



D PHYSICS

CSIR-NET,GATE , ALL SET, JEST, IIT-JAM, BARC

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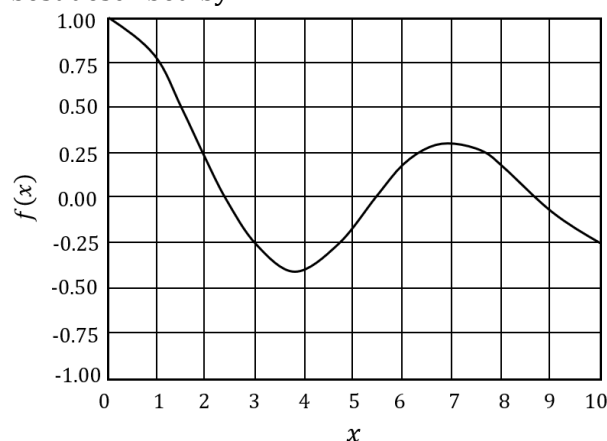
Part-B

1. A 2×2 matrix A has eigenvalues $e^{in/5}$ and $e^{i\pi/6}$.

The smallest value of 'n' such that $A^n = 1$ is:

- (a) 20 (b) 30
(c) 60 (d) 120

2. The graph of the function $f(x)$ as shown below is best described by



- (a) The Bessel function $J_0(x)$
(b) $\cos x$
(c) $e^{-x} \cos x$
(d) $\frac{1}{x} \cos x$

3. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is

- (a) $1/8$ (b) $5/8$
(c) $3/16$ (d) $5/16$

4. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} +$

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ is}$$

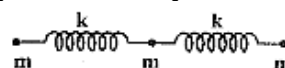
- (a) $\frac{bci+caj+abk}{\sqrt{a^2+b^2+c^2}}$ (b) $\frac{ai+bj+ck}{\sqrt{a^2+b^2+c^2}}$
(c) $\frac{bi+cj+ak}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{i+j+k}{\sqrt{3}}$

5. A solid cylinder of height H , radius R and density ρ , floats vertically on the surface of a liquid of density ρ_0 . The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

- (a) $\frac{\rho g}{\rho_0 H}$ (b) $\frac{\rho}{\rho_0} \sqrt{\frac{g}{H}}$
(c) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

6. Three particles of equal mass ' m ' are connected by two identical massless springs of stiffness constant ' k ' as shown in the figure.

If x_1, x_2 and x_3 denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:



- (a) $\frac{1}{2}k(x_1^2 + x_2^2 + x_3^2)$
(b) $\frac{1}{2}k[x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$
(c) $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 + 2x_2(x_1 + x_3)]$
(d) $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$

7. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass ' m '. Then

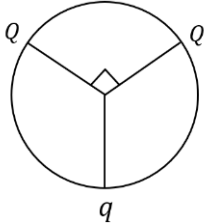
- (a) $\frac{dE}{dp} = \text{constant}$ (b) $p = mv$
(c) $v = \frac{cp}{\sqrt{p^2 + m^2 c^2}}$ (d) $E = mc^2$

8. A binary star system consists of two stars S_1 and S_2 , with masses m and $2m$ respectively separated by a distance ' r '. If both S_1 and S_2 individually

follow circular orbits around the centre of the mass with instantaneous speeds v_1 and v_2 respectively, the ratio of speeds v_1/v_2 is:

- (a) $\sqrt{2}$ (b) 1
(c) $\frac{1}{2}$ (d) 2

9. Three charges are located on the circumference of a circle of radius ' R ' as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge ' q ' is symmetrically placed with respect to the charges Q . If the electric field at the centre of the circle is zero, what is the magnitude of Q ?



- (a) $q/\sqrt{2}$ (b) $\sqrt{2}q$
(c) $2q$ (d) $4q$

10. Consider a hollow charged shell of inner radius ' a ' and outer radius ' b '. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (where k is a constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is:

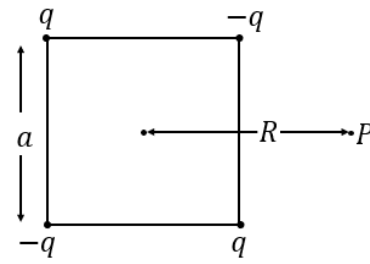
- (a) $\frac{k(b-a)}{\epsilon_0 r^2}$ for $r > a$
(b) $\frac{k(b-a)}{\epsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\epsilon_0 r^2}$ for $r > b$
(c) $\frac{k(r-a)}{\epsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\epsilon_0 r^2}$ for $r > b$
(d) $\frac{k(r-a)}{\epsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\epsilon_0 a^2}$ for $r > b$

11. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1 = \hat{i}E_0 \cos \omega t$ and $\vec{E}_2 = \hat{j}E_0 \cos (\omega t + \phi)$ where ϕ is the phase difference. The intensity of the resulting wave is given by $\frac{\epsilon_0}{2} \langle E^2 \rangle$, where $\langle E^2 \rangle$ is the time average of E^2 . The total intensity is

- (a) 0 (b) $\epsilon_0 E_0^2$
(c) $\epsilon_0 E_0^2 \sin^2 \phi$ (d) $\epsilon_0 E_0^2 \cos^2 \phi$

12. Four charges (two $+q$ and two $-q$) are kept fixed at the four vertices of a square of side ' a ' as shown

At the point P which is at a distance R from the centre ($R \gg a$), the potential is proportional to



- (a) $\frac{1}{R}$ (b) $\frac{1}{R^2}$
(c) $\frac{1}{R^3}$ (d) $\frac{1}{R^4}$

13. A point charge ' q ' of mass ' m ' is kept at a distance ' d ' below a grounded infinite conducting sheet which lies in the xy -plane. What is the value of ' d ' for which the charge remains stationary?

- (a) $q/4\sqrt{mg\pi\epsilon_0}$
(b) $q/\sqrt{mg\pi\epsilon_0}$
(c) There is no finite value of ' d '
(d) $\sqrt{mg\pi\epsilon_0}/q$

14. The wave function of a state of the hydrogen atom is given by

$\psi = \psi_{200} + 2\psi_{21} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$
where $\psi_{n/l/m}$ denotes the normalized eigenfunction of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state ψ is:

- (a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$
(c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

15. The energy eigenvalues of a particle in the potential $V(x) = \frac{1}{2}m\omega^2 x^2 - ax$ are

- (a) $E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{a^2}{2m\omega^2}$
(b) $E_n = \left(n + \frac{1}{2}\right)\hbar\omega + \frac{a^2}{2m\omega^2}$
(c) $E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{a^2}{m\omega^2}$
(d) $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

16. If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}(a^2 - x^2)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty Δp in its momentum is

- (a) $\frac{2\hbar}{5a}$ (b) $\frac{5\hbar}{2a}$
(c) $\frac{\sqrt{10}\hbar}{a}$ (d) $\frac{\sqrt{5}\hbar}{\sqrt{2}a}$

17. Given the usual canonical commutation relations, the commutator $[A, B]$ of $A = i(xp_y - yp_x)$ and $B = (yp_z + zp_y)$ is :

- (a) $\hbar(xp_z - p_xz)$ (b) $-\hbar(xp_z - p_xz)$
(c) $\hbar(xp_z + p_xz)$ (d) $-\hbar(xp_z + p_xz)$

18. The entropy of a system, S , is related to the accessible phase space volume Γ by $S = k_i \ln \Gamma(E, N, V)$ where E , N and V are the energy, number of particles and volume respectively.

From this one can conclude that Γ

- (a) does not change during evolution to equilibrium
(b) Oscillates during evolution to equilibrium
(c) Is a maximum in equilibrium
(d) Is a minimum in equilibrium

19. Let ΔW be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about ΔW is correct?

- (a) ΔW is a perfect differential if the process is isothermal
(b) ΔW is a perfect differential if the process is adiabatic
(c) ΔW is always a perfect differential
(d) ΔW cannot be a perfect differential.

20. Consider a system of three spins S_1, S_2 and S_3 each of which can take values $+1$ and -1 . The energy of the system is given by $E = -J[S_1 S_2 + S_2 S_3 + S_3 S_1]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,

- (a) J and 1 (b) $-3J$ and 1
(c) $-3J$ and 2 (d) $-6J$ and 2

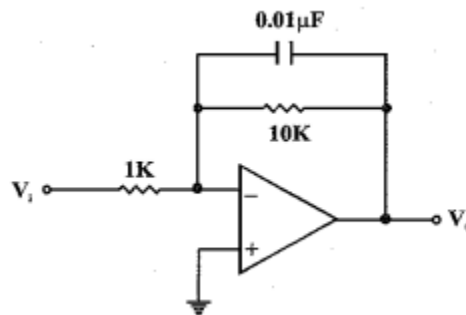
21. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

- (a) $14\pi^2 \hbar^2 / mL^2$ (b) $91\pi^2 \hbar^2 / mL^2$
(c) $7\pi^2 \hbar^2 / mL^2$ (d) $3\pi^2 \hbar^2 / mL^2$

22. A live music broadcast consists of a radio-wave of frequency 7MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10KHz. The spectrum of modulated output will be zero outside the frequency band

- (a) 7.00MHz to 7.01MHz
(b) 6.99MHz to 7.01MHz
(c) 6.99MHz to 7.00MHz
(d) 6.995MHz to 7.005MHz

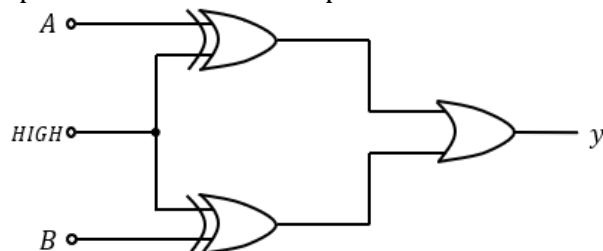
23. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_o is the output signal.



The magnitude of the gain and the phase shift, respectively, are close to the values

- (a) $5\sqrt{2}$ and $\frac{\pi}{2}$ (b) $5\sqrt{2}$ and $-\frac{\pi}{2}$
(c) 10 and zero (d) 10 and π

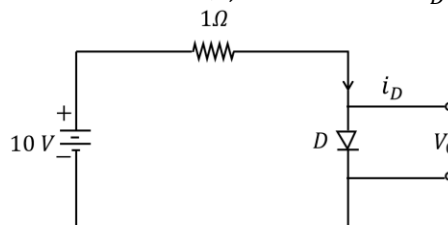
24. The logic circuit shown in the figure below, implements the Boolean expression



- (a) $y = \overline{A} \cdot \overline{B}$ (b) $y = \overline{A} \cdot \overline{B}$
(c) $y = A \cdot B$ (d) $y = A + B$

25. A diode D as shown in the circuit as an $i - v$ relation which can be proximated by

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \leq 0 \end{cases}$$



The value of v_D in the circuit is:

- (a) $(-1 + \sqrt{11})V$ (b) 8 V
(c) 5 V (d) 2 V

PART-C

26. The Taylor expansion of the function $\ln(\cosh x)$, where 'x' is real, about the point $x = 0$ starts with the following terms:

- (a) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \dots$
(b) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \dots$

- (c) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$
 (d) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$

27. Given a 2×2 unitary matrix U satisfying $U'U = U' = I$ with $\det U = e^{iq}$, one can construct a unitary matrix $V(V'V = VV' = 1)$ with $\det V = 1$ from it by

- (a) Multiplying U by $e^{-i/2}$
 (b) Multiplying any single element of U by $e^{-i\varphi}$
 (c) Multiplying any row or column of U by $e^{-i\varphi/2}$
 (d) Multiplying U by $e^{-i\varphi}$.

28. The value of the integral $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$, where C is a closed contour defined by the equation $2|z| - 5 = 0$, traversed in the anti-clockwise direction, is:

- (a) $-16\pi i$ (b) $16\pi i$
 (c) $8\pi i$ (d) $2\pi i$

29. A function $f(x)$ obeys the differential equation $\frac{d^2 f}{dx^2} - (3 - 2i)f = 0$ and satisfies the conditions $f(0) = 1$ and $f(x) \rightarrow 0$ as $x \rightarrow \infty$. The value of $f(\pi)$ is:

- (a) $e^{2\pi}$ (b) $e^{-2\pi}$
 (c) $-e^{-2\pi}$ (d) $-e^{2\pi}$

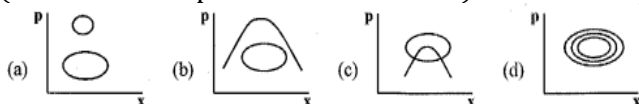
30. A planet of mass 'm' moves in the gravitational field of the Sun (mass M). If the semi-major and semi-minor axes of the orbit are 'a' and 'b' respectively, the angular momentum of the planet is:

- (a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$
 (c) $\sqrt{\frac{2GMm^2ab}{(a-b)}}$ (d) $\sqrt{\frac{2GMm^2ab}{(a+b)}}$

31. The Hamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length l is $H = \frac{p_\theta^2}{2m\ell^2} + mg\ell(1 - \cos \theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is:

- (a) $-\frac{2g}{\ell} p_\theta \sin \theta$ (b) $-\frac{g}{\ell} p_\theta \sin 2\theta$
 (c) $\frac{g}{\ell} p_\theta \cos \theta$ (d) $\ell p_\theta^2 \cos \theta$

32. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)?

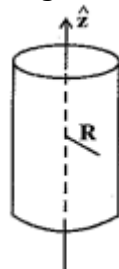


33. Two bodies of equal mass 'm' are connected by a massless rigid rod of length 'l' lying in the xy-

plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω , its angular momentum is

- (a) $m\ell^2\omega/4$ (b) $m\ell^2\omega/2$
 (c) $m\ell^2\omega$ (d) $2m\ell^2\omega$

34. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I .



The vector potential \vec{A} at a distance R from the axis.

- (a) Is constant inside and varies as R outside the solenoid.
 (b) Varies as R inside and is constant outside the solenoid.
 (c) Varies as $1/R$ inside and as R outside the solenoid.
 (d) Varies as R inside and as $1/R$ outside the solenoid.

35. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x, y, z) is given by

$$\hat{A} = \frac{\mu_0 K}{4c} (ct - z)^2 \hat{i}$$

The magnetic field \vec{B} is;

- (a) $\frac{\mu_0 Kt}{2} \hat{j}$ (b) $-\frac{\mu_0 Kz}{2c} \hat{j}$
 (c) $-\frac{\mu_0 K}{2c} (ct - z) \hat{i}$ (d) $-\frac{\mu_0 K}{2c} (ct - z) \hat{j}$

36. When a charged particle emits electromagnetic radiation, the electric field \vec{E} and the Poynting vector $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ at a large distance 'r' from the emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for 'n' and 'm' are correct?

- (a) $n = 1$ and $m = 1$ (b) $n = 2$ and $m = 2$
 (c) $n = 1$ and $m = 2$ (d) $n = 2$ and $m = 4$

37. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential $V(x)$ are -4 and -1, respectively, (in units in which $\hbar = 1$). If the corresponding wavefunctions are related by $\psi_1(x) = \psi_0(x) \sinh x$, then the ground state eigenfunction

is

- (a) $\psi_0(x) = \sqrt{\text{sech}x}$ (b) $\psi_0(x) = \text{sech } x$
 (c) $\psi_0(x) = \text{sech}^2 x$ (d) $\psi_0(x) = \text{sech}^3 x$

38. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass ' m ' confined in an infinite square well potential

$$V(x) = \begin{cases} 0 & -a < x < a \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

- (a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$
 (c) $2ba$ (d) ba

39. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator.

The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is :

- (a) $\Delta x = \sqrt{\hbar/2 m\omega}$ (b) $\Delta x = \sqrt{\hbar/m\omega}$
 (c) $\Delta x = \sqrt{2\hbar/m\omega}$ (d) $\Delta x = \sqrt{\hbar/4 m\omega}$

40. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \alpha \delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction $\psi(x) = Ae^{-bx^2}$ with b as the variational parameter?

[Hint: $\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)$]

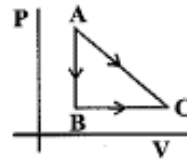
- (a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$
 (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

41. The free energy difference between the superconducting and the normal states of a material is given by $\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$, where ψ is an order parameter and α and β are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of ΔF in the superconducting state is

- (a) $-\alpha^2/\beta$ (b) $-\alpha^2/2\beta$
 (c) $-3\alpha^2/2\beta$ (d) $-5\alpha^2/2\beta$

42. A given quantity of gas is taken from the state $A \rightarrow C$ reversibly, by two paths, $A \rightarrow C$ directly and $A \rightarrow B \rightarrow C$ as shown in the figure below. During the $A \rightarrow C$ the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process $A \rightarrow B \rightarrow C$ the work done by the gas is

30 J, the heat absorbed is:



- (a) 20 J (b) 80 J
 (c) 220 J (d) 280 J

43. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2 J. In a configuration with r spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^N C_r$; r varies from 0 to N . The partition function is

- (a) $\left(\frac{J}{k_B T}\right)^N$ (b) $e^{-NJ/k_B T}$
 (c) $\left(\sinh \frac{J}{k_B T}\right)^N$ (d) $\left(\cosh \frac{J}{k_B T}\right)^N$

44. A magnetic field sensor based on the Hall effect is to be fabricated by implanting. As into a Si film of thickness $1\mu\text{m}$. The specifications require a magnetic field sensitivity of 500mV/ Tesla at an excitation current of 1 mA. The implanatation dose is to be adjusted such that the average carrier density, after activation, is

- (a) $1.25 \times 10^{26} \text{ m}^{-3}$ (b) $1.25 \times 10^{22} \text{ m}^{-3}$
 (c) $4.1 \times 10^{21} \text{ m}^{-3}$ (d) $4.1 \times 10^{20} \text{ m}^{-3}$

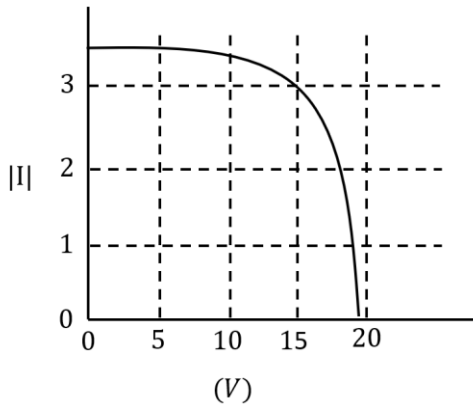
45. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are ω_0^{LP} and ω_0^{HP} , respectively, the condition required to implement the band-pass and band-reject filters are respectively.

- (a) $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$ and $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$
 (b) $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$ and $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$
 (c) $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$ and $\omega_0^{\text{HP}} < \omega_0^{\text{LP}}$
 (d) $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$ and $\omega_0^{\text{HP}} > \omega_0^{\text{LP}}$

46. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.

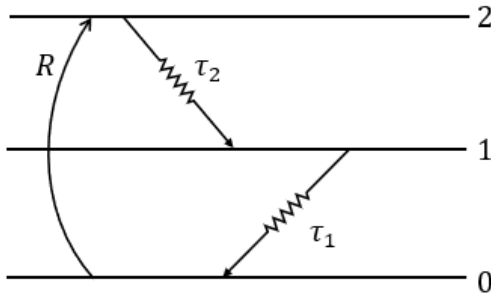
(v)

If the solar cell is to power a load of 5Ω , the power drawn by the load is:



- (a) 97 W (b) 73 W
(c) 50 W (d) 45 W

47. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is $10^{20} \text{ atoms cm}^{-3} \text{ s}^{-1}$ and the decay routes are as shown with $\tau_{21} = 20 \text{ ns}$ and $\tau_1 = 1 \mu\text{s}$, the equilibrium populations of states 2 and 1 are, respectively,

- (a) 10^{14} cm^{-3} and $2 \times 10^{12} \text{ cm}^{-3}$
(b) $2 \times 10^{12} \text{ cm}^{-3}$ and 10^{14} cm^{-3}
(c) $2 \times 10^{12} \text{ cm}^{-3}$ and $2 \times 10^6 \text{ cm}^{-3}$
(d) zero and 10^{20} cm^{-3} .

48. Consider a hydrogen atom undergoing a $2P \rightarrow 1S$ transition. The lifetime t_s of the $2P$ state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV . Assuming that the refractive index of the medium $n_6 = 1$, the ratio of the Einstein coefficients for stimulated emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

(a) $0.683 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$
(b) $0.146 \times 10^{-12} \text{ J s m}^{-3}$
(c) $6.83 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$
(d) $1.463 \times 10^{-12} \text{ J s m}^{-3}$

49. Consider a He-Ne laser cavity consisting of two mirrors of reflectivity's $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance $d = 20 \text{ cm}$ and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes δv and the width Δv_p of each mode of the

laser cavity are:

- (a) $\delta v = 75 \text{ kHz}, \Delta v_p = 24 \text{ kHz}$
(b) $\delta v = 100 \text{ kHz}, \Delta v_p = 100 \text{ kHz}$
(c) $\delta v = 750 \text{ MHz}, \Delta v_p = 2.4 \text{ MHz}$
(d) $\delta v = 2.4 \text{ MHz}, \Delta v_p = 750 \text{ MHz}$

50. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

- (a) $\hbar\omega/2$ (b) $\hbar\omega$
(c) $3\hbar\omega/2$ (d) 0

51. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a)$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

- (a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$
(c) $\frac{\hbar^2}{2\beta a^2}$ (d) $\frac{\hbar^2}{3\beta a^2}$

52. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3 , is

- (a) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ (b) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$
(c) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ (d) $\frac{1}{a}$

53. The muon has mass $105 \text{ MeV}/c^2$ and mean lifetime $2.2 \mu\text{s}$ in its rest frame. The mean distance traversed by muon of energy $315 \text{ MeV}/c^2$ before decaying is approximately

- (a) $3 \times 10^5 \text{ km}$ (b) 2.2 km
(c) $6.6 \mu\text{m}$ (d) 1.98 km

54. Consider the following particles: the proton p , the neutron n , the neutral pion π^0 and the delta resonance Δ^+ . When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

- (a) π^0, n, p, Δ^* (b) p, n, Δ^+, π^0
(c) p, n, π^0, Δ^+ (d) Δ^+, n, π^0, p

55. The single particle energy difference between the p-orbitals (i.e. $p_{3/2}$ and $p_{1/2}$) of the nucleus $^{114}_{50}\text{Sn}$ is 3 MeV . The energy difference between the states in its $1f$ orbital is

- (a) -7 MeV (b) 7 MeV
(c) 5 MeV (d) -5 MeV

❖ ANSWER KEY

21. c	22. a	23. d	24. a	25. d
26. d	27. c	28. d	29. a	30. c
31. *	32. c	33. a	34. d	35. a
36. d	37. c	38. c	39. b	40. c
41. a	42. b	43. d	44. a	45. d
46. b	47. a	48. a	49. c	50.
51. a	52. c	53. b	54. d	55. d
56. c	57. c	58. d	59. a	60. c
61. b	62. b	63. d	64. b	65. b
66. d	67. b	68. a	69. c	70. c
71. d	72. a	73. d	74. c	75. b