❖ CSIR-UGC-NET/JRF- JUNE – 2016 PHYSICAL SCIENCES BOOKLET - [A]

➢ PART-B

1. The radius of convergence of the Taylor series expansion of the function $\frac{1}{\cosh(x)}$ around $x = 0$, is

(a) ∞ (b) π

(c) $\frac{\pi}{2}$ (d) 1

2. The value of the contour integral $\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$ around the unit circle C traversed in the anti-clockwise direction, is

(a) 0 (b) 2

(c) $-\frac{8}{\sqrt{3}}$ (d) $-\tanh\left(\frac{1}{2}\right)$

3. The Gauss hypergeometric function $F(a, b, c; z)$, defined by the Taylor series expansion around $z = 0$ as

$$F(a, b, c; z) = \sum_{n=0}^{\infty} \frac{a(a+1)\ldots(a+n-1)b(b+1)\ldots(b+n-1)}{c(c+1)\ldots(c+n-1)n!} z^n$$

satisfies the equation relation

(a) $\frac{d}{dz}F(a, b, c; z) = \frac{c}{ab}F(a-1, b-1, c-1; z)$

(b) $\frac{d}{dz}F(a, b, c; z) = \frac{c}{ab}F(a+1, b+1, c+1; z)$

(c) $\frac{d}{dz}F(a, b, c; z) = \frac{ab}{c}F(a-1, b-1, c=1; z)$

(d) $\frac{d}{dz}F(a, b, c; z) = \frac{ab}{c}F(a+1, b+1, c+1; z)$

4. Let X and Y be two independent random variables, each of which follow a normal distribution with the same standard deviation σ, but with means $+\mu$ and $-\mu$, respectively. Then the sum $X + Y$ follows a

(a) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$

(b) normal distribution with mean 0 and standard deviation 2σ

(c) distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation 2σ

(d) normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$

5. Using dimensional analysis, Planck defined a characteristic temperature T_P from powers of the gravitional constant G, Planck's constant h, Boltzmann constant k_k and the speed of light c in vacuum. The expression for T_P is proportional to

(a) $\sqrt{\frac{hc^5}{k_B^2 G}}$ (b) $\sqrt{\frac{hc^3}{k_B^2 G}}$

(c) $\sqrt{\frac{G}{hc^4 k_b^2}}$ (d) $\sqrt{\frac{hk_\beta^2}{Gc^3}}$

6. Let (x, t) and (x', t') be the coordinate systems used by the observers O and O', respectively. Ob. server O' moves with a velocity $v = \beta c$ along their common positive x-axis. If $x_+ = x + ct$ and $x_- = x - ct$ are the linear combinations of the coordinates, the Lorentz transformation relating O and O' takes the form

(a) $x'_+ = \frac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$ and $x'_- = \frac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$,

(b) $x'_+ = \sqrt{\frac{1+\beta}{1-\beta}}x_+$ and $x'_- = \sqrt{\frac{1-\beta}{1+\beta}}x_-$

(c) $x'_+ = \frac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$ and $x'_- = \frac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$,

(d) $x'_+ = \sqrt{\frac{1-\beta}{1+\beta}}x_+$ and $x'_- = \sqrt{\frac{1+\beta}{1-\beta}}x_-$

7. A ball of mass m, initially at rest, is dropped from a height of 5 meters. If the coefficient of restitution is 0.9 , the speed of the ball just before it hits the floor the second time is approximately (take $g = 9.8\ \mathrm{m/s^2}$)

(a) 9.80 m/s (b) 9.10 m/s
(c) 8.91 m/s (d) 7.02 m/s

8. Four equal charges of $+Q$ each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a(\ll R)$ from the center. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

(a) $\sqrt{\frac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ (b) $\sqrt{\frac{Q^2}{\pi\varepsilon_0 R^3 m}}$
(c) $\sqrt{\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 R^5 m}}$ (d) $\sqrt{\frac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

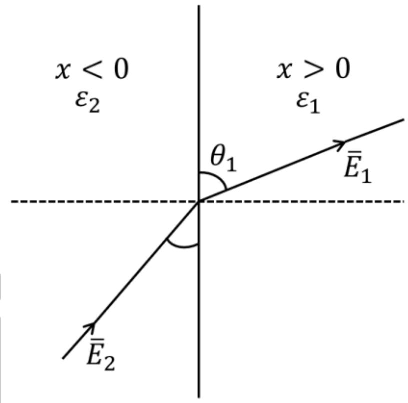
9. The Hamiltonian of a system with generalized coordinate and momentum (q, p) is $H = p^2 q^2$. A solution of the Hamiltonian equation of motion is (in the following A and B are constants)

(a) $p = Be^{-2At}, q = \frac{A}{B}e^{2At}$
(b) $p = Ae^{-2At}, q = \frac{A}{B}e^{-2At}$
(c) $p = Ae^{At}, q = \frac{A}{B}e^{-At}$
(d) $p = 2Ae^{-A^2t}, q = \frac{A}{B}e^{A^2t}$

10. Two parallel plate capacitors, separated by distances x and $1,1x$ respectively, have a dielectric material of dielectric constant 3.0 inserted between the plates, and are connected to a battery of voltage V. The difference in charge on the second capacitor compared to the first is

(a) +66% (b) +20%
(c) −3.3% (d) −10%

11. The half space regions $x > 0$ and $x < 0$ are filled with dielectric media of dielectric constants ε_1 and ε_2 respectively. There is a uniform electric field in each part. In the right half, the electric field makes an angle θ_1 to the interface. The corresponding angle θ_2 in the left half satisfies



(a) $\varepsilon_1 \sin\ \theta_2 = \varepsilon_2 \sin\ \theta_1$
(b) $\varepsilon_1 \tan\ \theta_2 = \varepsilon_2 \tan\ \theta_1$
(c) $\varepsilon_1 \tan\ \theta_1 = \varepsilon_2 \tan\ \theta_2$
(d) $\varepsilon_1 \sin\ \theta_1 = \varepsilon_2 \sin\ \theta_2$

12. The x - and z-components of a static magnetic field in a region are $B_x = B_0(x^2 - y^2)$ and $B_z = 0$, respectively. Which of the following solutions for its y-component is consistent with the Maxwell equations?

(a) $B_y = B_0 xy$
(b) $B_y = -2B_0 xy$
(c) $B_y = -B_0(x^2 - y^2)$
(d) $B_y = B_0\left(\frac{1}{3}x^3 - xy^2\right)$

13. A magnetic field $\boldsymbol{B}$ is $B\hat{z}$ in the region $x > 0$ and zero elsewhere. A rectangular loop, in the xy plane, of sides l (along the x-direction) and h (along the y-direction) is inserted into the $x > 0$ region from the $x < 0$ region at a constant velocity $\mathbf{v} = \mathrm{v}\hat{x}$. Which of the following values of l and h will generate the largest EMF ?

(a) $l = 8, h = 3$ (b) $l = 4, h = 6$
(c) $l = 6, h = 4$ (d) $l = 12, h = 2$

14. The state of a particle of mass m in a one-dimensional rigid box in the interval 0 to L is given by the normalised wavefunction

$$\psi(x) = \sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right) + \frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right)$$

If its energy is measured, the possible outcomes and the average value of energy are, respectively.

(a) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{50}\frac{h^2}{mL^2}$
(b) $\frac{h^2}{8mL^2}, \frac{h^2}{2mL^2}$ and $\frac{19}{40}\frac{h^2}{mL^2}$
(c) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{19}{10}\frac{h^2}{mL^2}$
(d) $\frac{h^2}{8mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{200}\frac{h^2}{mL^2}$

15. If $\hat{L}_x, \hat{L}_y$ and $\hat{L}_x$ are the components of the angular momentum operator in three dimensions, the commutator $[\hat{L}_x, \hat{L}_x\hat{L}_y\hat{L}_z]$ may be simplified to

(a) $i\hbar L_x(\hat{L}_z^2 - \hat{L}_y^2)$
(b) $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$
(c) $i\hbar L_i(2\hat{L}_i^2 - \hat{L}_y^2)$
(d) 0

16. Suppose that the Coulomb potential of the hydrogen atom is changed by adding an inverse-square term such that the total potential is $V(\vec{r}) = -\frac{ze^2}{r} + \frac{g}{r^2}$, where g is a constant. The energy eigenvalues $E_{m/N}$ in the modified potential

(a) depend on n and l, but not on m
(b) depend on n but not on l and m
(c) depend on n and m, but not on l
(d) depend explicitly on all three quantum numbers n, l and m

17. The eigenstates corresponding to eigenvalues E_1 and E_2 of a time-independent Hamiltonian are $|1\rangle$ and $|2\rangle$ respectively. If at $t = 0$, the system is in a state $|\psi(t=0)\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle$ the value of $\langle\psi(t) \mid \psi(t)\rangle$ at time t will be

(a) 1
(b) $\frac{(E_1\sin^2\theta + E_2\cos^2\theta)}{\sqrt{E_1^2+E_2^2}}$
(c) $e^{iE_1t/\hbar}\sin\theta + e^{iE_2/\hbar}\cos\theta$
(d) $e^{-iE_1//\hbar}\sin^2\theta + e^{-iE_2t/\hbar}\cos^2\theta$

18. The specific heat per molecule of a gas of diatomic molecules at high temperatures is

(a) $8k_B$
(b) $3.5k_B$
(c) $4.5k_B$
(d) $3k_B$

19. When an ideal monatomic gas is expanded adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio T/T_0 is

(a) $\frac{1}{3}$
(b) $\left(\frac{1}{3}\right)^{2/3}$
(c) $\left(\frac{1}{3}\right)^{1/3}$
(d) 3

20. A box of volume V containing N molecules of an ideal gas, is divided by a wall with a hole into two compartments. If the volume of the smaller compartment is $V/3$, the variance of the number of particles in it, is

(a) $\frac{N}{3}$
(b) $\frac{2N}{9}$
(c) $\sqrt{N}$
(d) $\frac{\sqrt{N}}{3}$

21. A gas of non-relativistic classical particles in one-dimension is subjected to a potential $V(x) = \alpha|x|$, where α is a constant). The partition function is $\left(\beta = \frac{1}{k_BT}\right)$

(a) $\sqrt{\frac{4m\pi}{\beta^3\alpha^2h^2}}$
(b) $\sqrt{\frac{2m\pi}{\beta^3\alpha^2h^2}}$
(c) $\sqrt{\frac{8m\pi}{\beta^3\alpha^2h^2}}$
(d) $\sqrt{\frac{3m\pi}{\beta^3\alpha^2h^2}}$

22. The dependence of current I on the voltage V of a certain device is given by

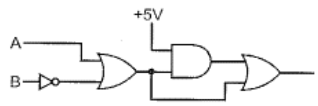
$$I = I_0\left(1 - \frac{V}{V_0}\right)^2$$

where I_0 and V_0 are constants. In an experiment the current I is measured as the voltage V applied across the device is increased. The parameters V_0 and $\sqrt{I_0}$ can be graphically determined as

(a) the slope and the y-intercept of the $I - V^2$ graph
(b) the negative of the ratio of the y-intercept and the slope, and the y-intercept of the $I - V^2$ graph

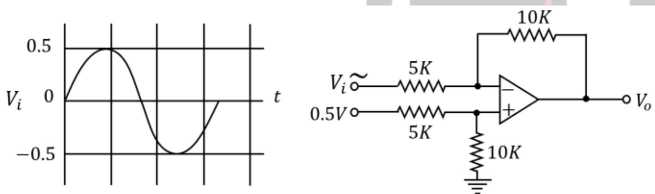
(c) the slope and the y-intercept of the $\sqrt{I}-V$ graph
(d) the negative of the ratio of the y-intercept and the slope, and the y-intercept of the $\sqrt{I}-V$ graph

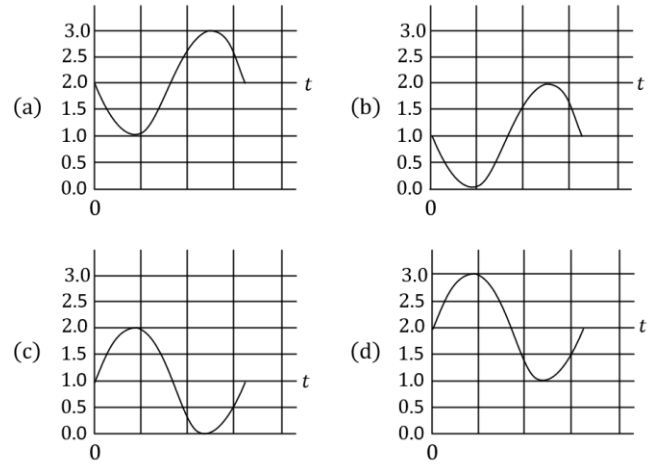
23. In the schematic figure given below, assume that the propagation delay of each logic gate is t_{gume} .
The propagation delay of the circuit will be maximum when the logic inputs A and B make the transition



(a) $(0,1) \rightarrow (1,1)$
(b) $(1,1) \rightarrow (0,1)$
(c) $(0,0) \rightarrow (1,1)$
(d) $(0,0) \rightarrow (0,1)$

24. Given the input voltage V_i, which of the following waveforms correctly represents the output voltage V_0 in the circuit shown below ?





25. The intensity distribution of a red LED on an absorbing layer of material is a Gaussian centered at the wavelength $\lambda_0 = 660$ nm and width 20 nm. If the absorption coefficients varies with wavelength as $\alpha_0 - K(\lambda - \lambda_0)$, where α_0 and K are positive constants, the light emerging from the absorber will be
(a) blue shifted retaining the Gaussian intensity distribution
(b) blue shifted with an asymmetric intensity distribution
(c) red shifted retaining the Gaussian intensity distribution
(d) red shifted with an asymmetric intensity distribution

➢ PART - C

26. What is the Fourier transform $\int dx e^{ikx} f(x)$ of $f(x) = \delta(x) + \sum_{n=1}^{\infty} \frac{d^n}{dx^n}\delta(x)$, where $\delta(x)$ is the Dirac delta-function?

(a) $\frac{1}{1-ik}$ (b) $\frac{1}{1+ik}$
(c) $\frac{1}{k+i}$ (d) $\frac{1}{k-i}$

27. The integral equation $\phi(x,t) = \lambda \int dx'dt'$

$$\int \frac{d\omega dk}{(2\pi)^2} \frac{e^{-ik(x-x')+i\omega(t-t')}}{\omega^2 - k^2 - m^2 + i\varepsilon} \phi^3(x',t')$$

is equivalent to the differential equation

(a) $\left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} - m^2 + i\varepsilon\right)\phi(x,t) = -\frac{1}{6}\lambda\phi^3(x,t)$

(b) $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2 - i\varepsilon\right)\phi(x,t) = \lambda\phi^2(x,t)$
(c) $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2 - i\varepsilon\right)\phi(x,t) = -3\lambda\phi^2(x,t)$
(d) $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2 - i\delta\right)\phi(x,t) = -\lambda\phi^3(x,t)$

28. A part of the group multiplication table for a six element group $G = \{e, a, b, c, d, f\}$ is shown below. (In the following e is the identity element of G).

s.	e	a	b	c	d	$\vec{f}$
e	e	a	b	c	d	f
a	a	b	e	d		
b	b	e	x	f	y	z
c	c					
d	d					
f	f					

The entries x, y and z should be
(a) $x = a, y = d$ and $z = c$
(b) $x = c, y = a$ and $z = d$
(c) $x = c, y = d$ and $z = a$
(d) $x = a, y = c$ and $z = d$

29. In finding the roots of the polynomial $f(x) = 3x^3 - 4x - 5$ using the iterative Newton-Raphson method, the initial guess is taken to be $x = 2$. In the next iteration its value is nearest to
(a) 1.671 (b) 1.656
(c) 1.559 (d) 1.551

30. For a particle of energy E and momentum p (in a frame F), the rapidity y is defined as $y = \frac{1}{2}\ln\left(\frac{E+p_3c}{E-p_3c}\right)$. In a frame F' moving with velocity $v = (0,0,\beta c)$ with respect to F, the rapidity y' will be
(a) $y' = y + \frac{1}{2}\ln(1-\beta^2)$
(b) $y' = y - \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$
(c) $y' = y + \ln\left(\frac{1+\beta}{1-\beta}\right)$
(d) $y' = y + 2\ln\left(\frac{1+\beta}{1-\beta}\right)$

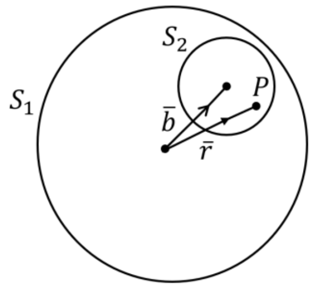
31. A canonical transformation $(q,p) \to (Q,P)$ is made through the generating function $F(q,P) = q^2P$ on the Hamiltonian $H(q,p) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4}q^4$, where α and β are constants. The equations of motion for (Q,P) are
(a) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\beta Q$
(b) $\dot{Q} = \frac{4P}{\alpha}$ and $P = \frac{-\beta Q}{2}$
(c) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\frac{2P^2}{Q} - \beta Q$
(d) $\dot{Q} = \frac{2P}{\alpha}$ and $\dot{P} = -\beta Q$

32. The Lagrangian of a system moving in three dimensions is

$$L = \frac{1}{2}m\dot{x}_1^2 + m(\dot{x}_2^2 + \dot{x}_3^2) - \frac{1}{2}kx_1^2 - \frac{1}{2}k(x_2 + x_3)^2$$

The independent constant(s) of motion is/are
(a) energy alone
(b) only energy, one component of the linear momentum and one component of the angular momentum
(c) only energy and one component of the linear momentum
(d) only energy and one component of the angular momentum

33. Consider a sphere S_1 of radius R which carries a uniform charge of density ρ. A smaller sphere S_2 of radius $a < \frac{R}{2}$ is cut out and removed from it. The centers of the two spheres are separated by the vector $\vec{b} = \hat{n}\frac{R}{2}$, as shown in the figure.



The electric field at a point P inside S_2 is

(a) $\frac{\rho R}{3\varepsilon_0}\hat{n}$ (b) $\frac{\rho R}{3\varepsilon_0 a}(\vec{r}-\hat{n}a)$

(c) $\frac{\rho R}{6\varepsilon_0}\hat{n}$ (d) $\frac{\rho a}{3\varepsilon_0 R}\vec{r}$

34. The values of the electric and magnetic fields in a particular reference frame (in Gaussian units) are $\boldsymbol{E} = 3\hat{x} + 4\hat{y}$ and $\boldsymbol{B} = 3\hat{z}$, respectively. An inertial observer moving with respect to this frame measures the magnitude of the electric field to be $|E'| = 4$. The magnitude of the magnetic field $|B'|$ measured by him is

(a) 5 (b) 9

(c) 0 (d) 1

35. A loop of radius a, carrying a current I, is placed in a uniform magnetic field $\boldsymbol{B}$. If the normal to the loop is denoted by $\hat{n}$, the force F and the torque T on the loop are

(a) $\boldsymbol{F} = 0$ and $\boldsymbol{T} = \pi a^2 I\hat{n} \times \boldsymbol{B}$

(b) $\boldsymbol{F} = \frac{\mu_0}{4\pi} I \times \boldsymbol{B}$ and $\boldsymbol{T} = 0$

(c) $\boldsymbol{F} = \frac{\mu_0}{4\pi} I \times \boldsymbol{B}$ and $\boldsymbol{T} = I\hat{n} \times \boldsymbol{B}$

(d) $\boldsymbol{F} = 0$ and $\boldsymbol{T} = \frac{1}{\mu_0\varepsilon_0} I\boldsymbol{B}$

36. A wavelength has a square cross-section of side $2a$. For the TM modes of wavevector k, the transverse electromagnetic modes are obtained in terms of a function $\psi(x, y)$ which obeys the equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega^2}{c^2} - k^2\right)\right]\psi(x, y) = 0$$

with the boundary condition $\psi(\pm a, y) = \psi(x, \pm a) = 0$. The frequency ω of the lowest mode is given by

(a) $\omega^2 = c^2\left(k^2 + \frac{4\pi^2}{a^2}\right)$

(b) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{a^2}\right)$

(c) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{2a^2}\right)$

(d) $\omega^2 = c^2\left(k^2 + \frac{\pi^2}{4a^2}\right)$

37. Consider a particle of mass m in a potential $V(x) = \frac{1}{2}m\omega^2x^2 + g\cos\, kx$. The change in the ground state energy, compared to the simple harmonic potential $\frac{1}{2}m\omega^2x^2$, to first order in g is

(a) $g\exp\left(-\frac{k^2\hbar}{2m\omega}\right)$ (b) $g\exp\left(\frac{k^2\hbar}{2m\omega}\right)$

(c) $g\exp\left(-\frac{2k^2\hbar}{m\omega}\right)$ (d) $g\exp\left(-\frac{k^2\hbar}{4m\omega}\right)$

38. The energy levels for a particle of mass m in the potential $V(x) = \alpha|x|$, determined in the WKB approximation $\sqrt{2m}\int_a^b \sqrt{E - V(x)}dx = \left(n + \frac{1}{2}\right)\hbar\pi$, where a, b are the turning points and $n = 0,1,2, \dots$), are

(a) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(b) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(c) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

(d) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

39. A particle of mass m moves in one-dimension under the influence of the potential $V(x) = -\alpha\delta(x)$, where α is a positive constant. The uncertainty in the product $(\Delta x)(\Delta p)$ in its ground state is

(a) $2\hbar$ (b) $\frac{\hbar}{2}$

(c) $\frac{\hbar}{\sqrt{2}}$ (d) $\sqrt{2}\hbar$

40. The ground state energy of a particle of mass m in the potential $V(x) = \frac{\hbar^2\beta}{6m}x^4$, estimated using the normalized trial wavefunction $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$ is [Use $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^2 e^{-\alpha x^2} = \frac{1}{2\alpha}$ and $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx x^4 e^{-\alpha x^2} = \frac{3}{4\alpha^2}$].

(a) $\frac{3}{2m}\hbar^2\beta^{1/3}$
(b) $\frac{8}{3m}\hbar^2\beta^{1/3}$
(c) $\frac{2}{3m}\hbar^2\beta^{1/3}$
(d) $\frac{3}{8m}\hbar^2\beta^{1/3}$

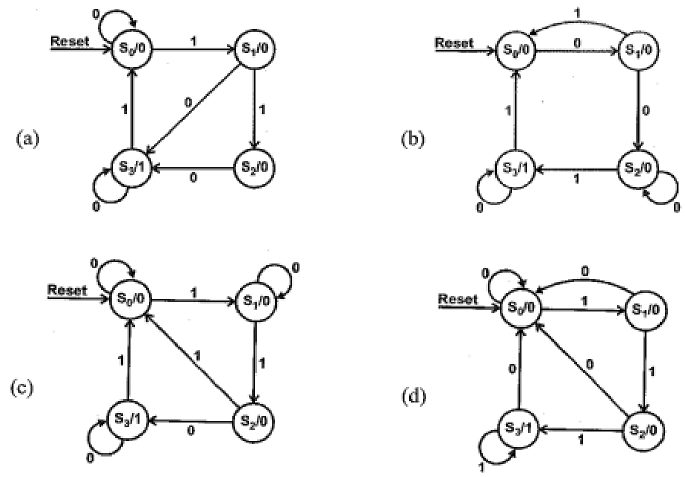
41. Consider a gas of Cs atoms at a number density of 10^{12} atoms/cc. When the typical inter-particle distance is equal to the thermal de-Broglie wavelength of the particles, the temperature of the gas is nearest to (Take the mass of a Cs atom to be 22.7×10^{-26} kg).
(a) 1×10^{-9} K (b) 7×10^{-5} K
(c) 1×10^{-3} K (d) 2×10^{-8} K

42. The internal energy $E(T)$ of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$. Then the entropy $S(T)$, as a function of temperature, is
(a) $\frac{1}{2}aT^2 + \frac{1}{4}bT^4$ (b) $2aT^2 + 4bT^4$
(c) $2aT + \frac{4}{3}bT^3$ (d) $2aT + 2bT^3$

43. A radioactive element X decays to Y, which in turn decays to a stable element Z. The decay constant from X to Y is λ_1, and that from Y to Z is λ_2. If, to begin with, there are only N_0 atoms of X, at short times ($t \ll 1/\lambda_1$ as well as $1/\lambda_2$) the number of atoms of Z will be
(a) $\frac{1}{2}\lambda_1\lambda_2 N_0 t^2$ (b) $\frac{\lambda_1\lambda_2}{2(\lambda_1+\lambda_2)}N_0 t$
(c) $(\lambda_1 + \lambda_2)^2 N_0 t^2$ (d) $(\lambda_1 + \lambda_2)N_0 t$

44. Two completely overlapping semi-circular parallel plates comprise a capacitive transducer. One of the plates is rotated by an angle of 10° relative to their common center. Ignoting edge effects, the ratio, $I_n : I_0$ of sensitivity of the transducer in the new configuration with respect to the original one, is
(a) 8: 9 (b) 11: 12
(c) 17: 18 (d) 35: 36

45. The state diagram that detects three or more consecutive 1's in a surial bit stream is



46. The decay constants f_p of the heavy pseudoscalar mesons, in the heavy quark limit, are related to their masses m_p by the relation $f_P = \frac{a}{\sqrt{m_p}}$, where a is an empirical parameter to be determined. The values $m_p = 6400 \pm 160$MeV and $f_p = 180 \pm 15$MeV correspond to uncorrelated measurement of a meson. The error on the estimate of a is
(a) $175(\text{MeV})^{3/2}$ (b) $900(\text{MeV})^{3/2}$
(c) $1200(\text{MeV})^{3/2}$ (d) $2400(\text{MeV})^{3/2}$

47. Consider electrons in graphene, which is a planar monatomic layer of carbon atoms. If the dispersion relation of the electrons is taken to be $\varepsilon(k) = ck$ (where c is constant). over the entire k-space, then the Fermi energy ε_F depends on the number density of electrons ρ as
(a) $\varepsilon_F \propto \rho^{1/2}$ (b) $\varepsilon_F \propto \rho$
(c) $\varepsilon_F \propto \rho^{2/3}$ (d) $\varepsilon_F \propto \rho^{1/3}$

48. Suppose the frequency of phonons in a one-dimensional chain of atoms is proportional to the wavevector. If n is the number density of atoms and c is the speed of the phonons, then the Debye frequency is
(a) $2\pi cn$ (b) $\sqrt{2}\pi cn$
(c) $\sqrt{3}\pi cn$ (d) $\frac{\pi cn}{2}$

49. The band energy of an electron in a crystal for a particular k-direction has the form $\varepsilon(k) = A - B\cos\ 2ka$, where A and B are positive constants and $0 < ka < \pi$. The electron has a holelike behaviour over the following range of k :

(a) $\frac{\pi}{4} < ka < \frac{3\pi}{4}$ (b) $\frac{\pi}{2} < ka < \pi$

(c) $0 < ka < \frac{\pi}{4}$ (d) $\frac{\pi}{2} < ka < \frac{3\pi}{4}$

50. The ground state electronic configuration of ^{22}Ti is $[\mathrm{Ar}]3d^2 4s^2$. Which state, in the standard spectroscopic notations, is not possible in this configuration?

(a) $^1F_3$ (b) $^1S_0$

(c) $^t\mathrm{D}_2$ (d) $^3P_0$

51. In a normal Zeeman effect experiment using a magnetic field of strength 0.3 T, the splitting between the components of a 660 nm spectral line is

(a) 12pm (b) 10pm

(c) 8pm (d) 6pm

52. The separation between the energy levels of a two-level atom is 2eV. Suppose that 4×10^{20} atoms are in the ground state and 7×10^{20} atoms are pumped into the excited state just before lasing starts. How much energy will be released in a single laser pulse ?

(a) 24.6 J (b) 22.4 J

(c) 98 J (d) 48 J

53. In the large hadron collider (LHC), two equal energy proton beams traverse in opposite directions along a circular path of length 27 km. If the total center of mass energy of a proton-proton pair is 14 TeV, which of the following is the best approximation for the proper time taken by a proton to traverse the entire path?

(a) 12 ns (b) $1.2\mu s$

(c) 1.2 ns (d) $0.12\mu s$

54. Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S({}^{27}_{13}\mathrm{Al}) : E_S({}^{64}_{30}\mathrm{Zn})$ is

(a) 2: 3 (b) 4: 3

(c) 5: 3 (d) 3: 2

55. According to the shell model, the nuclear magnetic moment of the ${}^{27}_{13}$Al nucleus is (Given that for a proton $g_l = 1, g_s = 5.586$, and for a neutron $g_l = 0, g_s = -3.826$).

(a) $-1.913\mu_N$ (b) $14.414\mu_N$

(c) $4.793\mu_N$ (d) 0

❖ ANSWER KEY

21. c	22. c	23. d	24. d	25. a
26. d	27. c	28. c	29. a	30. d
31. c	32. b	33. b	34. a	35. a
36. a	37. a	38. b,c	39. b	40. b
41. c	42. d	43. d	44. b	45. d
46. b	47. d	48. d	49. b	50. b
51. b	52. a	53. c	54. c	55. a
56. c	57. d	58. b	59. c	60. d
61. d	62. c	63. a	64. c	65. d
66. c	67. a	68. a	69. a,d	70. a
71. d	72. d	73. a	74. b	75.