

(a) $\frac{1}{2}(x^2 + y^2)$ (b) $\frac{1}{2}(x^2 - y^2)$ (c) x + y (d) x - y

9. Consider three inertial frames of reference A, B, and C. The frame B moves with a velocity c/2 with respect to A and C moves with a velocity c/10 with respect to B in the same direction. The velocity of C as measured in A is

(a) $\frac{3c}{7}$	(b) $\frac{4c}{7}$
(c) $\frac{c}{7}$	(d) $\frac{\sqrt{3}c}{7}$

- **10.** A plane electromagnetic wave is travelling along the positive *z*-direction. The maximum electric field along the *x*-direction is 10 V/m. The approximate maximum values of the power per unit area and the magnetic induction *B*, respectively, are (a) 3.3×10^{-7} watts /m² and 10 tesla (b) 3.3×10^{-7} watts//m² and 3.3×10^{-4}
 - tesla
 - (c) 0.265 watts $/m^2$ and 10 tesla
 - (d) 0.265 watts/m² and 3.3×10^{-8} tesla
- **11.** Suppose the *yz*-plane forms a chargeless boundary between two media of permittivities $\varepsilon_{\text{kett}}$ and $\varepsilon_{\text{right}}$ where $\varepsilon_{\text{lett}}$: $\varepsilon_{\text{right}} = 1$: 2. If the uniform electric field on the left is $\vec{E}_{\rm ten} = c(\hat{\imath} + \hat{\jmath} + \hat{k})$ (where *c* is a constant), then the electric field on the right \vec{E}_{right} is (a) $c(2\hat{i} + \hat{j} + \hat{k})$ (b) $c(\hat{i} + 2\hat{j} + 2\hat{k})$ (c) $c(\frac{1}{2}\hat{i} + \hat{j} + \hat{k})$ (d) $c(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k})$
- **12.** A proton moves with a speed of 300 m/s in a circular orbit in the *xy*-pland in a magnetic field 1 tesla along the positive *z*-direction. When an electric field of 1 V/m is applied along the positive y-direction, the center of the circular orbit
 - (a) remains stationary

(b) moves at 1 m/s along the negative xdirection

(c) moves at 1 m/s along the positive zdirection

(d) moves at 1 m/s along the positive xdirection

13. Which of the following transformations $(V, \vec{A}) \rightarrow (V', \vec{A'})$ of the electrostatic potential V and the vector potential \vec{A} is a gauge transformation? (a) $(V' = V + ax, \vec{A'} = \vec{A} + at\hat{k})$ (b) $(V' = V + ax, \vec{A}' = \vec{A} - at\hat{k})$ (c) $\left(V' = V + a x \vec{A}' = \vec{A} + a t \hat{i}\right)$

(d)
$$(V' = V + ax, \vec{A'} = \vec{A} \cdots a\hat{i})$$

- **14.** The ratio of the energy of the first excited state E_1 , to that of the ground state E_0 of a particle in a three-dimensional rectangular box of sides L, L and L/2 is (a) 3:2 (b) 2:1 (c) 4:1 (d) 4:3
- **15.** The waveform of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p) =$ $\int \psi(x) e^{-ipx/\hbar} dx$ in the momentum representation. If the action of an operator \hat{T} on $\psi(x)$ is given by $\hat{T}\psi(x) = \psi(x+a)$, where a is a constant, then $\hat{T}\phi(p)$ is given by (a) $-\frac{i}{\hbar}ap\phi(p)$ (b) $e^{-iqp/\hbar}\phi(p)$ (c) $e^{+iap/h}\phi(p)$ (d) $\left(1+\frac{i}{\hbar}ap\right)\phi(p)$

16. If *L_i* are the components of the angular momentum operator \vec{L} , then the operator $\Sigma i = 1,2,3 \left[\left[\vec{L}, L_i \right], L_i \right]$ equals

	L .	-	
(a) <i>L</i>			(b) 2 <i>L</i>
(c) 3 <i>L</i>			(d) $-\vec{L}$

17. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where k(t) is a time dependent parameter. Then $\frac{d}{dt}\langle V\rangle$, the rate of change of the expression value $\langle V \rangle$ of the potential energy, is (a) $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{k}{2m} \langle xp + px \rangle$ (b) $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{1}{2m} \langle p^2 \rangle$

(c)
$$\frac{k}{2m} \langle xp + px \rangle$$

(d) $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle$

18. A system of *N* distinguishable particles, each of which can be in one of the two energy levels 0 and ε , has a total energy $n\varepsilon$, where *n* is an integer. The entropy of the system is proportional to

(a) $N\ell nn$ (b) $n\ln N$ (c) $\ln \left(\frac{N!}{n!}\right)$ (d) $\ln \left(\frac{N!}{n!(N-n)!}\right)$

- **19.** A system of *N* non-intersecting classical particles, each of mass *m* is in a two-dimensional harmonic potential of the form $V(r) = \alpha(x^2 + y^2)$ where α is a positive constant. The canonical partition function of the system at temperature *T* is $\left(\beta = \frac{1}{k_BT}\right)$.
 - (a) $\left[\left(\frac{\alpha}{2m}\right)^2 \frac{\pi}{\beta}\right]^N$ (b) $\left(\frac{2m\pi}{\alpha\beta}\right)^{2N}$ (c) $\left(\frac{\alpha\pi}{2m\beta}\right)^N$ (d) $\left(\frac{2m\pi^2}{\alpha\beta^2}\right)^N$
- **20.** In a two-state system, the transition rate of a particle from state 1 to state 2 is t_{12} , and the transition rate from state 2 to state 1 is t_{21} . In the steady state, the probability of finding the particle in state 1 is

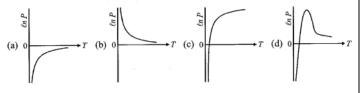
(b) $\frac{t_{12}}{t_{12}+t_{21}}$ (d) $\frac{t_{12}-t_{21}}{t_{12}-t_{21}}$

(a)
$$\frac{t_{21}}{t_{12}+t_{21}}$$

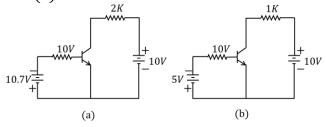
(c) $\frac{t_{12}t_{21}}{t_{12}+t_{21}}$

21. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation $\frac{dP}{dT} \approx \frac{Q_l}{Tv_{van}}$

(Clausius-Clayperon equation), where v_{vp} is the volume per particle in the vapour phase, and Q_l is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct?



22. Consider the circuits shown in Figures (a) and (b) below.



If the transistors in Figures (a) and (b) have current gain (β_{tik}) of 100 and 10 respectively, then they operate in the (a) active region and saturation region respectively

(b) saturation region and active region respectively

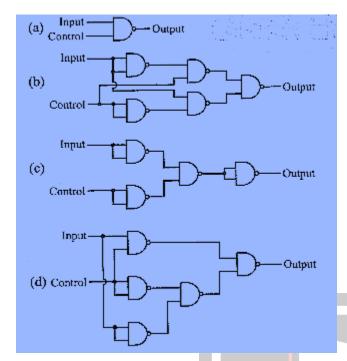
(c) saturation region in both cases

(d) active region in both cases

23. The viscosity η of a liquid is given by Poiseuille's formula $\eta = \frac{\pi P a^4}{8 I V}$. Assume that land V can be measured very accurately, but the pressure P has an rms error of 1% and the radius a has an independent rms error of 3%. The rms error of the viscosity is closest to

a) 2%	(b) 4%
c) 12%	(d) 13%

24. Which of the following circuits behaves as a controlled inverter?



25. The concentration of electrons, *n* and holes, *p* for an intrinsic semiconductor at a temperature *T* can be expressed as n = p = $AT^{3/2} \exp\left(-\frac{E_g}{2k_BT}\right)$, where E_g is the band gap and A is a constant. If the mobility of both types of carries is proportional to $T^{-3/2}$, then the log of the conductivity is a linear function of T^{-1} with slope (b) $\frac{E_g}{k_g}$ (d) $\frac{-E_g}{k_p}$

(a)
$$\frac{E_g}{(2k_B)}$$

(c) $\frac{-E_g}{(2k_B)}$

> PART C

26. Three real variables *a*, *b* and *c* are each randomly chosen from a uniform probability distribution in the interval [0,1]. The probability that a + b > 2c is

(a) $\frac{3}{4}$	(b) $\frac{2}{3}$
(c) $\frac{1}{2}$	$(d)\frac{1}{4}$

27. The rank-2 tensor $x_i x_i$, where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set)

- containing (a) 4 and 2 elements (b) 5 and 1 elements (c) 3, 2 and 1 elements (d) 4, 1 and 1 elements
- **28.** Consider the differential equation $\frac{dy}{dx} = x^2 x^2$ *y* with the initial condition y = 2 at x = 0. Let $y_{(1)}$ and $y_{(1)}$ 2) be the solutions at x = 1obtained using Euler's forward algorithm with step size 1 and 1/2 respectively. The value of $(y_{(1)} - y_{(1/2)})/y_{(1/2)}$ is $(a) - \frac{1}{2}$ (b) -1 $(c)\frac{1}{c}$ (d) 1
- **29.** Let f(x, t) be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at t = $0, f(x, 0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x, 0) = 0$ for all x, then f(x, t) for all future times t > 0 is described by (a) $e^{-(x^2-v^2t^2)}$ (b) $e^{-(x-v)^2}$ (c) $\frac{1}{4}e^{-(x-v)^2} + \frac{3}{4}e^{-(x+w)^2}$ (d) $\frac{1}{2}\left[e^{-(x-w)^2} + e^{-(x+v)^2}\right]$
- **30.** Let *q* and *p* be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical?

A:
$$Q_1 = \frac{1}{\sqrt{2}}q^2$$
 and $P_1 = \frac{1}{\sqrt{2}}p^2$
B: $Q_2 = \frac{1}{\sqrt{2}}(p+q)$ and $P_2 = \frac{1}{\sqrt{2}}(p-q)$

- (a) neither A nor B (b) both A and B (d) only B (c) only A
- **31.** The differential cross-section for scattering by a target is given by $\frac{d\sigma}{d\Omega}(\theta, \varphi) = a^2 +$ $b^2 \cos^2 \theta$. If N is the flux of the incoming particles, the number of particles scattered per unit time is
 - (a) $\frac{4\pi}{3}N(a^2 + b^2)$ (b) $4\pi N\left(a^2 + \frac{1}{6}b^2\right)$ (c) $4\pi N\left(\frac{1}{2}a^2 + \frac{1}{3}b^2\right)$ (d) $4\pi N\left(a^2 + \frac{1}{3}b^2\right)$

32. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$?

$$(a) \xrightarrow{p_1} (b) \xrightarrow{p_1} (c) \xrightarrow{p_1$$

- **33.** Consider a rectangular wave guide with transverse dimensions 2 m × 1 m driven with an angular frequency $\omega = 10^9$ rad/s. Which transverse electric (TE) modes will propagate in this wave guide ? (a) TE₁₀, TE₀₁ and TE₂₀
 - (b) TE_{10} , TE_{11} and TE_{20}
 - (c) TE_{01} , TE_{10} and TE_{11}^{20}
 - (d) TE_{01} , TE_{10} and TE_2

34. A rod of length *L* carries a total charge *Q* distributed uniformly. If this is observed in a frame moving with a speed *v* along the rod, the charge per unit length (as measured by the moving observer) is

(a)
$$\frac{Q}{L} \left(1 - \frac{v^2}{c^2} \right)$$
 (b) $\frac{Q}{L} \sqrt{1 - \frac{v^2}{c^2}}$
(c) $\frac{Q}{L\sqrt{1 - \frac{v^2}{c^2}}}$ (d) $\frac{0}{1\left(1 - \frac{v^2}{v^2}\right)}$

35. The electric and magnetic fields in the charge free region z > 0 are given by

$$\vec{E}(\vec{r},t) = E_0 e^{-k_1 z} \cos(k_2 x - \omega t) \hat{j}$$
$$\vec{B}(\vec{r},t)$$
$$= \frac{E_0}{\omega} e^{-k_1 z} [k_1 \sin(k_2 x - \omega t) \hat{i}]$$

$$+k_2\cos(k_2x-\omega t)\hat{k}$$

where ω , $k_1 \& k_2$ are positive constants. The average energy flow in the *x*-direction is

(a)
$$\frac{E_0^2 k_2}{2\mu_0 \omega} e^{-2k_1 z}$$
 (b) $\frac{E_0^2 k_2}{\mu_0 \omega} e^{-2k_1 z}$
(c) $\frac{E_0^2 k_1}{2\mu_0 \omega} e^{-2k_1 z}$ (d) $\frac{1}{2} c \varepsilon_0 E_0^2 e^{-2k_0 z}$

36. A uniform magnetic field in the positive *z*-direction passes through a circular wire loop

of radius 1 cm and resistance 1Ω lying in the *xy*-plane. The field strength is reduced from 10 tesla to 9 tesla in 1 s. The charge transferred across any point in the wire is approximately (a) 3.1×10^{-4} coulomb

- (b) 3.4×10^4 coulomb
- (c) 4.2×10^{-4} coulomb
- (d) 5.2×10^{-4} coulomb
- **37.** The Dirac Hamiltonian $H = c\vec{\alpha} \cdot \vec{p} + \beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2c^2 + m^2c^4$. The classical energy-momentum relation of a particle of charge *q* in a electromagnetic potential (ϕ, \vec{A})
 - is $(E q\phi)^2 = c^2 \left(\vec{p} \frac{q}{c}\vec{A}\right)^2 + m^2 c^4$. Therefore, the Dirac Hamiltonian for an electrom in an electromagnetic field is (a) $c\vec{\alpha} \cdot \vec{p} + \frac{e}{c}\vec{A} \cdot \vec{A} + \beta mc^2 - e\phi$ (b) $c\vec{\alpha} \cdot \left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi$ (c) $c \left(\vec{\alpha} \cdot \vec{p} + e\phi + \frac{e}{c}|\vec{A}|\right) + \beta mc^2$ (d) $c\vec{\alpha} \cdot \left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 - e\phi$
- **38.** A particle of mass *m* is in a potential $V = \frac{1}{2}m\omega^2 x^2$, where ω is a constant. Let $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{\hat{p}\hat{p}}{m\omega}\right)$. In the Heisenberg picture $\frac{d\hat{a}}{dt}$ is given by (a) $\omega \hat{a}$ (b) $-i\omega \hat{a}$ (c) $\omega \hat{a}^t$ (b) $-i\omega \hat{a}$
- **39.** A particle of energy *E* scatters off a repulsive spherical potential

$$V(r) = \begin{cases} V_0 & \text{for } r < a \\ 0 & \text{for } r \ge a \end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering, cross-

section is $\sigma = 4\pi a^2 \left(\frac{1}{ka} \tanh ka - 1\right)^2$, where $k^2 = \frac{2m}{\hbar^2} (V_0 - E) > 0$. In the limit $V_0 \to \infty$ the ratio of σ to the classical scattering crosssection off a sphere of radius *a* is (a) 4 (b) 3 (c) 1 (d) 1/2 **40.** Two different sets of orthogonal basis vectors $\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \}$ and $\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \}$ are given for a two-dimensional real vector space. The matrix representation of a linear operator \hat{A} in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$(b)\begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}$
$(c)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\\ 1 & -1 \end{pmatrix}$	$(\mathbf{d}) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0\\ 1 & 1 \end{pmatrix}$

41. A large number N of Brownian particles in one-dimension start their diffusive motion from the origin at time t = 0. The diffusion coefficients is D. The number of particles crossing a point at a distance L from the origin, per unit time, depends on L and time t as

(a) $\frac{N}{\sqrt{4\pi Dt}} e^{-l^{\hbar}/(4Dt)}$ (b) $\frac{NL}{\sqrt{4\pi Dt}} e^{-4Dtl^{t}}$ (c) $\frac{N}{\sqrt{16\pi Dt^{3}}} e^{-l^{2}/(40r)}$ (d) Ne^{-4Dti L²}

42. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength *J*. The partition function of the system at

temperature *T* is given by $\left(\beta = \frac{1}{k_{c}T}\right)$:

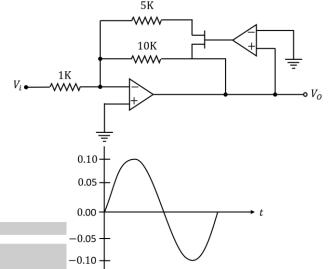
- (a) $2e^{3,\beta/} + 6e^{-\beta\rho}$ (b) $2e^{-3\beta J} + 6e^{\beta J}$ (c) $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$ (d) $(2\cosh \beta J)^3$
- **43.** An ideal Bose gas in *d*-dimensions obeys the dispersion relation $s(\hat{k}) = Ak^s$, where *A* and *s* are constants. For Bose-Einstein condensation to occur, the occupancy of excited states

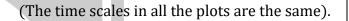
$$N_e = c \int_0^\infty \frac{\varepsilon^{(d-s)/s}}{e^{\beta(\varepsilon-\mu)} - 1} d\varepsilon$$

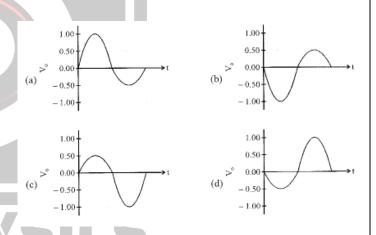
where *c* is a constant, should remain finite even for $\mu = 0$. This can happen if

(a) $\frac{d}{s} < \frac{1}{4}$	(b) $\frac{1}{4} < \frac{d}{s} < \frac{1}{2}$
$(c)\frac{d}{s} > 1$	$(d)\frac{1}{2} < \frac{d}{s} < 1$

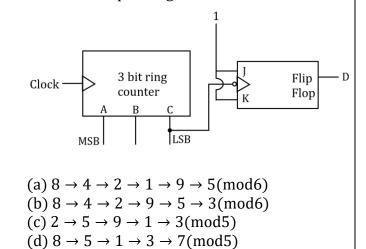
44. For the circuit and the input sinusoidal waveform shown in the figures below, which is the correct waveform at the output?



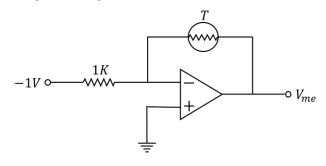




45. For the logic circuit given below, the decimal count sequence and the modulus of the circuit corresponding to ABCD are



46. In the circuit given below, the thermistor has a resistance $3k\Omega$ at 25° C. Its resistance decreases by 150Ω per °C upon heating. The output voltage of the circuit at 30°C is



(b) -2.25 V

(d) 3.75 V

(a) −3.75 V	
(c) 2.25 V	

- **47.** The low-energy electronic excitations in a two-dimensional sheet of graphene is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to (b) $E^{1/2}$ (d) E^2 (a) *E* (c) $E^{1/2}$
- **48.** X-ray of wavelength $\lambda = a$ is reflected from the (111) plane of a simple cubic lattice. If the lattice constant is a, the corresponding Bragg angle (in radian) is $(b) \frac{\pi}{\frac{4}{8}}$ $(d) \frac{\pi}{\frac{8}{8}}$

(a) $\frac{\pi}{6}$		
$(c)\frac{\frac{0}{\pi}}{3}$		
C) 3		

- 49. The critical magnetic fields of a superconductor at temperatures 4 K and 8 K are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately (b) 10.6 K (a) 8.4 K (c) 12.9 K (d) 15.0 K
- **50.** A diatomic molecule has vibrational states with energies $E_v = \hbar \omega \left(v + \frac{1}{2}\right)$ and rotational states with energies $E_i = Bj(j + 1)$, where vand *j* are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $j \leq 2$ and subject to the selection rules $\Delta v = +1$ and

 $\Delta i = \pm 1$. Then the largest allowed energy of transition is

- (a) ħω 3B (b) $\hbar \omega - B$ (c) $\hbar \omega + 4B$ (d) $2\hbar\omega + B$
- **51.** Of the following term symbols of the np^2 atomic configurations, 1 S₀, 3 P₀, 3 P₁, 3 P₂ and $^{1}D_{2}$ which is the grounded state?
 - (a) ${}^{^{3}}P_{0}$ (c) ${}^{^{3}}P_{2}$ (b) ${}^{1}S_{0}$ (d) ${}^{3}P_{1}$
- **52.** A He-Ne laser operates by using two energy levels of Ne separated by 2.26eV. Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be 1/20, is approximately (the Boltzmann constant $k_b = 8.6 \times 10^{-5} \text{eV/K}$). (a) 10¹⁰ K . . . (b) 10⁸ K (c) 10°K (d) 10^4 K
- **53.** Let us approximate the nuclear potential in the shell model by a 3-dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta l = 0and l = 1 respectively, which of the following two nuclei have magic numbers of protons and neutrons?
 - (a) ${}^{4}_{2}$ He and ${}^{16}_{8}$ O (b) ${}^{2}_{1}$ D and ${}^{8}_{4}$ Be (c) ${}^{4}_{2}$ He and ${}^{8}_{4}$ Be (d) ${}^{4}_{2}$ He and ${}^{12}_{6}$ Be
- **54.** The charm quark is assigned a charm quantum number C = 1. How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarks?

(a)
$$I_3 + \frac{1}{2}(B - S - C)$$

(b) $I_3 + \frac{1}{2}(B - S + C)$
(c) $I_3 + \frac{1}{2}(B + S - C)$
(d) $I_3 + \frac{1}{2}(B + S + C)$

- **55.** The reaction ${}_{1}^{2}D + {}_{1}^{2}D \rightarrow {}_{2}^{4}He + \pi^{0}$ cannot proceed via strong interactions because it violates the conservations of
 - (a) angular momentum (b) electric charge
 - (c) baryon number (d) isospin

✤ ANSWER KEY

1. c	2. a	3. b	4. a	5. a
6. c	7. b	8. a	9. b	10. d
11. c	12. d	13. d	14. a	15. c
16. b	17. a	18. d	19. d	20. a
21. c	22. b	23. c	24. b	25. c
26. c	27. b	28. b	29. d	30. d
31. d	32. a	33. a	34. c	35. a
36. d	37. d	38. b	39. a	40. c
41. a	42. a	43. c	44. b	45. b
46. c	47. a	48. c	49. b	50. c
51. a	52. d	53. a	54. d	55. d

