

- 9. The parabolic coordinates (ξ, η) are related to the Cartesian coordinates (x, y) by $x = \xi \eta$ and $y = \frac{1}{2}(\xi^2 - \eta^2)$. The Lagrangian of a twodimensional simple harmonic oscillator of mass *m* and angular frequency ω is (a) $\frac{1}{2}m[\dot{\xi}^2 + \dot{\eta}^2 - \omega^2(\xi^2 + \eta^2)]$ (b) $\frac{1}{2}m(\xi^2 + \eta^2) \left[(\dot{\xi}^2 + \dot{\eta}^2) - \frac{1}{4}\omega^2(\xi^2 + \eta^2)\right]$ (c) $\frac{1}{2}m(\xi^2 + \eta^2) (\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{2}\omega^2\xi\eta)$ (d) $\frac{1}{2}m(\xi^2 + \eta^2) (\xi^2 + \dot{\eta}^2 - \frac{1}{4}\omega^2)$
- **10.** A conducting circular disc of radius r and resistivity ρ rotates with an angular velocity ω in a magnetic field B perpendicular to it. A voltmeter is connected as shown in the figure below.

Assuming its internal resistance to be infinite, the reading on the voltmeter

- (a) depends on ω , *B*, *r* and ρ
- (b) depends on ω , *B* and *r*, but not on ρ
- (c) is zero because the flux through the loop is not changing
- (d) is zero because a current flows in the direction of B



11. The charge per unit length of a circular wire of radius *a* in the *xy*-plane, with its center at the origin, is $\lambda = \lambda_0 \cos \theta$, where λ_0 is a constant and the angle θ is measured from the positive *x*-axis. The electric field at the center of the circle is

(a)
$$\vec{E} = -\frac{\lambda_0}{4\varepsilon_0 a} \hat{i}$$
 (b) $\vec{E} = \frac{\lambda_0}{4\varepsilon_0 a} \hat{i}$
(c) $\vec{E} = -\frac{\lambda_0}{4\varepsilon_0 a} \hat{j}$ (d) $\vec{E} = \frac{\lambda_0}{4\pi\varepsilon_0 a} \hat{k}$

12. A screen has two slits, each of width *w*, with their centres at a distance 2*w* apart. It is illuminated by a monochromatic plane wave

travelling along the *x*-axis.



The intensity of the interference pattern, measured on a distant screen, at an angle $\theta = n\lambda/w$ to the *x* axis is

- (a) zero for n = 1,2,3 ...
- (b) maximum for n = 1,2,3 ...
- (c) maximum for $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

(d) zero for n = 0 only

13. The electric field of an electromagnetic wave is $\vec{E}(z,t) = E_0 \cos (kz + \omega t)\hat{i} + 2E_0 \sin (kz + \omega t)\hat{j}$, where ω and k are positive constants. This represents

(a) a linearly polarised wave travelling in the positive *z*-direction

(b) a circularly polarised wave travelling in the negative *z*-direction

(c) an elliptically polarised wave travelling in the negative *z*-direction

(d) an unpolarised wave travelling in the positive *z*-direction

14. Consider the two lowest normalized energy eigenfunctions $\psi_0(x)$ and $\psi_1(x)$ of a one dimensional system. They satisfy $\psi_0(x) = \psi_0^*(x)$ and $\psi_1(x) = \alpha \frac{d\psi_0}{dx}$, where α is a real constant. The expectation value of the momentum operator in the state ψ_1 is (a) $-\frac{\hbar}{x^2}$ (b) 0 0

(c)
$$\frac{h}{\alpha^2}$$
 (d) $\frac{2\hbar}{\alpha^2}$

15. Consider the operator $a = x + \frac{d}{dx}$ acting on smooth functions of *x*. The commutator $[a, \cos x]$ is

(a) $-\sin x$ (b) $\cos x$ (c) $-\cos x$ (d) 0 **16.** Let $a = \frac{1}{\sqrt{2}}(x + ip)$ and $a^{\dagger} = \frac{1}{\sqrt{2}}(x - ip)$ be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and *h* have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle \psi | x | \psi \rangle$ in the state $|\psi\rangle = \exp(\lambda a^{\dagger} - \lambda^* a) |0\rangle$, is

(a)
$$|\lambda|$$

(b) $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$
(c) $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$
(d) $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

17. Consider the operator $\vec{\pi} = \vec{p} - q\vec{A}$, where \vec{p} is the momentum operator, $\vec{A} = (A_x, A_y, A_z)$ is the vector potential and q denotes the electric charge. If $\vec{B} = (B_x, B_y, B_z)$ denotes the magnetic field, the z – component of the vector operator $\vec{\pi} \times \vec{\pi}$ is (a) $iq\hbar B_z + q(A_x p_y - A_y p_x)$ (b) $-iq\hbar B_z - q(A_x p_y - A_y p_x)$ (c) $-iq\hbar B_z$ (d) $iq\hbar B_z$

18. Consider a gas of *N* classical particles in a two-dimensional square box of side *L*. If the total energy of the gas is *E*, the entropy (apart from an additive constant) is

(a) $Nk_B \ln \left(\frac{L^2 E}{N}\right)$	(b) $Nk_B \ln \left(\frac{LE}{N}\right)$
(c) $2Nk_B \ln\left(\frac{L\sqrt{E}}{N}\right)$	(d) $L^2 k_B \ln \left(\frac{E}{N}\right)$

19. Consider a continuous time random walk. If a step has taken place at time t = 0, the probability that the next step takes place between t and t + dt is given by btdt, where b is a constant. What is the average time between successive steps ?

(a)
$$\sqrt{\frac{2\pi}{b}}$$
 (b) $\sqrt{\frac{\pi}{b}}$
(c) $\frac{1}{2}\sqrt{\frac{\pi}{b}}$ (d) $\sqrt{\frac{\pi}{2b}}$

20. The partition function of a two-level system

governed by the Hamiltonian $H = \begin{bmatrix} \gamma & -\delta \\ -\delta & -\gamma \end{bmatrix}$ is

- (a) $2\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$ (b) $2\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)$ (c) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)+\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$ (d) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2+\delta^2}\right)-\sinh\left(\beta\sqrt{\gamma^2+\delta^2}\right)\right]$
- **21.** A silica particle of radius 0.1μ m is put in a container of water at T = 300 K. The densities of silica and water are 2000 kg/m³ and 1000 kg/m³, respectively. Due to thermal fluctuations, the particle is not always at the bottom of the container. The average height of the particle above the base of the container is approximately

a) 10 ⁻³ m	(b) 3×10^{-4} m
c) 10 ⁻⁴ m	(d) 5×10^{-5} m

22. Which of the following circuits implements the Boolean function F(A, B, C): $\Sigma(1,2,4,6)$?



23. A pair of parallel glass plates separated by a distance *d* is illuminated by white light as shown in the figure below. Also shown is the graph of the intensity of the reflected light light *I* as a function of the wavelength λ recorded by a spectrometer.



Assuming that the interference takes place only between light reflected by the bottom surface of the top plate and the top surface of bottom plate, the distance d is closest to (a) 12μ m (b) 24μ m (c) 60μ m (d) 120μ m

24. The I - V characteristics of a device is $I = I_s \left[\exp \left(\frac{aV}{T} \right) - 1 \right]$, where T is the temperature and a and I_s are constants independent of T and V. Which one of the following plots is correct for a fixed applied voltage V?



25. The active medium in a blue LED (Light Emitting Diode) is a $Ga_x In_{1-x}$ N alloy. The band gaps of GaN and InN are 3.5eV and 1.5eV respectively. If the band gap of $Ga_x In_{1-x}$ N varies approximately linearly with x, the value of x required for the emission of blue light of wavelength 400 nm is (take $hc \approx 1200$ eV-nm)

> PART-C

26. A stable asymptotic solution of the equation $x_{n+1} = 1 + \frac{3}{1+x_n}$ is x = 2. If we take $x_n = 2 + \varepsilon_n$ and $x_{n+1} = 2 + \varepsilon_{n+1}$, where ε_n and ε_{n+1} are both small, the ratio $\varepsilon_{n+1}/\varepsilon_n$ is approximately

(a)
$$-\frac{1}{2}$$
 (b) $-\frac{1}{4}$
(c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

27. The 2 × 2 identity matrix *I* and the Pauli matrices σ^x , σ^y , σ^z do not form a group under matrix multiplication. The minimum number of 2 × 2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is

(a) 20	(b) 8
(c) 12	(d) 16

- 28. Given the values sin 45° = 0.7071, sin 50° 0.7660, sin 55° = 0.8192 and sin 60° = 0.8660, the approximate value of sin 52°, computed by Newton's forward difference method, is

 (a) 0.804
 (b) 0.776
 (c) 0.788
 (d) 0.798
- **29.** Let f(x, t) be a solution of the heat equation $\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at t = 0 is $f(x, 0) = e^{-x^2}$ for $-\infty < x < \infty$. Then for all t > 0, f(x, t) is given by [Usoful integral : $\int_{0}^{\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{2}}$]

+2Dt

π

$$\begin{array}{l} \text{(b)} \frac{1}{\sqrt{1+Dt}} e^{-\frac{x^2}{1+Dt}} & \text{(b)} \frac{1}{\sqrt{1+2Dt}} e^{-\frac{x^2}{1+Dt}} \\ \text{(c)} \frac{1}{\sqrt{1+4Dt}} e^{-\frac{x^2}{1+4Dt}} & \text{(d)} e^{\frac{x^2}{1+Dt}} \end{array}$$

30. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is

(a)
$$\theta = \frac{\pi}{2}$$
 (b) $\theta = \pi$
(c) $0 < \theta < \frac{\pi}{2}$ (d) $\frac{\pi}{2} < \theta \le$

31. Consider circular orbits in a central force potential $V(r) = -\frac{k}{r^n}$, where k > 0 and 0 < n < 2. If the time period of a circular orbit of radius *R* is T_1 and that of radius 2*R* is T_2 , then T_2/T_1 is $(2) 2^{\frac{n}{2}}$ (b) $2^{\frac{2}{n}n}$

(a) 2^{2}	(b) 2₃
(c) $2^{\frac{n}{2}+1}$	(d) 2 ^{<i>n</i>}

32. Consider a radioactive nucleus that is travelling at a speed c/2 with respect to the lab frame. It emits γ -rays of frequency v_0 in its rest frame. There is a stationary detector (which is not on the path of the nucleus) in the lab. If a γ -ray photon is emitted when the nucleus is closest to the detector, its observed frequency at the detector is

(a)
$$\frac{\sqrt{3}}{2} v_0$$
 (b) $\frac{1}{\sqrt{3}} v_0$
(c) $\frac{1}{\sqrt{2}} v_0$ (d) $\sqrt{\frac{2}{3}} v_0$

- **33.** Suppose that free charges are present in a material of dielectric constant $\varepsilon = 10$ and resistivity $\rho = 10^{11}\Omega m$. Using Ohm's law and the equation of continuity for charge, the time required for the charge density inside the material to decay by 1/e is closest to (a) 10^{-6} s (b) 10^{6} s (c) 10^{12} s (d) 10 s
- **34.** A particle with charge -q moves with a uniform angular velocity ω in a circular orbit of radius a in the xy-plane, around a fixed charge +q, which is at the centre of the orbit at (0,0,0). Let the intensity of radiation at the point (0,0,R) be I_1 and at (2R,0,0) be I_2 . The ratio I_2/I_1 , for $R \gg a$, is

(a) 4	(b) $\frac{1}{4}$
$(c)\frac{1}{8}$	(d) 8

35. A parallel plate capacitor is formed by two circular conducting plates of radius a separated by a distance d, where $d \ll a$. It is being slowly charged by a current that is nearly constant. At an instant when the current is *I*, the magnetic induction between the plates at a distance a/2 from the centre of the plate, is

(a)
$$\frac{\mu_0 I}{\pi a}$$
 (b) $\frac{\mu_0 I}{2\pi a}$
(c) $\frac{\mu_0 I}{a}$ (d) $\frac{\mu_0 I}{4\pi a}$

36. Two uniformly charged insulating solid spheres A and B, both of radius a, carry total charges +Q and -Q, respectively. The spheres are placed touching each other as shown in the figure.



If the potential at the center of the sphere A is V_A and that at the center of B is V_B , then the difference $V_A - V_B$ is

$(a) \frac{q}{q}$	(b) $\frac{-Q}{-Q}$
$(a) \frac{1}{4\pi\varepsilon_0 a}$	$(D) \frac{1}{2\pi\varepsilon_0 a}$
$(c) \frac{q}{q}$	(d) $-\tilde{q}$
$\left(C \right) \frac{1}{2\pi \varepsilon_0 a}$	$(\mathbf{u}) \frac{1}{4\pi\varepsilon_0 a}$

37. A particle is scattered by a central potential $V(r) = V_0 r e^{-\mu r}$, where V_0 and μ are positive constants.

If the momentum transfer \vec{q} is such that $q = |\vec{q}| \gg \mu$, the scattering cross-section in the Born approximation, as $q \to \infty$, depends on q as

[You may use $\int x^n e^{ax} dx = \frac{d^n}{da^n} \int e^{ax} dx$] (a) q^{-8} (b) q^{-2} (c) q^2 (d) q^6

- **38.** A particle in one dimension is in a potential $V(x) = A\delta(x a)$. Its wavefunction $\psi(x)$ is continuous everywhere. The discontinuity in $d\psi/dx$ at x = a is (a) $\frac{2m}{\hbar^2}A\psi(a)$ (b) $A(\psi(a) - \psi(-a))$ (c) $\frac{\hbar^2}{2m}A$ (d) 0
- **39.** The dynamics of a free relativistic particle of mass *m* is governed by the Dirac Hamiltonian $H = c\vec{\alpha} \cdot \vec{p} + \beta mc^2$, where \vec{p} is the momentum operator and $\vec{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ and β are four 4×4 Dirac matrices. The acceleration operator can be expressed as $(a) \frac{2ic}{c} (c\vec{n} \vec{\alpha}H) \qquad (b) 2ic^2\vec{\alpha}\beta$

(a)
$$\frac{1}{\hbar}(cp - \alpha H)$$
 (b) $2ic^2\alpha\beta$
(c) $\frac{ic}{\hbar}H\ddot{\alpha}$ (d) $-\frac{2ic}{\hbar}(c\vec{p} + \vec{\alpha}H)$

40. A particle of charge q in one dimension is in a simple harmonic potential with angular frequency ω . It is subjected to a time dependent electric field $E(t) = Ae^{-(t/\tau)^2}$, where A and τ are positive constants and $\omega \tau \gg 1$. If in the distant past $t \to -\infty$ the particle was in its ground state, the probability that it will be in the first excited state as $t \to +\infty$ is proportional to

(a)
$$e^{-\frac{1}{2}(\omega\tau)^2}$$

(b) $e^{\frac{1}{2}(\omega\tau)^2}$
(c) 0
(d) $\frac{1}{(\omega\tau)^2}$

41. Consider a random walk on an infinite twodimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps ?

(a) $\frac{1}{36}$	(b) $\frac{1}{216}$
(c) $\frac{1}{36}$	(d) $\frac{1}{216}$
$(C)\frac{18}{18}$	$(u) \frac{1}{12}$

- **42.** An atom has a non-degenerate ground state and a doubly-degenerate oxcited state, The energy difference between the two states is ε . The specific heat at very low temperatures ($\beta \varepsilon \neq 1$) is given by
 - (a) $k_B(\beta \varepsilon)$ (b) $k_B e^{-\beta \varepsilon}$ (c) $2k_B(\beta \varepsilon)^2 e^{-\beta \varepsilon}$ (d) k_n
- **43.** The electrons in graphene can be thought of as a two-dimensional gas with a linear energy-momentum relation $E = |\vec{p}|v$, where $\vec{p} = (p_x, p_y)$ and v is a constant. If ρ is the number of electrons per unit area, the energy per unit area is proportional to (a) $\rho^{3/2}$ (b) ρ
 - (c) $\rho^{1/3}$ (d) p^2

44. In the circuit below, the input voltage V_i is 2 V, $V_{cc} = 16$ V, $R_2 = 2$ kΩ and $R_L = 10$ kΩ.



The value of R_f required to deliver 10 mW of power across R_L is (a) $12k\Omega$ (b) $4k\Omega$

$(a) 12k\Omega$	(b) 4kΩ		
(c) 8kΩ	(d) 14kΩ		

45. Two sinusoidal signals are sent to an analog multiplier of scale factor 1 V⁻¹ followed by a low pass filter (LPF).



If the roll-off frequency of the LPF is $f_c =$ 5 Hz, the output voltage V_{out} is (a) 5 V (b) 25 V (c) 100 V (d) 50 V

46. The resistance of a sample is measured as a function of temperature, and the data are shown below.

<i>T</i> (°C)	2	4	6	8
$R(\Omega)$	90	105	110	115

The slope of *R* vs *T* graphs, using a linear least-squares fit to the data, will be (a) $6\Omega/^{\circ}C$ (b) $4\Omega^{\circ}C$ (c) $2\Omega/^{\circ}C$ (d) $8\Omega/^{\circ}C$

47. Consider a one-dimensional chain of atoms with lattice constant *a*. The energy of an electron with wave-vector *k* is $\varepsilon(k) = \mu$ –

 $\gamma \cos (ka)$, where μ and γ are constants. If an electric field *E* is applied in the positive *x*-direction, the time dependent velocity of an electron is (in the following *B* is the constant)

(a) proportional to $\cos\left(B - \frac{eE}{\hbar}at\right)$

- (b) proportional to *E*
- (c) independent of *E*
- (d) proportional to $\sin \left(B \frac{eE}{\hbar}at\right)$
- **48.** A thin rectangular conducting plate of length *a* and width *b* is placed in the *xy*-plane in two different orientations, as shown in the figures below. In both cases a magnetic field *B* is applied in the *z* direction and a current flows in the *x*-direction due to the applied voltage *V*.



If the Hall voltage across the *y*-direction in the two cases satisfy $V_2 = 2V_1$, the ratio *a*: *b* must be

(a) 1:2	(b) 1:√2
(c) 2: 1	(d) $\sqrt{2}$: 1

49. Consider a hexagonal lattice with basis vectors as shown in the figure below.
 v ↑



If the lattice spacing is a = 1, the reciprocal lattice vectors are

(a) $\left(\frac{4\pi}{3}, 0\right)$, $\left(-\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$ (b) $\left(\frac{4\pi}{3}, 0\right)$, $\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$

(c) $\left(0, \frac{4\pi}{\sqrt{3}}\right), \left(\pi, \frac{2\pi}{\sqrt{3}}\right)$ (d) $\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right), \left(-2\pi, \frac{2\pi}{\sqrt{3}}\right)$

50. In the L-S coupling scheme, the terms arising from two non-equivalent p-electrons are

(a) ³ S, ¹P, ³P, ¹D, ³D (b) ¹ S, ³ S, ¹P, ¹D

- (c) ¹ S, ³ S, ³ P, ³ D
- (d) ¹ S. ³ S. ¹P. ³P. ¹D. ³D
- **51.** The total spin of a hydrogen atom is due to the contribution of the spins of the electron and the proton. In the high temperature limit, the ratio of the number of atoms in the spin-1 state to the number in the spin- 0 state is (a) 2 (b) 3 (c) $\frac{1}{2}$ (d) 1/3
- **52.** A two level system in a thermal (black body) environment can decay from the excited state by both spontaneous and thermally stimulated emission. At room temperature (300 K), the frequency below which thermal emission dominates over spontaneous emission is nearest to (b) 10⁸ Hz (a) 10^{13} Hz (d) 10^{11} Hz (c) 10⁵ Hz
- **53.** What should be the minimum energy of a photon for it to split an α -particle at rest into a tritium and a proton? (The masses of ${}_{2}^{4}$ He, ${}_{1}^{3}$ H and ${}_{1}^{H}$ H are 4.0026amu, 3.0161amu and 1.0073amu, respectively, and 1 amu \approx 938MeV). (a) 32.2MeV (b) 3MeV (c) 19.3MeV (d) 931.5MeV
- **54.** Which of the following reaction(s) is/are allowed by the conservation laws? (i) $\pi^+ + n \rightarrow \Lambda^0 + K^+$ (ii) $\pi^- + p \rightarrow \Lambda^0 + K^0$ (a) Both (i)and(ii) (b) Only (i) (d) Neither(i)nor(ii) (c) Only (ii)
- 55. A particle, which is a composite state of three quarks *u*, *d* and *s*, has electric charge, spin and strangeness respectively, equal to

(a) $1, \frac{1}{2}, -1$ (b) 0, 0, -1(c) $0, \frac{1}{2}, -1$ (d) $-1, -\frac{1}{2}, +1$

✤ ANSWER KEY

1. c	2. d	3. b	4. c	5. b
6. d	7. a	8. d	9. b	10. b
11. a	12. b	13. a	14. b	15.
16. d	17. d	18. a	19. d	20. b
21. c	22. b	23. a	24. d	25. b
26. c	27. d	28. c	29. c	30. a
31. c	32. a	33. d	34. c	35. d
36. c	37. a	38. a	39. a	40. a
41. c	42. c	43. a	44. c	45. b
46. b	47. d	48. d	49. a	50. d
51. b	52. a	53. c	54. a	55. c