

(d)Al three evolves linearly

**8.** The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height 6 km are  $T_1$  and  $T_2$ , respectively. If the radius of earth is approximately 6000 km, then  $\frac{(T_2-T_1)}{T_1}$  is

 $(b) - 10^{-3}$ 

 $(d)10^{-3}$ 

closest to	
$(a) - 10^{-4}$	
(c)10	

9. A particle of rest mass m is moving with a velocity  $v\hat{k}$ , with respect to an inertial frame S. The energy of the particle as measured by an observer S', who is moving with a uniform velocity u1 with respect to S (in terms of  $\gamma_u = 1/\sqrt{1 - u^2/c^2}$  and  $\gamma_v = 1/\sqrt{1 - v^2/c^2}$  ) is (a)  $\gamma_u \gamma_v m(c^2 - uv)$ 

(a) 
$$\gamma_{u}\gamma_{v}mc^{2}$$
  
(b)  $\gamma_{u}\gamma_{v}mc^{2}$   
(c)  $\frac{1}{2}(\gamma_{u} + \gamma_{v})mc^{2}$   
(d)  $\frac{1}{2}(\gamma_{u} + \gamma_{v})m(c^{2} - uv)$ 

- **10.** An electromagnetic wave is incident from vacuum normally on a planar surface of a non-magnetic medium. If the amplitude of the electric field of the incident wave is  $E_0$ and that of the transmitted wave is  $2E_0/3$ , then neglecting any loss, the refractive index of the medium is
  - (a) 1.5
  - (c) 2.4
  - (d) 2.7
- **11.** A part of an infinitely long wire, carrying a current I, is bent in a semicircular arc of radius r (as shown in the figure).



(b) 2.0

The magnetic field at the centre 0 of the arc is

$(a)\frac{\mu_0 I}{4r}$	$(b)\frac{\mu_0}{4\pi}$
$(c)\frac{\mu_0 I}{2r}$	$(d)\frac{\mu_0}{2\pi}$

**12.** Two positive and two negative charges of magnitude q are placed on the alternate vertices of a cube of side a (as shown in the figure).



The electric dipole moment of this charge configuration is

(a)—2qa <b>k</b>	(b)2qa <b>ƙ</b>
$(c)2qa(\mathbf{\hat{i}} + \mathbf{\hat{j}})$	$(d)2qa(\hat{i} - \hat{j})$

13. The electric and magnetic fields in an inertial frame are  $\mathbf{E} = 3a\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$  and  $\mathbf{B} = \frac{5a}{c}\hat{\mathbf{k}}$ ,

where a is a constant. A massive charged particle is released from rest. The necessary and sufficient condition that there is an inertial frame, where the trajectory of the particle is a uniform-pitched helix, is

(a)  $1 < a < \sqrt{2}$ (b) - 1 < a < 1

 $(c) - a^2 > 1$ 

 $(d)a^2 > 2$ 

**14.** If the expectation value of the momentum of a particle in one dimension is zero, then its (nonnormalizable) wavefunction may be of the form (a)sin kx (b) e<sup>ikx</sup>cos kx (c)e<sup>ikx</sup>sin kx (d)sin kx +  $e^{ikx}$ cos kx

**15.** In terms of a complete set of orthonormal basis kets  $|n\rangle$ ,  $n = 0, \pm 1, \pm 2, \cdots$ , the Hamiltonian is  $H = \sum_{n} (E|n) \langle n| + \epsilon |n + \epsilon |n|$  $1 \langle n | + \epsilon | n \rangle \langle n + 1 | \rangle$ where E and  $\epsilon$  are constants. The state  $|\phi\rangle = \sum_n e^{in\phi} |n\rangle$  is an eigenstate with energy (a)E +  $\epsilon \cos \varphi$ (b) $E - \epsilon \cos \theta$ 

 $(c)E + 2\epsilon \cos \varphi$ 

- 16. The momentum space representation of the Schrödinger equation of a particle in a potential  $V(\vec{r})$ 
  - is  $(|\mathbf{p}|^2 + \beta (\nabla_p^2)^2) \psi(\mathbf{p}, t) = i\hbar \frac{\partial}{\partial t} \psi(\mathbf{p}, t)$ , where  $(\nabla_p)_i = \frac{\partial}{\partial p_i}$ , and  $\beta$  is a constant. The potential is (in the following V<sub>0</sub> and a are constants)

(a) 
$$V_0 e^{-r^2/a^2}$$
 (b)  $V_0 e^{-r^4/a^2}$   
(c)  $V_0 \left(\frac{r}{a}\right)^2$  (d)  $V_0 \left(\frac{r}{a}\right)^4$ 

**17.** Consider the Hamiltonian  $H = AI + B\sigma_x + C\sigma_y$ , where A, B and C are positive constants, I is the 2 × 2 identity matrix and  $\sigma_x$ ,  $\sigma_y$  are Pauli matrices. If the normalized eigenvector corresponding to its largest

energy eigenvalue is  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ y \end{pmatrix}$ , then y is (a) $\frac{B+iC}{\sqrt{B^2+C^2}}$  (b) $\frac{A-iB}{\sqrt{A^2+B^2}}$ (c) $\frac{A-iC}{\sqrt{A^2+C^2}}$  (d) $\frac{B-iC}{\sqrt{B^2+C^2}}$ 

- **18.** If the average energy  $\langle E \rangle_T$  of a quantum harmonic oscillator at a temperature T is such that  $\langle E \rangle_T = 2 \langle E \rangle_{T \to 0}$ , then T satisfies (a) cot  $h\left(\frac{\hbar\omega}{k_BT}\right) = 2$  (b) cot  $h\left(\frac{\hbar\omega}{2k_BT}\right) = 2$  (c) cot  $h\left(\frac{\hbar\omega}{k_BT}\right) = 4$  (d) coth  $\left(\frac{\hbar\omega}{2k_BT}\right) = 4$
- **19.** A thermally isolated container, filled with an ideal gas at temperature T, is divided by a partition, which is clamped initially, as shown in the figure below.



The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction. The final pressure at equilibrium is (a)5P/3 (b)5P/4 (c)3P/5 (d)4P/5

- **20.** A walker takes steps, each of length L, randomly in the directions along east, west, north and south. After four steps its distance from the starting point is d. The probability that  $d \le 3L$  is (a)63/64, (b)57/64 (c)59/64 (d)55/64
- **21.** An elastic rod has a low energy state of length  $L_{max}$  and high energy state of length  $L_{min}$ . The best schematic representation of the temperature (T) dependence of the mean equilibrium length L(T) of the rod, is



**22.** The circuit containing two n-channel MOSFETs shown below, works as



(a)a buffer(b)an inverter(c)a non-inverting amplifier(d)a rectifier

**23.** The figure below shows a circuit with two transistors,  $Q_1$  and  $Q_2$ , having current gains  $\beta_1$  and  $\beta_2$  respectively.



The collector voltage V<sub>C</sub> will be closest to (a)0.9 V (b)2.2 V (c)2.9 V (d)4.2 V

**24.** Four students  $(S_1, S_2, S_3 \text{ and } S_4)$  make multiple measurements on the length of a table. The binned data are plotted as histograms in the following figures



**25.** A high impedance load network is connected in the circuit as shown below



The forward voltage drop for silicon diode is 0.7 V and the Zener voltage is 9.10 V. If the input voltage ( $V_{in}$ ) is sine wave with an amplitude of 15 V (as shown in the figure above), which of the following waveform qualitatively describes the output voltage ( $V_{out}$ ) across the load?



- 26. A bucket contains 6 red and 4 blue balls. A ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

  (a)4/11
  (b)2/11
  (c)1/4
  (d)3/4
- **27.** The value of the integral  $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2+1} dx$ , for

α > 0, is	
(a)πe <sup>α</sup>	(b) $\pi e^{-\alpha}$
$(c)\pi e^{\alpha/2}$	$(d)\pi e^{-\alpha/2}$

**28.** The Laplace transform L[f](y) of the function f(x) =



perpendicular to it is 33:32. If  $T_0$  is the time taken by earth to make one rotation around

(a) $V_0 e^{-\frac{t\sigma}{4\varepsilon}}$  (b) $V_0 e^{-\frac{t\sigma}{2\varepsilon}}$ 

difference at a later time t is

$$(c)V_0e^{-\frac{3t\sigma}{4\epsilon}}$$

$$(d)V_0e^{-\frac{t\sigma}{\epsilon}}$$

**35.** A stationary magnetic dipole  $\mathbf{m} = \mathbf{m}\mathbf{\hat{k}}$  is placed above an infinite surface (z = 0) carrying a uniform surface current density  $\mathbf{\kappa} = \kappa \mathbf{\hat{i}}$ . The torque on the dipole is Options:-

$(a)\frac{\mu_0}{2}m\kappa\hat{\mathbf{i}},$	$(b) - \frac{\mu_0}{2} m\kappa i$
$(c)\frac{\tilde{\mu_0}}{2}m\kappa\hat{j}$	$(d) - \frac{\tilde{\mu_0}}{2} m\kappa_1^2$

**36.** Two parallel conducting rings, both of radius R, are separated by a distance R. The planes of the rings are perpendicular to the line joining their centres, which is taken to be the x-axis.



If both the rings carry the same current i along the same direction, the magnitude of the magnetic field along the x axis is best represented by



**37.** At time t = 0, a particle is in the ground state of the Hamiltonian  $H(t) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 + \lambda xsin \frac{\omega t}{2}$  where  $\lambda, \omega$  and m are positive constants. To  $O(\lambda^2)$ , the probability that at  $t = \frac{2\pi}{\omega}$ , the particle would be in the first excited state of H(t = 0) is

(a) 
$$\frac{9\lambda^2}{16m\hbar\omega^3}$$
 (b)  $\frac{9\lambda^2}{8m\hbar\omega^3}$   
(c)  $\frac{16\lambda^2}{9m\hbar\omega^3}$  (d)  $\frac{8\lambda^2}{9m\hbar\omega^3}$ 

**38.** To first order in perturbation theory, the energy of the ground state of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{\hbar\omega}{\sqrt{512}}\exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

(treating the third term of the Hamiltonian as a perturbation) is

$$(a)^{\frac{15}{32}}\hbar\omega$$
 $(b)^{\frac{17}{32}}\hbar\omega$ 
 $(c)^{\frac{19}{32}}\hbar\omega$ 
 $(d)^{\frac{21}{32}}\hbar\omega$ 

**39.** The energy/energies E of the bound state(s) of a particle of mass m in one dimension in the potential V(x) =  $\begin{cases} \infty, & x \le 0 \\ -V_0, & 0 < x < a \text{ (where } V_0 > 0\text{) is/are} \\ 0, & x \ge a \\ \text{determined by} \end{cases}$  $(a) \cot^2 \left( a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = \frac{E-V_0}{E}$ 

$$(b)\tan^{2}\left(a\sqrt{\frac{2m(E+V_{0})}{\hbar^{2}}}\right) = -\frac{E}{E+V_{0}}$$
$$(c)\cot^{2}\left(a\sqrt{\frac{2m(E+V_{0})}{\hbar^{2}}}\right) = -\frac{E}{E+V_{0}}$$
$$(d)\tan^{2}\left(a\sqrt{\frac{2m(E+V_{0})}{\hbar^{2}}}\right) = \frac{E-V_{0}}{E}$$

**40.** The energy levels of a system, which is in equilibrium at temperature  $T = 1/(k_B\beta)$ , are 0,  $\epsilon$  and  $2\epsilon$ . If two identical bosons occupy these energy levels, the probability of the total energy being  $3\epsilon$ , is

$$(a) \frac{e^{-3\beta\epsilon}}{1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta}}$$
$$(b) \frac{e^{-3\beta\epsilon}}{1+2e^{-\beta}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$$
$$(c) \frac{e^{-3\beta\epsilon}}{e^{-\beta}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$$



**44.** A high frequency voltage signal  $V_i =$  $V_m$ sin  $\omega t$  is applied to a parallel plate deflector as shown in the figure  $D_{-}$ Ż An electron beam is passing through the deflector along the central line. The best (a qualitative representation of the intensity (0 I(t) of the beam after it goes through the narrow circular aperture D, is (a) I(t)(a)2330 and 2 (c)2350 and 3 ωt 2 3 (b)  $I(t)^{\prime}$ ωt 2 3 (c)  $I(t)^{t}$ ωt  $(v_{\mu} - v_{H})/v_{H}$  is (a)0.001 (d) (c)-0.01 I(t)ωt

**45.** An amplifier with a voltage gain of 40 dB without feedback is used in an electronic circuit. A negative feedback with a fraction 1/40 is connected to the input of this amplifier. The net gain of the amplifier in the circuit is closest to

(a)40 dB	(b)37 dB		
(c)29 dB	(d)20 dB		

**46.** A receiver operating at  $27^{\circ}$ c has an input resistance of  $100\Omega$ . The input thermal noise voltage for this receiver with a bandwidth of 100kHz is closest to

a)0.4nV	(b)0.6pV	
c)40mV	(d)0.4µV	

**47.** The Raman rotational-vibrational spectrum of nitrogen molecules is observed using an incident radiation of wavenumber  $12500 \text{ cm}^{-1}$ . In the first shifted band, the wavenumbers of the observed lines (in cm<sup>-1</sup>) are 10150,10158,10170,10182 and 10190. The values of vibrational frequency and rotational constant (in cm<sup>-1</sup>), respectively, are (a)2330 and 2 (b)2350 and 2

(d)2330 and 3

- **48.** The electronic configuration of  ${}^{12}C$  is  $1s^2 2s^2 2p^2$ . Including LS coupling, the correct ordering of its energies is (a)E( ${}^{3}P_2$ ) < E( ${}^{3}P_1$ ) < E( ${}^{3}P_0$ ) < E( ${}^{1}D_2$ ) (b)E( ${}^{3}P_0$ ) < E( ${}^{3}P_1$ ) < E( ${}^{3}P_2$ ) < E( ${}^{1}D_2$ ) (c)E( ${}^{1}D_2$ ) < E( ${}^{3}P_2$ ) < E( ${}^{3}P_1$ ) < E( ${}^{3}P_0$ ) (d)E( ${}^{3}P_1$ ) < E( ${}^{3}P_0$ ) < E( ${}^{3}P_2$ ) < E( ${}^{1}D_2$ )
- **49.** In the absorption spectrum of H-atom, the frequency of transition from the ground state to the first excited state is  $v_H$ . The corresponding frequency for a bound state of a positively charged muon ( $\mu^+$ )and an electron is  $v_{\mu}$ . Using  $m_{\mu} = 10^{-2}$  kg,  $m_e = 10^{-30}$  kg and  $m_p \gg m_e$ ,  $m_{\mu}$ , the value of  $(v_{\mu} v_H)/v_H$  is

(b)0.001
(d)0.01

**50.** The energies of a two-level system are  $\pm E$ . Consider an ensemble of such noninteracting systems at a temperature T. At low temperatures, the leading term in the specific heat depends on T as

(a)
$$\frac{1}{T^2}e^{-E/k_BT}$$
 (b) $\frac{1}{T^2}e^{-\frac{2E}{k_BT}}$   
(c) $T^2e^{-E/k_BT}$  (d) $T^2e^{-\frac{2E}{k_BT}}$ 

**51.** The Figures (i), (ii) and (iii) below represent an equilateral triangle, a rectangle and a regular hexagon, respectively.



Which of these can be primitive unit cells of a Bravais lattice in two dimensions? (a)Only (i) and (iii) but not (ii), (b)Only (i) and (ii) but not (iii), (c)Only (ii) and (iii) but not (i), (d)All of them

**52.** The Hamiltonian for a spin- 1/2 particle in a magnetic field  $\mathbf{B} = B_0 \hat{k}$  is given by  $\mathbf{H} = \lambda \mathbf{S} \cdot \mathbf{B}$ , where **S** is its spin (in units of  $\hbar$ ) and  $\lambda$  is a constant. If the average spin density is  $\langle \mathbf{S} \rangle$  for an ensemble of such non- interacting particles, then  $\frac{d}{dt} \langle S_x \rangle$ 

 $(a)\frac{\lambda}{\hbar}B_{0}\langle S_{x}\rangle \qquad (b)\frac{\lambda}{\hbar}B_{0}\langle S_{y}\rangle \\ (c)-\frac{\lambda}{\hbar}B_{0}\langle S_{x}\rangle \qquad (d)-\frac{\lambda}{\hbar}B_{0}\langle S_{y}\rangle$ 

## **53.** The tensor component of the nuclear force may be inferred from the fact that deuteron nucleus

$${}_{1}^{2}H$$

(a)has only one bound state with total spin S = 1

(b)has a non-zero electric quadrupole moment in its ground state

(c) Is stable while triton  ${}_{1}^{3}$ H is unstable

- (d)Is the only two nucleon bound state
- **54.** The elastic scattering process  $\pi^- p \rightarrow \pi^- p$ may be treated as a hard-sphere scattering. The mass of  $\pi^-$ ,  $m_\pi \simeq \frac{1}{6}m_p$ , where  $m_p \simeq$ 938MeV/c<sup>2</sup> is the mass of the proton. The total scattering cross-section is closest to (a)0.01 milli-barn (b)1 milli-barn, (c) 0.1 barn, (d) 10 barn,

55. Thermal neutrons may be detected most efficiently by a
(a)Li<sup>6</sup> loaded plastic scintillator
(b)Geiger-Müller counter
(c)inorganic scintillatorCaF2
(d) silicon detector

## ANSWER KEY

1. c	2. d	3. d	4. a	5. b
6. b	7. a	8. d	9.	10. b
11. a	12. b	13. c	14. a	15. c
16. d	17. a	18.	19. a	20. d
21. d	22. b	23. b	24.	25. b
26. b	27. b	28. d	29. a	30. d
31. b	32. a	33. b	34. d	35. a
36. a	37. d	38. b	39. c	40. b
41. d	42. d	43. b	44. a	45. c
46. d	47. a	48. b	49. c	50. b
51. c	52. d	53. b	54. c	55. a