D PHYSICS

CSIR-NET, GATE, SET, JEST, IIT-JAM, BARC, TIFR

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Paper & Answer Key

PHYSICAL SCIENCE

❖ PART- B

1. A particle of rest mass m_0 and energy E collides with another particle at rest, with the same rest mass. What is the minimum value of E so that after the collision, there may be four particles of rest mass m_0 ?

(a)$4m_0c^2$ (b)$3m_0c^2$ (c)$7m_0c^2$ (d)$16m_0c^2$

2. Given the sum of the infinite series

$$\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}\cdots=\frac{\pi^4}{90}$$

the sum of the infinite series

$$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$$

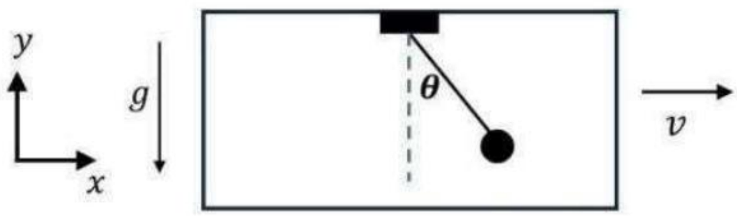
would be

(a) $\frac{\pi^4}{128}$ (b) $\frac{\pi^4}{144}$ (c) $\frac{\pi^4}{120}$ (d) $\frac{\pi^4}{96}$

3. Consider a particle in a one-dimensional infinite potential well between $0 \leq x \leq$ L. If a small perturbation, $V(x)=\lambda\cos\left(\frac{\pi x}{L}\right)$, (where $\lambda \ll 1$) is applied, the firstorder energy correction to the ground state is

(a)λ (b)0 (c)$-\lambda$ (d)2λ

4. The point of support of a simple pendulum, of mass m and length l, is attached to the roof of a taxi as shown in the figure. The taxi is moving with uniform velocity v. The Lagrangian for the pendulum is



(a)$L=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mv^2+mlv\cos\,\theta\dot{\theta}-mgl\cos\,\theta$

(b)$L=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mv^2+mlv\cos\,\theta\dot{\theta}+mgl\cos\,\theta$

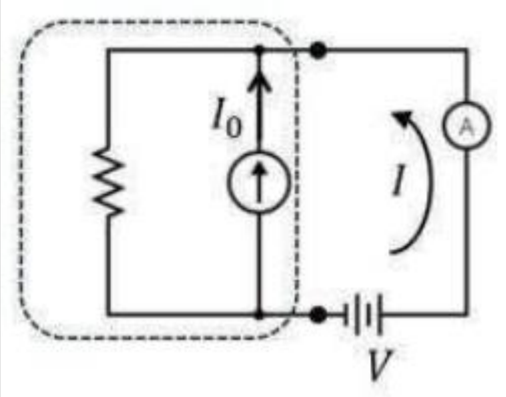
(c) $L=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mv^2+mlv\sin\ \theta\dot{\theta}+mgl\cos\ \theta$

(d) $L=\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mv^2+mlv\sin\ \theta\dot{\theta}-mgl\cos\ \theta$

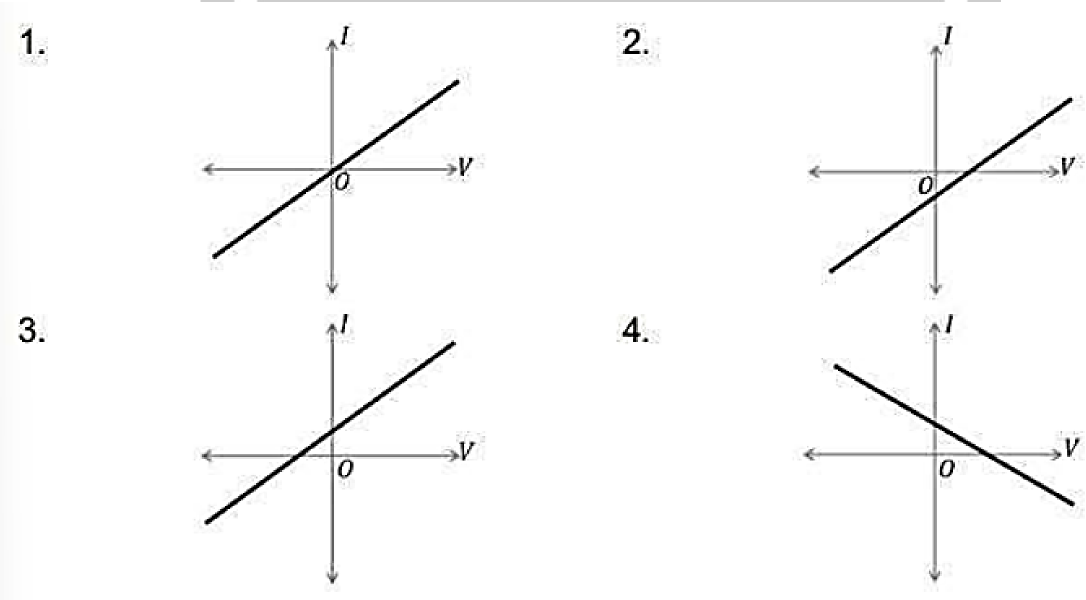
5. A DC motor operating at a voltage V and a current I is used to lift a mass m to a height h. The percentage uncertainty in the measurement of time t is 5% and that for the other parameters ($V, I, m,$ and h) are 1% each. If the measurements are independent and the errors are random, the uncertainty in the estimation of the efficiency $\left(\frac{\text{output power}}{\text{input power}}\right)$ of the motor is closest to

(a) 3.1% (b) 5.4% (c) 4.8% (d) 6.3%

6. A circuit component consists of a resistor in parallel with an ideal current source. The $I-V$ characteristics of the component was measured using a variable voltage source and an ammeter ' A '.



The arrow in the figure indicates the positive direction of current. The I-V characteristics of the component is best represented by



7. If I is an $n \times n$ identity matrix and $\mathrm{adj}(2I) = 2^k I$, then k is equal to

(a) 1 (b) n (c) $n-1$ (d) 2

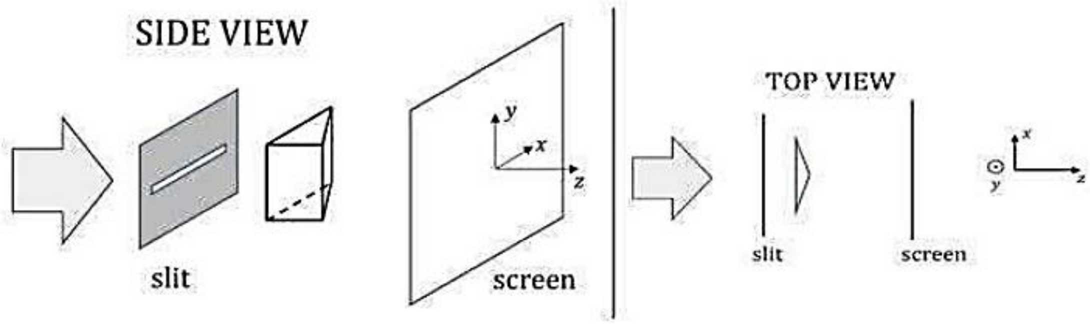
8. A certain elementary particle is created in the upper atmosphere. It then moves downward with speed $v = 0.9999c$ with respect to an observer on earth. Its lifetime in its rest frame is 2×10^{-6}sec. The distance (in the earth's frame) travelled by the elementary particle before it decays is closest to

(a) 0.6 km (b) 42 km (c) 12 km (d) 72 km

9. For an ideal Bose gas, the density of states is given by $\rho(E) = CE^2$, where C is a positive constant. Assume that the number of bosons is not conserved. The variation of the specific heat of the gas with temperature T is closest to

(a) T^2 (b) T^3 (c) T (d) T^4

10. A narrow horizontal slit is illuminated by an extended sodium lamp. A thin Fresnel biprism with its edge aligned perpendicular to the slit is positioned, as shown in the figure.



Given that the length of the slit is larger than the base of the biprism, the pattern of illumination on the screen is best described by

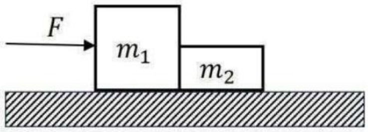
(a) Fringes in both x and y direction.

(b) Almost uniform illumination.

(c) Horizontal fringes periodic only along the x-axis.

(d) Horizontal fringes periodic only along the y-axis.

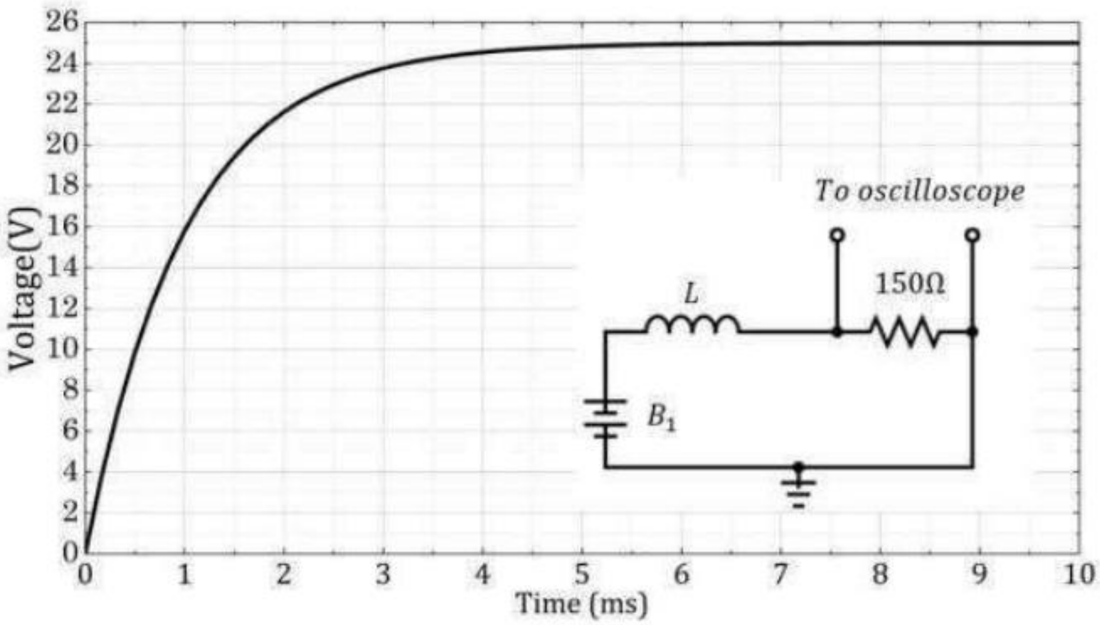
11. Two blocks m_1 and m_2 are in contact on a frictionless horizontal table. A horizontal force is applied to one of the blocks, as shown in the figure.



If $m_1 = 2$ kg, $m_2 = 1$ kg, and $F = 3$ N, the force of contact between the blocks is

(a) 1 N (b) $2N$ (c) 1.5 N (d) $3N$

12. An ideal inductor L is connected in series to a 150Ω resistor as shown in the circuit (inset). When the circuit is driven by a battery B_1, the voltage across the resistor as a function of time, as measured by an oscilloscope, is shown in the plot.



Based on this observation, the estimated value of L is closest to

(a) 50 mH (b) 300 mH (c) 450 mH (d) 150 mH

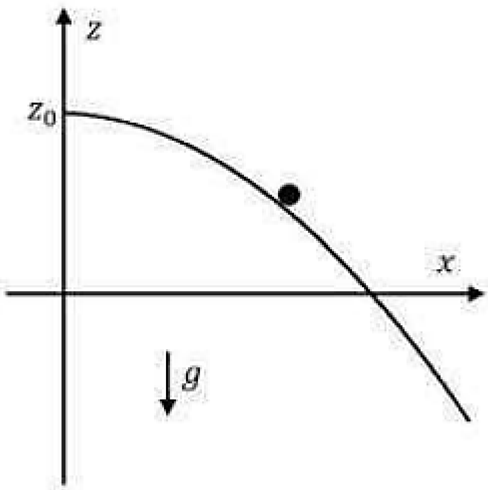
13. Two non-interacting identical spin $-\frac{1}{2}$ particles, each of mass m, are placed in a two-dimensional infinite square well of side L. The single-particle spatial wavefunction is given by

$$\varphi_{n_x,n_y}(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$$

where n_x and n_y are positive integers. If the particles are in a total spin state $|j = 1, m = 0\rangle$, the lowest possible energy eigenvalue is

(a) $\frac{5\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{mL^2}$ (c) $\frac{2\hbar^2\pi^2}{mL^2}$ (d) $\frac{7\hbar^2\pi^2}{2mL^2}$

14. A frictionless track is defined by $z = z_0 - \frac{x^2}{4z_0}$, as shown in the figure.



A particle is constrained to slide down the track under the action of gravity. The tangential acceleration at position (x, z) would be

(a) $\frac{2gx}{\sqrt{x^2 + 4z_0^2}}$ (b) $\frac{gx}{\sqrt{x^2 + 4z_0^2}}$ (c) $\frac{gx}{2z_0}$ (d) $g\sqrt{\frac{x(x + z_0)}{x^2 + 4z_0^2}}$

15. An isolated box of volume V contains 5 identical, but distinguishable and noninteracting particles. The particles can either be in the ground state of zero energy or in an excited state of energy ε. The ground state is non-degenerate while the excited state is doubly degenerate. There is no restriction on the number of particles that can be put in a given state. The number of accessible microstates corresponding to the macrostate of the system with energy $E = 2\varepsilon$ are

(a) 10 (b) 20 (c) 40 (d) 30

16. The following table shows the relationship between an independent quantity x and an experimentally measured quantity y.

x	0	1	2	3	4	5
y	0.1	2.1	8.1	17.9	32.2	49.7

The relationship between x and y is best represented by

(a) $y \propto x^3$ (b) $y \propto e^x$ (c) $y \propto x^2$ (d) $y \propto \sqrt{x}$

17. An electron is in the spin state $|\psi\rangle = \frac{1}{5}\binom{3i}{4}$ in the $\hat{S}_z$ basis. A measurement of $\hat{S}_x$ is made on this state. The probabilities of getting $\hbar/2$ and $-\hbar/2$ are

(a) $\frac{1}{3}, \frac{2}{3}$ (b) $\frac{1}{4}, \frac{3}{4}$ (c) $\frac{1}{2}, \frac{1}{2}$ (d) $\frac{3}{7}, \frac{4}{7}$

18. A grating spectrometer in vacuum, illuminated by 500 nm light, gives first-order spectrum at an angle of 20°. When the vacuum chamber is filled with Argon gas at pressure P, this angle

(a) increases, due to increase in the refractive index of the medium

(b) decreases, due to increase in the refractive index of the medium

(c) decreases, due to decrease in the frequency of light in argon gas

(d) increases, due to decrease in the frequency of light in argon gas

19. A sphere with uniform charge and mass density, having total charge Q and mass M, rotates about an axis through its center with angular velocity ω. The ratio of its magnetic dipole moment to its angular momentum is

(a) $\frac{2Q}{M}$ (b) $\frac{Q}{M}$ (c) $\frac{Q}{2M}$ (d) $\frac{Q}{4M}$

20. A particle of mass m is in a cubic box of side a. The potential inside the box $(0 \le x \le a, 0 \le y \le a, 0, \le z \le a\ )$ is zero and infinite outside. If the particle is in an energy eigenstate with $E = \frac{7\pi^2\hbar^2}{ma^2}$, a possible wavefunction is

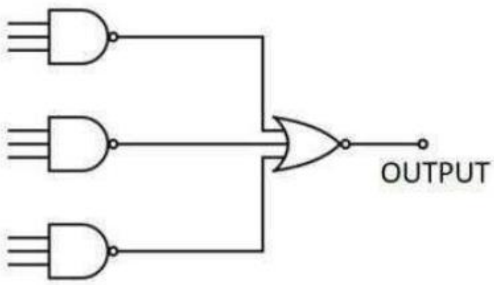
(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{a}\right) \sin\left(\frac{2\pi z}{a}\right)$

(b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{a}\right) \sin\left(\frac{\pi z}{a}\right)$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{a}\right) \sin\left(\frac{3\pi z}{a}\right)$

(d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{a}\right) \sin\left(\frac{2\pi z}{a}\right)$

21. The output of the following circuit is always found to be zero.



Such an observation can be due to

(a) Two of the inputs of any one of the NAND gates being accidentally shorted to each other

(b) One of the inputs to the NOR gate being accidentally grounded

(c) One of the inputs to one of the NAND gates being accidentally grounded

(d) Two of the inputs of the NOR gate being accidentally shorted to each other

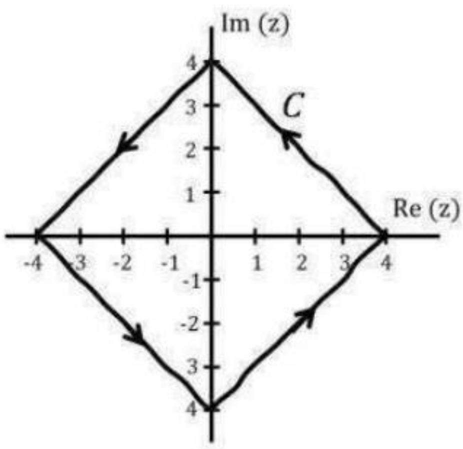
22. A system comprises of N distinguishable atoms ($N \gg 1$). Each atom has two energy levels ω and $3\omega(\omega > 0)$. Let ε_{eq} denote the average energy per particle when the system is in thermal equilibrium, the upper limit of ε_{eq} is

(a) $\frac{3\omega}{2}$ (b) 3ω (c) $\frac{5\omega}{2}$ (d) 2ω

23. The value of the integral (where k is a constant),

$$\frac{1}{2\pi i}\oint_C \frac{5}{(z-2)^2}\sin(kz)\,dz$$

over the closed contour C as shown below, is

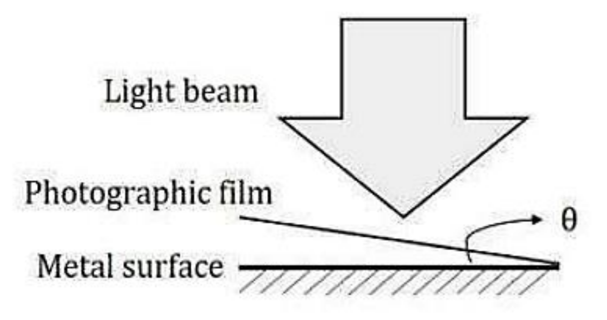


(a)$5k\cos(2k)$ (b)$5k\sin(2k)$ (c)$5\cos(2k)$ (d)$-5k^2\sin(2k)$

24. A spherical cavity of volume V is filled with thermal radiation at temperature T. The cavity expands adiabatically to 8 times its initial volume. If σ is Stefan's constant and c is the speed of light in vacuum, what is the closest value of the work done in the process?

(a)$8\left(\frac{\sigma T^4 V}{c}\right)$ (b)$4\left(\frac{\sigma T^4 V}{c}\right)$ (c)$\frac{1}{2}\left(\frac{\sigma T^4 V}{c}\right)$ (d)$2\left(\frac{\sigma T^4 V}{c}\right)$

25. When a photographic film is exposed to light, the electric field of light causes the film to turn dark after chemical processing. A photographic film of thickness 50 nm is kept inclined to a shiny metal surface at an angle of $\theta = 0.01$ radian, as shown in the figure.



After exposing this film to a linearly polarized beam of light of wavelength 500 nm incident normally to the metal surface, it developed periodic bright bands. We can explain this observation as the proof of

(a)Interference between the incident wave and the wave reflected from the surface of the metal.

(b)Diffraction pattern produced by the photographic film.

(c)Interference of light due to the presence of photographic film.

(d)Polarization of light due to photographic film.

❖ PART- C

26. The masses of proton, neutron, Polonium and Lead nuclei are as follows:

$$m_p = 1.007825\text{a. u}, m_n = 1.008665\text{a} \cdot \text{u}$$
$$m(\,^{210}_{84}\text{Po}) = 209.982876\text{a. u}, m(\,^{206}_{82}\text{ Pb}) = 205.974455\text{a. u.}$$

Binding energy of $^4_2$He is 28.80 MeV and $1\text{a. u} = 931.99\,\frac{\text{MeV}}{\text{c}^2}$.

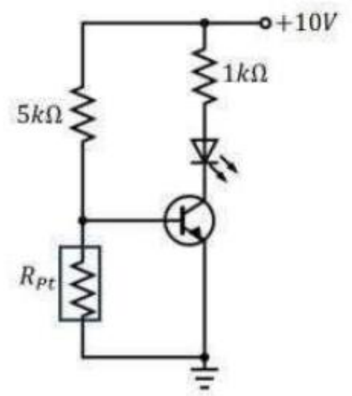
The binding energies of $^{210}_{84}$Po, $^{206}_{82}$ Pb and the Q value of the α-decay of $^{210}_{84}$Po are closest to

(a) 1645.21MeV, 1622.33MeV, 5.92MeV
(b) 1645.21MeV, 1622.33MeV, −5.92MeV
(c) 1545.21MeV, 1522.33MeV, −5.92MeV
(d) 1645.21MeV, 1522.33MeV, 5.92MeV

27. An LED is required to glow brightly when the temperature sensed by a Platinum resistance thermometer exceeds a certain value. In the circuit shown below, the resistance of the Pt thermometer (in ohms) varies as

$$\text{R}_{\text{Pt}}(\text{T}) = 100 + 0.4\,\text{T}$$

where T is temperature in degree Celsius. The transistor turns on when $V_{BE} > 0.7$ V and it has a very high current gain. The temperature at which the LED would start glowing is closest to



(a) 850°C
(b) 400°C
(c) 500°C
(d) 700°C

28. A spherical cavity of radius r_0 has an impenetrable wall. A quantum particle of mass m inside the cavity is in its ground state. The pressure exerted on the cavity wall is

(a) $\dfrac{\pi\hbar^2}{4mr_0^5}$
(b) $\dfrac{\pi\hbar^2}{mr_0^5}$
(c) $\dfrac{\pi^2\hbar^2}{2mr_0^5}$
(d) $\dfrac{\pi^2h^2}{4mr_0^5}$

29. Bose condensation experiments are carried out on two samples A and B of an ideal Bose gas. The same gas species is used in both. The condensate densities achieved at a given temperature below the critical temperature are $n_{\mathrm{A}} = 1.80 \times 10^{14}\ \mathrm{cm}^{-3}$ and $n_{\mathrm{B}} = 1.44 \times 10^{15}\ \mathrm{cm}^{-3}$, respectively. If P_{A} and P_{B} are the pressures of the two gas samples, the ratio $\frac{P_{\mathrm{A}}}{P_{\mathrm{B}}}$ is

(a) 1 (b) $\left(\frac{1}{8}\right)^{\frac{3}{2}}$ (c) $\left(\frac{1}{8}\right)^{\frac{2}{3}}$ (d) 8

30. Consider a free fermion gas in a hypercubic infinite potential well in hypothetical 4-dimensional space. What will be the expression for ground state energy per particle in term of the Fermi energy E_F ? (Ignore spin degeneracy of the fermions)

(a) $\frac{4}{5}E_F$ (b) $\frac{2}{3}E_F$ (c) $\frac{1}{3}E_F$ (d) $\frac{2}{5}E_F$

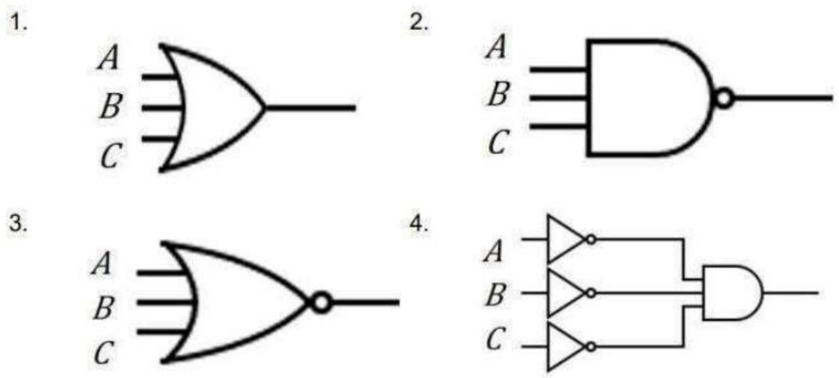
31. The lattice spacing in a simple cubic lattice is given to be 5Å. The number of lattice points per square nanometer in the lattice plane with Miller index (212) is closest to

(a) 7.5 (b) 3 (c) 1.33 (d) 0.66

32. The logic circuit that will have the output

$$Y = (A + B)\overline{(\bar{A}(\bar{B} + \bar{C}))} + \bar{A}(B + C)$$

is



33. The complex integral $\int_c z^4 \exp\left(\frac{1}{2z}\right) dz$, where C is the unit circle centered around the origin traversed counter-clock-wise, equals

(a) $\frac{\pi i}{120}$ (b) $\frac{\pi i}{960}$ (c) 0 (d) $\frac{\pi i}{1920}$

34. A particle of mass m is bound in one dimension by the potential $V(x) = V_0\delta(x)$ with $V_0 < 0$. If the probability of finding it in the region $|x| < a$ is 0.25 , then a is

(a) $\frac{\hbar^2}{4mV_0}\ln\frac{3}{4}$ (b) $\frac{\hbar^2}{2mV_0}\ln\frac{3}{4}$ (c) $\frac{\hbar^2}{4mV_0}\ln\frac{1}{4}$ (d) $\frac{\hbar^2}{2mV_0}\ln\frac{1}{4}$

35. Naturally occurring uranium is a mixture of the $^{238}U(99.28\%)$ and $^{235}U(0.72\%)$ isotopes. The life times are $\tau(^{235}U) = 1 \times 10^9$ years and $\tau(^{238}U) = 6.6 \times 10^9$ years. What is the closest value of the age of the solar system if one assumes that at its creation both isotopes were present in equal quantities?

(a) 6.2×10^9 years (b) 5.8×10^9 years

(c) 4.7×10^9 years (d) 7.2×10^9 years

36. Consider a spherical region of radius $\frac{R}{2}$ centered at the origin of the coordinate system. Three point charges each of magnitude Q are placed at $(0,0,R)$, $(0,R,0)$ and $(\sqrt{2}R,0,0)$. What is the magnitude of the average electric field over the spherical region due to these charges in units of $\frac{Q}{4\pi\epsilon_0 R^2}$?

(a) $\frac{3}{5}$ (b) 0 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$

37. The hyperfine splitting of the ground state of the hydrogen atom is given as

$$\Delta E \propto \frac{g_p g_e}{m_p m_e a^3}$$

where g_p and g_e are the nuclear and electron Landê g factors respectively, and a is the orbital radius of the ground state. It is given that g (proton) $= 5.59$. In Hydrogen, transition between these split levels corresponds to radiation of wavelength 21 cm . If the proton is replaced by a positron, the corresponding wavelength would be

(a) 2.6 mm (b) 3.2 mm (c) 3.2 cm (d) 2.6 cm

38. For the transformation

$$Q = \ln(1 + q^{1/2}\cos p), P = 2q^{1/2}(1 + q^{1/2}\cos p)\sin p$$

the generating function is

(a) $-(e^Q - 1)^2\cot p$ (b) $(e^Q - 1)^2\cot p$

(c) $(e^Q - 1)^2\tan p$ (d) $-(e^Q - 1)^2\tan p$

39. A particle of mass m, moving in one-dimension is subjected to the potential

$$V(x) = \begin{cases} V_0\delta(x-a) & 0 < x < 2a \\ \infty & \text{otherwise} \end{cases}$$

The energy eigenvalues E satisfy

(a) $\tan\frac{a\sqrt{2mE}}{\hbar} = \frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$

(b) $\tanh\frac{a\sqrt{2mE}}{\hbar} = \frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$

(c) $\tan\frac{a\sqrt{2mE}}{\hbar} = -\frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$

(d) $\tanh\frac{a\sqrt{2mE}}{\hbar} = -\frac{\hbar}{V_0}\sqrt{\frac{2E}{m}}$

40. Consider the Bromine ion Br^+ in its ground state. The atomic number of Br is 35. The fine structure term symbol $\left({}^{2S+1}L_J\right)$ under the LS coupling scheme for the lowest energy state of this ion would be

(a) ${}^3P_2$ (b) ${}^3P_0$ (c) ${}^1D_2$ (d) ${}^4S_{3/2}$

41. A hydrogen atom, excited to electronic configuration $3S_{1/2}$ (nL_j notation), relaxes to the ground state via electric dipole transitions. Considering only fine structure and ignoring hyperfine structure, the maximum number of emitted spectral lines is

(a) 3 (b) 6 (c) 1 (d) 4

42. Gamma function with argument z is defined as

$$\Gamma[z] = \int_0^\infty dt t^{z-1}e^{-t}$$

where z is a complex variable and $\mathrm{Re}z \geq 0$. $\Gamma[z]$ has

(a) a branch point at $z = 0$

(b) a simple pole at $z = 0$

(c) a removable singularity at $z = 0$

(d) an essential singularity at $z = 0$

43. Consider N mutually non-interacting electrons moving in a crystal where the ionic potential seen by an electron satisfies the condition $V(\vec{r}) = V(\vec{r} + \vec{R})$, where $\vec{R}$ is one of the lattice translation vectors. The energy eigenstates of the electrons are labelled as $\psi_{\vec{k}}(\vec{r})$ where $\vec{k}$ is a vector in the first Brillouin zone. Which of the following is true?

(a)$\left|\psi_{\vec{k}}(\vec{r})\right|$ is constant.

(b)$\psi_{\vec{k}}(\vec{r})$ is also an eigenstate of the momentum operator.

(c)$\psi_{\vec{k}}(\vec{r}) = \psi_{\vec{k}}(\vec{r}+\vec{R})$

(d)$\left|\psi_{\vec{k}}(\vec{r})\right| = \left|\psi_{\vec{k}}(\vec{r}+\vec{R})\right|$

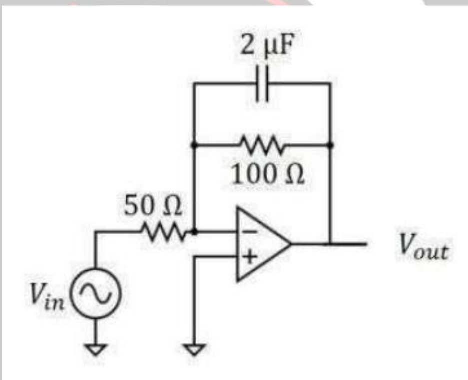
44. The constant B which makes e^{-ax^2} an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Bx^2\right)$ is

(a)$4a^2$ (b)0 (c)$2a$ (d)1

45. In the circuit shown below, the input voltage (in volts) is given by

$$V_{in}(t) = 0.1\sin(\omega_1 t) + \sin(\omega_2 t)$$

where $\omega_1 = 5 \times 10^2\ \mathrm{s}^{-1}$ and $\omega_2 = 5 \times 10^4\ \mathrm{s}^{-1}$.



The time varying part of the output voltage $V_{\mathrm{out}}(t)$ (in volts) is closest to

(a)$-0.2\sin(\omega_1 t) - 2\sin(\omega_2 t)$

(b)$-0.2\sin(\omega_1 t) + 0.2\cos(\omega_2 t)$

(c)$2\cos(\omega_1 t) + 0.2\cos(\omega_2 t)$

(d)$2\cos(\omega_1 t) - 2\sin(\omega_2 t)$

46. The Lagrangian of a system is

$$L = \frac{15}{2}m\dot{x}^2 + 6m\dot{x}\dot{y} + 3m\dot{y}^2 - mg(x+2y)$$

Which one of the following is conserved?

(a)$12\dot{x} + 3\dot{y}$ (b)$12\dot{x} - 3\dot{y}$ (c)$3\dot{x} - 12\dot{y}$ (d)$3\dot{x} + 3\dot{y}$

47. A static charge distribution produces an electric field

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{e^{-br}}{r^3}\vec{r},$$

where $Q, b > 0$ are constants. The charge density of the distribution is given by

(a) $\frac{Q}{4\pi}\left[-\frac{b}{2r^2}\right]$

(b) $\frac{Q}{4\pi}e^{-b}\left[-\frac{b}{r^2} - 4\pi\delta(\vec{r})\right]$

(c) $\frac{Q}{4\pi}e^{-br}\left[-\frac{2b}{r^2}\right]$

(d) $\frac{Q}{4\pi}e^{-br}\left[-\frac{b}{r^2} + 4\pi\delta(\vec{r})\right]$

48. An electron enters a region of uniform electric and magnetic fields $\vec{E}_0$ and $\vec{B}_0$. Its velocity, $\vec{E}_0$ and $\vec{B}_0$ are mutually perpendicular to each other. Initially, E_0 is so adjusted that the electron suffers no deflection. E_0 is then switched off and the electron moves in a circular path of radius R. The speed of the electron and its charge to mass ratio would be

(a) $\frac{2E_0}{B_0}, \frac{E_0}{2B_0^2 R}$

(b) $\frac{2E_0}{B_0}, \frac{E_0}{B_0^2 R}$

(c) $\frac{E_0}{B_0}, \frac{E_0}{B_0^2 R}$

(d) $\frac{E_0}{B_0}, \frac{2E_0}{B_0^2 R}$

49. For the decay of the Δ-baryons, the ratio of the decay rates

$$\frac{\Gamma(\Delta^- \to n\pi^-)}{\Gamma(\Delta^0 \to p\pi^-)}$$

is best approximated by

(a) $\frac{3}{2}$

(b) 3

(c) 1

(d) $\frac{2}{3}$

50. For a system of two electrons, define an operator

$$\hat{A} = \frac{3}{a^2}\left(\hat{\vec{S}}_1 \cdot \vec{a}\right)\left(\hat{\vec{S}}_2 \cdot \vec{a}\right) - \hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2$$

where $\vec{a}$ is an arbitrary vector, and $\hat{\vec{S}}_1$ and $\hat{S}_2$ are spin operators. The eigenvalues of $\hat{A}$ (in units of $\hbar^2$) are

(a) $-1, 1, \frac{3}{2}, \frac{3}{2}$

(b) $-1, -\frac{1}{2}, -\frac{1}{2}, 0$

(c) $\frac{1}{2}, 1, \frac{3}{2}, \frac{3}{2}$

(d) $0, \frac{1}{2}, \frac{1}{2}, -1$

51. A class has 60% boys and 40% girls. In an examination 8% of the boys and 12% of the girls got an ' A ' grade. If a randomly selected student had an ' A ' grade, what is the probability that the student is male?

(a) 0.7 (b) 0.6 (c) 0.4 (d) 0.5

52. Magnetization M as a function of applied magnetic field H for two different solid samples at temperature T are shown below. These samples are known to be superconducting below their respective critical temperatures (T_C).

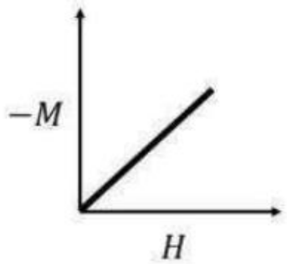


Fig. 1

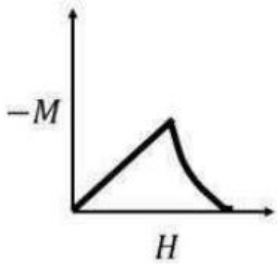


Fig. 2

The correct set of statements is

(a) Fig. 1: Type I superconductor above T_C;
Fig. 2: Type II superconductor below T_C and upto upper critical field;

(b) Fig. 1: Type II superconductor below T_C and upto upper critical field;
Fig. 2: Type II superconductor below T_C and upto lower critical field.

(c) Fig. 1: Type I superconductor below T_C and below critical field;
Fig. 2: Type II superconductor below T_C upto upper critical field;

(d) Fig. 1: Type I superconductor below T_C and below critical field;
Fig. 2: Type II superconductor below T_C and below lower critical field.

53. Energy of two Ising spins $\left(s = \pm\frac{1}{2}\right)$ is given by

$$E = s_1 s_2 + s_1 + s_2$$

At temperature T, the probability that both spins take the value $-\frac{1}{2}$ is 16 times the probability that both take the value $+\frac{1}{2}$. At the same temperature, what is the probability that the spins take opposite values?

(a) $\frac{16}{25}$ (b) $\frac{8}{25}$ (c) $\frac{8}{33}$ (d) $\frac{16}{33}$

54. Eigenstates of a system are specified by two non negative integers n_1 and n_2. The energy of the system is given by

$$E_n = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)2\hbar\omega$$

If $\alpha \equiv \exp\left(-\frac{\hbar\omega}{k_B T}\right)$, what is the probability that at temperature T the energy of the system will be less than $4\hbar\omega$?

(a) $(1-\alpha^2)(1-\alpha)(2+\alpha+2\alpha^2)$
(b) $(1-\alpha)^2(1-\alpha)(2+\alpha+\alpha^2)$
(c) $(1-\alpha^2)(1+\alpha)(1+\alpha+2\alpha^2)$
(d) $(1-\alpha)^2(1+\alpha)(1+\alpha+2\alpha^2)$

55. A non-relativistic particle of mass m and charge q is moving in a magnetic field $\vec{B}(x,y,z)$. If $\vec{v}$ denotes its velocity and $\{\dots\}_{P.B.}$ denotes the Poisson Bracket, then $\epsilon_{ijk}\{v_i, v_j\}_{\text{P.B.}}$ is equal to

(a) $-\frac{q}{m^2}B_k($
(b) 0
(c) $\frac{2q}{m^2}B_k$
(d) $\frac{q}{m^2}B_k$

Answer Key									
1. c	2. d	3. b	4. b	5. b	6. b	7. c	8. b	9. b	10.b
11.a	12.d	13.d	14.b	15.c	16.c	17.c	18.b	19.c	20.c
21.c	22.d	23.a	24.d	25.a	26.a	27.d	28.a	29.a	30.b
31.c	32.a	33.d	34.b	35.b	36.d	37.a	38.d	39.c	40.a
41.d	42.b	43.d	44.a	45.b	46.a	47.d	48.c	49.b	50.d
51.d	52.c	53.d	54.d	55.c					