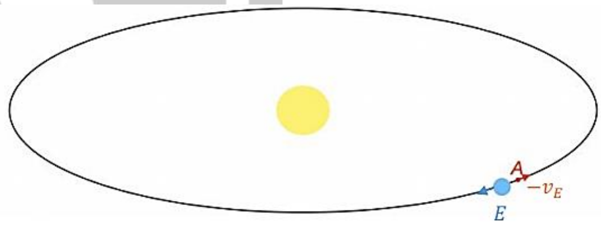
[TIFR 2024]



(a) $\frac{L}{c}\sqrt{\frac{1+v_x/c}{1-v_x/c}}$ (b) $\frac{L}{c}\sqrt{1-\frac{v_x^2}{c^2}}$

(c) $\frac{L}{c}\sqrt{\frac{1-v_x/c}{1+v_x/c}}$ (d) $\frac{L}{c}\frac{1}{\sqrt{1-v_x^2/c^2}}$

26. The Figure below shows a rocket (red arrow) launched from the earth which is now at a point A where the Earth's gravitational field is negligible. The rocket thrusters have stopped. In the rest frame of the Sun, the velocity of the rocket at A is same in magnitude but opposite in direction to that of the earth was, when it was at the same point. Which of the following statements is correct?



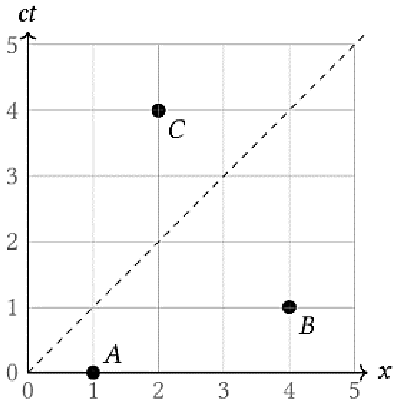
(a) The rocket will move exactly on the earth's elliptical orbit shown in the figure and eventually collide with the earth

(b) The rocket will eventually escape the Sun's gravitational field

(c) The rocket will eventually reverse its direction and follow the earth

(d) The rocket will turn towards the sun and eventually collide with it

27. Consider the following spacetime diagram which indicates three events A, B and C for an inertial observer. Which of the following statements is true?



(a) It is always possible to find an inertial observer for whom events A and B are simultaneous. However, no inertial observer can be found for whom events A and C are simultaneous.

(b) It is always possible to find an inertial observer for whom events A and C are simultaneous. However, no inertial observer can be found for whom events A and B are simultaneous.

(c) It is always possible to find an inertial observer for whom events A and B are simultaneous. Similarly, an inertial observer can also be found for whom events A and C are simultaneous.

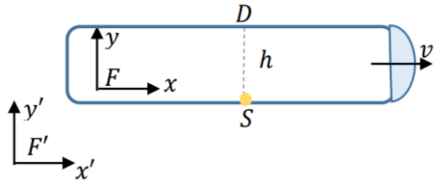
(d) It is impossible to find an inertial observer for whom events A and B are simultaneous. Similarly, no inertial observer can be found for whom events A and C are simultaneous.

28. Consider two relativistic particles, each with mass m and momentum of magnitude p, colliding head-on. As a result of the collision, two heavier particles are produced, each with mass αm, where $\alpha > 1$. The minimum value of p required for this collision to occur is:

(a) $\sqrt{\alpha^2 - 1}mc$ (b) $(\alpha - 1)mc$

(c) $2\alpha mc$ (d) $(\sqrt{\alpha} - 1)^2 mc$

29. A spaceship is moving with a constant relativistic velocity $v\hat{x}'$ with respect to an inertial frame F'. In the frame F moving with spaceship, light is emitted from the source S and is detected at the detector D with displacement $h\hat{y}$ from S. In the frame F', what is the time t' taken for the light to reach from S to D ?



(a) $\dfrac{\left(\frac{h}{c}\right)}{\sqrt{1 - v^2/c^2}}$ (b) $\left(\frac{h}{c}\right)\sqrt{1 - v^2/c^2}$

(c) $\left(\frac{h}{c}\right)\sqrt{\dfrac{1 - v/c}{1 + v/c}}$ (d) $\left(\frac{h}{c}\right)$

Answers key				
CSIR-NET				
1. b/a	2. b	3. b	4. b	5. b
6. c	7. d	8. a	9. d	10. c
11. b	12. c	13. c	14. b	15. d
16. b	17. d	18. a	19. c	20. b
21. b	22. b	23. a	24. c	25. d
26. a	27. a	28. b	29. a	30. c
31. b	32. a	33. a	34. a	35. c
GATE				
1.	2.	3.	4.	5. **c**
6. a	7. c	8. c	9. c	10. c
11. b	12.	13. c	14. d	15. a
16. b	17. b	18. c	19. d	20. c
21. b	22. c	23. a	24. b	25. a
26. c	27. d	28. d	29. 30	30. b
31. c	32. c	33. b	34. b	35. a
36.	37.	38. c	39. 0.28	40. 0.18
41. c	42. 5	43. 4.77	44. 2	45.
46. c	47. a			
JEST				
1. d	2. c	3.	4. a	5. b
6. b	7. b	8.	9. c	10. a
11. b	12. b	13. 000	14. 250	15. c
16. 002	17. d	18. b	19. c	20. a
TIFR				
1. a	2. b	3. a	4. a	5. b
6. c	7. b	8. d	9. a	10. a
11. a	12. a	13. c	14. b	15. 194
16.	17.	18. b	19. b	20. a
21. b	22. c	23. a	24. b	25. a
26. a	27. a	28. a	29. a	

GATE Q.36: 1.72 TO 1.74
GATE Q.37: 140TO1.45

❖ CSIR-NET PYQ's

1. The acceleration due to gravity (g) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately: **[CSIR JUNE 2011]**

(a) 1.1 (b) 1.3

(c) 2.3 (d) 5.2

2. Two gravitating bodies A and B with masses m_A and m_B, respectively, are moving in circular orbit. Assume that $m_B \gg m_A$ and let the radius of the orbit of body A be R_A. If the body A is losing mass adiabatically, its orbital radius R_A is proportional to

[CSIR JUNE 2011]

(a) $1/m_A$ (b) $1/m_A^2$

(c) m_A (d) m_A^2

3. A planet of mass m moves in the inverse square central force field of the Sun of mass M. If the semi-major and semi-minor axes of the orbit are a and b, respectively, the total energy of the planet is: **[CSIR DEC 2011]**

(a) $-\frac{GMm}{a+b}$ (b) $-\mathrm{GMm}\left(\frac{1}{a}+\frac{1}{b}\right)$

(c) $-\frac{\mathrm{GMm}}{a}\left(\frac{1}{b}-\frac{1}{a}\right)$ (d) $-GMm\frac{(a-b)}{(a+b)^2}$

4. Two particles of identical mass move in circular orbits under a central potential $\mathrm{V(r)}=\frac{1}{2}\mathrm{kr}^2$. Let ℓ_1
and ℓ_2 be the angular momenta and r_1, r_2 be the radii of the orbits respectively. If $\ell_1/\ell_2=2$, the value of r_1/r_2 is:

[CSIR DEC 2011]

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$

(c) 2 (d) ½

5. A binary star system consists of two stars S_1 and S_2, with masses m and $2m$ respectively separated by a distance ' r '. If both S_1 and S_2 individually
follow circular orbits around the centre of the mass with instantaneous speeds v_1 and v_2 respectively, the ratio of speeds $\mathrm{v}_1/\mathrm{v}_2$ is:

[CSIR DEC 2012]

(a) $\sqrt{2}$ (b) 1

(c) ½ (d) 2

6. A planet of mass ' m ' moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are ' a ' and ' b ' respectively, the angular momentum of the planet is:

[CSIR DEC 2012]

(a) $\sqrt{2\mathrm{GMm}^2(\mathrm{a}+\mathrm{b})}$ (b) $\sqrt{2\mathrm{GMm}^2(\mathrm{a}-\mathrm{b})}$

(c) $\sqrt{\frac{2\mathrm{GMm}^2\mathrm{ab}}{(\mathrm{a}-\mathrm{b})}}$ (d) $\sqrt{\frac{2\mathrm{GMm}^2\mathrm{ab}}{(\mathrm{a}+\mathrm{b})}}$

7. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, $V(r)=-\mathrm{k}/\mathrm{r}$, where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is

[CSIR JUNE 2013]

(a) $\mathrm{mk}^2/\sqrt{2}\,\mathrm{L}^3$ (b) $\mathrm{mk}^2/\mathrm{L}^3$

(c) $\sqrt{2}\mathrm{mk}^2/\mathrm{L}^3$ (d) $\sqrt{3}\mathrm{mk}^2/\mathrm{L}^3$

8. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is

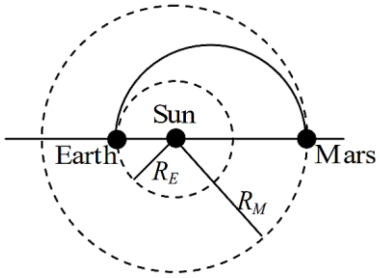
[CSIR DEC 2014]

(a) 64 km (b) 48 km

(c) 32 km (d) 16 km

9. The probe Mangalyaan was sent recently to explore the planet Mars. The inter-planetary parts of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and Mars are approximately circular with radii R_E and R_M respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is a distance $r(R_E \ll r \ll R_M)$ from the Sun, neglecting the effect of Earth and Mars, is

[CSIR DEC 2014]



(a) $\sqrt{2GM\dfrac{(R_E+R_M)}{r(R_E+R_M-r)}}$

(b) $\sqrt{2GM\dfrac{(R_E+R_M-r)}{r(R_E+R_M)}}$

(c) $\sqrt{2GM\dfrac{R_E}{rR_M}}$

(d) $\sqrt{\dfrac{2GM}{r}}$

10. A particle moves in three-dimensional space in a central potential $V(r)=kr^4$, where k is a constant. The angular frequency ω for a circular orbit depends on its radius R as

[CSIR DEC 2015]

(a) $\omega \propto R$ (b) $\omega \propto R^{-1}$

(c) $\omega \propto R^{1/4}$ (d) $\omega \propto R^{-2/3}$

11. Consider circular orbits in a central force potential $V(r)=-\frac{k}{r^n}$, where $k>0$ and $0<n<2$. If the time period of a circular orbit of radius R is T_1 and that of radius $2R$ is T_2, then T_2/T_1 is

[CSIR DEC 2016]

(a) $2^{\frac{n}{2}}$ (b) $2^{\frac{2}{3}n}$

(c) $2^{\frac{n}{2}+1}$ (d) 2^n

12. Consider a set of particles which interact by a pair potential $V=ar^6$, where r is the inter-particle separation and $a>0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is

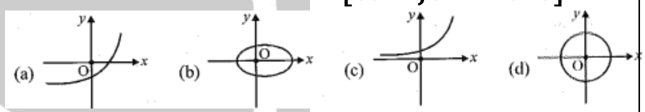
[CSIR DEC 2017]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

13. Which of the following figures best describes the trajectory of a particle moving in a repulsive central potential $V(r)=\frac{\alpha}{r}(\alpha>0$ is a constant $)$?

[CSIR JUNE 2018]



14. A particle of mass m moves in a central potential $V(r)=-\frac{k}{r}$ in an elliptic orbit

$$r(\theta)=\frac{a(1-e^2)}{1+e\cos\theta}$$

, where $0\le\theta\le 2\pi$ and a and e denote the semi-major axis and eccentricity, respectively. If its total energy is $E=-\frac{k}{2a}$, the maximum kinetic energy is

[CSIR JUNE 2018]

(a) $E(1-e^2)$ (b) $E\dfrac{(e+1)}{(e-1)}$

(c) $\dfrac{E}{(1-e^2)}$ (d) $E\dfrac{(1-e)}{(1+e)}$

15. In the attractive Kepler problem described by the central potential $V(r)=-\frac{k}{r}$, where k is a positive constant), a particle of mass m with a non-zero angular momentum can never reach the centre due to the centrifugal barrier. If we modify the potential to

$$V(r)=-\frac{k}{r}-\frac{\beta}{r^3}$$

one finds that there is a critical value of the angular momentum ℓ_c below which there is no centrifugal barrier. This value of ℓ_c is

[CSIR DEC 2018]

(a) $[12\ \mathrm{km}^2\beta]^{1/2}$ (b) $[12\ \mathrm{km}^2\beta]^{-1/2}$

(c) $[12\ \mathrm{km}^2\beta]^{1/4}$ (d) $[12\ \mathrm{km}^2\beta]^{-1/4}$

16. Assume that the earth revolves in a circular orbit around the sun. Suppose the gravitational constat G varies slowly as a function of time. In particular, it decreases to half its initial value in the course of one million years. Then during this time the

[CSIR JUNE 2019]

(a) radius of the earth's orbit will increase by a factor of two.

(b) total energy of the earth remains constant.

(c) orbital angular momentum of the earth will increase.

(d) radius of the earth's orbit remains the same.

17. An object is dropped on a cushion from a height 10 m above it. On being hit, the cushion is depressel by 0.1 m. Assuming that the cushion provides a constant resistive force, the deceleration of the object after hitting the cushion, in terms of the acceleration due to gravity g, is

[CSIR JUNE 2019]

(a) 10 g (b) 50 g

(c) 100 g (d) g

18. A particle, thrown with a speed v from the earth's surface, attains a maximum height h (measured from the surface of the earth). If v is half the escape velocity and R denotes the radius of earth, then $\frac{h}{R}$ is

[CSIR FEB 2022]

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

19. A particle in two dimensions is found to trace an orbit $r(\theta) = r_0\theta^2$. If it is moving under the influence of a central potential $V(r) = c_1 r^{-a} + c_2 r^{-b}$, where r_0, c_1 and c_2 are constants of appropriate dimensions, the values of a and b, respectively, are

[CSIR FEB 2022]

(a) 2 and 4 (b) 2 and 3

(c) 3 and 4 (d) 1 and 3

20. A satellite of mass m orbits around earth in an elliptic trajectory of semi-major axis α. At a radial distance $r = r_0$, measured from the centre of the earth, the kinetic energy is equal to half the magnitude of the total energy. If M denotes the mass of the earth and the total energy is $-\frac{GMm}{2a}$, the value of $\frac{r_0}{a}$ is nearest to

[CSIR FEB 2022]

(a) 1.33 (b) 1.48

(c) 1.25 (d) 1.67

21. The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height 6 km are T_1 and T_2, respectively. If the radius of earth is approximately 6000 km, then

$$\frac{(\mathrm{T}_2 - \mathrm{T}_1)}{\mathrm{T}_1}$$

is closest to **[CSIR SEP 2022]**

(a) -10^{-4} (b) -10^{-3}

(c) 10 (d) 10^{-3}

22. A particle in one dimension executes oscillatory motion in a potential $V(x) = A|x|$, where $A > 0$ is a constant of appropriate dimension. If the time period T of its oscillation depends on the total energy E as E^{α}, then the value of α is

[CSIR FEB 2022]

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

23. The minor axis of Earth's elliptical orbit divides the area within it into two halves. The eccentricity of the orbit is 0.0167 . The difference in time spent by Earth in the two halves is closest to

[CSIR JUNE 2023]

(a) 3.9 days (b) 4.8 days

(c) 12.3 days (d) 0 days

24. The trajectory of a particle moving in a plane is expressed in polar coordinates (r, θ) by the equation $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$ where the parameters r_0, β and ω are positive. Let v_r and a_r denote the velocity and acceleration, respectively, in the radial direction. For this trajectory

[CSIR JUNE 2023]

(a) $a_r < 0$ at all times irrespective of the values of the parameters

(b) $a_r > 0$ at all times irrespective of the values of the parameters

(c) $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters

(d) $\frac{dv_r}{dt} > 0$ however, $a_r = 0$ for some choices of parameters

25. A particle of mass m is moving in a stable circular orbit of radius r_0 with angular momentum L. For a potential energy $V(r) = \beta r^k (\beta > 0$ and $k > 0)$, which of the following options is correct?

[CSIR DEC 2023]

(a) $k = 3, r_0 = \left(\frac{3L^2}{5m\beta}\right)^{1/5}$

(b) $k = 2, r_0 = \left(\frac{L^2}{2m\beta}\right)^{1/4}$

(c) $k = 2, r_0 = \left(\frac{L^2}{4m\beta}\right)^{1/4}$

(d) $k = 3, r_0 = \left(\frac{5L^2}{3m\beta}\right)^{1/5}$

26. A particle moves in a circular orbit under a force field given by

$$\vec{F}(\vec{r}) = -\frac{k}{r^2}\hat{r}$$

. where k is a positive constant. If the force changes suddenly to

$$\vec{F}(\vec{r}) = -\frac{k}{2r^2}\hat{r}$$

, the shape of the new orbit would be

[CSIR DEC 2023]

(a) parabolic (b) circular

(c) elliptical (d) hyperbolic

27. A particle of mass m is moving in a 3-dimensional potential

$$\phi(r) = -\frac{k}{r} - \frac{k'}{3r^3} \quad k, k' > 0.$$

For the particle with angular momentum l, the necessary condition to have a stable circular orbit is

[CSIR DEC 2023]

(a) $kk' < \frac{l^4}{4m^2}$ (b) $kk' > \frac{l^4}{4m^2}$

(c) $kk' < \frac{l^4}{m^2}$ (d) $kk' > \frac{l^4}{m^2}$

28. A body of mass m is acted upon by a central force $\vec{f}(\vec{r}) = -k\vec{r}$, where k is a positive constant. If the magnitude of the angular momentum is l, then the total energy for a circular orbit is

[CSIR JUNE 2024]

(a) $2\sqrt{\frac{kl^2}{m}}$ (b) $\frac{1}{2}\sqrt{\frac{kl^2}{m}}$

(c) $\frac{3}{2}\sqrt{\frac{kl^2}{m}}$ (d) $\sqrt{\frac{kl^2}{m}}$

29. A particle of mass m is moving in a potential $V(r) = -\frac{k}{r}$, where k is a positive constant. If $\vec{L}$ and $\vec{p}$ denote the angular momentum and linear momentum respectively, the value of α for which $\vec{A} = \vec{L} \times \vec{p} + \alpha mk\hat{r}$ is a constant of motion, is

[CSIR JUNE 2024]

(a) -2 (b) -1

(c) 2 (d) 1

❖ GATE PYQ's

Common Data for Q. 1 and Q. 2

Consider a comet of mass m moving in a parabolic orbit around the Sun. The closest distance berween the comet and the Sun is b, the mass of the Sun is M and the universal gravitation constant is G.

1. The angular momentum of the comet is

[GATE 2004]

(a) $M\sqrt{Gmb}$ (b) $b\sqrt{GmM}$

(c) $G\sqrt{mMb}$ (d) $m\sqrt{2GMb}$

2. Which one of the following is TRUE for the above system?

[GATE 2004]

(a) The acceleration of the comet is maximum when it is closest to the Sun

(b) The linear momentum of the comet is a constant

(c) The comet will return to the solar system after a specified period

(d) The kinetic energy of the comet is a constant

3. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is

[GATE 2006]

(a) circular (b) elliptical

(c) parabolic (d) hyperbolic

4. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a positive constant. In a stationary state, the expectation value of the kinetic energy $\langle T\rangle$ of the particle is

[GATE 2006]

(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$

(c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

5. A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. This orbit is

[GATE 2007]

(a) a larger circle (b) a smaller circle

(c) an ellipse (d) a parabola

6. For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

If

$$A(p,q) = \frac{p+iq}{\sqrt{2}}$$

and $H(p,q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p,q)H(p,q)\}$ is given

[GATE 2008]

(a) $iA(p,q)$ (b) $A^*(p,q)$

(c) $-iA^*(p,q)$ (d) $-iA(p,q)$

7. The Lagrangian of a diatomic molecule is given by

$$L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}x_1x_2$$

, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k > 0$. The normal frequencies are

[GATE 2009]

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$

(c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm\left(\frac{k}{2m}\right)^{1/2}$

8. In a central force field, the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by

$$\frac{1}{r} = \frac{m}{L^2}(1 + \varepsilon\cos\,\theta)$$

where, ε is the eccentricity of the particle's motion? Which one of the following choices for ε gives rise to a parabolic trajectory?

[GATE 2012]

(a) $\varepsilon = 0$ (b) $\varepsilon = 1$

(c) $0 < \varepsilon < 1$ (d) $\varepsilon > 1$

9. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$, where k is a positive constant. The orbital angular momentum of the planet is

[GATE 2014]

(a) $2r_0$ km (b) $\sqrt{2r_0km}$

(c) r_0 km (d) $\sqrt{r_0km}$

10. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic average potential and total energies respectively, then which one of the following options is correct?

[GATE 2015]

(a) $V=-2T; E=-T$

(b) $V=-T; E=0$

(c) $V=-T/2; E=T/2$

(d) $V=-3T/2; E=-T/2$

11. A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0)=10\ \mathrm{ms}^{-1}$. If the air exerts a frictional force of the form, $f=-kv$, then for $k=0.05\mathrm{Nm}^{-1}$ s, the velocity (in ms^{-1}) at time $t=0.2s$ is__________ (upto two decimal places) (use $g=10\ \mathrm{ms}^{-2}$ and $e=2.72$)

[GATE 2015]

12. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If $\vec{F}_i$ and $\vec{F}_{co}$ denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

[GATE 2015]

(a) $\vec{F}_c$ is radially outward and $\vec{F}_{Co}=\vec{F}_c$

(b) $\vec{F}_c$ is radially inward and $\vec{F}_{co}=-2\vec{F}_c$

(c) F_c is radially outward and $F_{C_0}=-2F_c$

(d) $\vec{F}_c$ is radially outward and $\vec{F}_{Co}=2\vec{F}_c$

13. A particle moving under the influence of a central force $\vec{F}(\vec{r})=-k\vec{r}$ (where $\vec{r}$ is the position vector of the particle and k is a positive constant) has non-zero angular momentum. Which of the following curves is a possible orbit for this particle?

[GATE 2016]

(a) A straight line segment passing through the origin.

(b) An ellipse with its center at the origin.

(c) An ellipse with one of the foci at the origin.

(d) A parabola with its vertex at the origin.

14. A person weighs w_p at Earth's north pole and w_e at the equator. Treating the Earth as a perfet sphere of radius 6400 km, the value $100\times\left(w_p-w_e\right)/w_p$ is . (up to two deciaml places). (Take $g=10\ \mathrm{ms}^{-2}$)

[GATE 2017]

15. An interstellar object has speed v at the point of its shortest distance R from a star of much larger mass M. Given $v^2=2GM/R$, the trajectory of the object is

[GATE 2018]

(a) circle

(b) ellipse

(c) parabola

(d) hyperbola

16. The spin-orbit interaction term of an electron moving in a central field is written as $f(r)\vec{l}\cdot\vec{s}$, where r is the radial distance of the electron from the origin. If an electron moves inside a uniformly charged sphere, then

[GATE 2019]

(a) $f(r)=$ constant

(b) $f(r)\propto r^{-1}$

(c) $f(r)\propto r^{-2}$

(d) $f(r)\propto r^{-3}$

17. A particle is moving in a central force field given by $\vec{F}=-\frac{k}{r^3}\hat{r}$, where $\hat{r}$ is the unit vector pointing away from the center of the field. The potential energy of the particle is given by

[GATE 2020]

(a) $\frac{k}{r^2}$

(b) $\frac{k}{2r^2}$

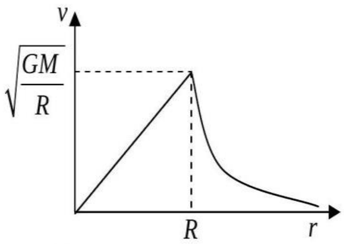
(c) $-\frac{k}{r^2}$

(d) $-\frac{k}{2r^2}$

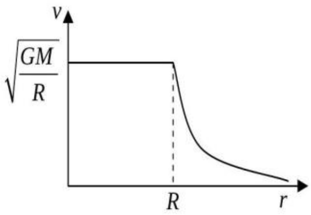
18. Consider a spherical galaxy of total mass M and radius R, having a uniform matter distribution. In this idealized situation, the orbital speed v of a star of mass $m(m\ll M)$ as a function of the distance r from the galactic centre is best described by (G is the universal gravitational constant)

[GATE 2021]

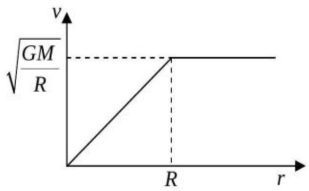
(a)



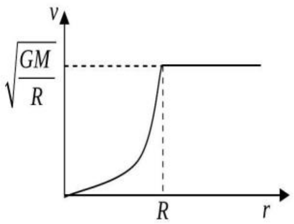
(b)



(c)



(d)



19. A particle of unit mass moves in a potential

$$V(r) = -V_0 e^{-r^2}$$

. If the angular momentum of the particle is

$$L = 0.5\sqrt{V_0}$$

, then which of the following statements are true? **[GATE 2022]**

(a) There are two equilibrium points along the radial coordinate

(b) There is one stable equilibrium point at r_1 and one unstable equilibrium point at $r_2 > r_1$

(c) There are two stable equilibrium points along the radial coordinate

(d) There is only one equilibrium point along the radial coordinate

20. Lagrangian of a particle of mass m is

$$L = \frac{1}{2}m\dot{x}^2 - \lambda x^4$$

, where λ is a positive constant. If the particle oscillates with total energy E, then the time period of oscillations is

$$a\int_0^{\left(\frac{E}{\lambda}\right)^{\frac{1}{4}}} \frac{dx}{\sqrt{\left(\frac{2}{m}\right)(E - \lambda x^4)}}$$

The value of a is (in integer). **[GATE 2024]**

❖ JEST PYQ's

1. A binary system consists of two stars of equal mass ' m ' orbiting each other in a circular orbit under the influence of gravitational forces. The period of the orbit is τ. At $t = 0$, the motion is stopped and the stars are allowed to fall to towards each other. After what time t, expressed in terms of τ, do they collide? The following integral may be useful

$$(x = r^{1/2})\int \frac{x^2 dx}{\sqrt{x^2 - a^2}} = \frac{x}{2}\sqrt{x^2 - a^2} + \frac{\alpha}{2}\sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$$

[JEST 2012]

(a) $\sqrt{2}\tau$ (b) $\frac{\tau}{\sqrt{2}}$

(c) $\frac{\tau}{2\sqrt{2}}$ (d) $\frac{\tau}{4\sqrt{2}}$

2. A plant orbits a massive star in a highly elliptical orbit, i.e. the total energy E is close to zero. The initial distance of closest approach is R_0. Energy is dissipated through tidal motions until the orbit is circularized with a final radius of R_f. Assume that orbital angular momentum is conserved during the circularization process. Then

[JEST 2012]

(a) $R_f = R_0/2$ (b) $R_f = \sqrt{2}R_0$

(c) $R_f = R_0$ (d) $R_f = R_0$

3. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet

[JEST 2012]

(a) attains a constant velocity

(b) moves with a constant acceleration

(c) moves with a constant deceleration

(d) executes simple harmonic motion

4. A spherical planet of radius R has a uniform density ρ and does not rotate. If the planet is made up of some liquid, the pressure at any point r from the center is

[JEST 2013]

(a) $\frac{4\pi\rho^2 G}{3}(R^2 - r^2)$ (b) $\frac{4\pi\rho G}{3}(R^2 - r^2)$

(c) $\frac{2\pi\rho G}{3}(R^2 - r^2)$ (d) $\frac{\rho G}{3}(R^2 - r^2)$

5. If, in a Kepler potential, the pericenter distance of a particle in a parabolic orbit is $\mathbf{r}_\mathrm{p}$ while the radius of the circular orbit with the same angular momentum is r_c, then

[JEST 2013]

(a) $r_C = 2r_P$ (b) $r_C = r_P$

(c) $2r_C = r_P$ (d) $r_C = \sqrt{2r_P}$

6. The free fall time of a test mass on an object of mass M from a height $2R$ to R is

[JEST 2014]

(a) $(\pi/2 + 1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$

(c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

7. How is your weight affected if the Earth suddenly doubles in radius, mass remaining the same?

[JEST 2015]

(a) Increases by a factor of 4

(b) Increases by a factor of 2

(c) Decreases by a factor of 4

(d) Decreases by a factor of 2

8. A classical particle with total energy E moves under the influence of a potential $V(x, y) = 3x^3 + 2x^2y + 2xy^2 + y^3$. The average potential energy, calculated over a long time is equal to,

[JEST 2015]

(a) $\frac{2E}{3}$ (b) $\frac{E}{3}$

(c) $\frac{E}{5}$ (d) $\frac{2E}{5}$

9. The central force which results in the orbit $r = a(1 + \cos\,\theta)$ for a particle is proportional to:

[JEST 2016]

(a) r

(b) r^2

(c) r^{-2}

(d) None of the above

10. Consider a particle of mass m moving under the effect of and attractive central potential given as $\mathrm{V} = -kr^{-3}$, where $\mathrm{k} > 0$. For a given angular momentum $\mathrm{L}, \mathrm{r}_0 = 3\,\mathrm{km}/\mathrm{L}^2$ corresponds to the radius of the possible circular orbit and the corresponding energy is $\mathrm{E}_0 = \mathrm{L}^2/(6mr_0^2)$. The particle is released from $\mathrm{r} > \mathrm{r}_0$ with an inward velocity, energy $\mathrm{E} = \mathrm{E}_0$ and angular momentum L. how long will be particle take to reach r_0 ?

[JEST 2018]

(a) Zero (b) $2mr_0^2L^{-1}$

(c) $\sqrt{2}mr_0^2L^{-1}$ (d) Infinite

11. In a fixed target elastic scattering experiment, a projectile of mass m, having initial velocity v_0, and impact parameter b, approaches the scatterer. It experiences a central repulsive force

$$f(r) = \frac{k}{r^2}(k > 0)$$

. What is the distance of the closest approach d ?

[JEST 2019]

(a) $d = \left(b^2 + \frac{k}{mv_\theta^2}\right)^{\frac{1}{2}}$

(b) $d = \left(b^2 - \frac{k}{mv_0^2}\right)^{\frac{1}{2}}$

(c) $d = b$

(d) $d = \sqrt{\dfrac{k}{mv_0^2}}$

12. A particle of mass m carrying angular momentum I moves in a central potential

$$V(r) = -\frac{ke^{-ar}}{r}$$

, where k, a are positive constants. If the particle undergoes circular motion, what is the equation determining its radius r_0 ?

[JEST 2020]

(a) $\dfrac{l^2}{mr_0} = kar_0 e^{-aa_0}$

(b) $\dfrac{l^2}{mr_0} = ke^{-ar_0}(1 + ar_0)$

(c) $\dfrac{l^2}{2mr_0} = ke^{-ar_0}$

(d) $\dfrac{l^2}{2mr_0} = ke^{-ar_0}(1 + ar_0)$

13. A particle of mass m is placed in a potential well $U(x) = cX^n$, where c is a positive constant and n is an even positive integer. If the particle is in equilibrium at constant temperature, which one of the following relations between average kinetic energy $\langle K \rangle$ and average potential energy $\langle U \rangle$ is correct?

[JEST 2020]

(a) $\langle K \rangle = \dfrac{2}{n}\langle U \rangle$

(b) $\langle K \rangle = \langle U \rangle$

(c) $\langle K \rangle = \dfrac{n}{2}\langle U \rangle$

(d) $\langle K \rangle = 2\langle U \rangle$

14. Consider a particle with total energy E is oscillating in a potential $U(x) = A|x|^n$ with $A > 0$ and $n > 0$ in one dimension. Which one of the following gives the relation between the time-period of oscillation T and the total energy E :

[JEST 2020]

(a) $T \propto E^{1/n-1/2}$

(b) $T \propto E^0$

(c) $T \propto E^n$

(d) $T \propto E^{1/n}$

15. A particle of mass m having a non-zero angular momentum of magnitude l is subject to a central force potential $V(\vec{r}) = k\ln(r)$, where k is a constant and $r = |\vec{r}|$. What is the radius R at which it will have a circular orbit? Will the circular orbit be stable or unstable?

[JEST 2021]

(a) $R = \dfrac{l}{\sqrt{2km}}$, unstable orbit

(b) $R = \dfrac{l}{\sqrt{2km}}$, stable orbit

(c) $R = \dfrac{l}{\sqrt{km}}$, unstable orbit

(d) $R = \dfrac{l}{\sqrt{km}}$, stable orbit

16. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where, k is a constant. The power delivered to the particle by the force acting on it is

[JEST 2022]

(a) $2\pi mk^{\frac{3}{2}}r^2$

(b) mk^2r^2t

(c) $\dfrac{1}{2}mk^2r^2t$

(d) 0

17. A particle moving in a central force field centered at $r = 0$, follows a trajectory given by $r = e^{-\alpha\theta}$ where, (r, θ) is the polar coordinate of the particle and $\alpha > 0$ is a constant. The magnitude of the force is proportional to

[JEST 2022]

(a) r^{-3}

(b) r^2

(c) r^{-1}

(d) r^3

18. A particle of mass 1 kg, angular momentum $L = \sqrt{2}$ kg m^2/s and total energy $E = 3$ J is subjected to a central force field $\vec{F} = -k\vec{r}$ where $k = 2$ kg/s^2. Which of the following statements is true? [Note: The centres of all the circles in the options below are at the origin.] =

[JEST 2023]

(a) The particle is constrained to be in the region outside the circle with radius

$$R = \sqrt{\frac{3+\sqrt{5}}{2}}$$

(b)Theparticleisboundedwithintheannular region described by the two circles with radii

$$r_1 = \sqrt{\frac{5-\sqrt{3}}{2}} \text{ and } r_2 = \sqrt{\frac{5+\sqrt{3}}{2}}.$$

(c) The particle is bounded within the annular region described by the two circles with radii

$$r_1 = \sqrt{\frac{3-\sqrt{5}}{2}} \text{ and } r_2 = \sqrt{\frac{3+\sqrt{5}}{2}}.$$

(d)Theparticleisconstrainedtobeinthe region outside the circle with radius

$$R = \sqrt{\frac{5+\sqrt{3}}{2}}$$

19. A satellite of mass 2000 kg is placed in an elliptic orbit around Earth with semi major axis (a) Assume that the total energy of the orbiting satellite is E and the angular momentum is L. Through a series of manoeuvres, the elliptic orbit is changed to a circular orbit with radius A. For the orbit change described, which of the following is true?

[JEST 2024]

(a) E does not change, but L changes.

(b) E changes, but L does not change.

(c) Both E and L change.

(d) Neither E nor L changes.

20. Consider a particle of mass m and nonzero angular momentum ℓ subjected to a central force potential $V(r) = k\ln\, r$, where k is a positive constant. What is the radius R at which it can have a circular orbit? Will the circular orbit be stable or unstable?

[JEST 2024]

(a)$R = \frac{\ell}{\sqrt{2km}}$ and unstable.

(b)$R = \frac{\ell}{\sqrt{km}}$ and unstable.

(c)$R = \frac{\ell}{\sqrt{2km}}$ and stable.

(d)$R = \frac{\ell}{\sqrt{km}}$ and stable.

21. A point mass m constrained to move along the X-axis is under the influence of gravitational attraction from two point particles each of mass M fixed at the points $(x = 0, y = a)$ and $(x = 0, y = -a)$. Find the time period of small oscillations of the mass m in units of $\pi\sqrt{\frac{a^3}{8GM}}$, where G is the universal gravitational constant.

[JEST 2024]

❖ TIFR PYQ's

1. A small meteor approaches the Earth. When it is at a large distance, it has velocity v_∞ and impact parameter b. If R_e is the radius of the Earth and v_0 is the escape velocity, the condition for the meteor to strike the Earth is

[TIFR 2010]

(a) $b < R_e\sqrt{1-(v_0/v_\infty)^2}$

(b) $b > R_e\sqrt{1+(v_0/v_\infty)^2}$

(c) $b < R_e\sqrt{1+(v_0/v_\infty)^2}$

(d) $b = R_e(v_0/v_\infty)$

2. Consider a spherical planet, rotating about an axis passing through its centre. The velocity of a point on its equator is $v_{eq.}$. If the acceleration due to gravity g measured at the equator is half of the value of g measured at one of the poles, then the escape velocity for a particle shot upwards from that pole will be

[TIFR 2012]

(a) $v_{eq}/2$ (b) $v_{eq}/\sqrt{2}$

(c) $\sqrt{2}v_{eq}$ (d) $2v_{eq}$

3. Two planets A and B move around the Sun in elliptic orbits with time periods T_A and T_B respectively. If the eccentricity of the orbit of B is ε and its distance of closest approach to the Sun is R, then the maximum possible distance between the planets is [Eccentricity of an ellipse:

[TIFR 2013]

$\varepsilon = \dfrac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}}$]

(a) $\dfrac{1+\varepsilon^2}{1-\varepsilon^2}\left(1+\dfrac{T_A^{3/2}}{T_B^{3/2}}\right)R$

(b) $\sqrt{\dfrac{1+\varepsilon}{1-\varepsilon}\left(1+\dfrac{T_A^{3}}{T_B^{3}}\right)}R$

(c) $\dfrac{1+\varepsilon}{1-\varepsilon}\left(1+\dfrac{T_A^{2/3}}{T_B^{2/3}}\right)R$

(d) $\sqrt{\dfrac{1+\varepsilon^2}{1-\varepsilon^2}}\left(1+\dfrac{T_A^{2/3}}{T_B^{2/3}}\right)R$

4. Which of the following classic experiments provides unambiguous proof that the Earth is a non-inertial frame of reference with respect to the fixed stars?

[TIFR 2013]

(a) Fizeau's rotating wheel experiment

(b) Foucault's pendulum experiment

(c) Newton's coin-and-feather experiment

(d) Michelson-Morley experiment

5. Assume that the Earth is a uniform sphere of radius R, rotating about its axis with a uniform angular velocity ω. A rocket is launched from the Equator in a direction due North. If it keeps on flying at a uniform speed v (neglecting air resistance), the highest latitude that can be achieved is

[TIFR 2014]

(a) $\dfrac{\pi}{2}$

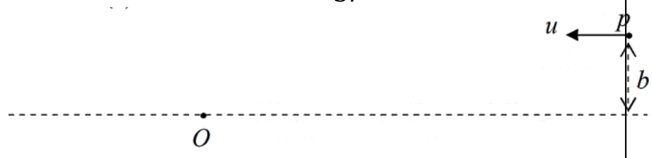
(b) $\dfrac{\pi}{2} - (\pi - 2)\dfrac{\omega R}{v}$

(c) $\dfrac{\pi}{2} - (\pi + 2)\dfrac{\omega R}{v}$

(d) $\dfrac{\pi}{2}\left(1 - \dfrac{2\omega R}{v}\right)$

6. A particle P of mass m moves under the influence of a central potential, centred at the origin O, of the form

$$V(r) = -\frac{k}{3r^3}$$



where k is a positive constant If the particle P comes in from infinity with initial velocity u and impact parameter b (see figure), then the largest value of b for which the particle gets captured by the potential is

[TIFR 2014]

(a) $\left(\dfrac{3k^2}{m^2u^4}\right)^{1/6}$

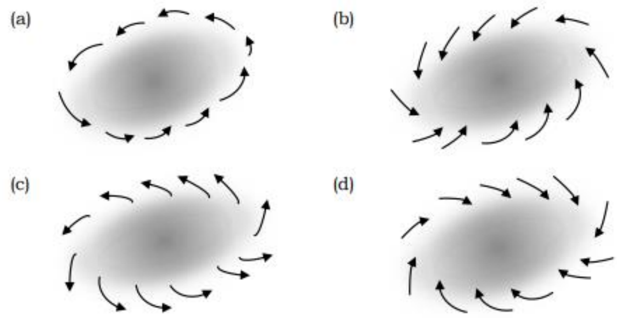
(b) $\left(\dfrac{k}{3mu}\right)^{1/3}$

(c) $\left(\dfrac{2k^2}{m^2u^4}\right)^{1/6}$

(d) $\left(\dfrac{2k}{3mu}\right)^{1/3}$

7. In the Earth's atmosphere, a localized low-pressure region (shaded in diagrams) develops somewhere in the southern hemisphere. Which one of the following diagrams represents the correct air flow pattern as observed from a satellite?

[TIFR 2015]



8. A particle moves under the influence of a central potential in an orbit $r = k\theta^4$, where k is a constant and r is the distance from the origin. It follows that the angle θ varies with time t as

[TIFR 2015]

(a) $\theta \propto t^{1/9}$

(b) $\theta \propto t^{1/8}$

(c) $\theta \propto t^{1/7}$

(d) $\theta \propto t^{1/6}$

9. Solar radiation tends to push any particle inside solar system away from the Sun. Consider a spherical dust particle of specific gravity 6.0 and no angular momentum about the Sun. What should be its minimum radius so that it does not escape from the solar system? Take the solar luminosity to be 3.8×10^{26} W.

[TIFR 2015]

(a) $10^{-6}\mu$m (b) 0.01μm

(c) 0.1μm (d) 10μm

10. On a planet having the same mass and diameter as the Earth, it is observed that objects become weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth). **[TIFR 2016]**

11. In a simple stellar model, the density ρ of a spherical star of mass M varies according to the distance r from the centre according to

$$\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$$

where R is the radius of the star. The gravitational potential energy of this star (in terms of Newton's constant G_N) will be

[TIFR 2016]

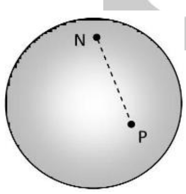
(a) $-G_N M^2/4\pi R$ (b) $-3G_N M^2/5R$

(c) $-5G_N M^2/7R$ (d) $-3G_N M^2/7R$

12. A space telescope in orbit around the Earth discovers a new planet, which is observed to move around the Sun by an angle of 4.72 milliradians in a year. Assuming a circular orbit, estimate the distance, in A.U., of the planet from the Sun.

[TIFR 2017]

13. Consider a sphere of radius R, with the north pole N marked as shown in the figure below.



The r.m.s. distance (straight line cutting through the sphere) of a point P on the sphere from this north pole N is given by

[TIFR 2020]

(a) R (b) $2\sqrt{2/5}R$

(c) $\sqrt{4\pi}R$ (d) $\sqrt{2}R$

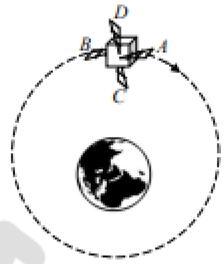
14. Consider a satellite orbiting the Earth in a circular orbit, as sketched in the figure on the right (not to scale). The satellite has four small thruster rockets, whose exhaust gases come out along

(A) the forward direction,

(B) the backward direction,

(C) radially inward towards the Earth's centre, and

(D) radially outward from the Earth's centre, as indicated in the figure.



If the satellite wants to increase its speed, while remaining in a circular orbit, and has fuel enough to keep only one thruster rocket in operation, it should fire the rocket marked

(a) A (b) B

(c) C (d) D

15. Consider two planets P_1 and P_2 which can be modeled as uniform spheres of radii R_1 and R_2 respectively, and of the same material with the same density and other physical properties. If the maximum possible height of a conical mountain (of the same material) on these planets is denoted by h_1 and h_2 respectively ($h_1 \ll R_1$, $h_2 \ll R_2$), then the ratio h_1/h_2 is

[TIFR 2020]

(a) R_2/R_1 (b) R_1/R_2

(c) $R_2^{2/3}/R_1^{2/3}$ (d) $R_1^{2/3}/R_2^{2/3}$

16. A planet is moving around a star of mass M_0 in a circular orbit of radius R. The star starts to lose its mass very slowly (adiabatically), and after some time, it reaches a mass $M(M<M_0)$. If the motion of the planet is still circular at that time, the radius of its orbit will become

[TIFR 2021]

(a) $R\left(\frac{M_0}{M}\right)^2$ (b) $R\left(\frac{M}{M_0}\right)^2$

(c) $R\left(\frac{M_0}{M}\right)^{1/2}$ (d) $R\left(\frac{M}{M_0}\right)$

17. A star moves in an orbit under the influence of massive but invisible object with the effective one-dimensional potential

$$V(r) = -\frac{1}{r} + \frac{L^2}{2r^2} - \frac{L^2}{r^3}$$

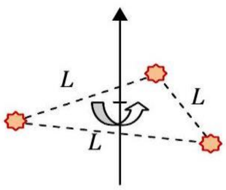
where L is the angular momentum of the star. There would be two possible circular orbits of the star if **[TIFR 2021]**

(a) $L^2 > 12$ (b) $L^2 > 6$

(c) $L^2 > 3$ (d) $L^2 > 9$

18. Three stars, each of mass M, are rotating under gravity around a fixed common axis such that they are always at the vertices of an equilateral triangle of side L (see figure).

The time period of rotation of this triple star system is **[TIFR 2021]**



(a) $\frac{2\pi L^{3/2}}{3\sqrt{G_N M}}$ (b) $\frac{2\pi L^{3/2}}{\sqrt{3G_N M}}$

(c) $\frac{\pi L^{3/2}}{\sqrt{3G_N M}}$ (d) $\frac{\pi L^{3/2}}{3\sqrt{G_N M}}$

19. A particle of mass m moves under the action of a central potential $V(r) = -\frac{e^2}{r}$

where e is a constant. Two vectors which remain conserved during the motion are

(i) the angular momentum $\vec{L} = \vec{r} \times \vec{p}$

(ii) the Runge-Lenz vector $\vec{K} = \vec{p} \times \vec{L} - me^2\hat{r}$ (where $\hat{r} = \vec{r}/r$) The conserved energy E of the particle can be written as

[TIFR 2022]

(a) $\frac{m^2e^4 - K^2}{2mL^2}$ (b) $\frac{K^2 - m^2e^4}{2mL^2}$

(c) $\frac{2mL^2}{K^2 - m^2e^4}$ (d) $\frac{2mL^2}{m^2e^4 - K^2}$

20. A spectrographic method to search for exoplanets is by measuring its velocity along the line of sight, using the Doppler shift in the spectrum. If a star of mass M and a planet of mass m are moving around their common centre of mass, this component of velocity will vary periodically with an amplitude.

$$A = \left(\frac{2\pi G_N}{T}\right)^{1/3} \frac{m}{M^{2/3}}$$

For a particular planet-star system, if the time period is $T = (12 \pm 0.3)$ years, and A and M are measured with an accuracy of 3% each, then the error in the measurement of the mass m is

[TIFR 2022]

(a) 6.3% (b) 8.5%

(c) 5.8% (d) 3.7%

21. A satellite used to make Google Earth images carries on board a telescope which must be designed, when operating at a wavelength λ, to be able to resolve objects on the ground of length as small as δ. If the satellite goes around the Earth in a circular orbit with uniform speed v, the minimum diameter $D_{\min}$ of the telescope mirror can be determined in terms of R, the radius of the Earth, and g, the acceleration due to gravity at the surface, to be

[TIFR 2022]

(a) $\frac{1.22\lambda}{\delta}\left(\frac{gR^2}{v^2} - R\right)$

(b) $\frac{1.22\lambda}{\delta}\frac{gR^2}{v^2}\left(1 + \frac{R}{\lambda}\right)$

(c) $\frac{1.22\lambda}{\delta}\frac{gR^2}{\lambda v^2}$

(d) $\frac{1.22\lambda}{\delta}\sqrt{\frac{gR^3}{v^2}}$

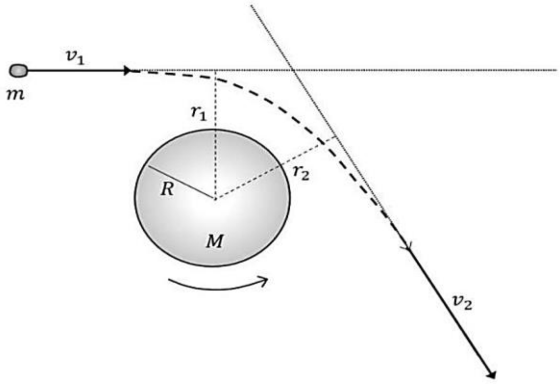
22. From the knowledge that you already have about the length of one year and the fact that the Sun subtends 0.5° in the sky, the average density of the Sun can be computed in $\mathrm{kg-m^{-3}}$ as

[TIFR 2022]

(a) 1.7×10^3 (b) 7.5×10^3

(c) 1.7×10^2 (d) 7.5×10^2

23. A spherical planet of mass M, radius R and uniform density is rotating anticlockwise about an axis passing through its centre, which, in the figure below, is normal to the plane of the paper. The duration of a 'day' on this planet is T.



A small asteroid of mass m approaches the above planet from far away with a uniform speed v_1 along a straight line at a perpendicular distance r_1 from the centre of the planet (see figure). This path gets distorted by the gravitational field of the planet, and the asteroid leaves with a final uniform speed v_2 along a straight line at a perpendicular distance r_2 from the centre of the planet.

It is observed that after the passage of the asteroid, the length of the day on the planet has changed by $\delta T =$ **[TIFR 2023]**

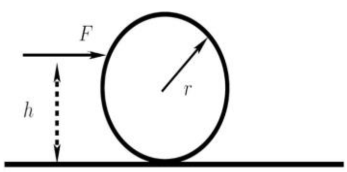
(a) $\dfrac{4\pi}{5}\dfrac{MR^2}{m(v_2r_2 - v_1r_1)}$

(b) $\dfrac{5T^2}{4\pi}\dfrac{m(v_2r_2 - v_1r_1)}{MR^2}$

(c) $\dfrac{5}{4\pi}\dfrac{MR^2}{m(v_2r_1 - v_1r_2)}$

(d) 0

24. A horizontal constant force F is applied on a uniform disc placed on a horizontal surface. The mass of the disc is m, and the radius is r. The point of application of F is at a height $h(< 2r)$ from the surface. The disc starts from rest at $t = 0$ and rolls without slipping. What is the speed of the centre of the disc at time t ? **[TIFR 2024]**



(a) $\dfrac{2Fht}{3mr}$ (b) $\dfrac{Ft}{m}$

(c) $\dfrac{2Ft}{3m}$ (d) $\dfrac{3Fht}{2mr}$

25. Consider a universe that always expands with a scale factor a that increases with time as $a(t) = Ct^{2/3}$ where C is a constant. Its expansion rate at time t is defined by the Hubble parameter

$$H(t) = \frac{1}{a(t)}\frac{da(t)}{dt}$$

The current value of $H(t)$ in the universe is given by $H_0 = 975 \text{ km s}^{-1}\text{Mpc}^{-1}$ where $1\text{Mpc} = 3.1 \times 10^{22}$ m. What is the approximate age of this universe? **[TIFR 2024]**

(a) 10^{11} years (b) 10^{7} years

(c) 10^{9} years (d) 10^{13} years

26. In an infinite fluid of density ρ there are two spherical gas bubbles of radii r_1 and r_2 respectively. The gas has density $\rho_g < \rho$. The centres of the bubbles are separated by a distance $R \gg r_1, r_2$. If the space has no other forces than gravity, the bubbles will: **[TIFR 2024]**

(a) Move towards each other due to an attractive gravitational force

$$F = G(\rho - \rho_g)^2\left(\frac{4\pi}{3}\right)^2\frac{r_1^3r_2^3}{R^2}$$

(b) Move towards each other due to an attractive gravitational force

$$F = G(\rho - \rho_g)^2\left(\frac{4\pi}{3}\right)\frac{r_1^3r_2^3}{R^2}$$

(c) Move away from each other due to a repulsive gravitational force

$$F = G(\rho - \rho_g)^2\left(\frac{4\pi}{3}\right)^2\frac{r_1^3r_2^3}{R^2}$$

(d) Move away from each other due to a repulsive gravitational force

$$F = G(\rho - \rho_g)^2\left(\frac{4\pi}{3}\right)\frac{r_1^3r_2^3}{R^2}$$

27. A relativistic particle moving under the central force of gravity experiences the following effective potential:

$$V_{\text{eff}}(r) = -\frac{GMm}{r} + \frac{l^2}{2mr^2} - \frac{GMl^2}{mc^2r^3}$$

where the last term is the relativistic correction to the Newtonian formula. The smallest radius at which a stable circular orbit can exist for some value of the angular momentum l is given by:

[TIFR 2025]

(a) $\frac{6GM}{c^2}$

(b) $\frac{3GM}{c^2}$

(c) $\frac{2GM}{c^2}$

(d) There are no stable circular orbits

Answers key				
CSIR-NET				
1. c	2. b	3.	4. a	5. a
6.	7. b	8. c	9. b	10. a
11. c	12. c	13. c	14. b	15. c
16. a	17. c	18. b	19. b	20. a
21. d	22. b	23. a	24. d	25. b
26. a	27. a	28. d	29. d	
GATE				
1. d	2. a	3. d	4. a	5. c
6. a	7. d	8. b	9. d	10. a
11. 4.9	12. d	13. b	14. 0.33	15. c
16. a	17. d	18. a	19. a,b	20. 4
JEST				
1. d	2. d	3. a	4. c	5. a
6. a	7. c	8. d	9. d	10. d
11. a	12. b	13. c	14. a	15. d
16. b	17. a	18. c	19. a	20. d
21. 4				
TIFR				
1.	2. d	3. b	4. b	5. b
6. a	7. d	8. a	9. c	10. 0.85
11. c	12.	13. d	14. d	15. a
16. a	17. a	18. b	19. b	20. a
21. a	22. a	23. b	24. a	25. c
26. a	27. a			

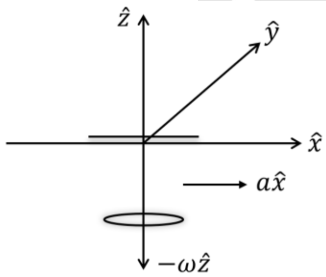
❖ CSIR-NET PYQ's

1. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass ' m ' horizontally with speed ' v '. The acceleration of the bullet, in the reference frame of the shooter, is

[CSIR JUNE 2012]

(a) $2v\Omega$ to his right
(b) $2v\Omega$ to his left
(c) $v\Omega$ to his right
(d) $v\Omega$ to his left

2. A disc of mass m is free to rotate in a plane parallel to the xy-plane with an angular velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.



The car accelerates in the positive x-direction with an acceleration $a > 0$. Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?

[CSIR DEC 2017]

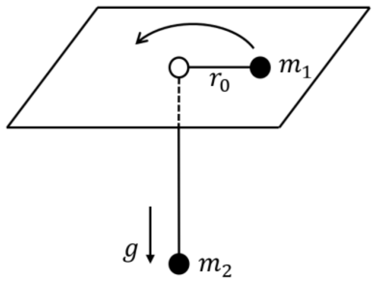
(a) only the x and the z coordinates change
(b) only the y and the z coordinates change
(c) only the x and the y coordinates change
(d) all the three coordinates change

3. A turn-table is rotating with a constant angular velocity ω_0. In the rotating frame fixed to the turn: table, a particle moves radially outwards at a constant speed v_0. The acceleration of the particle in the $r\theta$-coordinates, as seen from an inertial frame, the origin of which is at the centre of the turt: table, is

[CSIR JUNE 2019]

(a) $-r\omega_0^2\hat{r}$
(b) $2r\omega_0^2\hat{r} + v_0\omega_0\hat{\theta}$
(c) $r\omega_0^2\hat{r} + 2v_0\omega_0\hat{\theta}$
(d) $-r\omega_0^2\hat{r} + 2v_0\omega_0\hat{\theta}$

4. Two particles of masses m_1 and m_2 are connected by a massless thread of length ℓ as shown in figure below.



The particle of mass m_1 on the plane undergoes a circular motion with radius r_0 and angular momentum L. When a small radial displacement ε (where $\varepsilon < r_0$) is applied, its radial coordinate is found to oscillate about r_0. The frequency of the oscillations is

[CSIR JUNE 2019]

(a) $\sqrt{(m_2 g}$
(b) $\sqrt{\dfrac{7m_2 g}{(m_1+m_2)r_0}}$
(c) $\sqrt{\dfrac{3m_2 g}{\left(m_1+\frac{m_2}{2}\right)r_0}}$
(d) $\sqrt{\dfrac{3m_2 g}{(m_1+m_2)r_0}}$

5. The Hamiltonian of a quantum particle of mass m is

$$H = \frac{p^2}{2m} + \alpha|x|^r$$

, where α and r are positive constants. The energy E_n of the n^{th} level, for large n, depends on n as

[CSIR JUNE 2019]

(a) n^{2r}
(b) n^{+2}
(c) $n^{L(r+2)}$
(d) $n^{2n(r+2)}$

6. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius a is placed on it at a distance r from the centre of the table, its linear velocity will be

[CSIR JUNE 2020]

(a) $-r\omega\hat{r} + a\omega\hat{\theta}$
(b) $r\omega\hat{r} + a\omega\hat{\theta}$
(c) $a\omega\hat{r} + r\omega\hat{\theta}$
(d) 0 (zero)

7. A rod pivoted at one end is rotating clockwise 25 times a second in a plane. A video camera which records at a rate of 30 frames per second is used to film the motion. To someone watching the video, the apparent motion of the rod will seem to be **[CSIR JUNE 2020]**

(a) 10 rotations per second in the clockwise direction

(b) 10 rotations per second in the anticlockwise direction

(c) 5 rotations per second in the clockwise direction

(d) 5 rotations per second in the anticlockwise direction

8. Gauge factor of a strain gauge is defined as the ratio of the fractional change in resistance $\left(\frac{\Delta R}{R}\right)$ to the fractional change in length $\left(\frac{\Delta L}{L}\right)$. A metallic strain gauge with a gauge factor 2 has a resistance of 100Ω under unstrained condition. An aluminum foil with Young's modulus $Y = 70\text{GN/m}^2$ is installed on the metallic gauge. Keeping the foil within its elastic limit, a stress of 0.2GN/m^2 is applied on the foil. The change in the resistance of the gauge will be closest to

(a) 0.14Ω (b) 1.23Ω

(c) 0.28Ω (d) 0.56Ω

❖ GATE PYQ's

1. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed ω. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If r is the distance of the mass from the axis of rotation and v is its speed then the magnitude of the Coriolis force is

[GATE 2005]

(a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$

(c) $mv\omega$ (d) $2mv\omega$

2. Under a certain rotation of coordinate axes, a rank-1 tensor $v_a (a = 1,2,3)$ transforms according to the orthogonal transformation defined by the relations $v_1' = \frac{1}{\sqrt{2}}(v_1 + v_2); v_2' = \frac{1}{\sqrt{2}}(-v_1 + v_2); v_3' = v_3$. Under the same rotation a rank 2 tensor $T_{a,b}$ would transform such that

[GATE 2008]

(a) $T'_{1,1} = T_{1,1}T_{1,2}$

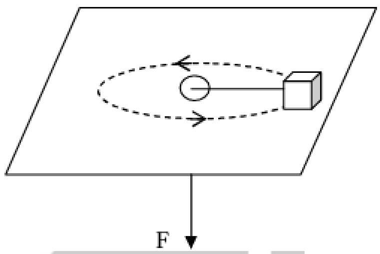
(b) $T'_{1,1} = T_{1,1}$

(c) $T_{1,1} = T_{1,1} + 2T_{2,2} - T_{2,1}$

(d) $T'_{1,1} = \frac{1}{2}(T_{1,1} + T_{2,2} + T_{1,2} + T_{2,1})$

3. A mass m is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius r_0 and angular speed ω_0 by an applied force $\vec{F}$ communicated through an inextensible thread that passes through a hole on the surface as shown in the figure. This force is then suddenly doubled. The magnitude of the radial velocity of the mass

[GATE 2008]



(a) Increases till the mass falls into the hole

(b) Decreases till the mass falls into the hole

(c) Remains constant

(d) Becomes zero at a radius r_1 where $0 < r_1 < r_0$

4. The electric field $E(r,t)$ at a point r at time t in a metal due to the passage of electrons can be described by the equation

$$\nabla^2 \vec{E}(\vec{r},t) = \frac{1}{c^2}\left[\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} + \omega'^2 \vec{E}(\vec{r},t)\right]$$

where ω' is a characteristic associated with the metal and c is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp[i(\vec{k}\cdot\vec{r} - \omega t)]$ is given by

[GATE 2001]

(a) $\omega^2 = c^2k^2 - \omega'^2$ (b) $\omega^2 = c^2k^2 + \omega'^2$

(c) $\omega = ck - \omega'$ (d) $\omega = ck + \omega'$

5. A circular conducting loop C_1 of radius 2 m is located in the XOY plane such that its centre is at (0,0,0). Another circular conducting loop C_2 of radius 2 m is located at (0,0,4) such that the plane of C_2 is parallel to the XOY plane. A current of 5 A is flowing in each of these loops such that the positive Z-axis lies to the left of the directions of the currents. Find the magnetic induction $\vec{B}$ produced at (0,0,0), neglecting the mutual induction of the loops. [GATE 2001]

❖ JEST PYQ's

1. Two objects of unit mass are thrown up vertically with a velocity of 1 ms^{-1} at latitudes 45^0 N and 45°S, respectively. The angular velocity of the rotation of Earth is given to be $7.29 \times 10^{-5}\ \text{s}^{-1}$. In which direction will the objects deflect when they reach their highest point (due to Coriolis force)? Assume zero air resistance.

[JEST 2019]

(a) to the east in Northern hemisphere and west in Southern Hemisphere

(b) to the west in Northern hemisphere and east in Southern Hemisphere

(c) to the east in both hemispheres

(d) to the west in both hemispheres

2. A cylindrical rigid block has principal moments of inertia I about the symmetry axis and $2I$ about each of the perpendicular axes passing through the center of mass. At some instant, the components of angular momentum about the center of mass in the body-fixed principal axis frame is (l, l, l), with $l > 0$. What is the cosine of the angle between the angular momentum and the angular velocity?

[JEST 2024]

(a) $\frac{2}{\sqrt{6}}$ (b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5}{3\sqrt{3}}$

❖ TIFR PYQ's

1. A body is dropped from rest at a height h above the surface of the Earth at a latitude λ_N in the northern hemisphere. If the angular velocity of rotation of the Earth is ω, the lateral displacement of the body at its point of impact on the Earth's surface will be

[TIFR 2021]

(a) $\left(\frac{2h^3\omega^2}{9g}\right)^{1/2} \cos\lambda_N$

(b) $\left(\frac{8h^3\omega^2}{9g}\right)^{1/2} \sin\lambda_N$

(c) $\left(\frac{8h^3\omega^2}{9g}\right)^{1/2} \cos\lambda_N$

(d) $\left(\frac{2h^3\omega^2}{9g}\right)^{1/2} \sin\lambda_N$

Answers key				
CSIR-NET				
1. a	2. d	3. d	4. d	5. d
6. d	7. d	8. d		
GATE				
1. b	2. d	3. d	4. b	5.
JEST				
1. d	2. b			
TIFR				
1. c				