❖ CSIR-UGC-NET/JRF- DEC - 2017 PHYSICAL SCIENCES BOOKLET - [A]

➢ PART-B

1. Consider the differential equation $\frac{dy}{dx} + ay = e^{-bt}$ with the initial condition $y(0) = 0$. Then the Laplace transform $Y(s)$ of the solution $y(t)$ is

(a) $\frac{1}{(s+a)(s+b)}$ (b) $\frac{1}{b(s+a)}$
(c) $\frac{1}{a(s+b)}$ (d) $\frac{e^{-a}-e^{-b}}{b-a}$

2. Consider the matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution, and the corresponding normalized solution (up to a sign) is

(a) $b = 2c$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$
(b) $c = 2b$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, 1, -2)$
(c) $c = b + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(2, -1, -1)$
(d) $b = c + 1$ and $(x, y, z) = \frac{1}{\sqrt{6}}(1, -2, 1)$

3. Consider the real function $f(x) = \frac{1}{(x^2+4)}$. The Taylor expansion of $f(x)$ about $x = 0$ converges

(a) for all values of x
(b) for all values of x except $x = \pm 2$
(c) in the region $-2 < x < 2$
(d) for $x > 2$ and $x < -2$

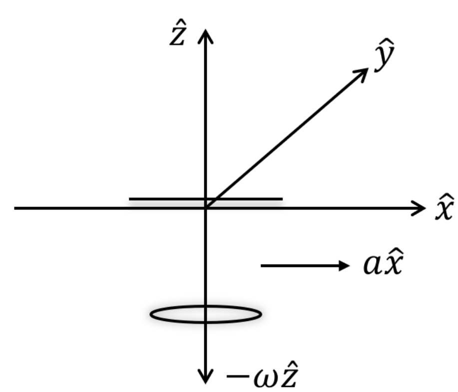
4. Let A be a non-singular 3×3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $(A^T)^{-1}$. Which of the following is true?

(a) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 1$
(b) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 1, \vec{u} \cdot \vec{c} = 0$
(c) $\vec{u} \cdot \vec{a} = 1, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$
(d) $\vec{u} \cdot \vec{a} = 0, \vec{u} \cdot \vec{b} = 0, \vec{u} \cdot \vec{c} = 0$

5. The number of linearly independent power series solutions, around $x = 0$, of the second order linear differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ is

(a) 0 (this equation does not have a power series solution)
(b) 1
(c) 2
(d) 3

6. A disc of mass m is free to rotate in a plane parallel to the xy-plane with an angular velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.



The car accelerates in the positive x-direction with an acceleration $a > 0$. Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?
(a) only the x and the z coordinates change
(b) only the y and the z coordinates change
(c) only the x and the y coordinates change
(d) all the three coordinates change

7. A cyclist, weighing a total of 80 kg with the bicycle, pedals at a speed of 10 m/s. She stops pedalling at an instant which is taken to be $t = 0$. Due to the velocity dependent frictional force, her velocity is found to vary as

$$v(t) = \frac{10}{\left(1 + \frac{t}{30}\right)} \text{m/s}$$

where t is measured in seconds. When the velocity drops to 8 m/s, she starts pedalling again to maintain a constant speed. The energy expended by her in one minute at this (new) speed, is
(a) 4 kJ (b) 8 kJ
(c) 16 kJ (d) 32 kJ

8. A light signal travels from a point A to a point B, both within a glass slab that is moving with uniform velocity (in the same direction as the light) with speed 0.3c with respect to an external observer. If the refractive index of the slab is 1.5 , then the observer will measure the speed of the signal as
(a) $0.67c$ (b) $0.81c$
(c) $0.97c$ (d) c

9. A monoatomic gas of volume V is in equilibrium in a uniform vertical cylinder, the lower end of which is closed by a rigid wall and the order by a frictionless piston. The piston is pressed lightly and released. Assume that the gas is a poor conductor of heat and the cylinder and piston are perfectly insulating. If the cross-sectional area of the cylinder is A, the angular frequency of small oscillations of the piston about the point of equilibrium, is
(a) $\sqrt{\frac{5gA}{(3V)}}$ (b) $\sqrt{\frac{4gA}{(3V)}}$
(c) $\frac{5}{3}\sqrt{\frac{gA}{V}}$ (d) $\sqrt{\frac{7gA}{(5V)}}$

10. The normalized wavefunction of a particle in three dimensions is given by

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{8\pi a^3}} e^{-r/2a}$$

where $a > 0$ is a constant. The ratio of the most probable distance from the origin to the mean distance from the origin, is [You may use $\int_0^\infty dx x^n e^{-x} = n!$].
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

11. The state vector of a one-dimensional simple harmonic oscillator of angular frequency ω, at time $t = 0$, is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |2\rangle]$, where $|0\rangle$ and $|2\rangle$ are the normalized ground state and the second excited state, respectively. The minimum time t after which the state vector $|\psi(t)\rangle$ is orthogonal to $|\psi(0)\rangle$, is
(a) $\frac{\pi}{2\omega}$ (b) $\frac{2\pi}{\omega}$
(c) $\frac{\pi}{\omega}$ (d) $\frac{4\pi}{\omega}$

12. The normalized wavefunction in the momentum space of a particle in one dimension is $\phi(p) = \frac{\alpha}{p^2+\beta^2}$, where α and β are real constants. The uncertainty Δx in measuring its position is
(a) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^2}$ (b) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^3}$
(c) $\frac{\hbar}{\sqrt{2}\beta}$ (d) $\sqrt{\frac{\pi}{\beta}\frac{\hbar\alpha}{\beta}}$

13. Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator

$$\left[\frac{1}{2m}p^2 + \beta x^2, \frac{1}{m}p^2 + \gamma x^2\right],$$

where β and γ are constants, is zero if
(a) $\gamma = \beta$ (b) $\gamma = 2\beta$
(c) $\gamma = \sqrt{2\beta}$ (d) $2\gamma = \beta$

14. Two point charges $+3Q$ and $-Q$ are placed at $(0,0,d)$ and $(0,0,2d)$ respectively, above an infinite grounded conducting sheet kept in the xy-plane. At a point $(0,0,z)$, where $z \gg d$, the electrostatic potential of this charge configuration would approximately be

(a) $\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2d}{z^2}Q$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{3d}{z^2}Q$ (d) $-\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$

15. A rectangular piece of dielectric material is inserted partially into the (air) gap between the plates of a parallel plate capacitor. The dielectric piece will

(a) remain stationary where it is placed.

(b) be pushed out from the gap between the plates.

(c) be drawn inside the gap between the plates and its velocity does not change sign.

(d) execute an oscillatory motion in the region between the plates.

16. An electromagnetic wave is travelling in free space (of permittivity E_0) with electric field $\vec{E} = \hat{k}E_0 \cos\ q(x-ct)$. The average power (per unit area) crossing planes parallel to $4x+3y=0$ will be

(a) $\frac{4}{5}\varepsilon_0 cE_0^2$ (b) $\varepsilon_0 cE_0^2$

(c) $\frac{1}{2}\varepsilon_0 cE_0^2$ (d) $\frac{16}{25}\varepsilon_0 cE_0^2$

17. A plane electromagnetic wave from within a dielectric medium (with $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$) is incident on its boundary with air, at $z=0$. The magnetic field in the medium is $\ddot{H} = \hat{\jmath}H_0 \cos\ (\omega t - kx - k\sqrt{3}z)$, where ω and k are positive constants.

The angles of reflection and refraction are, respectively,

(a) 45° and 60° (b) 30° and 90°

(c) 30° and 60° (d) 60° and 90°

18. The dispersion relation of a gas of spin $-\frac{1}{2}$ fermions in two dimensions is $E = \hbar v|\vec{k}|$, where E is the energy, $\vec{k}$ is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is ε_F, the number of particles per unit area is

(a) $\frac{\varepsilon_F}{(4\pi v\hbar)}$ (b) $\frac{\varepsilon_F^3}{(6\pi^2 v^3\hbar^2)}$

(c) $\frac{\pi\varepsilon_F^{3/2}}{(3v^3\hbar^3)}$ (d) $\frac{\varepsilon_F^2}{(2\pi v^2\hbar^2)}$

19. The relation between the internal energy U, entropy S, temperature T, pressure p, volume V, chemical potential μ and number of particles N of a thermodynamic system is $dU = TdS - pdV + \mu dN$. Thal U is an exact differential implies that

(a) $-\frac{\partial p}{\partial S}\Big|_{V,N} = \frac{\partial T}{\partial V}\Big|_{S,N}$

(b) $p\frac{\partial U}{\partial T}\Big|_{S,N} = S\frac{\partial U}{\partial V}\Big|_{s,n}$

(c) $p\frac{\partial U}{\partial T}\Big|_{S,N} = -\frac{1}{T}\frac{\partial U}{\partial V}\Big|_{S,\mu}$

(d) $\frac{\partial p}{\partial S}\Big|_{V;N} = \frac{\partial T}{\partial V}\Big|_{S,N}$

20. The number of microstates of a gas of N particles in a volume V and of internal energy U, is given by

$$\Omega(U,V,N) = (V-Nb)^N\left(\frac{aU}{N}\right)^{3N/2},$$

(where a and b are positive constants). Its pressure P, volume V and temperature T, are related by

(a) $\left(P + \frac{aN}{V}\right)(V-Nb) = Nk_BT$

(b) $\left(P - \frac{aN}{V^2}\right)(V-Nb) = Nk_BT$

(c) $PV = Nk_BT$

(d) $P(V-Nb) = Nk_BT$

21. Consider a system of identical atoms in equilibrium with blackbody radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1)$ are P_0 and P_1, respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\to1}$ and $W_{1\to0}^t$ denote, respectively, the squares of the matrix elements corresponding to the atomic transitions $|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$. Which of the fol lowing equations hold in equilibrium ?

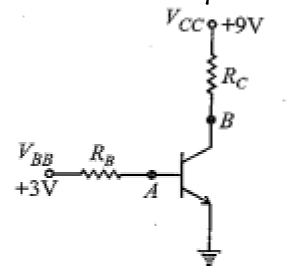
(a) $P_0 nW_{0\to1} = P_1 W_{1\to0}$

(c) $P_0 nW_{0\to1} = P_1 W_{1\to0} - P_1 nW_{1\to0}$

(b) $P_0 W_{0\to1} = P_1 nW_{1\to0}$

(d) $P_0 nW_{0\to1} = P_1 W_{1\to0} + P_1 nW_{1\to0}$

22. In the circuit below the voltages V_{BB} and V_{CC} are kept fixed, the voltage measured at B is a constant, but that measured at A fluctuates between a few μV to a few mV.



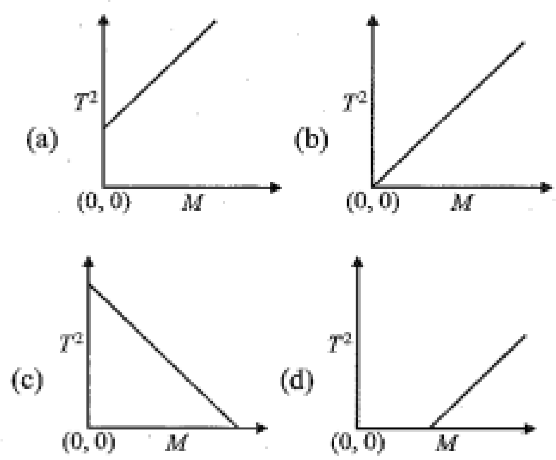
From these measurements it may be inferred that the

(a) base is open internally
(b) emitter is open internally
(c) collector resistor is open
(d) base resistor is open

23. The full scale voltage of an n-bit Digital-to-Analog Converter is V. The resolution that can be achieved in it is

(a) $\frac{V}{(2^n-1)}$ (b) $\frac{V}{(2^n+1)}$
(c) $\frac{V}{2^{2n}}$ (d) $\frac{V}{n}$

24. The spring constant k, of a spring of a mass m_s, is determined experimentally by loading the spring with mass M and recording the time period T, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is



25. A Zener diode with an operating voltage of 10 V at 25°C has a positive temperature co-efficient of 0.07% per °C of the operating voltage. The operating voltage of this Zener diode at 125°C is

(a) 12.0 V (b) 11.7 V
(c) 10.7 V (d) 9.3 V

➢ PART - *C*

26. Consider an element $U(\varphi)$ of the group SU(2), where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as $U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(\varphi)$. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be

(a) positive definite (b) real symmetric
(c) Hermitian (d) anti-hermitian

27. The differential equation $\frac{dy(x)}{dx} = \alpha x^2$, with the initial condition $y(0) = 0$, is solved using Euler's method. If $y_E(x)$ is the exact solution and $y_N(x)$ the numerical solution obtained using n steps of equal length, then the relative error $\left|\frac{(y_N(x)-y_E(x))}{y_E(x)}\right|$ is proportional to

(a) $\frac{1}{n^2}$ (b) $\frac{1}{n^3}$
(c) $\frac{1}{n^4}$ (d) $\frac{1}{n}$

28. The interval [0,1] is divided into n parts of equal length to calculate the integral $\int_0^1 e^{j2\pi x}dx$ using the trapezoidal rule. The minimum value of n for which the result is exact, is

(a) 2 (b) 3
(c) 4 (d) ∞

29. The generating function $G(t,x)$ for the Legendre polynomials $P_n(t)$ is

$$G(t,x) = \frac{1}{\sqrt{1-2xt+x^2}} = \sum_{n=0}^{\infty} x^n P_n(t), \text{ for } |x| < 1.$$

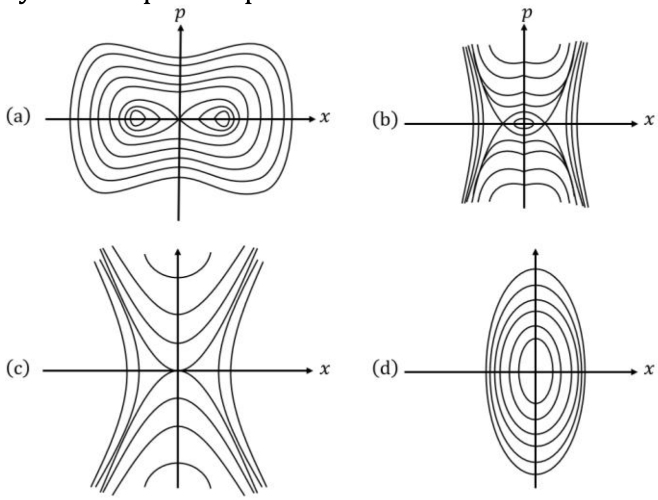
If the function $f(x)$ is defined by the integral equation

$$\int_0^x f(x')dx' = xG(1,x),$$

it can be expressed as

(a) $\sum_{n,m=0}^{\infty} x^{n+m} P_n(1) P_m\left(\frac{1}{2}\right)$
(b) $\sum_{n,m=0}^{\infty} x^{n+n} P_n(1) P_{m+1}(1)$
(c) $\sum_{n,m=0}^{\infty} x^{n-m} P_n\ (1)\ P_m\ (1)$
(d) $\sum_{n,m=0}^{\infty} x^{n-m} P_n(0) P_m(1)$

30. A particle moves in one dimension in a potential $V(x) = -k^2x^4 + \omega^2x^2$, where k and ω are constants. Which of the following curves best describes the trajectories of this system in phase space?



31. Let (x, p) be the generalized coordinate and momentum of a Hamiltonian system. If new variables (X, P) are defined by $X = x^{\alpha}\sinh(\beta p)$ and $P = x^{\gamma}\cosh(\beta p)$, where α, β and γ are constants, then the conditions for it to be a canonical transformation, are

(a) $\alpha = \frac{1}{2\beta}(\beta + 1)$ and $\gamma = \frac{1}{2\beta}(\beta - 1)$
(b) $\beta = \frac{1}{2\gamma}(\alpha + 1)$ and $\gamma = \frac{1}{2\alpha}(\alpha - 1)$
(c) $\alpha = \frac{1}{2\beta}(\beta - 1)$ and $\gamma = \frac{1}{2\beta}(\beta + 1)$
(d) $\beta = \frac{1}{2\gamma}(\alpha - 1)$ and $\gamma = \frac{1}{2\alpha}(\alpha + 1)$

32. Consider a set of particles which interact by a pair potential $V = ar^6$, where r is the inter-particle separation and $a > 0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

33. In an intertial frame S, the magnetic vector potential in a region of space is given by $\vec{A} = az\hat{\imath}$ (where a is a constant) and the scalar potential is zero. The electric and magnetic fields seen by an inertial observer moving with a velocity $v\hat{\imath}$ with respect to S, are respectively. [In the following $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$].

(a) 0 and $\gamma a\hat{\jmath}$ (b) $-va\hat{k}$ and $\gamma a\hat{\imath}$
(c) $r\gamma a\hat{k}$ and $r\gamma a\hat{\jmath}$ (d) $v\gamma a\hat{k}$ and $\gamma a\hat{\jmath}$

34. In the rest frame S_1 of a point particle with electric charge q_1, another point particle with electric charge q_2 moves with a speed v parallel to the x-axis at a perpendicular distance l. The magnitude of the electromagnetic force felt by q_1 due to q_2 when the distance between them is minimum, is [In the following $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}\left(1+\frac{v^2}{c^2}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}\left(1+\frac{v^2}{c^2}\right)$

35. A circular current carrying loop of radius a carries a steady current. A constant electric charge is kept at the centre of the loop. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$ respectively, at a distance d vertically above the centre of the loop satisfy

(a) $\vec{E} \perp \vec{B}$ (b) $\vec{E} = 0$
(c) $\vec{\nabla}(\vec{E} \cdot \vec{B}) = 0$ (d) $\forall \cdot (\vec{E} \times \vec{B}) = 0$

36. A phase shift of 30° is observed when a beam of particles of energy 0.1MeV is scattered by a target. When the beam energy is changed, the observed phase shift is 60°. Assuming that only s-wave scattering is relevant and that the cross-section does not change with energy, the beam energy is

(a) 0.4MeV (b) 0.3MeV
(c) 0.2MeV (d) 0.15MeV

37. The Hamiltonian of a two-level quantum system is $H = \frac{1}{2}\hbar\omega\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$. A possible initial state in which the probability of the system being in that quantum state does not change with time, is

(a) $\begin{pmatrix}\cos\frac{\pi}{4}\\ \sin\frac{\pi}{4}\end{pmatrix}$ (b) $\begin{pmatrix}\cos\frac{\pi}{8}\\ \sin\frac{\pi}{8}\end{pmatrix}$
(c) $\begin{pmatrix}\cos\frac{\pi}{2}\\ \sin\frac{\pi}{2}\end{pmatrix}$ (d) $\begin{pmatrix}\cos\frac{\pi}{6}\\ \sin\frac{\pi}{6}\end{pmatrix}$

38. Consider a one-dimensional infinite square well

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 < x < a, \\ \infty & \quad \text{otherwise} \end{cases}$$

If a perturbation

$$\Delta V(x) = \begin{cases} V_0 & \text{for} \quad 0 < x < \dfrac{a}{3}, \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state, to first order in ΔV, is nearest to

(a) V_0 (b) $0.16V_0$
(c) $0.2V_0$ (d) $0.33V_0$

39. The energy eigenvalues E_n of a quantum system in the potential $V = cx^6$ (where $c > 0$ is a constant), for large values of the quantum number n, varies as

(a) $n^{4/3}$ (b) $n^{3/2}$
(c) $n^{5/4}$ (d) $n^{6/5}$

40. Consider a quantum system of non-interacting bosons in contact with a particle bath. The probability of finding no particle in a given single particle quantum state is 10^{-6}. The average number of particles in that state is of the order of

(a) 10^{9} (b) 10^{6}
(c) 10^{9} (d) 10^{12}

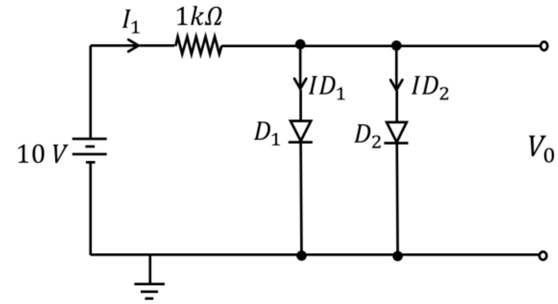
41. A closed system having three non-degenerate energy levels with energies $E = 0, \pm\varepsilon$, is at temperature T. For $\varepsilon = 2k_BT$, the probability of finding the system in the state with energy $E = 0$, is

(a) $\frac{1}{(1+2\cosh\ 2)}$ (b) $\frac{1}{(2\cosh\ 2)}$
(c) $\frac{1}{2}\cosh\ 2$ (d) $\frac{1}{\cosh\ 2}$

42. Two non-degenerate energy levels with energies 0 and ε are occupied by N non-interacting particles at a temperatures T. Using classical statistics, the average internal energy of the system is

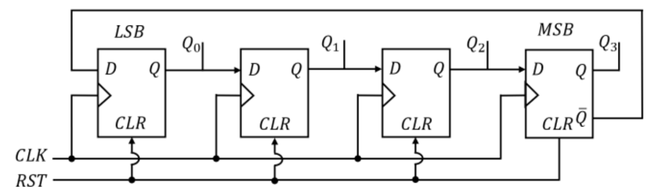
(a) $\frac{N\varepsilon}{(1+e^{\varepsilon/k_BT})}$ (b) $\frac{N\varepsilon}{(1-e^{\varepsilon/k_nT})}$
(c) $N\varepsilon e^{-\varepsilon/k_BT}$ (d) $\frac{3}{2}Nk_BT$

43. In the circuit below, D_1 and D_2 are two silicon diodes with the same characteristics. If the forward voltage drop of a silicon diode is 0.7 V, then the value of the current $I_1 + I_D$, is



(a) 18.6 mA (b) 9.3 mA
(c) 13.95 mA (d) 14.65 mA

44. The circuit below comprises of D-flip flops. The output is taken from Q_3, Q_2, Q_1 and Q_0, as shown in the figure.



The binary number given by the string $Q_3Q_2Q_1Q_0$ changes for every clock pulse that is applied to the CLK input. If the output is initialized at 0000 , then the corresponding sequence of decimal numbers that repeats itself, is

(a) 3,2,1,0
(c) 1,3,7,15,12,14,0
(b) 1,3,7,14,12,8
(d) 1,3,7,15,14,12,8,0

45. Two physical quantities T and M are related by the equation

$$T = \frac{2\pi}{a}\sqrt{\frac{M+b}{2}},$$

where a and b are constant parameters. The variation of T as a function of M was recorded in an experiment to determine the value of a graphically. Let m be the slope of the straight line when T^2 is plotted vs. M, and δm be the uncertainty in determining it. The uncertainty in determining a is

(a) $\frac{a}{2}\left(\frac{\delta m}{m}\right)$ (b) $a\left(\frac{\delta m}{m}\right)$
(c) $\frac{b}{2a}\left(\frac{\delta m}{m}\right)$ (d) $\frac{2\pi}{a}\left(\frac{\delta m}{m}\right)$

46. The sensitivity of a hot cathode pressure gauge is 10mbar^{-1}. If the ratio between the numbers of the impinging charged particles to emitted electrons is 1: 10, then the pressure is

(a) 10 mbar (b) 10^{-1}mbar

(c) 10^{-2}mbar (d) 10^{2}mbar

47. The Zeeman shift of the energy of a state with quantum numbers L, S, J and m_J is

$$H_Z = \frac{m_J \mu_B B}{J(J+1)}(\langle L \cdot J \rangle + g_S \langle S \cdot J \rangle)$$

where B is the applied magnetic field, g_S is the g-factor for the spin and $\mu_B/h =$ 1.4MHz^{-1}G^{-1}, where h is the Planck constant. The approximate frequency shift of the $S = 0, L = 1$ and $m_J = 1$ state, at a magnetic field of 1G, is

(a) 10MHz (b) 1.4MHz

(c) 5MHz (d) 2.8MHz

48. The separations between the adjacent levels of a normal multiplet are found to be 22 cm^{-1} and 33 cm 1. Assume that the multiplet is described well by the $L - S$ coupling scheme and the Lande's interval rule, namely $E(J) - E(J-1) = AJ$, where A is a constant. The term notations for this multiplet is

(a) $^3P_{0,1,2}$ (b) $^3F_{2,3,4}$

(c) $^3G_{3,4,5}$ (d) $^3D_{1,2,3}$

49. If the fine structure splitting between the 2^2P_{32} and $2\,^2P_{12}$ levels in the hydrogen atom is 0.4 cm^{-1}, the corresponding splitting in Li^{2+} will approximately be

(a) 1.2 cm^{-1} (b) 10.8 cm^{-1}

(c) 32.4 cm^{-1} (d) 36.8 cm^{-1}

50. A crystal of MnO has NaCl structure. It has a paramagnetic to anti-ferromagnetic transition at 120 K. Below 120 K, the spins within a single [111] plane are parallel but the spins in adjacent [111] planes are anti-parallel. If neutron scattering is used to determine the lattice constants, respectively, d and d', below and above the transition temperature of MnO then

(a) $d = \frac{d'}{2}$ (b) $d = \frac{d'}{\sqrt{2}}$

(c) $d = 2d'$ (d) $d = \sqrt{2}d'$

51. A metallic nanowire of length l is approximated as a one-dimensional lattice of N atoms with lattice spacing a. If the dispersion of electrons in the lattice is given as $E(k) = E_0 - 2t\cos\ ka$, where E_0 and t are constants, then the density of states inside the nanowire depends on E as

(a) $N^3\sqrt{\frac{t^2}{E-E_0}}$ (b) $\sqrt{\left(\frac{E-E_0}{2t}\right)^2 - 1}$

(c) $N^3\sqrt{\frac{E-E_0}{t^2}}$ (d) $\frac{N}{\sqrt{(2t)^2-(E-E_0)^2}}$

52. Consider a two-dimensional material of length l and width w subjected to a constant magnetic field B applied perpendicular to it. The number of charge carriers per unit area may be expressed as

$$n = \frac{k|q|B}{(2\pi\hbar)},$$

where k is a positive real number and q is the carrier charge. Then the Hall resistivity ρ_{xy} is

(a) $\frac{2\pi hk}{q^2}\sqrt{\frac{l}{w}}$ (b) $\frac{2\pi\hbar}{kq^2}\sqrt{\frac{w}{l}}$

(c) $\frac{2\pi\hbar}{kq^2}$ (d) $\frac{2\pi\hbar k}{q^2}$

53. The spin-parity assignments for the ground and first excited states of the isotrope $^{57}_{28}Ni$, in the singlo particle shell model, are

(a) $\left(\frac{1}{2}\right)^-$ and $\left(\frac{3}{2}\right)^-$ (b) $\left(\frac{5}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

(c) $\left(\frac{3}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ (d) $\left(\frac{3}{2}\right)^-$ and $\left(\frac{5}{2}\right)^-$

54. The first excited state of the rotational spectrum of the nucleus $^{238}_{92}U$ has an energy 45keV above the ground state. The energy of the second excited state (in keV), is

(a) 150 (b) 120

(c) 90 (d) 60

55. Which of the following processes is not allowed by the strong interaction but is allowed by the weal interaction?

(a) $K^0 + \pi^0 \to \bar{K}^0 + \pi^+ + \pi^-$

(b) $p + n \to d + p + \bar{p}$

(c) $\Delta^+ + K^0 \to p + n$

(d) $p + \Delta^+ \to \bar{n} + \Delta^{++}$

❖ ANSWER KEY

1. a	2. d	3. c	4. c	5. c
6. d	7. b	8. b	9. a	10. d
11. a	12. c	13. b	14. b	15. d
16. a	17. b	18. d	19. a	20. d
21. d	22. d	23. a	24. a	25. c
26. d	27. d	28. a	29. b	30. b
31. c	32. c	33. d	34. b	35. d
36. b	37. b	38. d	39. b	40. b
41. a	42. a	43. c	44. d	45. a
46. c	47. b	48.	49. c	50. c
51. d	52. c	53. d	54. a	55. a