D PHYSICS

CSIR-NET,GATE , ALL SET, JEST, IIT-JAM, BARC

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❖ CSIR-NET-JUNE Question With Solution-2023

➢ Part-B

1. A uniform circular disc on the xy plane with its canter at the origin has a moment of inertia I_0 about the x-axis. If the disc is set in rotation about the origin with an angular velocity $\omega = \omega_0(\hat{\jmath} + \hat{k})$ the direction of its angular momentum is along

(a) $-\hat{\imath} + \hat{\jmath} + \hat{k}$ (b) $-\hat{\imath} + \hat{\jmath} + 2\hat{k}$

(c) $\hat{\jmath} + 2\hat{k}$ (d) $\hat{\jmath} + \hat{k}$

2. The locus of the curve $\text{Im}\left(\frac{\pi(z-1)-1}{z-1}\right) = 1$ in the complex z-plane is a circle centered at $(x_0 y_0)$ and R-respectively are

(a) $\left(1, \frac{1}{2}\right)$ and $\frac{1}{2}$ (b) $\left(1, -\frac{1}{2}\right)$ and $\frac{1}{2}$

(c) $(1,1)$ and 1 (d) $(1, -1)$ and 1

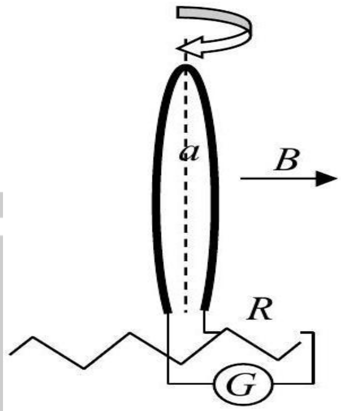
3. The value of $\langle L_x{}^2 \rangle$ in the state $|\varphi\rangle$ for which $L_x{}^2|\varphi\rangle = 6\hbar^2|\varphi\rangle$ and $L_z|\varphi\rangle = 2\hbar|\varphi\rangle$ is

(a) 0 (b) $4\hbar^2$

(c) $2\hbar^2$ (d) $\hbar^2$

4. A small circular wire loop of radius a and number of turns N, is oriented with its axis parallel to the direction of the local magnetic field B.A resistance and Galvano meter are connected to the coil as shown in then figure

When the coil is flipped (i.e. the direction of its axis is reversed) the galvanometer measures the total charge Q that flow through it. If the induce emf through the coil $E_F = IR$ then Q is



(a) $\pi N a^2 B/2R$ (b) $\pi N a^2 B/R$

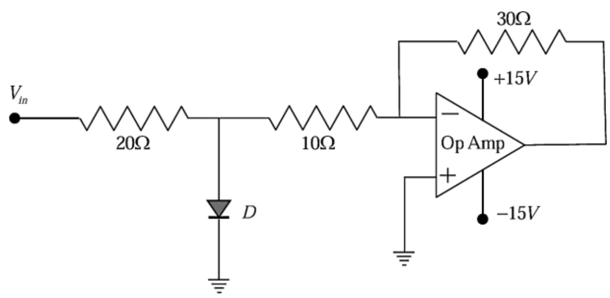
(c) $\sqrt{2}\pi N a^2 B/R$ (d) $2\pi N a^2 B/R$

5. The dispersion relation of a gas of non-interacting bosons in two dimensions is $E(k) = c\sqrt{k}$ where c is a positive constant. At low temperatures, the leading dependence of the specific heat on temperature T is

(a) T^4 (b) T^3

(c) T^2 (d) $T^{3/2}$

6. In the circuit below, there is a voltage drop of 0.7 V across the diode in forward bias while no current flows through it in reverse bias.



In V_{in} is a sinusoidal signal of frequency 50 Hz with rms value of 1 V the maximum current that flows through the diode is closest to

(a) $1A$ (b) 0.14 A

(c) 0 A (d) 0.07 A

7. The trajectory of a particle moving in a plane is expressed in polar coordinates (r, θ) by the equation $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$ where the parameters r_0, β and ω are positive. Let v_r and a_r denote the velocity and acceleration, respectively, in the radial direction. For this trajectory

(a) $a_r < 0$ at all times irrespective of the values of the parameters

(b) $a_r > 0$ at all times irrespective of the values of the parameters

(c) $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters

(d) $\frac{dv_r}{dt} > 0$ however, $a_r = 0$ for some choices of parameters

8. A long cylindrical wire of radius R and conductivity σ, lying along the z-axis, carries a uniform axial current density I. The Poynting vector on the surface of the wire is (in the following $\hat{\rho}$ and $\hat{\varphi}$ denote the unit vectors

(a) $\frac{I^2 R}{2\sigma}\hat{\rho}$ (b) $-\frac{I^2 R}{2\sigma}\hat{\rho}$

(c) $-\frac{I^2 \pi R}{4\sigma}\hat{\varphi}$ (d) $\frac{I^2 \pi R}{4\sigma}\hat{\varphi}$

9. A charged particle moves uniformly on the xy-plane along a circle of radius a centered at the origin. A detector is put at a distance d on the x axis is to detect the electromagnetic wave radiated by the particle along the x direction. If $d >> a$, the wave received by detector is

(a) unpolarized

(b) circularly polarized with the plane of polarization being the yz-plane

(c) linearly polarized along the y-direction

(d) linearly polarized along the z-direction

10. The single particle energies of a system of N non-interacting fermions of spin s(at $T = 0$) are $E_n = n^2 E_0, n = 1,2,3, \ldots$ .... The ratio of $\frac{\varepsilon_F\left(\frac{3}{2}\right)}{\varepsilon_F\left(\frac{1}{2}\right)}$ the Fermi energy of Fermions of spin $\frac{3}{2}$ and $\frac{1}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) 1

11. The Hamiltonian of a two-dimensional quantum harmonic oscillator is $H = \frac{p_x{}^2}{2m} + \frac{p_y{}^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 y^2$ where m and ω are positive constants. The degeneracy of the energy level $\frac{27}{2}\hbar\omega$ is

(a) 14 (b) 13

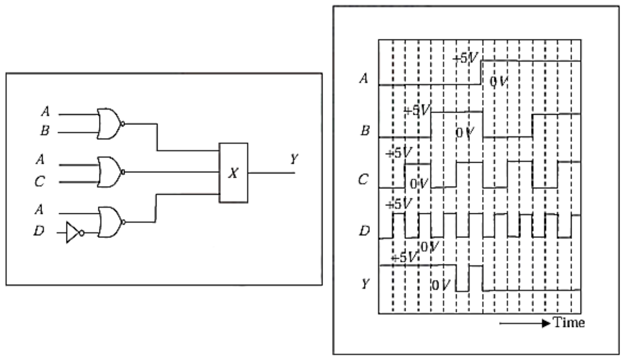
(c) 8 (d) 7

12. The minor axis of Earth's elliptical orbit divides the area within it into two halves. The eccentricity of the orbit is 0.0167 . The difference in time spent by Earth in the two halves is closest to

(a) 3.9 days (b) 4.8 days

(c) 12.3 days (d) 0 days

13. For the given logic circuit, the input waveforms A, B, C and D are shown as a function of time

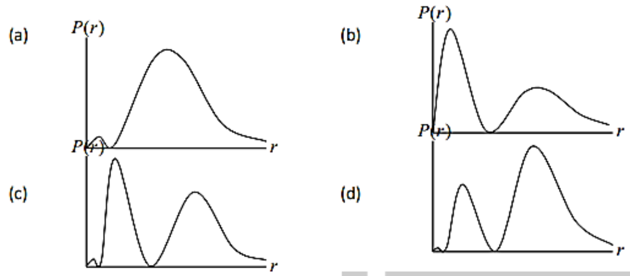


To obtain the output Y as shown in the figure, the logic gate X should be

(a) 1 an AND Gate (b) an OR gate

(c) a NAND gate (d) a NOR gate

14. The radial wavefunction of hydrogen atom with the principal quantum number $n = 2$ and the orbital quantum $R_{20} = N\left(1 - \frac{r}{2a}\right) e^{-\frac{r}{2a}}$ where N is the normalized constant. The best schematic representation of the probability density $p(r)$ for the electron to be between r and $r + dr$ is



15. A one-dimensional rigid rod is constrained to move inside a sphere such that its two ends are always in contact with the surface. The number of constraints on the Cartesian coordinates of the endpoints of the rod is

(a) 3 (b) 5
(c) 2 (d) 4

16. A DC motor is used to lift a mass M to a height H from the ground. The electric energy delivered to the motor is VIt, where V is the applied voltage, I is the current and t the time for which the motor runs. The efficiency e of the motor is the ratio between the work done by the motor and the energy delivered to it. If $M = 2.0 \pm 0.02$ kg, $h = 1.00 \pm 0.01$ m, $V = 10.0 \pm 0.1$ V, $I = 2.00 \pm 0.02$ A and $t = 300 \pm 15$ s, then the fractional error $|\delta e/e|$ in the efficiency of the motor is closest to

(a) 0.05 (b) 0.09
(c) 0.12 (d) 0.15

17. A particle in one dimension is in an infinite potential well between $-\frac{L}{2} \leq x \leq \frac{L}{2}$. For a perturbation $\varepsilon\cos\left(\frac{\pi x}{L}\right)$ where ε is a small constant, the change in the energy of the ground state, to first order in ε is $-\frac{L}{2} \leq x \leq \frac{L}{2}$

(a) $\frac{5\varepsilon}{\pi}$ (b) $\frac{10\varepsilon}{3\pi}$
(c) $\frac{8\varepsilon}{3\pi}$ (d) $4\,\frac{4\varepsilon}{\pi}$

18. The Hamiltonian of a two particle system is $H = p_1p_2 + q_1q_2$ where q_1 and q_2 are generalized coordinates and p_1 and p_2 are the respective canonical momenta. The Lagrangian of this system is

(a) $q_1q_2 + q_1q_2$ (b) $-q_1q_2 + q_1q_2$
(c) $-q_1q_2 - q_1q_2$ (d) $q_1q_2 - q_1q_2$

19. The value of the integral $I = \int_0^\infty e^{-x} x\sin(x)dx$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

20. The energy levels available to each electron in a system of N non-interacting electrons are $E_n = nE_0 n = 0,1,2,\cdots$. A magnetic field, which does not affect the energy spectrum, but completely polarizes the electron spins, is applied to the system. The change in the ground state energy of the system is

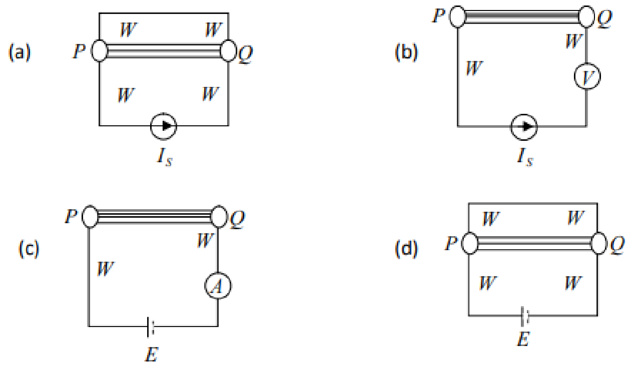
(a) $\frac{n^2E_0}{2}$ (b) n^2E_0
(c) $\frac{n^2E_0}{8}$ (d) $\frac{n^2E_0}{4}$

21. The matrix $M = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ satisfies the equation
$M^3 + \alpha M^2 + \beta M + 3 = 0$ if (α, β) are

(a) $(-2,2)$ (b) $(-3,3)$

(c) $(-6,6)$ (d) $(-4,4)$

22. A circuit needs to be designed to measure the resistance R of a cylinder PQ to the best possible accuracy, using an ammeter A, a voltmeter V, a battery E and a current source I_S (all assumed to be ideal). The value of R is known to be approximately 10Ω, and the resistance W of each of the connecting wires is close to 10Ω. If the current from the current source and voltage from the battery are known exactly, which of the following circuits provides the most accurate measurement of R ?



(a) B (b) A
(c) C (d) A

23. The electric potential on the boundary of a spherical cavity of radius R as a function of the polar angle θ is $V_0 \cos^2\frac{\theta}{2}$. The charge density inside the cavity is zero everywhere. The potential at a distance $\frac{R}{2}$ from the canter of the sphere is

(a) $\frac{V_0}{2}\left(1+\frac{\cos(\theta)}{2}\right)$ (b) $\frac{V_0}{2}\cos(\theta)$
(c) $\frac{V_0}{2}\left(1+\frac{\sin(\theta)}{2}\right)$ (d) $\frac{V_0}{2}\sin(\theta)$

24. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from $J1$ is

(a) 2/3 (b) 3/5
(c) 2/5 (d) 4/7

25. Two energy levels, 0 (non-degenerate) and ε (Doubly degenerate), are available to N noninteracting distinguishable particles. If U is the total energy of the system, for large values of N the entropy of the system is $k_B\left[N\ln N-\left(N-\frac{U}{\varepsilon}\right)\ln\left(N-\frac{U}{\varepsilon}\right)+X\right]$. In this expression X is

(a) $-\frac{U}{\varepsilon}\ln\left(\frac{U}{2\varepsilon}\right)$ (b) $-\frac{U}{\varepsilon}ln\left(\frac{2U}{\varepsilon}\right)$
(c) $-\frac{2U}{\varepsilon}ln\left(\frac{2U}{\varepsilon}\right)$ (d) $-\frac{U}{\varepsilon}\ln\left(\frac{U}{\varepsilon}\right)$

➢ Part-C

26. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from J1 is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$
(c) $\frac{2}{5}$ (d) $\frac{4}{7}$

27. Two random walkers A and B walk on a one-dimensional lattice. The length of each step taken by A is one, while the same for B is two, however, both move towards right or left with equal probability. If they start at the same point, the probability that they meet after 4 steps, is

(a) $\frac{9}{64}$ (b) $\frac{5}{32}$
(c) $\frac{11}{64}$ (d) $\frac{3}{16}$

28. Let the separation of the frequencies of the first Stokes and the first anti-Stokes lines in the pure rotational Raman Spectrum of the H_2 molecule be $\Delta v(H_2)$ while the corresponding quantity for D_2 is $\Delta v(D_2)$. The ratio $\frac{\Delta v(H_2)}{\Delta v(D_2)}$ is

(a) 0.6 (b) 1.2
(c) 1 (d) 2

29. A random variable Y obeys a normal distribution $P(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(Y-\mu)^2/2\sigma^2}$ The mean value of e^Y is

(a) $e^{\mu+\frac{\sigma^2}{2}}$ (b) $e^{\mu-\sigma^2}$

(c) $e^{\mu+\sigma^2}$ (d) $e^{\mu-\frac{\sigma^2}{2}}$

30. Two distinguishable non-interacting particles, each of mass m are in a one-dimensional infinite square well in the interval $[0, a]$. If x_1 and x_2 are position operators of the two particles, the expectation value $\langle x_1 x_2 \rangle$ in the state in which one particle is in the ground state and the other one is in the first excited state, is

(a) $\frac{a^2}{2}$ (b) $\frac{\pi^2 a^2}{2}$

(c) $\frac{a^2}{4}$ (d) $\frac{\pi^2 a^2}{4}$

31. In a one-dimensional system of N spins the allowed values of each spin are $\sigma_i = \{1,2, \ldots \ldots q\}$ where $q \geq 2$ is an integer. The energy of the system is $-J\sum \delta_{\sigma i, \sigma i+1}$ Where $j > 0$ is a constant. If periodic boundary conditions are imposed, the number of ground states of the

(a) q (b) Nq

(c) q^N (d) 1

32. An infinitely long solenoid of radius r_0 centred at origin which produces a time-dependent magnetic field $\frac{\alpha}{\pi {r_0}^2} \cos(\omega t)$ (where α and ω are constants) is placed along the z-axis. A circular loop of radius R, which carries unit line charge density is placed, initially at rest, on the xy-plane with its centre on the z-axis. If $R > r_0$, the magnitude of the angular momentum of the loop is

(a) $\alpha R(1 - \cos \omega t)$ (b) $\alpha R \sin(\omega t)$

(c) $\frac{\alpha R}{2}(1 - \cos 2\omega t)$ (d) $\frac{\alpha R}{2} \sin(2\omega t)$

33. Two electrons in thermal equilibrium at temperature $T = \frac{k_B}{\beta}$ can occupy two sites. The energy of the configuration in which they occupy the different sites is $JS_1 \cdot S_2$ (where J>0is a constant and S denotes the spin of an electron), while it is U if they are at the same site. If $U = 10J$, the probability for the system to be in the first excited state is

(a) $e^{-3\beta J/4}/\left(3e^{\beta J/4} + e^{-3\beta J/4} + 2e^{-10\beta}\right)$

(b) $3e^{-\beta J/4}/\left(3e^{-\beta/4} + e^{3\beta J/4} + 2e^{-10\beta J}\right)$

(c) $e^{-\beta J/4}/\left(2e^{-\beta J/4} + 3e^{3\beta/4} + 2e^{-10\beta}\right)$

(d) $3e^{-3\beta J/4}/\left(2e^{\beta J/4} + 3e^{-3\beta J/4} + 2e^{-10\beta J}\right)$

34. For the transformation $x \to X = \frac{\alpha p}{x}, p \to P = \beta x^2$ between conjugate pairs of a coordinate and its momentum, to be canonical, the constants α and β must satisfy

(a) $1 + \frac{1}{2}\alpha\beta = 0$ (b) $1 - \frac{1}{2}\alpha\beta =$

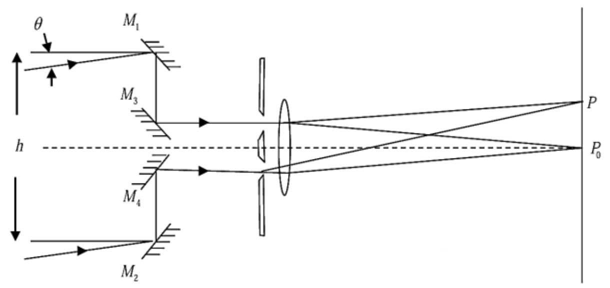
(c) $1 + 2\alpha\beta = 0$ (d) $1 - 2\alpha\beta = 0$

35. The bisection method is used to find a zero x_0 of the polynomial $f(x) = x^3 - x^2 - 1$. Since $f(1) = -1$, while $f(2) = 3$ the values $a = 1$ and $b = 2$ are chosen as the boundaries of the interval in which the x_0 lies. If the bisection method is iterated three times, the resulting value of x_0 is

(a) $\frac{15}{8}$ (b) $\frac{13}{8}$

(c) $\frac{11}{8}$ (d) $\frac{9}{8}$

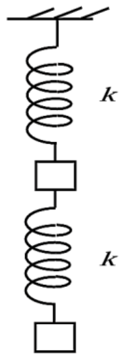
36. The angular width θ of a distant star can be measured by the Michelson radiofrequency stellar interferometer (as shown in the figure below).



The distance h between the reflectors M_1 and M_2 (assumed to be much larger than the aperture of the lens), is increased till the interference fringes (at P_0, P on the plane as shown) vanish for the first time. This happens for $h = 3$ m for a star which emits radiowaves of wavelength 2.7 cm. The measured value of θ (in degrees) is closest to

(a) 1.0 .63 (b) 2.0 .32
(c) 3.0 .52 (d) 4.0 .26

37. A system of two identical masses connected by identical springs, as shown in the figure, oscillates along the vertical direction.



The ratio of the frequencies of the normal modes is

(a) $\sqrt{3-\sqrt{5}}:\sqrt{3+\sqrt{5}}$ (b) $3-\sqrt{5}:3+\sqrt{5}$

(c) $\sqrt{5-\sqrt{3}}:\sqrt{5+\sqrt{3}}$ (d) $5-\sqrt{3}:5+\sqrt{3}$

38. The red line of wavelength 644 nm in the emission spectrum of Cd corresponds to a transition from the $^1D_2$ level to the $^1P_1$ level. In the presence of a weak magnetic field, this spectral line will split into (ignore hyperfine structure)

(a) 9 lines (b) 6 lines
(c) 3 lines (d) 2 lines

39. A neutral particle X^0 is produced in $\pi^- + p \to X^0 + n$ by s-wave scattering. The branching ratios of the decay of X^0 to $2\gamma, 3\pi$ and 2π are 0.38,0.30 and less than 10^{-3}, respectively. The quantum numbers J^{CP} of X^0 are

(a) 0^{-+} (b) 0^{+-}
(c) 1^{-+} (d) $4 \cdot 1^{+-}$

40. A lattice A consists of all points in three-dimensional space with coordinates (n_x, n_y, n_z) where n_x, n_y and n_z are integers with $n_x + n_y + n_z$ being odd integers. In another lattice B, $n_x + n_y + n_z$ are even integers. The lattices A and B are

(a) Both BCC `
(b) Both FCC
(c) BCC and FCC respectively
(d) FCC and BCC respectively

41. The charge density and current of an infinitely long perfectly conducting wire of radius a, which lies along the z-axis, as measured by a static observer are zero and a constant I, respectively. The charge density measured by an observer, who moves at a speed $v = \beta c$ parallel to the wire along the direction of the current, is

(a) $-\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (b) $-\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

(c) $\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (d) $\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

42. The electric and magnetic fields at a point due to two independent sources are $E_1 = E(\alpha\hat{\imath} + \beta\hat{\jmath}), B_1 = B\hat{k}$ and $E_2 = E\hat{\imath}, B_2 = -2B\hat{k}$, where α, β, E and B are constants. If the Poynting vector is along $\hat{\imath} + \hat{\jmath}$, then

(a) $\alpha + \beta + 1 = 0$ (b) $\alpha + \beta - 1 = 0$
(c) $\alpha + \beta + 2 = 0$ (d) $\alpha + \beta - 2 = 0$

43. The electron cloud (of the outermost electrons) of an ensemble of atoms of atomic number Z is described by a continuous charge density $\rho(r)$ that adjusts itself so that the electrons at the Fermi level have zero energy. If $V(r)$ is the local electrostatic potential, then $\rho(r)$ is

(a) $\frac{e}{3\pi^2\hbar^3}[2m_e eV(r)]^{3/2}$

(b) $\frac{Ze}{3\pi^2\hbar^3}[2m_e eV(r)]^{3/2}$

(c) $\frac{Ze}{3\pi^2\hbar^3}[Zm_e eV(r)]^{3/2}$

(d) $\frac{e}{3\pi^2\hbar^3}[m_e eV(r)]^{3/2}$

44. The matrix $R_{\hat{n}}(\theta)$ represents a rotation by an angle θ about the axis $\hat{n}$. The value of θ and $\hat{n}$ corresponding to the matrix

$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}$ respectively, are

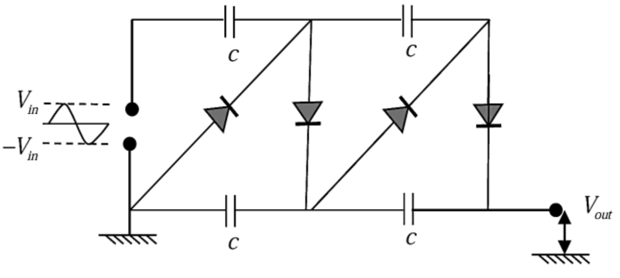
(a) $\pi/2$ and $\left(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$

(b) $\pi/2$ and $\left(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$

(c) π and $\left(0, -\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$

(d) π and $\left(0, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$

45. In the circuit shown below, four silicon diodes and four capacitors are connected to a sinusoidal voltage source of amplitude $V_{\text{in}} > 0.7$ V and frequency 1kHz. If the knee voltage for each of the diodes is 0.7 V and the resistances of the capacitors are negligible, the DC output voltage V_{out} after 2 seconds of starting the voltage source is closest to



(a) $4V_{\text{in}} - 0.7V$

(b) $4V_{\text{in}} - 2.8$ V

(c) $V_{\text{in}} - 0.7$ V

(d) $V_{\text{in}} - 2.8$ V

46. A layer of ice has formed on a very deep lake. The temperature of water, as well as that of ice at the ice-water interface, are 0°C whereas the temperature of the air above is −10°C. The thickness $L(t)$ of the ice increases with time t. Assuming that all physical properties of air and ice are independent of temperature, $L(t) \sim L_0 t^{\alpha}$ for large t. The value of α is

(a) $\frac{1}{4}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 4.1

47. The Hall coefficient R_H of a sample can be determined from the measured Hall voltage $V_H = \frac{1}{d}R_H BI + RI$ where d is the thickness of the sample, B is the applied magnetic field, I is the current passing through the sample and R is an unwanted offset resistance. A lock-in detection technique is used by keeping I constant with the applied magnetic field being modulated as $B = B_0 \sin \Omega t$, where B_0 is the amplitude of the magnetic field and Ω is frequency of the reference signal. The measured V_H is

(a) $B_0 \frac{R_H I}{d}$

(b) $\frac{B_0}{\sqrt{2}} \frac{R_H I}{d}$

(c) $\frac{I}{\sqrt{2}}\left(\frac{B_0 R_H I}{d} + R\right)$

(d) $I\left(\frac{B_0 R_H}{d} + R\right)$

48. A train of impulses of frequency 500 Hz, in which the temporal width of each spike is negligible compared to its period, is used to sample a sinusoidal input signal of frequency 100 Hz. The sampled output is

(a) Discrete with the spacing between the peaks being the same as the time period of the sampling signal

(b) a sinusoidal wave with the same time period as the sampling signal

(c) discrete with the spacing between the peaks being the same as the time period of the input signal

(d) a sinusoidal wave with the same time period as the input signal

49. The value of the integral $\int_{-\infty}^{\infty} dx 2^{-\frac{|x|}{\pi}} \delta(\sin\ x)$ where $\delta(x)$ is the Dirac delta function, is

(a) 3 (b) 0

(c) 5 (d) 1

50. The energy (in keV) and spin-parity values $E(J^P)$ of the low-lying excited states of a nucleus of mass number $A = 152$ are $122(2^+), 366(4^+), 707(6^+)$and $1125(8^+)$. It may be inferred that these energy levels correspond to a

(a) rotational spectrum of a deformed nucleus

(b) rotational spectrum of a spherically symmetric nucleus

(c) vibrational spectrum of a deformed nucleus

(d) vibrational spectrum of a spherically symmetric nucleus

51. Electrons polarized along the x-direction are in a magnetic field

$$B_1\hat{\imath} + B_2(\cos\ \omega t\hat{\jmath} + \sin\ \omega t\hat{k})$$

where $B_1 > B_2$ and ω are positive constants. The value of $\hbar\omega$ for which the polarization-flip process is a resonant one, is

(a) $2\mu_B|B_2|$ (b) $\mu_B|B_1|$

(c) $\mu_B|B_2|$ (d) $2\mu_B|B_1|$

52. The dispersion relation of electrons in three dimensions is $\varepsilon(k) = \hbar v_F k$, where v_F is the Fermi. If at low temperature $T << T_F$ the Fermi energy ε_F depends on the number density n as $\varepsilon_F(n) \sim n^{\alpha}$, the value of α is

(a) 1/3 (b) 2/3

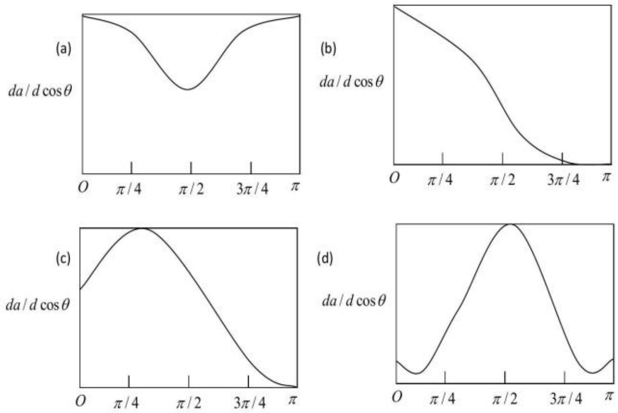
(c) 1 (d) 3/5

53. If the Bessel function of integer order n is defined as $J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{2k+n}$ then $\frac{d}{dx}[x^{-n}J_n(x)]$ is

(a) $-x^{n+1}J_{n+1}(x)$ (b) $-x^{n+1}J_{n-1}(x)$

(c) $-x^n J_{n-1}(x)$ (d) $-x^n J_{n+1}(x)$

54. The phase shifts of the partial waves in an elastic scattering at energy E are $\delta_0 = 12^0, \delta_1 = 4^0$ and $\delta_{l\geq 2} = 0^0$. The best qualitative depiction of θ dependence of the differential scattering cross-section $\frac{d\sigma}{d\cos(\theta)}$ is



55. Two operators A and B satisfy the commutation relations $[H, A] = -\hbar\omega B$ and $[H, B] = \hbar\omega A$ where ω is a constant and H is the Hamiltonian of the system. The expectation value $\langle A\rangle_{\varphi} t = \langle\varphi|A|\varphi\rangle$ in a state φ such that at time $t = 0$ $A_{\varphi}(0) = 0$ and $B_{\varphi}(0) = 0$ is

(a) sin (ωt) (b) sinh (ωt)

(c) cos (ωt) (d) cosh (ωt)

❖ ANSWER KEY

1. c	2. b	3. d	4. d	5. a
6. c	7. d	8. b	9. c	10. b
11. d	12. a	13. b	14. a	15. a
16. a	17. a	18. d	19. c	20. d
21. c	22. b	23. a	24. d	25. a
26. d	27. c	28. d	29. a	30. c
31. a	32. a	33. b	34. c	35. c
36. a	37. a	38. c	39. b	40. b
41. a	42. d	43. a	44.	45. b
46. c	47. b	48. a	49. a	50. a
51. d	52. a	53. a	54. b	55. b