❖ CSIR-UGC-NET/JRF- DEC - 2023 PHYSICAL SCIENCES BOOKLET - [A]

➢ PART-B

1. If z is a complex number, which among the following sets is neither open nor closed?

(a) $\{z | 0 \leq |z-1| \leq 2\}$

(b) $\{z | |z| \leq 1\}$

(c) $\{z \mid z \in (\mathbb{C} - \{3\}) \text{ and } |z| \leq 100\}$

(d) $\left\{z \mid z = re^{i\theta}, 0 \leq \theta \leq \frac{\pi}{4}\right\}$

2. The coordinates of the following events in an observer's inertial frame of reference are as follows:

Event 1: $t_1 = 0, x_1 = 0$: A rocket with uniform velocity $0.5c$ crosses the observer at origin along x axis

Event 2: $t_2 = T, x_2 = 0$: The observer sends a light pulse towards the rocket

Event 3: t_3, x_3 : The rocket receives the light pulse

The values of t_3, x_3 respectively are

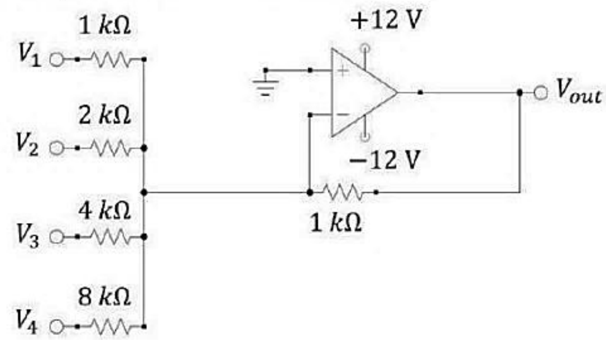
(a) $2T, cT$

(b) $2T, \frac{c}{2}T$

(c) $\frac{\sqrt{3}}{2}T, \frac{2}{\sqrt{3}}cT$

(d) $\frac{2}{\sqrt{3}}T, \frac{\sqrt{3}}{2}cT$

3. In the circuit shown below using an ideal opamp, inputs $V_j (j = 1,2,3,4)$ may either be open or connected to a -5 V battery.



The minimum measurement range of a voltmeter to measure all possible values of V_{out} is

(a) 10 V

(b) 30 V

(c) 3 V

(d) 1 V

4. A particle of mass m is moving in a stable circular orbit of radius r_0 with angular momentum L. For a potential energy $V(r) = \beta r^k (\beta > 0 \text{ and } k > 0)$, which of the following options is correct?

(a) $k = 3, r_0 = \left(\frac{3L^2}{5m\beta}\right)^{1/5}$

(b) $k = 2, r_0 = \left(\frac{L^2}{2m\beta}\right)^{1/4}$

(c) $k = 2, r_0 = \left(\frac{L^2}{4m\beta}\right)^{1/4}$

(d) $k = 3, r_0 = \left(\frac{5L^2}{3m\beta}\right)^{1/5}$

5. A one dimensional infinite long wire with uniform linear charge density λ, is placed along the z-axis. The potential difference $\delta V = V(\rho + a) - V(\rho)$, between two points at radial distances $\rho + a$ and ρ from the z-axis, where $a \ll \rho$, is closest to

(a) $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$ (b) $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$

(c) $\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$ (d) $\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$

6. In the measurement of a radioactive sample, the measured counts with and without the sample for equal time intervals are $C = 500$ and $B = 100$, respectively. The errors in the measurements of C and B are $|\Delta C| = 20$ and $|\Delta B| = 10$, respectively. The net error $|\Delta Y|$ in the measured counts from the sample $Y = C - B$, is closest to

(a) 22 (b) 10

(c) 30 (d) 43

7. Each allowed energy level of a system of non-interacting fermions has a degeneracy M. If there are N fermions and R is the remainder upon dividing N by M, then the degeneracy of the ground state is

(a) R^M (b) 1

(c) M (d) ${}^M C_R$

8. A quantum system is described by the Hamiltonian

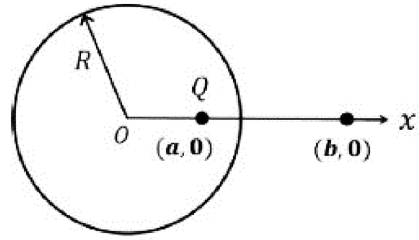
$$H = JS_z + \lambda S_x$$

where $S_i = \frac{\hbar}{2}\sigma_i$ and $\sigma_i (i = x, y, z)$ are the Pauli matrices. If $0 < \lambda \ll J$, then the leading correction in λ to the partition function of the system at temperature T is

(a) $\frac{\hbar\lambda^2}{2Jk_BT}\coth\left(\frac{J\hbar}{2k_BT}\right)$ (b) $\frac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\frac{J\hbar}{2k_BT}\right)$

(c) $\frac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\frac{J\hbar}{2k_BT}\right)$ (d) $\frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right)$

9. A conducting shell of radius R is placed with its centre at the origin as shown below. A point charge Q is placed inside the shell at a distance a along the x-axis from the centre.



The electric field at a distance $b > R$ along the x-axis from the centre is

(a) $\frac{Q}{4\pi\varepsilon_0 b^2}\hat{x}$

(b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} - \frac{aR}{(ab-R^2)^2}\right]\hat{\chi}$

(c) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} + \frac{aR}{(ab-R^2)^2}\right]\hat{x}$

(d) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b^2} - \frac{R^2}{a^2b^2}\right]\hat{x}$

10. The Beta function is defined as $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$.
Then $B(x, y+1) + B(x+1, y)$ can be expressed as

(a) $B(x, y-1)$ (b) $B(x+y, 1)$

(c) $B(x+y, x-y)$ (d) $B(x, y)$

11. The light incident on a solar cell has a uniform photon flux in the energy range of 1eV to 2eV and is zero elsewhere. The active layer of the cell has a bandgap of 1.5eV and absorbs 80% of the photons with energies above the bandgap. Ignoring non-radiative losses, the power conversion efficiency (ratio of the output power to the input power) is closest to

(a) 47% (b) 70%

(c) 23% (d) 35%

12. The Schrödinger wave function for a stationary state of an atom in spherical polar coordinates (r, θ, ϕ) is

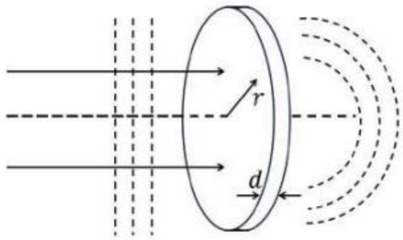
$$\psi = Af(r)\sin\theta\cos\theta e^{i\phi}$$

where A is the normalization constant. The eigenvalue of $\widehat{L_z}$ for this state is

(a) $2\hbar$ (b) h

(c) $-2\hbar$ (d) $-\hbar$

13. For a flat circular glass plate of thickness d, the refractive index $n(r)$ varies radially, where r is the radial distance from the centre of the plate. A coherent plane wavefront is normally incident on this plate as shown in the figure below.

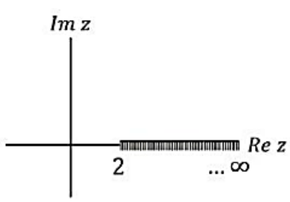


If the emergent wavefront is spherical and centered on the axis of the plate, then $n(r) - n(0)$ should be proportional to

(a)$r^{1/2}$ (b)r

(c)r^2 (d)$r^{3/2}$

14. The branch line for the function $f(z) = \sqrt{\frac{z^2-5z+6}{z^2+2z+1}}$ is

1.


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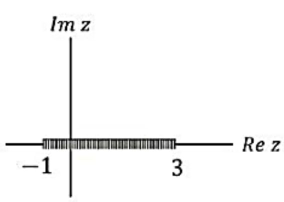

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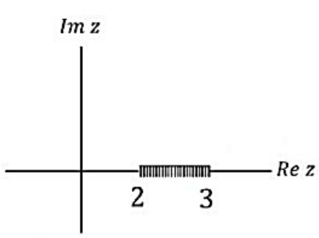

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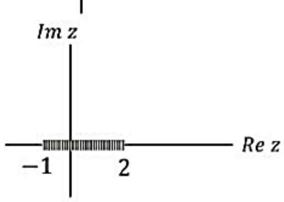


15. A particle moves in a circular orbit under a force field given by $\vec{F}(\vec{r}) = -\frac{k}{r^2}\hat{r}$. where k is a positive constant. If the force changes suddenly to $\vec{F}(\vec{r}) = -\frac{k}{2r^2}\hat{r}$, the shape of the new orbit would be

(a)parabolic (b)circular

(c)elliptical (d)hyperbolic

16. A system of N non-interacting classical spins, where each spin can take values $\sigma = -1,0,1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H = -\mu_B h\sigma + \Delta(1-\sigma^2),$$

where μ_B, Δ are positive constants with appropriate dimensions.

If M is the magnetization, the zero-field magnetic susceptibility per spin $\frac{1}{N}\frac{\partial M}{\partial h}\Big|_{h\to 0}$, at a temperature $T = 1/\beta k_B$ is given by

(a)$\beta\mu_B^2$ (b)$\frac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$

(c)$\beta\mu_B^2 e^{-\beta\Delta}$ (d)$\frac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

17. Four distinguishable particles fill up energy levels $0, \epsilon, 2\epsilon$. The number of available microstates for the total energy 4ϵ is

(a)20 (b)24

(c)11 (d)19

18. The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\varphi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\varphi)\chi_+\right].$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}h$. The expectation value of the z component of the total angular momentum in the above state is

(a)$-\frac{3}{4}\hbar$ (b)$\frac{3}{4}\hbar$

(c) $-\frac{9}{8}\hbar$ (d) $\frac{9}{8}\hbar$

19. A classical ideal gas is subjected to a reversible process in which its molar specific heat changes with temperature T as $C(T) = C_V + R\frac{T}{T_0}$. If the initial temperature and volume are T_0 and V_0, respectively, and the final volume is $2V_0$, then the final temperature is

(a) $T_0/\ln 2$ (b) $2T_0$

(c) $T_0/[1-\ln 2]$ (d) $T_0[1+\ln 2]$

20. A small bar magnet is placed in a magnetic field $B(\vec{r}) = B(x)\hat{z}$. The magnet is initially at rest with its magnetic moment along $\hat{y}$. At later times, it will undergo

(a) angular motion in the yz plane and translational motion along $\hat{y}$

(b) angular motion in the yz plane and translational motion along $\hat{x}$

(c) angular motion in the zx plane and translational motion along $\hat{z}$

(d) angular motion in the xy plane and translational motion along $\hat{z}$

21. The 1-dimensional Hamiltonian of a classical particle of mass m is

$$H = \frac{p^2}{2m}e^{-x/a} + V(x),$$

where a is a constant with appropriate dimensions. The corresponding Lagrangian is,

(a) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$

(b) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

(c) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$

(d) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

22. Let M be a 3×3 real matrix such that

$$e^{M\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where θ is a real parameter. Then M is given by

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

23. For three inputs A, B and C, the minimum number of 2-input NAND gates required to generate the output $Y = \overline{A+B} + \bar{C}$ is

(a) 3 (b) 4

(c) 7 (d) 6

24. A particle of unit mass subjected to the 1-dimensional potential

$$V(x) = \frac{2\alpha}{x^3} - \frac{3\beta}{x^2}$$

executes small oscillations about its equilibrium position, where α and β are positive constants with appropriate dimensions. The time period of small oscillations is

(a) $\frac{\pi\alpha^2}{\sqrt{6\beta^5}}$ (b) $\frac{\pi\alpha^2}{\sqrt{3\beta^5}}$

(c) $\frac{2\pi\alpha^2}{\sqrt{3\beta^5}}$ (d) $\frac{2\pi\alpha^2}{\sqrt{6\beta^5}}$

25. The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$, is

$$\hat{H} = \frac{\epsilon}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right].$$

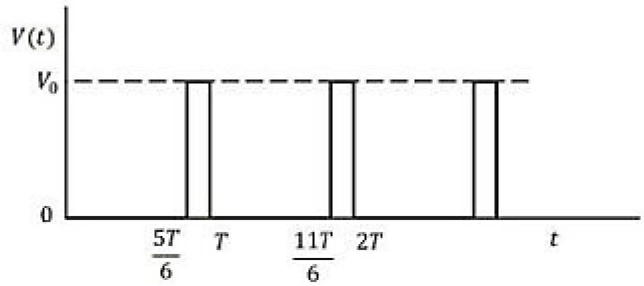
If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

(a) $3\epsilon, 2\epsilon, -\epsilon$ (b) $6\epsilon, 5\epsilon, 2\epsilon$

(c) $3\epsilon, 2\epsilon, \epsilon$ (d) $-3\epsilon, -2\epsilon, \epsilon$

➢ PART- C

26. An infinite waveform $V(t)$ varies as shown in the figure below



The lowest harmonic that vanishes in the Fourier series of $V(t)$ is

(a)2 (b)3

(c)6 (d)None

27. The ionization potential of hydrogen atom is 13.6eV, and λ_H and λ_D denote longest wavelengths in Balmer spectrum of hydrogen and deuterium atoms, respectively. Ignoring the fine and hyperfine structures, the percentage difference $y = \frac{\lambda_H - \lambda_D}{\lambda_H} \times 100$, is closest to

(a)1.0003% (b)−0.03%

(c)0.03% (d)−1.0003%

28. The function $f(z) = \frac{1}{(z+1)(z+3)}$ is defined on the complex plane. The coefficient of the $(z - z_0)^2$ term of the Laurent series of $f(z)$ about $z_0 = 1$ is

(a)$\frac{7}{64}$ (b)$\frac{7}{128}$

(c)$\frac{9}{64}$ (d)$\frac{9}{128}$

29. A transmission line has the characteristic impedance of $(50 + 1j)\Omega$ and is terminated in a load resistance of $(70 - 7j)\Omega$ (where $j^2 = -1$). The magnitude of the reflection coefficient will be closest to

(a)$\frac{5}{7}$ (b)$\frac{1}{2}$

(c)$\frac{1}{6}$ (d)$\frac{1}{7}$

30. Atmospheric neutrinos are produced from the cascading decays of cosmic pions $(\pi^{\pm})$to stable particles. Ignoring all other neutrino sources, the ratio of muon neutrino $(v_\mu + \bar{v}_\mu)$ flux to electron neutrino $(v_e + \bar{v}_e)$ flux in atmosphere is expected to be closest to

(a)2: 3 (b)1: 1

(c)1: 2 (d)2: 1

31. A quantum system is described by the Hamiltonian

$$H = -J\sigma_z + \lambda(t)\sigma_x,$$

where $\sigma_i (i = x, y, z)$ are Pauli matrices, J and λ are positive constants ($J \gg \lambda$) and

$$\lambda(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \lambda & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$$

At $t < 0$, the system is in the ground state. The probability of finding the system in the excited state at $t \gg T$, in the leading order in λ is

(a)$\frac{\lambda^2}{8J^2} \sin^2 \frac{JT}{\hbar}$ (b)$\frac{\lambda^2}{J^2} \sin^2 \frac{JT}{\hbar}$

(c)$\frac{\lambda^2}{4J^2} \sin^2 \frac{JT}{\hbar}$ (d)$\frac{\lambda^2}{16J^2} \sin^2 \frac{JT}{\hbar}$

32. The regular representation of two nonidentity elements of the group of order 3 are given by

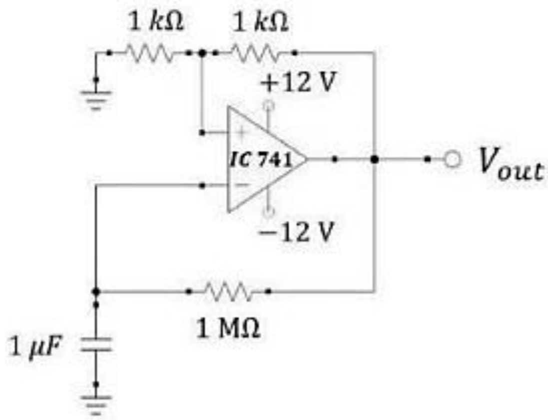
(a) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

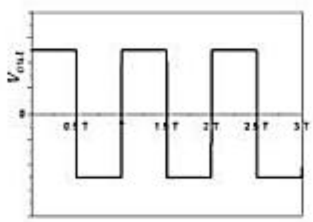
(c) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

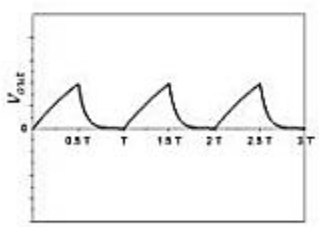
(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

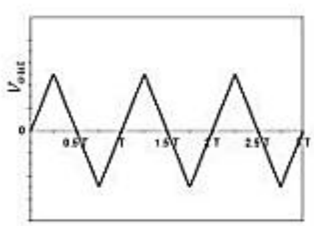
33. A circuit with operational amplifier is shown in the figure below.

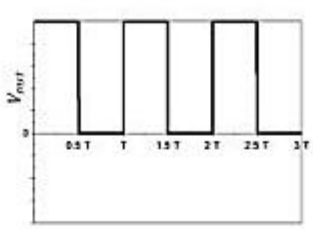


The output voltage waveform V_{out} will be closest to

1. 

2. 

3. 

4. 

Ans:1

34. The permittivity of a medium $\varepsilon(\vec{k},\omega)$, where ω and $\vec{k}$ are the frequency and wavevector, respectively, has no imaginary part. For a longitudinal wave, $\vec{k}$ is parallel to the electric field such that $\vec{k}\times\vec{E}=0$, while for a transverse wave $\vec{k}\cdot\vec{E}=0$. In the absence of free charges and free currents, the medium can sustain

(a)longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)>0$

(b)transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)<0$

(c)longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)=0$

(d)both longitudinal and transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)>0$

35. A solar probe mission detects a fractional wavelength shift $(\Delta\lambda/\lambda)$ of the spectral line $\lambda=630$ nm within a sunspot to be of the order of 10^{-5}. Assuming this shift is caused by the normal Zeeman effect (i.e., neglecting other physical effects), the estimated magnetic field (in tesla) within the observed sunspot is closest to

(a)3×10^{-5} (b)300

(c)0.3 (d)3×10^{5}

36. In a shell model description, neglecting Coulomb effects, which of the following statements for the energy and spin-parity is correct for the first excited state of $A=12$ isobars ${}^{12}_{5}$B, ${}^{12}_{6}$C, ${}^{12}_{7}$N ?

(a)same for ${}^{12}_{5}$B, ${}^{12}_{6}$C and ${}^{12}_{7}$N

(b)different for each ${}^{12}_{5}$B, ${}^{12}_{6}$C and ${}^{12}_{7}$N

(c)same for ${}^{12}_{6}$C and ${}^{12}_{7}$N, but different for ${}^{12}_{5}$B

(d)same for ${}^{12}_{5}$B and ${}^{12}_{7}$N, but different for ${}^{12}_{6}$C

37. An incident plane wave with wavenumber k is scattered by a spherically symmetric soft potential. The scattering occurs only in S - and P - waves. The approximate scattering amplitude at angles $\theta=\frac{\pi}{3}$ and $\theta=\frac{\pi}{2}$ are

$$f\left(\theta=\frac{\pi}{3}\right) \simeq \frac{1}{2k}\left(\frac{5}{2}+3i\right) \text{ and } f\left(\theta=\frac{\pi}{2}\right) \simeq \frac{1}{2k}\left(1+\frac{3i}{2}\right).$$

Then the total scattering cross-section is closest to

(a) $\frac{37\pi}{4k^2}$ (b) $\frac{10\pi}{k^2}$

(c) $\frac{35\pi}{4k^2}$ (d) $\frac{9\pi}{k^2}$

38. A photon inside the sun executes a random walk process. Given the radius of the sun $\approx 7\times10^8$ km and mean free path of a photon $\approx 10^{-3}$ m, the time taken by the photon to travel from the centre to the surface of the sun is closest to

(a) 10^6sec (b) 10^{24}sec

(c) 10^{12}sec (d) 10^{18}sec

39. The radius of a sphere oscillates as a function of time as $R + a\cos\,\omega t$, with $a < R$. It carries a charge Q uniformly distributed on its surface at all times. If P is the time averaged radiated power through a sphere of radius r, such that $r \gg R + a$ and $r \gg \frac{c}{\omega}$, then

(a) $P \propto \frac{Q^2\omega^4a^2}{c^3}$ (b) $P \propto \frac{Q^2\omega^2}{c}$

(c) $P = 0$ (d) $P \propto \frac{Q^2\omega^6a^4}{c^5}$

40. A particle of mass m is moving in a 3-dimensional potential

$$\phi(r) = -\frac{k}{r} - \frac{k'}{3r^3}\quad k, k' > 0.$$

For the particle with angular momentum l, the necessary condition to have a stable circular orbit is

(a) $kk' < \frac{l^4}{4m^2}$ (b) $kk' > \frac{l^4}{4m^2}$

(c) $kk' < \frac{l^4}{m^2}$ (d) $kk' > \frac{l^4}{m^2}$

41. In a quantum harmonic oscillator problem, $\hat{a}$ and $\hat{N}$ are the annihilation operator and the number operator, respectively. The operator $e^{\hat{N}}\hat{a}e^{-\hat{N}}$ is

(a) $\hat{a}$ (b) $e^{-1}\hat{a}$

(c) $e^{-(\hat{I}+\hat{a})}$ (d) $e^{\hat{a}}$

(where $\hat{I}$ is the identity operator)

42. A system of non-relativistic and non-interacting bosons of mass m in two dimensions has a density n. The Bose-Einstein condensation temperature T_c is

(a) $\frac{12nh^2}{\pi mk_B}$ (b) $\frac{3n\hbar^2}{\pi mk_B}$

(c) $\frac{6nh^2}{\pi mk_B}$ (d) 0

43. The solution $y(x)$ of the differential equation $y'' + \frac{y}{4} = \frac{x}{2}$, where $0 \le x \le \pi$, together with the boundary conditions $y(0) = y(\pi) = 0$ is

(a) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$

(b) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}n^2}$

(c) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$

(d) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2}$

44. The lattice constant of the bcc structure of sodium metal is 4.22Å. Assuming the mass of the electron inside the metal to be the same as free electron mass, the free electron Fermi energy is closest to

(a) 3.2eV (b) 2.9eV

(c) 3.5eV (d) 2.5eV

45. The ground state of ${}^{207}_{82}$Pb nucleus has spin-parity $J^\pi = \left(\frac{1}{2}\right)^-$, while the first excited state has $J^\pi = \left(\frac{5}{2}\right)^-$. For the transition from the first excited state to the ground state, possible multipolarities of emitted electromagnetic radiation are

(a) E2, E3 (b) M2, M3

(c) M2, E3 (d) E2, M3

46. A quantum particle of mass m is moving in a one dimensional potential

$$V(x) = V_0\theta(x) - \lambda\delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the

Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E \gg V_0 > \lambda$ and $\sqrt{2mE} \gg \frac{V_0 h}{\lambda}$ is

(a) $\frac{V_0^2}{4E^2}$ (b) $\frac{V_0^2}{8E^2}$

(c) $\frac{m\lambda^2}{2E\hbar^2}$ (d) $\frac{m\lambda^2}{4Eh^2}$

47. The collision time of the electrons in a metal in the Drude model is τ and their plasma frequency is ω_p. If this metal is placed between the plates of a capacitor, the time constant associated with the decay of the electric field inside the metal is

(a) $\tau + \frac{1}{\omega_p}$ (b) $\omega_p \tau^2$

(c) $\frac{1}{\omega_p^2 \tau}$ (d) $\frac{\tau}{1+\omega_p \tau}$

48. The work done on a material to change its magnetization M in an external field H is $dW = HdM$. Its Gibbs free energy is

$$G(T,H) = -\left(\gamma T + \frac{aH^2}{2T}\right),$$

where $\gamma, a > 0$ are constants. The material is in equilibrium at a temperature $T = T_0$ and in an external field $H = H_0$. If the field is decreased to $\frac{H_0}{2}$ adiabatically and reversibly, the temperature changes to

(a) $2T_0$ (b) $\frac{T_0}{2}$

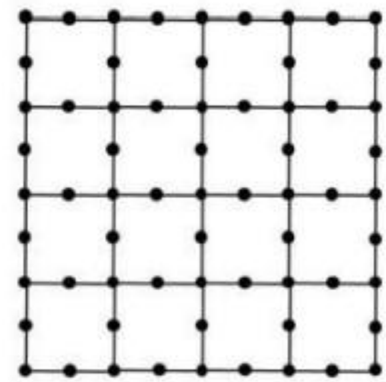
(c) $\left(\frac{a}{2\gamma}\right)^{\frac{1}{4}} \sqrt{H_0 T_0}$ (d) $\left(\frac{a}{\gamma}\right)^{\frac{1}{4}} \sqrt{H_0 T_0}$

49. Gauge factor of a strain gauge is defined as the ratio of the fractional change in resistance $\left(\frac{\Delta R}{R}\right)$ to the fractional change in length $\left(\frac{\Delta L}{L}\right)$. A metallic strain gauge with a gauge factor 2 has a resistance of 100Ω under unstrained condition. An aluminum foil with Young's modulus $Y = 70 \mathrm{GN/m^2}$ is installed on the metallic gauge. Keeping the foil within its elastic limit, a stress of $0.2 \mathrm{GN/m^2}$ is applied on the foil. The change in the resistance of the gauge will be closest to

(a) 0.14Ω (b) 1.23Ω

(c) 0.28Ω (d) 0.56Ω

50. In the section of an infinite lattice shown in the figure below, all sites are occupied by identical hard circular discs so that the resulting structure is tightly packed.



The packing fraction is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{16}$ (d) $\frac{9\pi}{16}$

51. Given the data points

x	1	3	5
y	4	28	92

using Lagrange's method of interpolation, the value of y at $x = 4$ is closest to

(a) 54 (b) 55

(c) 53 (d) 56

52. In the rotational-vibrational spectrum of an idealized carbon monoxide (CO) molecule, ignoring rotational-vibrational coupling, two transitions between adjacent vibrational levels with wavelength λ_1 and λ_2, correspond to the rotational transition from $J' = 0$ to $J'' = 1$, and $J' = 1$ to $J'' = 0$, respectively. Given that the reduced mass of CO is $1.2 \times^{-26}$ kg, equilibrium bond length of CO is 0.12 nm and vibrational frequency is 5×10^{13} Hz, the ratio of $\frac{\lambda_1}{\lambda_2}$ is closest to

(a) 0.9963 (b) 0.0963

(c) 1.002 (d) 1.203

53. A Lagrangian is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}\dot{z} + \dot{z}^2) - \alpha(2x + 3y + z).$$

The conserved momentum is

(a) $m[2\dot{x} + \dot{z}]$ (b) $m[2\dot{x} + \dot{y} + \dot{z}]$

(c) $m\left[\dot{x} + \frac{3}{2}\dot{y} + \frac{1}{2}\dot{z}\right]$ (d) $m[2\dot{x} + 3\dot{z}]$

54. A canonical transformation from the phase space coordinates (q, p) to (Q, P) is generated by the function

$$\psi(p, Q) = \frac{p^2}{2\omega}\tan\ 2\pi Q.$$

where ω is a positive constant. The function $\psi(p, Q)$ is related to $F(q, Q)$ by the Legendre transform $\psi = pq - F$, where F is defined by $dF = pdq - PdQ$. If the solution for (P, Q) is

$$P(t) = \frac{\omega}{4\pi}t^2, Q(t) = Q_0 = \text{ constant },$$

where t is time, then the solution for (p, q) variables can be written as

(a) $p = \frac{\omega t}{2\pi}\cos\ 2\pi Q_0, q = \frac{t}{2\pi}\sin\ 2\pi Q_0$

(b) $p = -\frac{\omega t}{2\pi}\cos\ 2\pi Q_0, q = \frac{t}{2\pi}\sin\ 2\pi Q_0$

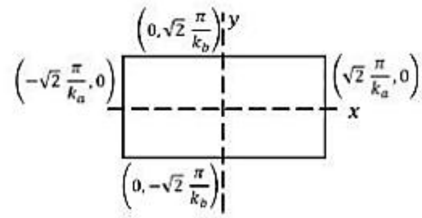
(c) $p = \frac{\omega t}{2\pi}\sin\ 2\pi Q_0, q = \frac{t}{2\pi}\cos\ 2\pi Q_0$

(d) $p = -\frac{\omega t}{2\pi}\sin\ 2\pi Q_0, q = \frac{t}{2\pi}\cos\ 2\pi Q_0$

55. A 2-dimensional resonant cavity supports a TM mode built from a function

$$\psi(x, y, t) = \sin(\vec{k}_a \cdot \vec{r} - \omega t) + \sin(\vec{k}_b \cdot \vec{r} - \omega t) + \sin(\vec{k}_a \cdot \vec{r} + \omega t) + \sin(\vec{k}_b \cdot \vec{r} + \omega t)$$

where $\vec{k}_a$ and $\vec{k}_b$ lie in the xy-plane and make angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ with the x-axis, respectively. If $0 < |\vec{k}_a| < |\vec{k}_b|$, then which of the following closely describes the outline of the cavity?

1.


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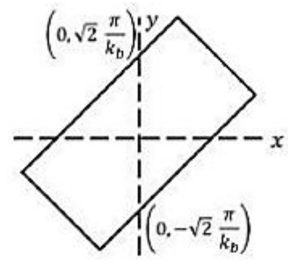
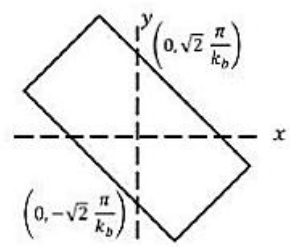
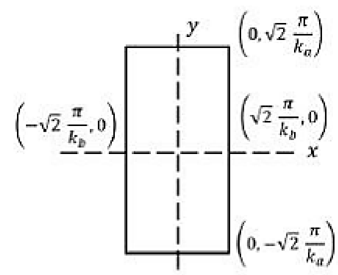


3.


4.


❖ ANSWER KEY

1. c	2. a	3. a	4. b	5. b
6. a	7. d	8. d	9. a	10. d
11. a	12. b	13. c	14. c	15. a
16. b	17. d	18. b	19. d	20. b
21. a	22. b	23. b	24. d	25. a
26. c	27. c	28. b	29. c	30. d
31. b	32. c	33. a	34. c	35. c
36. d	37. a	38. c	39. c	40. a
41. b	42. d	43. d	44. a	45. d
46. c	47. c	48. b	49. d	50. c
51. b	52. a	53. b	54. a	55. d